Universality of supersymmetric attractors

Sergio Ferrara*
Theory Division, CERN, 1211 Geneva 23, Switzerland

Renata Kallosh†
Physics Department, Stanford University, Stanford, California 94305-4060

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The macroscopic entropy-area formula for supersymmetric black holes in $N=2,4,8$ theories is found to be universal: in $d=4$ it is always given by the square of the largest of the central charges extremized in moduli space. The proof of universality is based on the fact that the doubling of unbroken supersymmetry near the black hole horizon requires that all central charges other than $Z=M$ vanish at the attractor point for $N=4,8$. The ADM mass at the extremum can be computed in terms of duality symmetric quartic invariants which are moduli independent. The extension of these results for $d=5$, $N=1,2,4$ is also reported. A duality symmetric expression for the energy of the ground state with spontaneous breaking of supersymmetry is provided by the power 1/2 (2/3) of the black hole area of the horizon in $d=4$ $(d=5)$. It is suggested that the universal duality symmetric formula for the energy of the ground state in supersymmetric gravity is given by the modulus of the maximal central charge at the attractor point in any supersymmetric theory in any dimension. [S0556-2821(96)06214-5]

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I. INTRODUCTION

The attractor behavior in the supersymmetric system was discovered in the context of $N=2$ extremal magnetic black holes [1]. It was explained in this work that unbroken supersymmetry leads to the fact that the area of the black hole horizon depends only on conserved charges. The idea was further developed for more general black hole solutions in [2,3]. In [3] the complete treatment of $d=4$, $N=2$ and $d=5$, $N=1$ supersymmetric attractors was presented as well as some particular examples of $N=4,8$ attractors in $d=4$. The main result of this work was that the area of the supersymmetric black hole horizon in the theories of $N=2$ supergravity interacting with arbitrary number of vector and hypermultiplets can be computed by extremizing the central charge of the theory in the moduli space. The summary of the attractor picture in $N=2$ theories is the following. There is only one central charge Z_{AB} where $A,B=1,2$ and the central charge Z_{AB} is complex and antisymmetric in A,B. Therefore we have only $Z_{12}=Z$. Unbroken supersymmetry of the $N=2$ black holes requires

$$M^2_{\mathrm{ADM}}(q,p,\phi)=|Z(q,p,\phi)|^2. \tag{1}$$

In gravitational theories the ADM mass in asymptotically flat spaces defines the energy of the space-time $M_{\mathrm{ADM}}=E$. Thus in $N=2$ theories one may have concluded that the area of the horizon is proportional to the square of the energy in its minimum:

$$\frac{A(q,p)}{4}=\pi E^2(q,p,\phi)\big|_{\partial E/\partial\phi=0}. \tag{2}$$

Vice versa, the minimal energy is given by the square root of the area:

$$E(q,p)=\left(\frac{A(q,p)}{4\pi}\right)^{1/2}. \tag{3}$$

In $d=5$ the minimal energy in $N=2$ theories was shown to be $E(q,p)\sim[A(q,p)]^{2/3}$ [3].

The purpose of this paper is to find out how much of this behavior carries over to higher extended supersymmetries, where the number of central charge eigenvalues is $N/2>1$. We would like to mention in this respect that the focus of interest to the problem as to whether the area always does not depend on moduli was stimulated by the work of Larsen and Wilczek [4].

The area formulas for theories with extended supersymmetries are build out of central charges on the basis of duality invariance and comparison with some known particular black hole solutions. For example, in $N=4$ theories there are two eigenvalues of the central charge matrix Z_1 and Z_2. The area of the horizon for the particular black holes in $N=4$ supergravity without matter multiplets is proportional to [5]

$$\left(\frac{A(q,p)}{4}\right)_{N=4}=\pi(|Z_1|-|Z_2|^2). \tag{4}$$

In $N=8$ theory the corresponding formula for the area depends on 4 eigenvalues of the central charge matrix and is given in agreement with some class of solutions by [6]

$$\left(\frac{A(q,p)}{4}\right)_{N=8}=\pi\left(\left|\left(\sum_i |Z_i|^4-2\sum_{i>j}|Z_i|^2|Z_j|^2 + 8|Z_1Z_2Z_3Z_4|\right)\right|\right)^{1/2}. \tag{5}$$

*Electronic address: ferraras@cernvm.cern.ch
†Electronic address: kallosh@physics.stanford.edu

Formula (5) corresponds to the Cremmer-Julia [7] E(7) quartic invariant,

$$\diamondsuit = \mathrm{Tr}(Z\overline{Z})^2 - \tfrac{1}{4}(\mathrm{Tr}Z\overline{Z})^2 + 4(\mathrm{Pf}Z + \mathrm{Pf}\overline{Z}), \tag{6}$$

in the normal frame for the central charge matrix [8]. Each central charge depends on moduli and electromagnetic charges.

One may have thought that the function to be extremized to get the area will be some combination of various central charges in $N>2$ theories generalizing the one central charge situation in $N=2$ theories. However, this is not the case, the result of our study is that the unbroken supersymmetry leads to the *universal formula for the area of the horizon* in all $d=4$ extended supersymmetric theories. First, let us note that the energy of the space-time, or ADM mass is equal to the largest eigenvalue of the central charge matrix from the requirement of unbroken supersymmetry[1]:

$$E(q,p,\phi) = M_{\mathrm{ADM}}(q,p,\phi) = \max|Z_C(q,p,\phi)|,$$
$$C = 1, \ldots, N/2. \tag{7}$$

We will find that the square of the minimal energy always defines the area of the horizon of the supersymmetric black holes:

$$\frac{A(q,p)}{4} = \pi E^2(q,p,\phi)|_{\partial E/\partial\phi=0}. \tag{8}$$

Let us relabel the central charges: the largest one $\max|Z_C(q,p,\phi)|$ will be called $|Z|$ so that

$$\max|Z_C(q,p,\phi)| \equiv |Z|,$$

and the remaining $(N/2-1)$ eigenvalues of the central charge matrix will be labelled by the index c which runs from 1 to $(N/2-1)$.

