Cosmological bounds to the magnetic moment of heavy τ neutrinos

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The magnetic moment of τ neutrinos in the MeV mass range may be large enough to modify the cosmological freeze-out calculation and determine the τ-neutrino relic density. In this paper we reexamine such a possibility. We calculate the evolution and freeze-out of the τ-neutrino number density as a function of its mass and magnetic moment. We then determine its relic density; then calculate its effect upon primordial nucleosynthesis including previously neglected effects. [S0556-2821(96)05214-9]

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Present experimental bounds to the electromagnetic properties of the τ neutrino are several orders of magnitude less stringent than the bounds to the corresponding properties for electron and muon neutrinos. For instance, while the upper limits to the diagonal magnetic moments of the electron and muon neutrinos are (μ_B is a Bohr magneton) $1.1\times10^{-9}\mu_B$ and $7.4\times10^{-10}\mu_B$, respectively [1], the experimental upper limit to the diagonal magnetic moment of the τ neutrino is $\mu_{\nu_\tau}<5.4\times10^{-7}\mu_B$ [2]. More stringent bounds on neutrino magnetic moments of order 10^{-10} to 10^{-12} are available from astrophysical constraints [3], mainly from the cooling of stars and from the study of SN 1987A. However, these bounds apply only if mass of the neutrino species does not exceed the stellar temperatures relevant for neutrino production. Furthermore, astrophysical constraints are model dependent since they assume that the outgoing wrong helicity neutrinos are completely sterile [4].

Big-bang nucleosynthesis (BBN) is a precious tool that has been employed to constrain many neutrino properties [5], so it is no surprise that BBN can be used to bound neutrino magnetic moments. In 1981, Morgan [6] showed that the ''sterile'' right-handed degree of freedom of Dirac neutrinos[1] can be populated through processes like $e\nu_L\rightarrow e\nu_R$ and $e^-e^+\rightarrow\nu_R\overline{\nu_L}$, mediated through a virtual photon coupled to the neutrino through its magnetic moment. The degree to which the right-handed neutrino is populated depends upon the strength of the above interactions, which in turn is proportional to the magnitude of the neutrino magnetic moment. Thus, endowing a neutrino species with a magnetic moment potentially leads to an unacceptable doubling of the contribution of that species to the energy density, jeopardizing BBN's successful predictions. If the neutrino species is relativistic at freeze-out, one must require that right-handed neutrinos decouple before the QCD phase transition so that their number density is diluted by the huge entropy shift associated with the transition. Such a requirement translates into an upper limit to the neutrino magnetic moment: $\mu_{\nu_\tau}\leqslant1$ to $2\times10^{-11}\mu_B$. However, in deriving this limit Morgan did not consider the possibility of nonzero neutrino masses. There are two main effects to be considered if one allows a nonzero mass for the neutrino species in question. First, the neutrino may not be relativistic at freeze-out, and its number density must be calculated by solving the Boltzmann equation. Furthermore, the scaling of the neutrino energy density with temperature depends upon the neutrino mass. Therefore, Morgan's useful limit applies only if the neutrinos are ultrarelativistic around freeze-out and BBN, that is if $m_{\nu_\tau}<0.1$ MeV.

The τ neutrino could be heavier than 0.1 MeV;[2] in fact, the present upper bound to the τ-neutrino mass is $m_{\nu_\tau}<24$ MeV [7]. Although somewhat model dependent, more stringent bounds on the neutrino masses can be determined from cosmological considerations. In particular, if the relic heavy-neutrino energy density today is sufficiently large, the predicted age of the universe will be less than observed. If the neutrino is stable and it is nonrelativistic today, the age limit ($\Omega h^2<1$) constrains the mass of any stable neutrino species to be less than the Cowsik-McClelland limit, $m_\nu<91.5$ eV [8]. Of course if the neutrino is unstable, the Cowsik-McClelland limit can be evaded [9]. But even in this case there are lifetime-dependent limits to the neutrino mass. If the heavy-neutrino lifetime is longer than a second or so, it can give an additional contribution to the energy density during nucleosynthesis and spoil the successful predictions of standard calculations. Using these kinds of considerations, BBN constraints to the τ-neutrino mass excludes the range $0.3<m_\nu<25$ MeV if it is a Dirac fermion, and the range $0.5<m_\nu<25$ MeV if it is a Majorana fermion [10]. (It was

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[1]If the neutrino has a magnetic moment it must be a Dirac fermion (we do not consider transitional magnetic moments in this paper).

