Quantum theory of nonrelativistic particles interacting with gravity

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We investigate the effects of the gravitational field on the quantum dynamics of nonrelativistic particles. We consider N nonrelativistic particles, interacting with the linearized gravitational field. Using the Feynman-Vernon influence functional technique, we trace out the graviton field to obtain a master equation for the system of particles to first order in G. The effective interaction between the particles as well as the self-interaction is in general non-Markovian. We show that the gravitational self-interaction cannot be held responsible for decoherence of microscopic particles due to the fast vanishing of the diffusion function. For macroscopic particles though, it leads to diagonalization to the energy eigenstate basis, a desirable feature in gravity-induced collapse models. We finally comment on possible applications. [S0556-2821(96)00214-7]

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I. INTRODUCTION

There has been recently considerable interest in the application of the influence functional technique [1] in the study of nonequilibrium systems in physics. In addition to quantum Brownian motion [2–5] for which the method was initially developed, it has been applied to the modeling of particle-field interactions [6], radiation damping [7], blackbody radiation [8], and most recently to noninertial detectors coupled to a scalar field [9]. It is one of the most powerful techniques to obtain master equations, when the coarse graining comes from the splitting of degrees of freedom to system and environment.

In this paper we apply the technique in another case: a system of N nonrelativistic particles coupled to linearized gravity. A motivation for this is the possibility that gravity induces decoherence on the particles' states. This is a suggestion made in different contexts on fundamental irreversibility in quantum mechanics [11–12]. The weakness of the coupling suggests that the probable decoherence time should be very large, but the particular form of the coupling (quadratic to momentum) and the possibility of persistent noise might give rise to relatively strong loss of coherence.

In addition, the model we present here can be generalized in a straightforward way to obtain a description of systems of quantum-mechanical detectors of gravitational waves.

Our model consists of nonrelativistic particles coupled to the linearized gravitational field, which is assumed to be initially in its vacuum state. We argue that a factorizing initial condition is, in contrast to quantum Brownian motion, well suited for our system. The modes of the graviton field are bounded in energy by an ultraviolet cutoff Λ, which on physical grounds should be much smaller than the Compton wavelength of the particles. In addition, we assume that the particles are almost stationary. Our analysis resembles, in a way, the one of [9]. Like those authors, we obtain correlation kernels describing a nonlocal interaction between the particles. The influence functional we construct is rather different from the ones considered in the literature, due to the

particular features of the gravitational coupling, which is quadratically coupled to momenta. The result of our analysis is the non-Markovian master equation (3.9) below. For the case of a single particle, it is simplified significantly [Eq. (4.1)]. We see that the dissipation and diffusion are determined solely from the Hamiltonian operator. We can interpret our results as a continuous monitoring of the energy of the particle by the gravitational environment.

The diffusion function which is responsible for decoherence vanishes at long times, and it turns out, that unless we consider macroscopically massive bodies, the rate of gravitationally induced decoherence is extremely small. This is a desirable result in connection with the gravity-induced collapse models.

II. THE MODEL

Consider N particles on a $(3+1)$-dimensional spacetime, moving on trajectories $(\mathbf{x}_n(\tau_n), t_n(\tau_n))$ parametrized by the proper times τ_n so that $t_n(\tau_n)$ is a strictly increasing function of τ_n [9]. We assume that the gravitational interaction is very weak and therefore work in the linearized approximation. That is, the metric is

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.1}$$

with $\eta_{\mu\nu}$ the Minkowski space metric.

We take the nonrelativistic limit for the particles, that is, we assume that there exists a frame with respect to which they are almost stationary, and therefore we can write their trajectories as $(a^i_{(n)} + x^i_{(n)}(t), t)$, having identified the global time coordinate t with the proper time of the particles. We assume that $|\mathbf{x}_{(n)}|$ is much smaller than the distance between any two particles $d_{nm} = |\mathbf{a}_{(n)} - \mathbf{a}_{(m)}|$. This is a good approximation as long as d_{nm} is much larger than the maximum wavelength of the graviton field that can be excited. Essentially, we consider the particles moving around some fixed sites coordinated by $\mathbf{a}_{(n)}$, so that their individual motion does not significantly change their distances. In any case, this approximation does not affect at all the discussion on the self-interaction of the particles through the gravitational field.