We will establish that the fixed point of attraction in the theories of extended supersymmetries with $N>2$ is given by the condition of the vanishing of all eigenvalues of the central charge matrix which are smaller than the largest one, defining the ADM mass:

$$|(Z_c)_{\mathrm{fix}}| = 0, \quad c = 1, \ldots, \left(\frac{N}{2} - 1\right). \tag{9}$$

The area formula therefore is always given by the extremum value of the central charge, or space-time energy, which is the point where the other charges vanish. Let for example in $N=4$ case the first eigenvalue is larger than the second $|Z_1| > |Z_2|$. We will find that the area formula for black holes in $N=4$ supergravity with $N=4$ vector multiplets is given by a duality symmetric formula which is also an extremum of the Arnowitt-Deser-Misner (ADM) mass defined by the vanishing of the next to the largest eigenvalue of the central charge.

$A(q,p)$ can be explicitly computed, by noticing that $\pi(|Z_1| - |Z_2|^2)_{Z_c=0} = \pi(|Z_1| - |Z_2|^2)$ at the matter attractor point (deduced by setting the gaugino variation $\delta\lambda^{ia}=0$ but not fixing the S dilaton). In $N=4$ we have that $|Z_2|=0$ forces the dilaton to take the fixed value $S=S_{\mathrm{fix}}$ and the matter scalars to the point $\delta\lambda_i^a(a=1,\ldots,n_v)=0$. So

$$(|Z|^2)|_{\mathrm{fix}} = (|Z_1| - |Z_2|^2)_{S=S_{\mathrm{fix}},\,\delta\lambda_i^a=0}. \tag{10}$$

But now we use the fact that $(|Z_1| - |Z_2|^2)_{\delta\lambda_i^a=0}$ is independent of S and this allows to give an explicit formula for $|Z_1|_{\mathrm{fix}}$ in terms of $2(6+n_v)$ charges:

$$|Z_1|^2_{\mathrm{fix}} = \tfrac{1}{2}\sqrt{q^2p^2 - (q\cdot p)^2}. \tag{11}$$

Similarly in $N=8$, the attractor point is the value of the 70 moduli $\phi=\phi_{\mathrm{fix}}$ for which $|Z_2|=|Z_3|=|Z_4|=0$. This means that the area formula is given in terms of the Cartan's quartic invariant J which depends only on charges and does not depend on moduli. The universality of the area formula comes from the fact that the Cremmer-Julia E(7) invariant at the attractor point on one hand has to depend only on charges and be E(7) symmetric, and therefore has to coincide with Cartan's invariant J; on the other hand at the attractor it is given by

$$\left(\frac{A(q,p)}{4}\right)_{N=8} = \pi\left(\left|\left(\sum_i |Z_i|^4 - 2\sum_{i>j}|Z_i|^2|Z_j|^2 + 8|Z_1Z_2Z_3Z_4|\right)\right|\right)^{1/2}_{|Z_2|=|Z_3|=|Z_4|=0} = |Z_1|^2_{\mathrm{fix}}, \tag{12}$$

therefore

$$|Z_1|^2_{\mathrm{fix}} = (\sqrt{\diamondsuit})_{\phi=\phi_{\mathrm{fix}}} = \sqrt{J}, \tag{13}$$

and this gives an explicit formula of $|Z_1|^2_{\mathrm{fix}}$ in terms of the 56 charges.

If Cremmer-Julia E(7) invariant $\diamondsuit$ is ϕ independent in the generic point of the moduli space, which seems likely, in this case we have also

$$\diamondsuit = J, \tag{14}$$

as conjectured by Cremmer and Julia [7]. In any case at the attractor point these two invariants coincide and this is the reason for the universality of our area formula.

Similar results are also obtained for $N=2,4$ at $d=5$.

We find it useful to introduce here an additional object $\Sigma(\rho)^I$ for any scalar field $\phi^I(r)$, which forms part of the black hole solution. It is typical for the attractor problem to have a pair of phase space variables $[x(t), y(t)=x'(t)]$. In our case the corresponding pair consists of the scalar field and the first derivative of the scalar field

$$\Sigma^I(\rho) \equiv \frac{\partial}{\partial\rho}\phi^I(\rho). \tag{15}$$

We have plotted the value of the dilaton $e^{-2\phi(\rho)}$ and the $\Sigma(\rho)$ for the dilaton $U(1)^2$ black hole. We are using here the radial variable $\rho=-1/r$ in terms of which the near horizon geometry is conformally flat. Figure 1(a) shows that starting

[1]For simplicity we consider only even N.

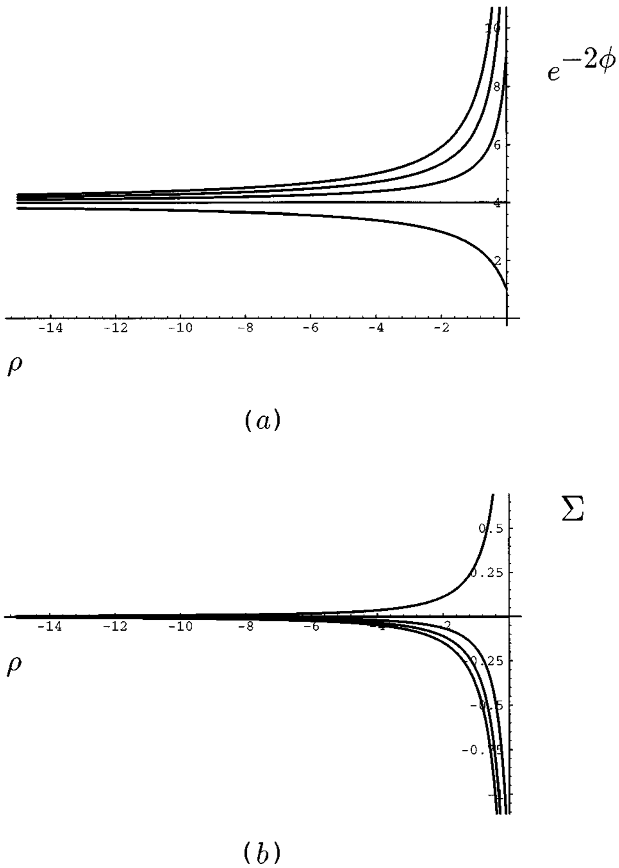

FIG. 1. Evolution of the dilaton field $e^{-2\phi}$ and of the effective dilaton charge $\Sigma(\rho) = -d\phi/d\rho$ for various initial conditions at $\rho=0$ $(r=\infty)$ to a common fixed point at $\rho=-\infty$ $(r=0)$.

with any initial conditions at $\rho=0$ $(r\to\infty)$ the field is driven to an attractor value at $\rho\to-\infty$ $(r=0)$. Figure 1(b) shows how the derivative evolves. The initial condition for the derivative of the scalar field at $\rho=0$ $(r\to\infty)$ is equal to the so-called scalar charge Σ . It is a function of the moduli at $\rho=0$ $(r\to\infty)$ and electromagnetic charges, therefore the scalar charge is also called a secondary hair of the black hole. When evolving into the core of the black hole, independently of the initial value of the scalar charge at $\rho=0$ $(r\to\infty)$, the derivative (i.e., the effective scalar charge) $\Sigma(\rho)$ goes to zero at $\rho=\to-\infty$ $(r=0)$.

