Initial conditions for smooth hybrid inflation

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We perform a numerical investigation of the field evolution in the smooth hybrid inflationary model. We find that for almost all the examined initial values we do get an adequate amount of inflation. Our results show that the model is ''natural'' and satisfactory. [S0556-2821(96)01014-4]

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The hybrid inflationary scenario [1] proposed by Linde in the context of nonsupersymmetric (SUSY) theories is a realization of chaotic inflation based on a coupled system of two scalar fields one of which may not be a gauge singlet. The great advantage of this scenario is that it produces the observed temperature fluctuations in the cosmic background radiation (CBR) with natural values of the coupling constants. However, inflation terminates abruptly and is followed by a ''waterfall'' regime during which topological defects can be easily produced. Recently, two of us have proposed [2] a variant of Linde's potential that can be derived in a wide class of SUSY grand unified theories (GUT's) based on semisimple gauge groups by utilizing the first nonrenormalizable contribution to the superpotential. Although one gets only a slight variation of Linde's potential, the cosmological scenario obtained is drastically different. Already, since the beginning of inflation, the system follows a particular valley of minima that leads to a particular point of the vacuum manifold. Thus, this inflationary scenario does not lead to production of topological defects. Also, the termination of inflation is not as abrupt as in the hybrid case. It is quite smooth and resembles more the cases of new or chaotic inflation. The main advantage of this smooth hybrid inflationary scenario is that the measured value of the temperature fluctuations of CBR can be reproduced with natural values of the parameters and with a GUT scale M_X, consistent with the unification of the minimal supersymmetric standard model (MSSM) gauge couplings. It is also remarkable that the scale controlling the nonrenormalizable terms in the superpotential turns out to be of order 10^{18} GeV. The spectral index of density fluctuations is close to unity.

For an inflationary scenario to be considered fully successful, one has to show that it is obtainable for a wide class of ''natural'' initial values of the fields and their time derivatives. In Ref. [2], two sets of initial conditions were studied semianalytically and it was argued that they lead to smooth hybrid inflation. These sets, however, cannot be considered completely ''natural'' since they require some or even a considerable discrepancy between the initial values of the fields. The purpose of this paper is to identify a wide class of ''natural'' initial conditions for smooth hybrid inflation, i.e., comparable initial values of the fields for which the system falls at the bottom of a particular valley of minima. Its subsequent evolution along this valley, then, produces smooth hybrid inflation. To this end, we solve numerically the evolution equations of the system for a wide class of initial conditions. The result is striking and unexpected. We find that, for almost all the examined initial conditions (except a narrow transition region), we do get smooth hybrid inflation with an adequate number of e foldings. This result together with the other advantages of this inflationary scenario makes it fully satisfying and ''natural.'' Our analysis also includes cases with all initial field values being much smaller than the Planck scale. In these cases, our results are expected to be less affected by replacing global with local supersymmetry.

The smooth hybrid inflationary scenario can be realized in the context of a SUSY GUT based on a gauge group G of rank $\geqslant 5$. We assume that G breaks spontaneously directly to the standard model (SM) group G_S at a scale $M_X \sim 10^{16}$ GeV and that below M_X the only SM nonsinglet states of the theory are the usual MSSM states. This guarantees the successful MSSM predictions for $\sin^2\theta_w$ and α_s. The theory could also possess some global symmetries. The symmetry breaking of G to G_S is obtained through a superpotential that includes the terms

$$W = s\left(-\mu^2 + \frac{(\phi\bar{\phi})^2}{M^2}\right). \tag{1}$$

Here, $\phi, \bar{\phi}$ is a conjugate pair of left-handed SM singlet superfields that belong to nontrivial representations of the gauge group G and reduce its rank by their vacuum expectation values (VEV's), s is a gauge singlet left-handed superfield, μ is a superheavy mass scale related to M_X, whereas M is a mass scale of the order of the ''compactification'' scale $M_c \sim 10^{18}$ GeV that controls the nonrenormalizable terms in the superpotential of the theory. The superpotential terms in Eq. (1) are the dominant couplings involving the superfields $s, \phi, \bar{\phi}$ consistent with a continuous R symmetry under which $W \to e^{i\theta}W, s \to e^{i\theta}s$, $\phi\bar{\phi} \to \phi\bar{\phi}$ and a discrete symmetry under which $\phi\bar{\phi}$ changes sign.