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We work in the transverse-traceless gauge for the linearized gravitational field ($h_{0\mu}=0$, $h_i^{ij}=0$, $h_i^i=0$). Under these approximations, the total action of the system for evolution from global time $t=0$ to $t=T$ reads

$$S_{\text{tot}}=S_{\text{gr}}+S_{\text{par}}+S_{\text{int}} \tag{2.2}$$

where

$$S_{\text{gr}}=\frac{1}{4\pi G}\int_0^T dt\int d^3x\; h_{\mu\nu,\rho}h^{\mu\nu,\rho}$$

$$=\frac{1}{4\pi G}\int_0^T dt\int d^3x(\dot{h}_{ij}\dot{h}^{ij}-h_{ij,k}h^{ij,k}), \tag{2.3}$$

$$S_{\text{par}}=\sum_n\int_0^T dt\,\tfrac{1}{2}\delta_{ij}\dot{x}^i_{(n)}\dot{x}^j_{(n)}, \tag{2.4}$$

$$S_{\text{int}}=\sum_n\int_0^T dt\; h_{ij}\dot{x}^i_{(n)}\dot{x}^j_{(n)}. \tag{2.5}$$

Note that we have set $\hbar=c=m=1$.

We expand the graviton field in normal modes:

$$h_{ij}(\mathbf{x},t)=\int\frac{d^3k}{(2\pi)^3}\sum_r(q^{(r)}_{1\mathbf{k}}\cos\mathbf{k}\cdot\mathbf{x}+q^{(r)}_{2\mathbf{k}}\sin\mathbf{k}\cdot\mathbf{x})A^{(r)}_{\mathbf{k}ij}. \tag{2.6}$$

The polarization matrices $A^{(r)}_{\mathbf{k}ij}$ ($r=1,2$) are traceless and transverse and can be chosen to satisfy

$$A^{(r)j}_{\mathbf{k}i}A^{(r')l}_{\mathbf{k}j}=\delta_{rr'}\left(\delta^i_l-\frac{k^ik_l}{k^2}\right), \tag{2.7}$$

$$\sum_r A^{(r)}_{\mathbf{k}ij}A^{(r)}_{\mathbf{k}kl}=\left(\delta_{(ij}-\frac{k_{(i}k_j}{\mathbf{k}^2}\right)\left(\delta_{k)l}-\frac{k_k)k_l}{\mathbf{k}^2}\right):=T^{ijkl}(\mathbf{k}). \tag{2.8}$$

The gravity part of the action, therefore, reads

$$S_{\text{gr}}=\frac{1}{2\pi G}\int_0^T\int\frac{d^3k}{(2\pi)^3}\sum_r[(\dot{q}^{(r)2}_{1\mathbf{k}}+\mathbf{k}^2q^{(r)2}_{1\mathbf{k}})$$

$$+(\dot{q}^{(r)2}_{2\mathbf{k}}+\mathbf{k}^2q^{(r)2}_{2\mathbf{k}})]. \tag{2.9}$$

This is just the action for two massless scalar fields propagating on Minkowski spacetime.

We now write the coupling part of the action,

$$S_{\text{int}}=\frac{1}{2}\int_0^T dt\int\frac{d^3k}{(2\pi)^3}\sum_r\sum_n(q^{(r)}_{1\mathbf{k}}\cos\mathbf{k}\cdot\mathbf{a}_{(n)}$$

$$+q^{(r)}_{2\mathbf{k}}\sin\mathbf{k}\cdot\mathbf{a}_{(n)})A^{(r)}_{\mathbf{k}ij}\dot{x}^i\dot{x}^j, \tag{2.10}$$

where within our approximations we ignored the $\mathbf{x}_{(n)}$ terms in the trigonometric functions.