The physical picture of this phenomenon was suggested to us by A. Linde. It reflects the fact that the values of electric and magnetic charges are protected by gauge invariance and the associated Gauss low. One can measure the actual values of electric and magnetic charges being far away from the black hole, which explains why these charges are called the black hole hair. The scalar charges which can be measured far away from the black hole are not protected by any conservation low. This is why the derivative of the scalar fields decreases when moving into the core of the black hole; see Fig. 2(b). Whereas the electric and magnetic charges are conserved and their presence and stability supports the existence of an infinite throat of the Bertotti-Robinson [9] geometry, the derivative of the scalar charge $\Sigma(\rho)\to 0$ at $\rho\to\infty$, i.e., it does not penetrate into the throat to keep its size (the area of the black hole horizon) minimal. This results in the minimal energy of the ground state. This picture may apply not only for supersymmetric black holes. One may try to find out if the analogous phenomenon takes place at least for near extreme black holes, as suggested to us by L. Susskind. The plausibility of this picture relies on the fact that we may be dealing here with the critical phenomena which have some specific range of applicability and in particular the attractor behavior of the system may well describe the nearby trajectories in the phase space of the system.

One should stress that the extremization of the energy which we study here is performed under condition that the scalar fields are constant as it is usually done in quantum field theory by looking for the minima of the energy in the class of configurations with constant scalars. This is the main feature of supersymmetric attractors, since we study the fixed points of the differential equations where the scalars have vanishing derivatives [1].

$$\phi'_{\text{fix}}(\rho)\to 0, \quad \rho\to\infty. \tag{16}$$

The massless black holes (solutions with ADM mass going to zero) which one would tend to associate with the minimum of the ADM energy, do not fit into the class of attractors with constant scalars in the fixed point and has to be studied separately.

From the point of view of supersymmetry, this distinction comes from the following fact. The graviphoton charge, which is a linear combination of moduli and electromagnetic charges, always represents the ADM mass for supersymmetric solutions, i.e., the energy of the space-time. The minimization in the class of configurations with constant scalars by supersymmetry requires all vector fields of the matter multiplets to vanish at the attractor. Their charges also are given by some linear combinations of moduli and electro-magnetic charges, which vanish at attractor. This leads to Bertotti-Robinson-type geometry and black holes with nonvanishing area of the horizon.

On the other hand one may be interested in the opposite situation when the graviphoton charge tends to zero, leading to a specific relation between the moduli and electromagnetic charges, corresponding to massless black holes. The matter multiplet charges do not vanish, otherwise the solution would be trivial. By supersymmetry it follows that the scalars do not tend to a constant near the black hole core, the geometry is very different from the Bertotti-Robinson-type geometry and this class of configurations has to be studied separately.

Also the configurations with the zero area which have doubled or quadrupled number of unbroken supersymmetries comparatively to those with nonvanishing area, do not have fixed points for the scalar fields near the horizon [10].

In what follows we will explain how the result described above follows from unbroken supersymmetry. In Sec. II we will explain the particular case of pure $N=4$ supergravity, the $U(1)^2$ model. We will present the complete theory of attractors in $N=4$ supergravity interacting with arbitrary number of $N=4$ vector multiplets in Sec. III. The area formula is presented in various forms which show the manifest S and T duality as well as moduli independence as the consequence of supersymmetry. On top of it the area formula is finally reduced to the universal one in terms of the minimization of the ADM mass in the moduli space. In Sec. IV we will analyze the simple $N=8$ model, an STU attractor, which presents in a nice way the gross features of the attractors with extended supersymmetries. The general $N=8$ attractor is described in Sec. V. Cartan's quartic invariant which reflects the E(7) symmetry of the theory is miraculously reduced to the simple and universal formula for the area as the minimum of the largest eigenvalue of the central charge. Section VI presents supersymmetric attractors in $d=5$. We present the area formulas in $N=1,2,4$ theories. In particular, the cubic E(6) invariant is described in connection with three central charges of the maximally extended $N=4$ theory in $d=5$. The universality of the area formula again follows from the vanishing of all eigenvalues of the central charges except the largest one. One more miracle of supersymmetry and we get the universal formula for the area in terms of the minimization of the ADM energy for all cases considered.

These results should apply to strings theories or generalization thereof (M theory?). A microscopic derivation is expected to correct the entropy formula for small values of charges. This is of course related to the fact that Einstein gravity is only the pointlike limit approximation of the more general theories.

Our study of supersymmetric black holes and their attractor behavior suggest the following interpretation: we have computed the exact energy of the ground state in supersymmetric gravity as the function of electric and magnetic charges. The presence of these charges (excitations of a superstrings) in the vacuum leads to spontaneous breaking of supersymmetry, resulting in a duality symmetric positive energy of the ground state associated with the nonvanishing area of the black hole horizon. The recent success in the calculation of the area of the horizon of some five- and four-dimensional black holes [11] from the point of view of counting string states with the use of D-brane technology [12] naturally fits into our interpretation. The nontrivial part of this picture is related to the fact that the extreme black holes with the nonvanishing area of the horizon have a conformal isometry which exchanges the two asymptotic regions. Therefore the calculation of the area of the horizon of such extreme black holes can be interpreted as the evaluation of the square of the energy of the ground state of this system. The corresponding conformal factor has one parameter, the area of the horizon, or the mass of the Bertotti-Robinson universe. The relevance of the computation of the area of the extreme black hole horizon to spontaneous supersymmetry breaking is explained in Sec. VII of the paper.