[2]We assume here that the mass of the muon neutrino is less than 0.1 MeV.

assumed in the above BBN analysis that the neutrino eventually decays after BBN; if it decays after decoupling but before or during BBN, the situation is more complicated [11].)

The Cowsik-McClelland limit and the nucleosynthesis considerations might be modified if one introduces new interactions that changes the neutrino annihilation cross section. This is the case if the neutrino has a large diagonal magnetic moment, because a large magnetic moment would increase ν-$\overline{\nu}$ annihilation (creation) into (by) $e^{\pm}$, keeping the neutrinos in equilibrium below the canonical (including only weak processes) neutrino decoupling temperature of about an MeV. If the neutrino mass is sufficiently small (much less than an electron mass) and remains coupled to electrons while the electrons annihilate, the neutrino number density will be *increased* because part of the electron's entropy will be shared with the neutrinos. However, if the neutrino mass is not much less m_e and it remains in equilibrium through magnetic-moment mediated interactions, its energy density will be Boltzmann suppressed before decoupling, weakening the BBN constraints.

In this paper we study how the interplay between the neutrino mass and magnetic moment modifies the cosmological constraints to τ-neutrino properties from the age of the universe and BBN. In addition to the mere extension of the upper limit on μ_{ν_τ} to larger neutrino masses, the main purpose of our letter is to give a final answer to the intriguing possibility that τ neutrinos with a large magnetic moment could form cold dark matter.

Giudice [12] first observed that if τ neutrinos are stable, have a mass in the range $m_{\nu_\tau}\sim 1$ to 10 MeV, and are endowed with a magnetic moment of $\mu_{\nu_\tau}\sim 10^{-6}\mu_B$, they would stay in equilibrium through their magnetic-moment interactions and would decouple when they are nonrelativistic. If the magnetic moment is large enough, their final abundance might give rise to a universe with $\Omega_\nu h^2 \simeq 1$. Although the latest experimental upper limit on μ_{ν_τ} [2] seems marginally at odds with Giudice's scenario, it is worthwhile to investigate this hypothesis further [13].

Giudice made use of the fact that Morgan's conclusions about doubling the effective τ-neutrino number density by populating the right-handed component cannot be applied directly to MeV-mass neutrinos. This is because their energy density is Boltzmann suppressed at freeze-out and during BBN. Thus, even including the right-handed components, τ neutrinos will not contribute so much to the energy density as to spoil BBN. We show that while this is approximately true, it is not exactly true. BBN is such a sensitive probe of the expansion rate of the universe at the temperatures of interest that even a small contribution to energy density is important. Therefore the contribution of the right-handed neutrino to BBN requires a careful treatment. We report the results of such an investigation in this communication.

There are three effects that must be carefully accounted for. (1) After a massive neutrino species decouples and becomes nonrelativistic its energy density grows relative to the energy density of a massless neutrino species [14]. Although one must solve the Boltzmann equation to compute the energy density of the heavy neutrino (see below), it is possible to estimate this effect by observing that $\rho_\nu(m_\nu \neq 0)/\rho_\nu(m_\nu = 0) \simeq (m_\nu/3.15T_\nu) r \propto t^{1/2}$ when $T \sim m_\nu$, where r is the ratio of the number density of massive neutrinos to massless neutrinos after freeze-out. (2) A neutrino species with a mass in the MeV range and with a magnetic moment close to the present experimental limit decouples when it is semirelativistic. Neither the relativistic nor the nonrelativistic cross section can be used, and a general treatment of the thermal-averaged annihilation cross section used in the Boltzmann equation for the neutrino abundance is required. (Giudice performed his analysis in the extreme nonrelativistic limit.) (3) Plasma effects must, at least *a priori*, be considered. Not only do thermal corrections to the amplitudes of the main processes involved in BBN have to be included [15], but more importantly, the mass corrections due to the electromagnetic coupling of the particles to the relativistic plasma must be accounted for. For example, the photon in the thermal bath becomes a *plasmon* and acquires an effective mass [16]. The plasmon mass has a double effect. In the first place, it affects the electromagnetic channel of the neutrino annihilation cross section. Although this is a second-order effect some resonance might enhance it dramatically [17]. Secondly, a plasmon mass gives rise to new processes that are kinematically forbidden in the vacuum. In our case the most relevant of these processes is the decay *plasmon* $\rightarrow \nu\overline{\nu}$, having a rate [18]