By using the collective index α to include the $\mathbf{k},r$ and the indexing of our oscillator by 1 or 2, we write

$$S_{\text{gr}}+S_{\text{int}}=\int_0^T\sum_\alpha\left[\frac{1}{2\pi G}(\dot{q}^2_\alpha+\omega^2_\alpha q^2_\alpha)+q_\alpha J_\alpha\right] \tag{2.11}$$

where

$$J^{(r)}_{\mathbf{k}1}=\cos\mathbf{k}\cdot\mathbf{a}_{(n)}A^{(r)}_{\mathbf{k}ij}\dot{x}^i\dot{x}^j \tag{2.12}$$

$$J^{(r)}_{\mathbf{k}1}=\sin\mathbf{k}\cdot\mathbf{a}_{(n)}A^{(r)}_{\mathbf{k}ij}\dot{x}^i\dot{x}^j \tag{2.13}$$

and $\omega_\mathbf{k}=|\mathbf{k}|$. This is just the action of a collection of forced harmonic oscillators. Therefore the total action is that of a collection of N nonrelativistic free particles interacting with a bath of harmonic oscillators, through couplings depending quadratically on the velocity.

The tracing out of the graviton modes can be done exactly since the path integral is a Gaussian with respect to them. We compute the influence functional:

$$\mathcal{F}[\mathbf{x}(t),\mathbf{x}'(t')]=\prod_{\alpha\beta}\int dq^\alpha_f\int dq'^\beta_f\int dq^\alpha_0\int dq^\beta_0\delta$$

$$\times(q_\alpha-q'_\beta)\int Dq_\alpha(t)Dq'_\beta(t')$$

$$\times\exp\{iS_{\text{gr}}[q_\alpha(t)]+iS_{\text{int}}[q_\alpha(t),\mathbf{x}(t)]$$

$$-iS_{\text{gr}}[q'_\alpha(t')]-iS_{\text{int}}[q'_\alpha(t'),\mathbf{x}'(t')]\}$$

$$\times\rho_0(q^\alpha_0,\mathbf{x}_0,q'^\beta_0,\mathbf{x}'_0) \tag{2.14}$$

where the integration is over the paths satisfying $q^\alpha(0)=q^\alpha_0$, $q^\alpha(T)=q^\alpha_f$, $q'^\alpha(0)=q'^\alpha_0$, and $q'^\alpha(T)=q'^\alpha_f$. Here ρ_0 is the density matrix of the total system. The path integrations can be carried out exactly, to obtain

$$\mathcal{F}[\mathbf{x}(t),\mathbf{x}'(t')]=\mathcal{N}(T)\exp\left[-\sum_\alpha\frac{i}{2\omega_\alpha}\int_0^T ds\int_0^s ds'\right.$$

$$\times(J_\alpha+J'_\alpha)(s)\sin\omega_\alpha(s-s')(J_\alpha-J'_\alpha)(s')$$

$$-\sum_\alpha\frac{1}{2\omega_\alpha}\int_0^T ds\int_0^s ds'(J_\alpha-J'_\alpha)(s)$$

$$\left.\times\cos\omega_\alpha(s-s')(J_\alpha-J'_\alpha)(s')\right]. \tag{2.15}$$

In deriving this we have assumed that at $t=0$ the states of the particles and of the graviton field were uncorrelated and that the field was in its vacuum state: i.e.,

$$\Psi[h_{ij}]=C\exp\left[\sum_\alpha\frac{i}{2\omega_\alpha}\mathbf{q}^2_\alpha\right]. \tag{2.16}$$

This initial condition is usually considered unphysical in quantum Brownian motion models. We believe that it is actually a quite good one for the case of gravity. Graviton modes are excited only by nonstationary particles. Therefore this initial condition reflects an operation on the particles of a very fast acceleration just before $t=0$.