II. ATTRACTOR IN PURE $N=4$ SUPERGRAVITY, $U(1)^2$ MODEL

The basic feature of the extended supersymmetry attractors can be easily understood already in the case of pure $N=4$ supergravity with one gravitational multiplet only. The bosonic fields in the SU(4) version consist of a complex axion-dilaton scalar, three vectors and three axial vectors and the metric. It was explained in [10] that near the horizon the unbroken supersymmetry of the $U(1)^2$ black hole is doubled. Instead of 1/4 of $N=4$ supersymmetry, near the horizon 1/2 of $N=4$ supersymmetry is restored. In the basis chosen in [5] we have the following situation near the horizon: for positive pq the unbroken $N=2$ supersymmetry consists of the third and the fourth ones, and for negative values of pq it is reverse, the first and the second supersymmetries are unbroken whereas the third and the fourth are broken. The dilatino transformations rules in the notation of [5] is

$$\tfrac{1}{2}\,\delta\Lambda_I=-\gamma^\mu\epsilon_I\partial_\mu\phi+\frac{1}{\sqrt{2}}\sigma^{\mu\nu}(e^{-\phi}F_{\mu\nu}\alpha_{IJ}-e^{\phi}\widetilde{G}_{\mu\nu}\beta_{IJ})^-\epsilon^J$$
$$=0. \tag{17}$$

The first term in Eq. (17) vanishes at the fixed point, since we are looking for $\phi'=0$. Thus at the attractor we get, for $pq>0$, $J=3,4$ as well as for $pq<0$, $J=1,2$ the second term in Eq. (17)

$$\Sigma_{\rm fix}\,\epsilon^J=\tfrac{1}{2}\,(e^{-\phi}|p|-e^{\phi}|q|)_{\rm fix}\,\epsilon^J=0, \tag{18}$$

which leads to the condition of the vanishing of the dilaton charge at the attractor:

$$\Sigma_{\rm fix}=\tfrac{1}{2}\,(e^{-\phi}|p|-e^{\phi}|q|)_{\rm fix}=0\Leftrightarrow e_{\rm fix}^{-2\phi}=\frac{|q|}{|p|}. \tag{19}$$

We may also rewrite the dilatino transformation rule at the attractor in the form

$$(Z_{IJ})_{\rm fix}\,\epsilon^J=0. \tag{20}$$

For $pq>0,\epsilon^3,\epsilon^4$ are nonvanishing, therefore at the attractor using also the gravitino transformation rule we learn that

$$Z_{34}=0,\quad |Z_{12}|=(M_{\rm ADM})_{Z_{34}=0}pq>0, \tag{21}$$

and for $pq<0,\epsilon^1,\epsilon^2$ are nonvanishing, therefore at the attractor

$$Z_{12}=0,\quad |Z_{34}|=(M_{\rm ADM})_{Z_{12}=0}pq<0. \tag{22}$$

III. ATTRACTOR IN $N=4$ SUPERGRAVITY WITH n_v VECTOR MULTIPLETS: GENERAL CASE

The geometry of the $N=4$ supergravity coupled to n_v matter vector multiplets[2] is based on the nonlinear sigma model $SU(1,1)/U(1)\times O(6,n_v)/O(6)\times O(n_v)$. The $SU(1,1)/U(1)$ manifold is parametrized by a complex scalar field S and the vector multiplet manifold by the coset representatives $L_\Lambda^A=(L_\Lambda^{ij},L_\Lambda^a)$:

$$L_\Lambda^{ij}=-L_\Lambda^{ij}=L_{\Lambda ij}^{*}=\tfrac{1}{2}\,\epsilon^{ijkl}L_{\Lambda kl}\,, \tag{23}$$

where $i,j=1,2,3,4,\Lambda=1,\dots,6+n_v,a=1,\dots,n_v$, and there are orthogonality relations

$$-L_\Lambda^aL_{a\Sigma}+L_\Lambda^{ij}L_{\Sigma ij}=\eta_{\Lambda\Sigma}\,,\quad L_\Lambda^aL_b^\Lambda=-\delta_b^a\,, \tag{24}$$

[2]We describe here the version of $N=4$ theory closely related to the one in [13]. The version here has the property of being symplectic covariant. The details of this construction will be presented elsewhere.

and

$$L^{ij}_{\Lambda}L^{\Lambda}_{kl}=\tfrac{1}{2}\,(\delta^{i}_{[k}\delta^{j}_{l]}),\quad L^{a}_{\Lambda}L^{\Lambda}_{ij}=0. \qquad (25)$$

The vector field (complexified) kinetic matrix is

$$\mathcal{N}_{\Lambda\Sigma}=(S-\overline{S})L^{ij}_{\Lambda}L_{\Sigma ij}+\overline{S}\,\eta_{\Lambda\Sigma}\,, \qquad (26)$$

and the symplectic sections are

$$(L^{\Lambda}_{ij}\,,\mathcal{N}_{\Lambda\Sigma}L^{\Sigma}_{ij}=SL_{\Lambda ij}),$$

$$(L^{\Lambda}_{a}\,,\mathcal{N}_{\Lambda\Sigma}L^{\Sigma}_{a}=\overline{SL}_{\Lambda a}). \qquad (27)$$

In terms of these sections the central charge Z_{ij} is

$$Z_{ij}=e^{K/2}[L^{\Lambda}_{ij}q_{\Lambda}-SL_{ij\Lambda}p^{\Lambda}], \qquad (28)$$

where $K=-\ln i(S-\overline{S})$ is the S-field Kahler potential. For the x-independent scalars $(S,L^{\Lambda}_{ij},L^{\Lambda}_{a})$ unbroken supersymmetry for the matter gaugino's $\delta\lambda^{a}_{i}=0$ requires at the attractor point

$$SL^{a}_{\Lambda}p^{\Lambda}-L^{a\Lambda}q_{\Lambda}=0, \qquad (29)$$

while unbroken $N=1$ supersymmetry for the dilatino χ^{i} requires that the central charge $|Z_2|<|Z_1|=M_{\rm ADM}$ given by

$$|Z_2|^2=\tfrac{1}{4}\,(Z_{ij}\overline{Z^{ij}}-\sqrt{(Z_{ij}\overline{Z^{ij}})^2-\tfrac{1}{4}\,|\epsilon^{ijkl}Z_{ij}Z_{kl}|^2}) \qquad (30)$$

should vanish. Equation (30) fixes the value of S at its attractor point:

$$Z_2=0. \qquad (31)$$

It can be proved using the symplectic formulation of $N=4$ theory given above that the quantity