The potential obtained from W in Eq. (1), in the supersymmetric limit, is

$$V = \left|\mu^2 - \frac{(\phi\bar{\phi})^2}{M^2}\right|^2 + 4|s|^2 \frac{|\phi|^2|\bar{\phi}|^2}{M^4}(|\phi|^2 + |\bar{\phi}|^2) + D \text{ terms}, \tag{2}$$

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. Vanishing of the D terms is achieved along the D flat directions where $|\bar{\phi}|=|\phi|$. The supersymmetric vacuum

$$\langle s\rangle=0, \quad \langle\phi\rangle\langle\bar{\phi}\rangle=\pm\mu M, \quad |\langle\bar{\phi}\rangle|=|\langle\phi\rangle| \tag{3}$$

lies on the particular D flat direction $\bar{\phi}^*=\pm\phi$. Restricting ourselves to this direction and performing appropriate gauge, discrete, and R transformations, we can bring the complex $s,\phi,\bar{\phi}$ fields on the real axis, i.e., $s\equiv\sigma/\sqrt{2}$, $\bar{\phi}=\phi\equiv(1/2)\chi$, where σ and χ are real scalar fields. The potential in Eq. (2) then takes the form

$$V(\chi,\sigma)=\left(\mu^2-\frac{\chi^4}{16M^2}\right)^2+\frac{\chi^6\sigma^2}{16M^4} \tag{4}$$

and the supersymmetric minima correspond to

$$|\langle\chi\rangle|=2(\mu M)^{1/2}, \quad \langle\sigma\rangle=0\,. \tag{5}$$

The mass acquired by the gauge bosons is $M_X=g(\mu M)^{1/2}$, where g is the GUT gauge coupling. For any fixed value of σ, the potential in Eq. (4), as a function of χ^2, has a local maximum at $\chi^2=0$ and an absolute minimum lying at

$$\chi^2\simeq\frac{4}{3}\,\frac{\mu^2M^2}{\sigma^2} \quad \text{for } \sigma^2\gg\mu M, \tag{6}$$

and $\chi^2\simeq4\mu M$, for $\sigma^2\ll\mu M$. The value of the potential along the maxima at $\chi^2=0$ is constant, $V_{\max}(\chi^2=0)=\mu^4$.

Assume for the moment that at some region of the Universe the scalar fields χ and σ, starting from appropriate initial conditions to be discussed below, evolve in such a way so they become almost uniform with values at the bottom of the valley of minima in Eq. (6). One can then show [2] that these fields, at subsequent times, move towards the supersymmetric minima in Eq. (5) following this valley and the system inflates till σ reaches the value

$$\sigma\simeq\sigma_0\equiv\left(\frac{2M_P}{9\sqrt{\pi}(\mu M)^{1/2}}\right)^{1/3}(\mu M)^{1/2}\gtrsim(\mu M)^{1/2}, \tag{7}$$

where $M_P=1.2\times10^{19}$ GeV is the Planck mass. The number of e foldings from the moment at which the σ field has the value σ till the end of inflation is given by

$$N(\sigma)\simeq\left(\frac{3\sqrt{2\pi}}{2\mu MM_P}\right)^2\sigma^6. \tag{8}$$