Substituting the expressions for the currents J_α into the influence functional we get

$$\mathcal{F}[\mathbf{x},\mathbf{x}'] = N(T)\exp\left[i\sum_{n,m}\int_0^T ds\int_0^s ds'\right.$$

$$\times(\dot{x}^i_{(n)}\dot{x}^j_{(n)}+\dot{x}'^i_{(n)}\dot{x}'^j_{(n)})(s)$$

$$\times\gamma^{ijkl}_{(n)(m)}(s-s')(\dot{x}^k_{(m)}\dot{x}^l_{(m)}-\dot{x}'^k_{(m)}\dot{x}'^l_{(m)})(s')$$

$$-\sum_{n,m}\int_0^T ds\int_0^s ds'(\dot{x}^i_{(n)}\dot{x}^j_{(n)}-\dot{x}'^i_{(n)}\dot{x}'^j_{(n)})(s)$$

$$\left.\times\eta^{ijkl}_{(n)(m)}(s-s')(\dot{x}^k_{(m)}\dot{x}^l_{(m)}-\dot{x}'^k_{(m)}\dot{x}'^l_{(m)})(s')\right].$$
$$(2.17)$$

The kernels $\gamma_{(n)(m)}$ and $\eta_{(n)(m)}$ are given by the expressions

$$\gamma^{ijkl}_{(n)(m)}(s)=\frac{G}{8\pi^2}\int\frac{d^3k}{|\mathbf{k}|}\sin|\mathbf{k}|s\,\cos\mathbf{k}\cdot(\mathbf{a}_n-\mathbf{a}_m)T^{ijkl}(\mathbf{k}),$$
$$(2.18)$$

$$\eta^{ijkl}_{(n)(m)}(s)=\frac{G}{8\pi^2}\int\frac{d^3k}{|\mathbf{k}|}\cos|\mathbf{k}|s\,\cos\mathbf{k}\cdot(\mathbf{a}_n-\mathbf{a}_m)T^{ijkl}(\mathbf{k}).$$
$$(2.19)$$

These are the dissipation and noise kernels, similar to the ones derived in [9] for the case of detectors minimally coupled to a scalar field. For $n\neq m$ they describe the dissipation and diffusion induced on the particle n from the particle m, while for $n=m$ they contain the effects of the self-interaction of the particle through its interaction with the gravitational field.

In order to keep them finite, we have to restrict the integration range to values of $|\mathbf{k}|$ smaller than a cutoff Λ. This is natural, since we do not expect the nonrelativistic particles to excite graviton modes with arbitrarily high energy. In fact, Λ should be much smaller than the Compton wavelength of the particle. This is in accordance with our previous approximations, since the distance between any particles remains much larger than their Compton wavelength.

In the particular case $n=m$ we can perform the angular integrations in spherical coordinates in the equations for the kernels and obtain

$$\gamma^{ijkl}_{(n)(n)}(s)=\frac{G}{15\pi}\,\delta^{ijkl}\int_0^\Lambda dk\,k\,\sin ks,\qquad(2.20)$$

$$\eta^{ijkl}_{(n)(n)}(s)=\frac{G}{15\pi}\,\delta^{ijkl}\int_0^\Lambda dk\,k\,\cos ks.\qquad(2.21)$$

We note that by taking the cutoff to infinity the dissipation kernel becomes essentially the derivative of a δ function, as in the well studied case of quantum Brownian motion with an Ohmic environment. The corresponding semiclassical equations for $t\gg\Lambda^{-1}$ can be found using the standard procedure [3,6,9]:

$$\ddot{x}^i+\frac{2G}{15}\,\delta_{ijkl}\dddot{x}^j\ddot{x}^k\dot{x}^l=(\ddot{x}^l\delta^{ik}+\ddot{x}^k\delta^{il})\xi_{kl},\qquad(2.22)$$

with $\xi_{kl}(t)$ a stochastic force determined by the correlator

$$\langle\xi_{ij}(t)\xi_{kl}(t')\rangle=\eta^{ijkl}(t-t').\qquad(2.23)$$

III. THE MASTER EQUATION

Having obtained an expression for the influence functional we can compute the reduced density matrix propagator:

$$J(\mathbf{x}_f,\mathbf{x}'_f,t|\mathbf{x}_0,\mathbf{x}'_0,0)=\int\int D\mathbf{x}\,D\mathbf{x}'\,\exp(iS_{\text{par}}[\mathbf{x}]$$
$$-iS_{\text{par}}[\mathbf{x}'])\mathcal{F}[\mathbf{x},\mathbf{x}'],\qquad(3.1)$$

where the integration is over all paths $\mathbf{x}(s)$, $\mathbf{x}'(s')$ satisfying $\mathbf{x}(0)=\mathbf{x}_0$, $\mathbf{x}'(0)=\mathbf{x}'_0$, $\mathbf{x}(t)=\mathbf{x}_f$, and $\mathbf{x}'(t)=\mathbf{x}_f$.