$$|Z_1|^2-|Z_2|^2=\tfrac{1}{2}\,\sqrt{(Z_{ij}\overline{Z^{ij}})^2-\tfrac{1}{4}\,|\epsilon^{ijkl}Z_{ij}Z_{kl}|^2} \qquad (32)$$

is S independent. This would be sufficient to prove the moduli independence of this expression in pure $N=4$ supergravity. However, in presence of $N=4$ vector multiplets this expression as a function of attractor variables corresponding to α,β charge vectors of string theory does depend on the scalars of these multiplets (the asymptotic value of the matrix $\mathcal{M}$ used in various black holes constructions). This was established in [14] [see Eq. (8.13) of this paper] starting with ten dimensional supersymmetry and using the Witten-Israel-Nestor construction. However our formalism shows that at the matter attractor point defined in Eq. (29) this expression does not depend on matter moduli anymore and becomes the function of charges only:

$$(|Z_1|^2-|Z_2|^2)_{|Z_2|=0}=\tfrac{1}{2}\,\sqrt{q^2p^2-(q\cdot p)^2}, \qquad (33)$$

where Lorentzian O$(6,n_v)$ norm for $q_{\Lambda}\,,p^{\Lambda}$ doublet is understood. It then follows that the area is

$$A=2\pi\sqrt{q^2p^2-(q\cdot p)^2}. \qquad (34)$$

Note that the $2(6+n_v)$ electric and magnetic charges form an SU(1,1) doublet and (6, n_v) Lorentzian vectors. This coincides with the minimum of the ADM mass,

$$A(q,p)=4\pi(M^2_{\rm ADM})_{\partial M/\partial\phi=0}, \qquad (35)$$

in the axion-dilaton and matter moduli space.

We would like to stress here that in [3] we have introduced the concept of attractor variables for the black hole solutions: variables in which the ADM mass depend on charges and moduli, however, the area depends only on charges. The area formula (34) in the attractor variables appeared in the recently revised version of Ref. [15]. Our choice of what are attractor variables is defined by the formulation of the theory with manifest symplectic symmetry. Simultaneously this form provides the proof of the independence of the area from the moduli and leads to the universality of the area formula in terms of the extrema of the ADM mass in the moduli space. On the other hand the heterotic area formula given in [15] as well as our proof of independence on all moduli at the attractor point applied to the expression for central charges in [14] provides the link to the properties of a string theory. Indeed the corresponding attractor variables are the conserved α,β charge vectors of string theory introduced into the black hole physics by Sen [16] a long time before it was realized that the area of supersymmetric black holes depends only on α,β by the reason of supersymmetry, as explained in this paper. Our $N=4$ results and formulas, since they entirely rely on general theory of $N=4$ supergravity coupled to vector multiplets, should equally apply to the heterotic string compactified on T_6 or to the type II string compactified on $K_3\times T_2$.

IV. STU ATTRACTOR IN $N=8$ SU(8) SUPERGRAVITY

In [3] we have described STU model in the attractor variables. Here, as the preparation to general $N=8$ attractor we would like to check whether the main principle of the minimization of the largest eigenvalue of the central charge will produce the moduli independent area. We will denote $e^{-\eta_0}=\mathrm{Im}S=s,\ e^{-\sigma_0}=\mathrm{Im}T=t,\ e^{-\rho_0}=\mathrm{Im}U=u$.

The ADM mass considered as a function of charges in generic point of the moduli space (s,t,u) is

$$M_{\rm ADM}=\frac{1}{4}\left(stu|q_1|+\frac{s}{tu}|q_3|+\frac{u}{st}|p_2|+\frac{t}{su}|p_4|\right). \qquad (36)$$

The variation of the mass over the moduli gives three attractor equations:

$$\left(stu|q_1|+\frac{s}{tu}|q_3|-\frac{u}{st}|p_2|-\frac{t}{su}|p_4|\right)=0,$$

$$\left(stu|q_1|-\frac{s}{tu}|q_3|+\frac{u}{st}|p_2|-\frac{t}{su}|p_4|\right)=0,$$

$$\left(stu|q_1|-\frac{s}{tu}|q_3|-\frac{u}{st}|p_2|+\frac{t}{su}|p_4|\right)=0. \qquad (37)$$

The solution of these equations puts the moduli into the fixed points where they become functions of charges:

$$(stu)_{\rm fix}|q_1|=\left(\frac{u}{st}\right)_{\rm fix}|p_2|=\left(\frac{s}{tu}\right)_{\rm fix}|q_3|=\left(\frac{t}{su}\right)_{\rm fix}|p_4|. \tag{38}$$

We get the useful relations

$$(s^2t^2)_{\rm fix}=\frac{|p_2|}{|q_1|},\quad (u^2t^2)_{\rm fix}=\frac{|q_3|}{|q_1|},\quad (s^2u^2)_{\rm fix}=\frac{|p_4|}{|q_1|}, \tag{39}$$

and

$$(stu)_{\rm fix}=\left(\frac{|p_2q_3p_4|}{|q_1|^3}\right)^{1/4}. \tag{40}$$

This allows to get the value of the ADM mass at the attractor:

$$(M_{\rm ADM})_{\rm fix}=(stu)_{\rm fix}|q_1|=|q_1p_2q_3p_4|^{1/4}. \tag{41}$$

We may now conjecture that the generalization of the four eigenvalues of the $N=8$ supergravity central charges [6] is

$$4Z_1=\left(stu|q_1|+\frac{s}{tu}|q_3|\right)+\left(\frac{u}{st}|p_2|+\frac{t}{su}|p_4|\right),$$

$$4Z_2=\left(stu|q_1|+\frac{s}{tu}|q_3|\right)-\left(\frac{u}{st}|p_2|+\frac{t}{su}|p_4|\right),$$

$$4Z_3=\left(stu|q_1|-\frac{s}{tu}|q_3|\right)+\left(\frac{u}{st}|p_2|-\frac{t}{su}|p_4|\right),$$

$$4Z_4=\left(stu|q_1|-\frac{s}{tu}|q_3|\right)-\left(\frac{u}{st}|p_2|-\frac{t}{su}|p_4|\right). \tag{42}$$

Now one can see that indeed the three attractor equations (37) mean exactly

$$Z_2=Z_3=Z_4=0,$$

$$Z_1|_{Z_2=Z_3=Z_4=0}=(M_{\rm ADM})_{\rm fix}=|q_1p_2q_3p_4|=E(p,q). \tag{43}$$

This example makes it natural to look for the general $N=8$ attractor expecting to get the area from the minimum in the moduli space of the largest of the four eigenvalues of the central charge matrix.