$$\Gamma_P = \frac{\mu_{\nu_\tau}^2}{16\pi}(\omega_O^2 - 4m_\nu^2)^{3/2}\frac{K_1(x_P)}{K_2(x_P)}, \tag{1}$$

where $x_P = m_P/T$ with m_P the temperature-dependent plasmon mass, $\omega_P \sim 0.1T$ is the plasma frequency, and $K_i(x)$ are the modified Bessel functions of order i. However, the threshold $\omega_P \geqslant 2m_\nu$, reduces the importance of the process *plasmon* $\rightarrow \nu\overline{\nu}$ during BBN for MeV-mass neutrinos. Analogously, since $2m_\nu > \omega_P$, screening effects induced by the plasma on the photon propagator turn out to be negligible. The relative unimportance of these considerations were verified by directly including them in our numerical calculations.

The cross section for the electromagnetic channel of the process $\nu\overline{\nu}\rightarrow e^- e^+$ is

$$\sigma_{\nu\overline{\nu}\rightarrow e^-e^+} = \frac{\alpha\mu_\nu^2}{6}\left(\frac{1-4m_e^2/s}{1-4m_\mu^2/s}\right)^{1/2} \times\left(1 + 8\frac{m_\nu^2}{s} + 2\frac{m_e^2}{s} + 16\frac{m_\nu^2 m_e^2}{s^2}\right), \tag{2}$$

where $\sqrt{s} > 2m_e$ is the total center-of-mass energy.

The weak contribution to the annihilation process of Eq. (2) can be neglected if the neutrino magnetic moment is larger than $10^{-10}(m_\nu/1\ \mathrm{MeV})\mu_B$. We will work within the limits of this assumption.[3]

[3]If the magnetic moment is larger than $10^{-10}(m_\nu/1\ \mathrm{MeV})\mu_B$, then neutrino annihilation will occur predominantly through photon exchange, rather than Z exchange. For $m_\nu \lesssim 100$ keV, considerations of stellar energy loss by neutrino pair emission limits the magnetic moment to be greater than about $10^{-11}\mu_B$. Thus we will consider neutrinos more massive than 100 keV.

Because both helicity eigenstates of the neutrino are symmetric with respect to electromagnetic interactions, we do not differentiate between them in our calculations. For this reason the processes $e\nu_{L(R)} \leftrightarrow e\nu_{R(L)}$ changes neither the total, nor the relative ν_L vs ν_R abundances.[4]

The Boltzmann equation for the abundance of the heavy neutrino is [19]

$$\frac{dY}{dx} = -\left(\frac{\pi}{45}\right)^{1/2} \frac{g_*^{1/2} m_\nu m_{\rm Pl}}{x^2} \langle \sigma v_{\rm Mol} \rangle (Y^2 - Y_{\rm eq}^2), \tag{3}$$

where $x = m_\nu / T$, $Y = n_\nu / s$ is the ratio of the ν_τ number density to the total entropy density of the universe, $v_{\rm M\o l}$ is the Mϕller invariant flux factor, and $m_{\rm Pl} = G_N^{-1/2}$ is the Planck mass. The parameter g_* is defined as

$$g_*^{1/2} = \frac{h_{\rm eff}}{g_{\rm eff}^{1/2}} \left(1 + \frac{1}{3} \frac{T}{h_{\rm eff}(T)} \frac{dh_{\rm eff}(T)}{dT}\right), \tag{4}$$

where the effective number of degrees of freedom for the energy density, $g_{\rm eff}(T)$, and for the entropy density, $h_{\rm eff}(T)$, are defined as

$$\rho = g_{\rm eff}(T) \frac{\pi^2}{30} T^4, \quad s = h_{\rm eff}(T) \frac{2\pi^2}{45} T^3. \tag{5}$$

Following [19], the thermal averaged cross section times the Mϕller velocity is

$$\langle \sigma v_{\rm M\o l} \rangle = \frac{1}{8 m_\nu^4 T K_2^2(x)} \int_{4m_\nu^2}^{\infty} \sigma(s)(s - 4m_\nu^2)\sqrt{s} K_1(\sqrt{s}/T) ds. \tag{6}$$

We have used the Maxwell-Boltzmann distribution to compute the thermal-averaged cross section (for a detailed review of computations in this approximation see, e.g., Ref. [20]). Although normally this is a very good approximation only for temperatures $T \lesssim 3 m_\nu$, we have checked that at the freeze-out temperature [the only temperature around which Eq. (6) plays a relevant role] the approximation is adequate.