The knowledge of the reduced density matrix propagator enables us to construct a master equation. Our system is characterized by the nonlocal dissipation and diffusion in the influence functional, and the coupling which is quadratic to the velocities. Because of the peculiarities of the latter, the general method of Hu, Paz, and Zhang [3] is not applicable here. Instead we compute the influence functional perturbatively (first order in G) and use the Feynman prescription for the determination of the master equation.

Our starting point is the density matrix propagator for the free particle under external forces $\mathbf{F}(s)$ and $\mathbf{F}'(s)$:

$$J^{(0)}[\mathbf{F},\mathbf{F}'](\mathbf{x}_f,\mathbf{x}'_f,t|\mathbf{x}_0,\mathbf{x}'_0,0)$$

$$=\frac{C}{t}\exp\left[\frac{i}{2t}(\mathbf{x}_f-\mathbf{x}_0)^2-\frac{i}{2t}(\mathbf{x}'_f-\mathbf{x}'_0)^2\right.$$

$$+\frac{i}{t}\mathbf{x}_0\int_0^t ds\,s\mathbf{F}(s)$$

$$-\frac{i}{t}\mathbf{x}'_0\int_0^t ds\,s\mathbf{F}'(s)+\frac{i}{t}\mathbf{x}_f\int_0^t ds(t-s)\mathbf{F}(s)$$

$$-\frac{i}{t}\mathbf{x}'_f\int_0^t ds(t-s)\mathbf{F}'(s)$$

$$+\frac{i}{t}\int_0^t ds\int_0^s ds'\,s'(t-s)\mathbf{F}(s)\mathbf{F}(s')$$

$$\left.-\frac{i}{t}\int_0^t ds\int_0^s ds'\,s'(t-s)\mathbf{F}'(s)\mathbf{F}'(s')\right].\qquad(3.2)$$

The perturbation expansion of the propagator is written then formally:

$$J(\mathbf{x}_f,\mathbf{x}'_f,t|\mathbf{x}_0,\mathbf{x}'_0,0)=\mathcal{F}\left[-i\frac{\delta}{\delta\mathbf{F}(s)},i\frac{\delta}{\delta\mathbf{F}'(s)}\right]J^{(0)}[\mathbf{F},\mathbf{F}']$$

$$\times(\mathbf{x}_f,\mathbf{x}'_f,t|\mathbf{x}_0,\mathbf{x}'_0,0)|_{\mathbf{F}=\mathbf{F}'=0}.\qquad(3.3)$$

To first order in G we obtain

$$J(\mathbf{x}_f,\mathbf{x}'_f,t|\mathbf{x}_0,\mathbf{x}'_0,0)=\frac{C}{t}\exp\Bigg[\sum_{(n)(m)}\Bigg(4G\,\delta_{ij}\delta_{kl}g^{ijkl}_{(n)(m)}+\frac{i}{2t}\,\delta_{ij}(x_f-x_0)^i_{(n)}(x_f-x_0)^j_{(n)}\delta_{mn}-\frac{i}{2t}\,\delta_{ij}(x'_f-x'_0)^i_{(n)}(x'_f-x'_0)^j_{(n)}\delta_{nm}$$

$$-\frac{G}{t}\,(3f-4ig)^{ijkl}_{(n)(m)}\delta_{ij}(x_f-x_0)^i_{(n)}(x_f-x_0)^j_{(n)}\delta_{nm}-\frac{G}{t}\,(3f+4ig)^{ijkl}_{(n)(m)}\delta_{ij}(x'_f-x'_0)^i_{(n)}(x_f-x_0)^j_{(n)}\delta_{nm}$$

$$-\frac{iG}{2t^2}\,f^{ijkl}_{(n)(m)}[(x_f-x_0)^i_{(n)}(x_f-x_0)^j_{(n)}(x_f-x_0)^k_{(m)}(x_f-x_0)^l_{(m)}-(x'_f-x'_0)^i_{(n)}(x'_f-x'_0)^j_{(n)}$$