V. $N=8$ ATTRACTOR: GENERAL CASE

$N=8$ theory has only one gravitational multiplet. Therefore all 28 vector fields are graviphotons, there are no vector fields which are not supersymmetric partners of the graviton. The 28 electric and 28 magnetic charges all together are in 56 fundamental representation of E(7). The black hole solutions of this theory with 1/8 of supersymmetry unbroken are known to have a nonvanishing area. The manifestly E(7) symmetric area formula is given by the unique quartic invariant of E(7). However, how can we find out if the area is independent on 70 moduli and depends only on 56 charges? Let us first analyze the supersymmetry transformation rules [7] near the attractor where all 70 moduli tend to a constant values:

$$\delta\Psi_{\mu A}=D_\mu\epsilon_A+Z_{AB\mu\nu}\gamma^\nu\epsilon^B, \tag{44}$$

$$\delta\chi_{ABC}=Z_{[AB\mu\nu}\sigma^{\mu\nu}\epsilon_{C]}\,. \tag{45}$$

Let us decompose $N=8$ into SU(4)×SU(4) as (4,1) + (1,4), keeping all fields of $N=8$ theory. Then $N=8$ supergravity multiplet will split into one gravitational multiplet of $N=4$ theory,

$$[(2)4(3/2)6(1)4(1/2)2(0)],$$

four spin 3/2 multiplets

$$4[(3/2)4(1)6+1(1/2)8(0)],$$

and six vector multiplets

$$6[(1)4(1/2)6(0)].$$

The eight-dimensional index A is split as $A=(i,a)$ where $i=1,\ldots,4$ and $a=1,\ldots,4$. The fermions are $\Psi_{\mu A}=(\Psi_{\mu i},\Psi_{\mu a})$ and $\chi_{ABC}=(\chi_{ijk},\chi_{iab},\chi_{aij},\chi_{abc})$. In $N=4$ theory χ_{ijk} is in spin 2 multiplet, χ_{aij},χ_{abc} belong to spin 3/2 multiplet and χ_{iab} to spin 1 multiplet. We may solve Eqs. (45) using the ansatz

$$\epsilon_a=0,\quad \epsilon_i=\{\epsilon_1,\epsilon_2\neq 0,\epsilon_3=\epsilon_4=0\}. \tag{46}$$

The transformation of four gravitino from the gravitational multiplets and of those from the spin 3/2 multiplets are

$$\delta\psi_{\mu i}=D_\mu\epsilon_i+Z_{ij\mu\nu}\gamma^\nu\epsilon^i, \tag{47}$$

$$\delta\psi_{\mu a}=D_\mu\epsilon_a+Z_{ai\mu\nu}\gamma^\nu\epsilon^i. \tag{48}$$

The central charge matrix can be put into the normal frame [8] by means of a SU(N) transformation. In this frame the off diagonal elements Z_{ai} are absent, $Z_{ai}=0$.

$$Z_{AB}=\begin{pmatrix} Z_{ij} & 0\\ 0 & Z_{ab}\end{pmatrix}. \tag{49}$$

Matrices Z_{ij} and Z_{ab} are diagonal:

$$Z_{ij}=\begin{pmatrix} z_1\sigma_2 & 0\\ 0 & z_2\sigma_2\end{pmatrix},\quad Z_{ab}=\begin{pmatrix} z_3\sigma_2 & 0\\ 0 & z_4\sigma_2\end{pmatrix}, \tag{50}$$

where

$$\sigma_2=i\begin{pmatrix} 0 & 1\\ -1 & 0\end{pmatrix}. \tag{51}$$

Thus the transformation of 4 gravitino from spin 3/2 multiplets vanishes $\delta\psi_{\mu a}=0$ due to the fact that 4 ϵ_a supersymmetries are broken and that in the normal frame the off diagonal elements of the central charge matrix Z_{ai} are absent. The variation of spin 1/2 fields vanishes

$$\delta\chi_{abc}=0,\quad \delta\chi_{aij}=Z_{ij}\epsilon_a+Z_{ia}\epsilon_j=0. \tag{52}$$

It remains to check the spin 1/2 transformations from the gravitational multiplet and from the vector multiplet of $N=4$ theory at the attractor point:

$$\delta\chi_{iab}=Z_{ab\mu\nu}\sigma^{\mu\nu}\epsilon_i\,, \tag{53}$$

$$\delta\chi_{ijk}=Z_{ij\mu\nu}\sigma^{\mu\nu}\epsilon^j. \tag{54}$$

The first of these two equations implies that $Z_{ab}=0$, i.e., $z_3=z_4=0$. The second one yields $z_2=0$, as in $N=4$. Thus we have proved that the attractor condition indeed requires all eigenvalues of the central charge which are smaller than the one equal to the ADM mass to vanish. This again leads us to the universal formula for the area, starting with an E(7) symmetric formula.

VI. ATTRACTORS IN $d=5$, $N=2,4$ THEORIES

The general formulas for $N=4$, $N=8$ derived in this paper have an obvious extension at $d=5$. In our previous paper [3] we gave the general area formula for an arbitrary $N=1$ theory in $d=5$ in terms of the symmetric constant d_{ABC}

$$A\sim Z_{\rm fix}^{3/2}=[d^{AB}(q)^{-1}q_Aq_B]^{3/4}, \tag{55}$$

where $d^{AB}(q)^{-1}$ is the inverse of the moduli dependent matrix $d_{AB}=d_{ABC}t^C$ computed at the attraction point $Z=Z_{\rm fix}$. There was only one central charge $Z=t^A(z)q_A$ ($d_{ABC}t^At^Bt^C=1$) and $\partial_iZ=0\Rightarrow Z=Z_{\rm fix}$.

For $N=2$, $N=4$ in $d=5$ we have two and three central charges respectively[3] [17]. Again the ADM mass is given by the largest eigenvalue. Let us call it Z and the other Z_c. As before,

$$A\sim(Z)_{\rm fix}^{3/2}. \tag{56}$$

For $N=2$ coupled to n_v matter multiplets there are $6+n_v$ electric charges. They are in the $(5,n_v)$ vector representation of O(5, n_v) + a singlet. The singlet is the charge of the vector dual to the $B_{\mu\nu}$ field.