The neutrino decoupling temperature T_F is here defined by the condition $Y(T_F) - Y_{\rm eq}(T_F) = 1.5 Y_{\rm eq}$, where

$$Y_{\rm eq} = \frac{n_\nu^{\rm eq}}{s} = \frac{45}{\pi^4} \frac{I_\nu(x)}{h_{\rm eff}(T)}, \tag{7}$$

with

$$I_\nu(x) = \int_1^\infty dz z \frac{\sqrt{z^2 - x^2}}{e^z + 1}. \tag{8}$$

We numerically solved Eq. (3) to compute the τ-neutrino abundance as function of T for fixed values of m_{ν_τ} and μ_{ν_τ}. Since the freeze-out temperature increases with μ_{ν_τ}, it is natural to expect that the final τ-neutrino abundance is suppressed as the magnetic moment increases.

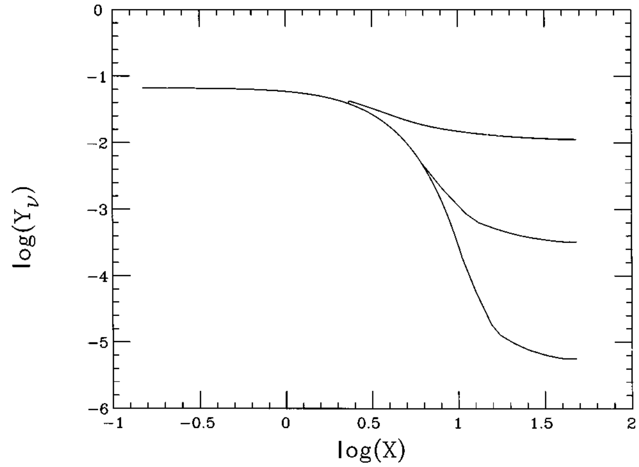

FIG. 1. τ-neutrino abundance vs the parameter $x = m_{\nu_\tau}/T$ is represented for different value of μ_{ν_τ}: from below, the three different curves refer to $\mu_\nu = 10^{-6}\mu_B$, $10^{-7}\mu_B$, and $10^{-8}\mu_B$. Here we have chose $m_\nu = 1$ MeV. The logarithms are base 10.

This is clearly visible in Fig. 1. Assuming the τ neutrinos to be stable we can easily check their effect on the dynamics of the universe for several values of m_{ν_τ} and μ_{ν_τ}. In particular, we first consider the contribution to the present energy density of the universe due to massive τ neutrinos:

$$\Omega_{\nu_\tau} h^2 = \frac{\rho_{\nu_\tau 0}}{\rho_C} h^2 = \frac{m_{\nu_\tau} S_0 Y_{\nu_\tau 0}}{1.054 \text{ MeV cm}^{-3}}, \tag{9}$$

where zero indicates quantities evaluated at the present time. Requiring $\Omega_{\nu_\tau} h^2 \leqslant 1$, we can verify which region of the parameter space μ_{ν_τ} versus m_{ν_τ} is compatible with the age constraint.

Of course if the τ neutrino is unstable, the cosmological age constraint discussed above does not apply. However, we can still use BBN to limit the properties of the τ neutrino provided that the lifetime, τ_{ν_τ}, is greater than about a second.

To evaluate the impact of the massive τ neutrino with a large magnetic moment on BBN we must know how it modifies the effective number of degrees of freedom of the energy density, $g_{\rm eff}(T)$, for $0.1 \lesssim T \lesssim 10$ MeV, since the light element relic abundances depend critically on the expansion rate of the universe during BBN, which in turn is parameterized by $g_{\rm eff}$ [5]:

$$H(T) = 1.66 g_{\rm eff}^{1/2}(T) \frac{T^2}{m_{\rm Pl}}. \tag{10}$$

The τ-neutrino contribution to $g_{\rm eff}$ is given by

$$g_{\nu_\tau}(T) = \rho_{\nu_\tau}(T) \left(\frac{30}{\pi^2}\right) T^{-4}, \tag{11}$$

where $\rho_{\nu_\tau} = s Y_{\nu_\tau} \sqrt{(3.15 T_{\nu_\tau})^2 + m_{\nu_\tau}^2}$, and T_{ν_τ} is computed by imposing entropy conservation. Figure 2 clearly demonstrates that $g_{\rm eff}$ grows as the decoupled τ neutrinos become nonrelativistic. This effect becomes less pronounced as the

[4]The $\nu_{L(R)}$ helicity eigenstates should not be confused with the chirality eigenstates. Since we ignore weak interactions, chirality does not play a role in our analysis.