$$\times(x'_f-x'_0)^k_{(m)}(x'_f-x'_0)^l_{(m)}]-\frac{G}{2t^2}\,g^{ijkl}_{(n)(m)}[(x_f-x_0)^i_{(n)}(x_f-x_0)^j_{(n)}(x_f-x_0)^k_{(m)}(x_f-x_0)^l_{(m)}$$

$$+(x'_f-x'_0)^i_{(n)}(x'_f-x'_0)^j_{(n)}(x'_f-x'_0)^k_{(m)}(x'_f-x'_0)^l_{(m)}-2(x_f-x_0)^i_{(n)}(x_f-x_0)^j_{(n)}(x'_f-x'_0)^k_{(m)}(x'_f-x'_0)^l_{(m)}]\Bigg)\Bigg]$$

$$(3.4)$$

where f and g are functions of time,

$$f^{ijkl}_{(n)(m)}(t)=\frac{1}{8\pi^2 t}\int_{|\mathbf{k}|<\Lambda}\frac{d^3k}{\mathbf{k}^2}\left(1-\frac{\sin|\mathbf{k}|t}{|\mathbf{k}|t}\right)$$

$$\times\cos\mathbf{k}\cdot(\mathbf{a}_n-\mathbf{a}_m)T^{ijkl}(\mathbf{k}),\qquad(3.5)$$

$$g^{ijkl}_{(n)(m)}(t)=\frac{1}{8\pi^2 t}\int_{|\mathbf{k}|<\Lambda}\frac{d^3k}{\mathbf{k}^2}\frac{1-\cos|\mathbf{k}|t}{|\mathbf{k}|t}$$

$$\times\cos\mathbf{k}\cdot(\mathbf{a}_n-\mathbf{a}_m)T^{ijkl}(\mathbf{k}),\qquad(3.6)$$

and, in particular,

$$f^{ijkl}_{(n)(n)}(t)=\frac{1}{15\pi t}\,\delta^{ijkl}\int_0^\Lambda dk\left(1-\frac{\sin kt}{kt}\right),\qquad(3.7)$$

$$g^{ijkl}_{(n)(n)}(t)=\frac{1}{15\pi t}\,\delta^{ijkl}\int_0^\Lambda dk\,\frac{1-\cos kt}{kt}.\qquad(3.8)$$

The standard prescription for the derivation of the master equation from the reduced density master propagator consists of taking its time derivative and using identities relating $\mathbf{x}_0$ and $\mathbf{x}'_0$ with the action of derivatives with respect to $\mathbf{x}_f$ and $\mathbf{x}'_f$. For the interested reader, we list the relevant identities in the Appendix.

After some calculations, the master equation turns out to be (inserting back $\hbar$, m, and c)

$$\frac{\partial}{\partial t}\rho=\sum_n\frac{i\hbar}{2m}[1-\delta m_{(n)}(t)]\left(\frac{\partial^2}{\partial\mathbf{x}_{(n)}{}^2}-\frac{\partial^2}{\partial\mathbf{x}'_{(n)}{}^2}\right)\rho$$

$$-\frac{i\hbar^4}{4m^2}\sum_{n,m}\alpha^{ijkl}_{(n)(m)}(t)\left(\frac{\partial^4}{\partial x^i_{(n)}\partial x^j_{(n)}\partial x^k_{(m)}\partial x^l_{(m)}}\right.$$

$$\left.-\frac{\partial^4}{\partial x'^i_{(n)}\partial x'^j_{(n)}\partial x'^k_{(m)}\partial x'^l_{(m)}}\right)\rho-\frac{\hbar^4}{4m^2}\sum_{n,m}\beta^{ijkl}_{(n)(m)}(t)$$

$$\times\left(\frac{\partial^4}{\partial x^i_{(n)}\partial x^j_{(n)}\partial x^k_{(m)}\partial x^l_{(m)}}+\frac{\partial^4}{\partial x'^i_{(n)}\partial x'^j_{(n)}\partial x'^k_{(m)}\partial x'^l_{(m)}}\right.$$

$$\left.-2\frac{\partial^4}{\partial x^i_{(n)}\partial x^j_{(n)}\partial x'^k_{(m)}\partial x'^l_{(m)}}\right)\rho.\qquad(3.9)$$