This implies that the value of the σ field when the present horizon crossed outside the inflationary horizon was

$$\sigma_H\simeq\left(\frac{9N_H}{2}\right)^{1/6}\sigma_0\,, \tag{9}$$

where N_H is the number of e foldings of the present horizon size during inflation. After the end of inflation, the σ and χ fields enter smoothly into an oscillatory phase about the global supersymmetric minimum of the potential in Eq. (5) with frequency $m_\sigma=m_\chi=2\sqrt{2}(\mu/M)^{1/2}\mu$. These fields should eventually decay into lighter particles and ''reheat'' the Universe. Taking $N_H=60$, $M_X=2\times10^{16}$ GeV, $g=0.7$ (consistent with the MSSM unification), the microwave background quadrupole anisotropy $(\Delta T/T)\simeq5\times10^{-6}$ from the Cosmic Background Explorer (COBE) implies that $M\simeq9.4\times10^{17}$ GeV and $\mu\simeq8.7\times10^{14}$ GeV. With these numbers, which we will use throughout this paper, one estimates the value of σ at the end of inflation [see Eq. (7)] $\sigma_0\simeq1.1\times10^{17}$ GeV $\simeq5.4M_X$ and its value when the present horizon size crossed outside the inflationary horizon [see Eq. (9)] $\sigma_H\simeq2.54\sigma_0\simeq2.7\times10^{17}$ GeV. An important advantage of the above smooth hybrid inflationary scenario is that inflation takes place at relatively low values of the field σ and, therefore, there is hope that it survives even in the context of supergravity theories although it may acquire drastic modifications.

We will now try to specify the initial conditions for the σ and χ fields that lead to the above-described inflationary scenario. In other words, we will try to identify the initial conditions for which the system falls at the bottom of the valley of minima in Eq. (6) with a value of $\sigma\geqslant\sigma_H$ so that its subsequent evolution along the valley produces an adequate amount of inflation. We assume that, after ''compactification'' at some initial cosmic time, a region emerges in the Universe where the scalar fields σ and χ happen to be almost uniform with negligible kinetic energies. (The initial values of σ and χ can always be transformed by appropriate gauge and R transformations to become positive.) The evolution of the system in this region is governed by the equations of motion

$$\ddot{\chi}+3H\dot{\chi}-\frac{\chi^3}{2M^2}\left(\mu^2-\frac{\chi^4}{16M^2}\right)+\frac{3\chi^5\sigma^2}{8M^4}=0, \tag{10}$$

$$\ddot{\sigma}+3H\dot{\sigma}+\frac{\chi^6\sigma}{8M^4}=0, \tag{11}$$

where overdots denote derivatives with respect to cosmic time and H is the Hubble parameter:

$$\begin{aligned}H&=\left(\frac{8\pi}{3}\right)^{1/2}M_P^{-1}\varrho^{1/2}\\&=\left(\frac{8\pi}{3}\right)^{1/2}M_P^{-1}\left(\frac{1}{2}\dot{\chi}^2+\frac{1}{2}\dot{\sigma}^2+V(\chi,\sigma)\right)^{1/2}\end{aligned} \tag{12}$$

(ϱ is the energy density).

The case where initially $\sigma\gg M_P\gg\chi$ was examined in Ref. [2]. Under these circumstances, the last term in Eq. (4) is initially the dominant contribution to the potential energy density of the system [assuming $\chi\gtrsim(\mu^2M^2/\sigma)^{1/3}$]. Also, Eq. (10) reduces to

$$\ddot{\chi}+3H\dot{\chi}+\frac{3\chi^5\sigma^2}{8M^4}\simeq0. \tag{13}$$

Let us, for the moment, assume that σ remains almost constant. The frequency of oscillations of the χ field is then much greater that H and χ initially performs damped oscillations over the maximum at $\chi=0$. The continuity equation

$$\dot{\varrho}=-3H(\varrho+p), \tag{14}$$

with p being the pressure averaged over one oscillation of χ, becomes

$$\dot{\varrho} = -3H\gamma\varrho, \tag{15}$$

where $\gamma = 3/2$ for a χ^6 potential [3]. This equation, together with the fact that ϱ is proportional to H^2, gives

$$H \simeq \frac{4}{9t}. \tag{16}$$

Comparing this result with Eq. (12) we can obtain the amplitude of the oscillating χ field

$$\chi_m \simeq \left(\frac{3}{8\pi}\right)^{1/6}\left(\frac{16M^2M_P}{9\sigma t}\right)^{1/3}. \tag{17}$$