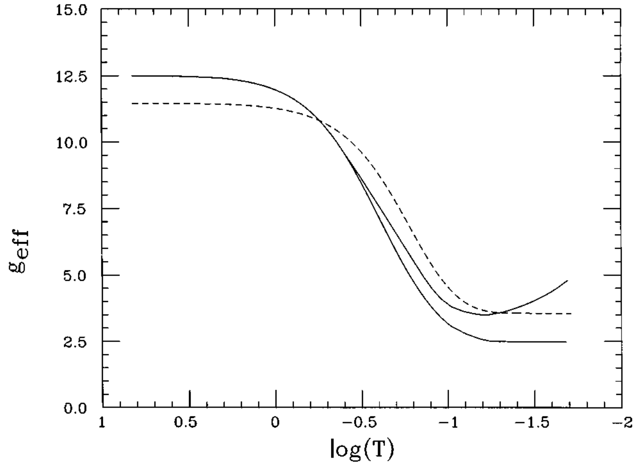

FIG. 2. The effective number of degrees of freedom in the energy density is shown as function of temperature for two chosen value of μ_ν and $m_\nu = 1$ MeV. The upper solid curve is for $\mu_{\nu_\tau} = 10^{-8}\mu_B$; the lower solid curve is for $\mu_{\nu_\tau} = 10^{-6}\mu_B$. The reader can compare our result with the result obtained for 3.4 standard massless neutrinos shown by the dashed line. The logarithm is base 10.

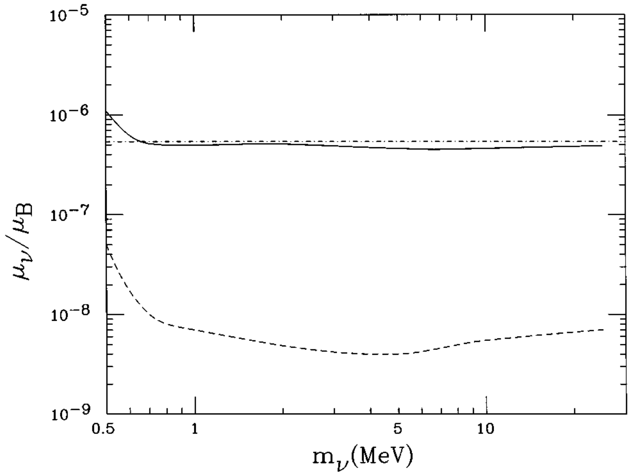

FIG. 4. Exclusion plot in the τ-neutrino magnetic moment–mass parameter space. The solid line provides the lower limit to μ_{ν_τ} coming from the requirement $\Omega h^2 \leq 1$. The dashed line provides the corresponding limit from BBN considerations. The dotted-dashed line represent the experimental upper limit.

magnetic moment is increased. Of course this is simply because increasing μ_{ν_τ} decreases T_F, leading to a τ-neutrino energy density more effectively Boltzmann suppressed before freeze-out. For this reason, values of the τ-neutrino magnetic moment larger than $10^{-8}\mu_B$ are not expected to have a large effect on BBN if $m_{\nu_\tau} > 0.1$ MeV.

To check this in detail and in order to be able to evaluate the effects of the neutrino mass and magnetic moment on light element production, we incorporated our results for the abundance as a function of temperature into the standard nucleosynthesis code [21]. In Fig. 3 our predictions for the relic ^{4}He abundance as a function of the τ-neutrino mass are shown for two values of the magnetic moment. As expected, the predicted abundance Y_P is suppressed with increasing