This is the main result of this paper: the master equation for N nonrelativistic particles interacting through linearized gravity. The gravitational field induces a renormalization in the mass of the particles, modifies the dynamics so that they become dissipative, and is responsible for noise. These three effects are contained in the functions $\delta m(t)$, $\alpha(t)$, and $\beta(t)$, respectively:

$$\delta m_{(n)}(t)=\frac{4G\hbar}{c^5}\,g^{ijkl}_{(n)(n)}\delta_{ij}\delta_{kl},\qquad(3.10)$$

$$\alpha^{ijkl}_{(n)(m)}(t)=\frac{4G}{\hbar c^5}\,\dot{f}^{ijkl}_{(n)(m)}t^2,\qquad(3.11)$$

$$\beta^{ijkl}_{(n)(m)}=\frac{4G}{\hbar c^5}\,(tg+\tfrac{1}{2}\dot{g}t^2)^{ijkl}_{(n)(m)}.\qquad(3.12)$$

IV. ONE PARTICLE

An interesting case is that of a single particle. Since then the functions in the master equation are totally symmetric in the spatial indices, we can, without loss of generality, consider it constrained to move in only one dimension. The master equation reads then in operator form

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[H_R,\rho] - i\alpha(t)[H_R^2,\rho] - \beta(t)[H_R,[H_R,\rho]]$$

(4.1)

and depends explicitly only on the renormalized Hamiltonian H_R. We can verify that this form of master equation (in particular the noise part) is particular to the free particle case. For a harmonic oscillator we would get an extra dissipation and diffusion term due to the coupling of the particle's position to the graviton oscillator Hamiltonian, and of form similar to the one derived in [3] for quadratic coupling to position.

The diffusion coefficient $\beta(t)$ exhibits a ''jolt'' for times of the order of Λ^{-1}. In quantum Brownian models this is a cause of rapid decoherence of the density matrix of the particle, and diagonalization in a basis determined by the coupling to the environment. Our particular form of the diffusion terms tempts us to propose that it should lead to diagonalization of the particle's density matrix in the energy eigenstate basis. But we have to take into account that the coupling is extremely weak and that after the jolt the diffusion coefficient falls to zero, quite slowly actually since it varies at most as $1/t$.

We can give an estimation of the decoherence in the energy by approximating $\beta(t)$ with a constant of the order of $G\Lambda^2/\hbar c$ for times of the order of Λ^{-1} and zero afterwards. We borrow some ideas from the quantum state diffusion picture of quantum mechanics [13–15]. At the times that $\beta(t)$ is constant, we have a unique unraveling of the density matrix into states evolving stochastically in a Hilbert space. It is straightforward to show [13] that an initial wave packet with energy spread ΔE_0 will emerge after the jolt with spread ΔE given by

$$\frac{1}{(\Delta E)^2} - \frac{1}{(\Delta E_0)^2} \sim \frac{G\Lambda}{\hbar c^5}.$$

(4.2)

For a single particle of mass m a good upper bound on $\hbar\Lambda$ is $Gm^3c/\hbar$: the classical gravitational self-energy of a mass distribution localized within the Compton wavelength of the particle. This means that

$$\frac{1}{(\Delta E)^2} - \frac{1}{(\Delta E_0)^2} \sim \frac{G^2 m^3}{\hbar^3 c^4}.$$

(4.3)

This is an extremely small quantity, when considering microscopic particles (even on atomic scales). On the other hand, for macroscopic and even mesoscopic particles the right hand side is quite large and we expect a localization of the particle in its energy eigenstates. For instance, a particle with mass $m = 10^{-8}$ g and irrespective of its initial configuration will emerge after 10^{-30} s localized in an energy eigenstate with spread of the order of 0.1 MeV, which is a tiny portion of its kinetic energy. But in this case, the gravity-induced decoherence is, in general, hidden beneath the effects of other types of environment [16]. In any case, this result is in good agreement with the assumptions of the gravitationally induced collapse models.