The general formula for Z at the attractor point coincides with the macroscopic formula given by Strominger and Vafa [11]:

$$Z|_{\rm fix}=(Q_HQ_F^2)^{1/3}, \tag{57}$$

where Q_H is the singlet charge and Q_F^2 is a Lorentzian $(5,n_\nu)$ norm of the other $5+n_\nu$ charges.

For the $N=4$ theory we have 27 charges which are in the 27 irreducible representation of E_6. The formula for Z is given by the cubic root of the unique E_6 invariant constructed out of the 27 dimensional representation of E_6, which is the central charge Z_{ij} ($i,j=1,\ldots,8$) [note that the 27 can be represented as a traceless Sp(8) symplectic matrix]:

$$Z|_{\rm fix}=(\triangle)^{1/3}=(q_{ij}\Omega^{jl}q_{lm}\Omega^{mn}q_{n\rho}\Omega^{\rho i})^{1/3}, \tag{58}$$

where q_{ij} is a 27 integer charge vector transforming under $E_6(\mathbf{Z})$ (integer valued E_6 group).

These $N=4$ results are expected to apply to the type II strings compactified on the five-torus or eleven-dimensional supergravity (or M theory) on the six-torus.

VII. EXACT TOTAL ENERGY OF THE GROUND STATE AND SPONTANEOUS BREAKING OF SUPERSYMMETRY

We would like to use here the experience from the study of supersymmetric black holes in string theory, accumulated in the community over the recent years and also some ideas from duality symmetric quantization of superstring theory [18] to learn about the properties of the exact Hamiltonian in quantum theories with local supersymmetry. It has been pointed out in [18] that the consistent quantization of κ-symmetry in the backgrounds with unbroken supersymmetry can be performed with the help of the supercharge of the background in which the extended κ-symmetric object can exist. The supercharge of the gravitational supersymmetric theory was defined by Teitelboim [19] as the surface integral in terms of the gravitino Ψ_μ field of the configuration, solving the field equations

$$\mathcal{Q}=\oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\Psi_\lambda\,. \tag{59}$$

The surface over which the integration has to be performed depends on the choice of configuration. In all cases it is the same surface the integration over which defines the ADM mass of a given system or the ADM mass per unit area (length). The on-shell backgrounds with some number of supersymmetries unbroken in bosonic sectors have the vanishing supersymmetry variation of the gravitino, when the parameters are Killing spinors:

$$\mathcal{Q}_k=\oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\delta_{\epsilon_k}\Psi_\mu=\oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\hat\nabla_\lambda\epsilon_k=0. \tag{60}$$

For anti-Killing spinors the supercharge is not vanishing. For the black hole multiplets it defines the so-called superhair of the black hole:

$$\mathcal{S}_{\rm superhair}\equiv\mathcal{Q}_{\overline{k}}=\oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\hat\nabla_\lambda\epsilon_{\overline{k}}. \tag{61}$$

The concept of the *superhair* was defined for the first time for extreme Reissner-Nordström black holes in [20] and studied more recently in the context of more general extreme black holes in [21].

In the theories with local supersymmetry the total-energy operator (Hamiltonian) [22] is defined via the quadratic combination of supercharges

$$P_0=(8\pi GhN)^{-1}\sum_{I=1}^{N}\sum_{A=1}^{4}\mathcal{Q}_{AI}^2. \tag{62}$$

The computation of the area of the horizon of the supersymmetric black holes performed above via the extremum of the ADM energy suggest the following interpretation: the ADM mass at the extremum in the moduli space is the value of the

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[3]The reason why in $N=4$ supergravity in $d=5$ there are only three central charges in the normal frame is due to the fact that the Z_{ij} ($i,j=1,\ldots,8$) central charge matrix is traceless $Z_{ij}\Omega^{ij}=0$ with respect to the Sp(8) metric $\Omega^{ij}=-\Omega^{ji}$. By reducing $d=5$ to $d=4$ on S_1 one gets a fourth charge from Kaluza-Klein vector $g_{\mu5}$.

total-energy operator (Hamiltonian) of the ground state. The ground state has a nonvanishing vacuum energy due to the presence of electric and magnetic charges, which cause spontaneous breaking of supersymmetry:

$$H_{\rm vac}=E(p,q)=\sqrt{\frac{A(p,q)}{4\pi}}. \tag{63}$$

The electric and magnetic charges are due to the excitation of the microscopical degrees of freedom of the string theory or alternatively due to the near horizon black hole geometry of Bertotti-Robinson, which exists only when the charges are nonvanishing. For the string theory interpretation of the ground state energy the appropriate formulas are: for the heterotic string compactified on T_6 (or type II compactified on $K_3\times T_2$) the SL(2,**Z**)×SO(6,22;**Z**) symmetric expression for the ground state energy is

$$E(p,q)_{\rm het}=\sqrt{\frac{A(p,q)}{4\pi}}=(q^2p^2-(q\cdot p)^2)^{1/4}, \tag{64}$$

where the 2 (6+22) electric and magnetic charges form SL(2,**Z**) doublets and (6,22) Lorentzian vectors. For type II string compactified on T_6 the vacuum energy is given by the E_7(**Z**) symmetric expression.

$$E(q,p)_{\rm II}=J^{1/4}, \tag{65}$$

where the quartic Cartan E(7) invariant is

$$J=q^{ij}p_{jk}q^{kl}p_{li}-\tfrac{1}{4}q^{ij}p_{ij}q^{kl}p_{kl}+\tfrac{1}{96}(\epsilon^{ijklmnop}p_{ij}p_{kl}p_{mn}p_{op} + \epsilon_{ijklmnop}q^{ij}q^{kl}q^{mn}q^{op}). \tag{66}$$

and the charges q^{ij} and p_{ij}, $i,j=1,\dots,8$ span the 56-dimensional space. Our interpretation is supported by the following space-time picture. The black hole configurations interpolate between two vacua, one trivial at asymptotic infinity and the second one described by the Bertotti-Robinson geometry, which is known to have an unbroken $N=2$ supersymmetry [23,24,10,3]. In particular, due to conformal flatness of the geometry and due to covariantly constant graviphoton field strength, the supersymmetry variation of the gravitino field strength vanishes without enforcing any linear combination of the supersymmetry parameter to vanish:

$$\delta_{\rm SUSY}(D_{[\mu}\psi_{\nu]})_{\rm BR}=0,\quad \epsilon^1\neq 0,\quad \epsilon^2\neq 0,$$

$$\delta_{\rm SUSY}(D_{[\mu}\psi_{\nu]})_{\rm triv}=0,\quad \epsilon^1\neq 0,\quad \epsilon^2\neq 0. \tag{67}$$