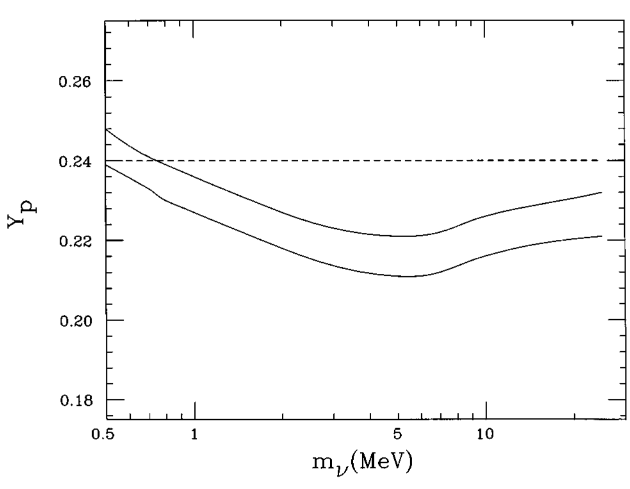

FIG. 3. The predicted ^{4}He relic abundance is represented as function of the τ-neutrino mass for two values of μ_ν. The upper curve is for $\mu_{\nu_\tau} = 10^{-8}\mu_B$, while the lower one is for $\mu_{\nu_\tau} = 10^{-6}\mu_B$. The dashed line corresponds to the observational upper limit.

μ_{ν_τ}. Increasing the mass above a few MeV increases Y_P, since the τ neutrinos then become nonrelativistic earlier. For small masses, Y_P grows with decreasing m_{ν_τ}. This is due both to the less effective Boltzmann suppression and to the entropy transfer from $e^\pm$ annihilation to the τ neutrinos.

In order to discriminate which region of the m_ν versus μ_ν parameter space is compatible with observations, we require that the predicted light element abundances do not exceed the observational limits [22]: $Y_P \leq 0.24$; $(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H} \leq 1.1 \times 10^{-4}$; and $^7\mathrm{Li}/\mathrm{H} \leq 1.7 \times 10^{-10}$. Since the baryon-to-photon ratio η is a free parameter, for every chosen pair of m_{ν_τ} and μ_{ν_τ} we fix it at the minimum value compatible with the $(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}$ upper limit. Then we check if the predicted ^{4}He relic abundance is consistent with the upper limit of 0.24. The ^{7}Li constraints turn out to be always less stringent than the limits coming from ^{4}He.

Our results are summarized in Fig. 4. As the reader can observe, the age-based constraints are much more stringent than BBN constraints if the τ neutrino is stable. In this case the border between the allowed and forbidden regions in the m_{ν_τ} vs μ_{ν_τ} parameter space from the age constraint almost coincides with the experimental limit line. This is a remarkable coincidence. In fact, the age limits are *lower* limits to μ_{ν_τ}, whereas the experimental limits are *upper* limits. This means that nearly the entire parameter space for 0.1 MeV $\lesssim m_{\nu_\tau}$ and $10^{-10}(m_{\nu_\tau}/1\ \mathrm{MeV}) \lesssim \mu_{\nu_\tau}$ (the very range to which our consideration applies) is excluded by our considerations. To be precise, a very small region between the experimental and the age-based constraints remains open. Even this region would be closed using a slightly larger value of h as recent observations suggest.

As a consequence, Giudice's hypothesis is definitely ruled out. Furthermore, our results improve the upper limit on the τ-neutrino magnetic moment by several orders of magnitude in the mass range we considered. We have checked that plasma-physics effects are subdominant. We have to stress that this limit is valid only if the τ neutrino is stable, as indeed Giudice assumed. Stability can be achieved by imposing some additional symmetries, e.g., individual lepton-number conservation. Of course, in any case some new physics beyond the standard model must be introduced in order to have such a large value of μ_{ν_τ}. Furthermore, a τ neutrino with mass larger than 1.1 MeV decaying according to a minimally extended standard model via the channel $\nu_\tau \rightarrow \nu_e e^+ e^-$ is incompatible with BBN. In fact, since experimental data constrains the ν_τ lifetime to be 1 s $\leqslant \tau_{\nu_\tau} \leqslant 10$ s, if $m_{\nu_\tau} > 1.1$ MeV, electrons and positrons produced from this decay would induce the photodestruction of light elements [23].

If the τ neutrino is unstable but the lifetime exceeds one second, a band of magnetic moment values, roughly $10^{-8}\mu_B \lesssim \mu_{\nu_\tau} \lesssim 10^{-6}\mu_B$, remains compatible with experimental and cosmological bounds. This confirms the result of Ref. [24] and extends it to a wider τ-neutrino mass range. It is understood that in the case the τ neutrino has to decay in some nonstandard way in order the decay products do not affect dramatically the light element relic abundances.

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