These features were more or less expected, since gravity couples very weakly and its strength increases with the mass of the interacting bodies. Still, there was the possibility that a persistent noise source might induce decoherence even in microscopic systems, despite the weakness of the coupling. Note that our analysis based on the linearized approximation does not rule out the possibility that highly nonlinear Planck scale processes [11,12] might be a source of noise, giving rise to decoherence at smaller mass scales.

The dissipation function $\alpha(t)$ approaches asymptotically a constant of the order of $G\Lambda/\hbar c^5$. The overall picture we get is that of a particle continuously dissipating energy and suffering at early times noise from the environment until it becomes correlated with the gravitational field.

V. CONCLUSIONS

We have studied the quantum theory of N nonrelativistic particles coupled to the linearized gravitational field using the influence functional formalism. Our main result was the master equation (3.9) containing information about nonlocal interaction between the particles. We should note that the gravitational field, being coupled quadratically to the velocities, gives a rather unusual expression for the influence functional. This results in a master equation where both dissipation and diffusion are determined uniquely by the Hamiltonian operator. This is in accordance with our intuitive feeling, that the gravitational field acts as if continuously ''measuring'' a particle's energy.

One of our motivations for this work was to establish whether we can consider the gravitational field as a source of fundamental decoherence in quantum mechanics. The answer comes out negative for microscopic systems, but systems with large mass seem to decohere with a fast rate, in the energy eigenstate basis. In addition, it might be interesting to examine the evolution of a single particle under the action of a particular matter distribution. The formalism we used can be extended with slight modifications to cover this case. We can, for instance, consider almost stationary cosmic dust and even a cosmological spacetime. The collective effect of matter plus gravity might give the strongest decoherence to the particle.

In addition, it would be of interest to study the response of a system of detectors to different initial conditions for the graviton field. The case where a number of modes is excited seems very interesting. The information on the state of the field should be encoded in the correlation kernels of the particles, from the time evolution of which we would be able to determine the presence of the graviton fields. This might give a nice toy model for detectors of gravitational waves.

APPENDIX

We give here the identities that enable us to compute perturbatively the master equation. We give the form for the case of one dimension and one particle. The generalization is straightforward.

$$(x_f - x_0)^2 J(x_f, x_f', t | x_0, x_0', 0)$$
$$= \left[-t^2(1 - 2iG(9f - 11ig)) \left(\frac{\partial^2}{\partial x_f^2} - \frac{i}{t} \right) \right.$$
$$- 2Gt^2 \left(\frac{\partial^2}{\partial x_f^2} + \frac{i}{t} \right) - 2Gt(3f - 4ig)$$
$$\left. + \frac{4G}{t}(f - ig)(x_f - x_0)^4 + \frac{4iG}{t} g(x_f - x_0)^2(x_f' - x_0')^2 \right]$$
$$\times J(x_f, x_f', t | x_0, x_0', 0) + \mathcal{O}(G^2), \tag{A1}$$

$$(x_f - x_0)^4 J(x_f, x_f', t | x_0, x_0', 0)$$
$$= \left[t^4 \frac{\partial^4}{\partial x_f^4} - 6it^3 \frac{\partial^2}{\partial x_f^2} - 3t^2 \right] J(x_f, x_f', t | x_0, x_0', 0) + \mathcal{O}(G), \tag{A2}$$

$$(x_f - x_0)^2 (x_f' - x_0')^2 J(x_f, x_f', t | x_0, x_0', 0) = \left[t^4 \frac{\partial^4}{\partial x_f^2 \partial x_f'^2} \right.$$
$$\left. + it^3 \frac{\partial^2}{\partial x_f^2} - it^3 \frac{\partial^2}{\partial x_f'^2} + t^2 \right] J(x_f, x_f', t | x_0, x_0', 0) + \mathcal{O}(G). \tag{A3}$$

We should keep in mind that eventually we keep terms to first order in G. The expressions for the primed quantities are obtained by permutation of primed with unprimed ones and complex conjugation.

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