At asymptotic infinity the trivially flat vacuum is also characterized by the unbroken space-time supersymmetry. Moreover, for the trivial vacuum the variation of the gravitino itself vanishes, since both spin connections as well as vector field strengths vanish at asymptotic infinity. Consider now the second vacuum, the near horizon configuration of the supersymmetric black holes. The space is only conformally flat, it is characterized by some electric and magnetic charges. The unique parameter, characterizing the geometry, the Bertotti-Robinson mass, is given by duality symmetric function of all available electric and magnetic charges. One can check that despite the fact that the supersymmetry transformation of the field strength of gravitino in this background vanishes the transformation of gravitino does not for nonvanishing charges $p,=q,p$. This causes the main difference with the trivial asymptotically flat vacuum with zero energy:

$$\delta_{\rm SUSY}(\psi_\nu)_{\rm BR}\neq 0,\quad \epsilon^1\neq 0,\quad \epsilon^2\neq 0, \tag{68}$$

$$\delta_{\rm SUSY}(\psi_\nu)_{\rm triv}=0,\quad \epsilon^1\neq 0,\quad \epsilon^2\neq 0. \tag{69}$$

The ground state energy, which is proportional to the square root of the area of the black hole horizon (or to the Bertotti-Robinson mass) is positive, duality symmetric and presents a nontrivial computation of an eigenvalue of the energy-operator of a ground state of quantum gravity system. The energy of the ground state

$$(P_0)_{\rm vac}=(8\pi GhN)^{-1}\sum_{I=1}^{N}\sum_{A=1}^{4}(Q_{AI})^2_{\rm vac} \tag{70}$$

is always non-negative, however, for the ground state to have a positive energy one has to require that the supercharge of the ground state is nonvanishing. This happens in our system since the ground state supercharge does not vanish in presence of the covariantly constant graviphoton field strength: This makes the calculation of the ground state energy of the theory with local supersymmetry consistent with the idea of spontaneous breaking of supersymmetry with the nonvanishing constant value of the vacuum supercharge.

VIII. DISCUSSION

In this paper we have established the universality of the black hole area formula for extended supersymmetric theories $N\geqslant 2$ in $d=4$. It is based on the fact that all central charges of the theory except the one which equals the ADM mass have to vanish near the black hole horizon by requirement of supersymmetry. In this way supersymmetry realizes the principle of lowest possible ground state. The fact that the ADM mass as a function of charges and moduli is equal to the central charge does not mean yet that it is a ground state. Since the central charge depends on conserved electric and magnetic charges and moduli in the generic point the energy is not the minimal one. One has to minimize it in the moduli space and this is how we get the minimal energy of the ground state.

This universality can be understood also from the fact that in supersymmetric theories with asymptotically flat spaces there exists a well defined universal expression for the Hamiltonian in terms of the sum over all supercharges [22]. The presence of an extended microscopic object like superstring introduces spontaneous breaking of the supersymmetry, from the point of view of the space-time Hamiltonian, since it has a nonvanishing value on the ground state of the system. The electric and magnetic charges which are interpreted as charges (q,p) defining the Bertotti-Robinson geometry and defining the size of its infinite throat $A(q,p)$, from the point of view of string theory are simply the conserved charge vectors of string theory$(\vec{\alpha},\vec{\beta})$.

If one accepts the point of view that the calculation of the area of the black hole horizon was a tool to get the ground state energy, it become clear that any supersymmetric black hole solution with particular area formula actually provides the calculation of the ground state energy and gives a specific example of the ground state energy calculation. However, the general expression for the energy of the ground state is simultaneously universal and duality invariant as explained in this paper,

$$E(q,p)=E(\vec{\alpha},\vec{\beta})=\left(\frac{A(q,p)}{4\pi}\right)^{1/2}. \tag{71}$$

This result can be associated with the fact discovered by Gaillard and Zumino [25] that the energy momentum tensor in supergravities is duality invariant whereas the off-shell Lagrangian is not. Our analysis also predicts that the most general S and T duality invariant area formulas in the heterotic theory compactified on T_6 [or type II, compactified on $K_3\times T_2$, given in Eq. (64) or with U duality for type II string theory compactified on T_6 as given in Eqs. (65)] should be reachable by the counting of string states, as it was already demonstrated in particular examples [11].

The space-time picture is that the most general four dimensional supersymmetric black holes with the nonvanishing area of the horizon, covering the singularities, interpolate between four dimensional Minkowski space-time $\mathbf{M}^4$, at spatial infinity $r\to\infty$ and $adS_2\times S^2$ down the infinite wormhole throat $r\to 0$ as was noticed by Gibbons [23] with respect to Reissner-Nordström geometry. In addition the near horizon geometry upon the change of variables

$$r=-\frac{E^2(p,q)}{\rho} \tag{72}$$

was also interpreted in [3] as the second conformally flat $\mathbf{M}^4$. The conformal factor relating these two asymptotic regions $\mathbf{M}^4(r)$ and $\mathbf{M}^4(\rho)$ was found to be equal to [3]

$$\frac{E(q,p)^2}{\rho^2}=\frac{r^2}{E(q,p)^2}. \tag{73}$$

Thus the total stability of this picture of the space-time geometry is constrained severely by the fact that the space-time energy of the ground state $E(q,p)$ is not vanishing. The existence of these charges is explained by the existence of the string states. Moreover, from the point of view of string theory the two $\mathbf{M}^4$ do not seem to be distinguishable. This is one possible explanation of the spontaneous supersymmetry breaking behind the nonvanishing of the ground state energy $E(q,p)\neq 0$. When the energy of the ground state vanishes and the area of the black hole horizon shrinks to zero this picture of two asymptotically $\mathbf{M}^4$ regions is not valid anymore, scalar fields do not stop evolving inside the throat. In the duality symmetric area formulas the vertices described in [5,6] are reached, double or quadruple number of supersymmetries is restored and singularities become naked unless the ground state energy $E(q,p)$ is nonvanishing. Our macroscopic formulas are supposed to be valid for large values of charges but one may expect corrections to them from microscopic physics, for example one can find some disagreement between microscopic and macroscopic calculation of the entropy for small charges [11]. Therefore the status of the zero entropy limit may be changed by microscopic physics.

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