Equation (11) averaged over one oscillation of χ then gives

$$\ddot{\sigma} + \frac{4}{3t}\dot{\sigma} + \frac{1}{27\pi t^2}\left(\frac{M_P}{\sigma}\right)^2 \sigma \simeq 0. \tag{18}$$

The solutions of this equation are of the form $\sigma = t^\alpha$, where α satisfies the quadratic equation

$$\alpha^2 + \frac{1}{3}\alpha + \frac{1}{27\pi}\left(\frac{M_P}{\sigma}\right)^2 \simeq 0. \tag{19}$$

For $\sigma \gg M_P$, the solutions are

$$\alpha \simeq -\frac{1}{3} + \frac{1}{9\pi}\left(\frac{M_P}{\sigma}\right)^2 \text{ and } \alpha \simeq -\frac{1}{9\pi}\left(\frac{M_P}{\sigma}\right)^2. \tag{20}$$

This means that σ quickly approaches an extremely slowly decreasing function of time and, thus, our starting assumption that it remains approximately constant is justified. When the amplitude of the χ field drops to about $(\mu^2 M^2/\sigma)^{1/3}$, the μ^4 term dominates the potential in Eq. (4) and the Hubble parameter becomes approximately constant and equal to $H = (8\pi/3)^{1/2}\mu^2/M_P$ and remains so thereafter till the end of inflation. The subsequent evolution of the system has been studied in detail in Ref. [2]. The overall conclusion is that, in a time interval

$$\Delta t \sim 6\pi\left(\frac{\sigma}{M_P}\right)^2 H^{-1}, \tag{21}$$

the χ field falls into the valley of minima in Eq. (6) and relaxes at the bottom of this valley whereas the σ field still remains unchanged and much greater than M_P. After that, the system follows the valley of minima towards the supersymmetric vacuum and, therefore, the smooth hybrid inflationary scenario is realized for initial values of the fields satisfying the inequality $\sigma \gg M_P \gg \chi$. However, these initial conditions cannot be considered totally satisfying because, assuming that the initial energy density is well below M_P^4 we see that there must be some discrepancy between the initial values of the fields. Also, the inclusion of supergravity is expected to invalidate the above discussion of initial conditions which involve values of the field $\sigma \gtrsim M_P$. To minimize the influence from supergravity one could start with field values much smaller than M_P and, as pointed out in Ref. [2], still obtain adequate inflation. However, this requires an initial energy density much smaller than M_P^4 and an initial value of χ ''unnaturally'' smaller than the initial value of σ.

For smooth hybrid inflation to be considered as a fully successful inflationary scenario, one must show that it is obtained for a wide class of initial conditions that are more ''natural'' than the ones just discussed. This can be done only numerically. To this end, we put $\hat{\chi} \equiv \chi/M_P$ and $\hat{\sigma} \equiv \sigma/M_P$ in Eqs. (10) and (11) which become

$$\hat{\chi}'' + 3\hat{H}\hat{\chi}' - \frac{\hat{\chi}^3}{32\hat{M}^4}(16\hat{\mu}^2\hat{M}^2 - \hat{\chi}^4) + \frac{3\hat{\chi}^5\hat{\sigma}^2}{8\hat{M}^4} = 0, \tag{22}$$

$$\hat{\sigma}'' + 3\hat{H}\hat{\sigma}' + \frac{\hat{\chi}^6\hat{\sigma}}{8\hat{M}^4} = 0, \tag{23}$$

where

$$\hat{H} \equiv \frac{H}{M_P} = \left(\frac{8\pi}{3}\right)^{1/2}\left[\frac{1}{2}(\hat{\chi}')^2 + \frac{1}{2}(\hat{\sigma}')^2 + \hat{V}(\hat{\chi},\hat{\sigma})\right]^{1/2} \tag{24}$$

with

$$\hat{V}(\hat{\chi},\hat{\sigma}) \equiv \frac{V(\chi,\sigma)}{M_P^4} = \left(\hat{\mu}^2 - \frac{\hat{\chi}^4}{16\hat{M}^2}\right)^2 + \frac{\hat{\chi}^6\hat{\sigma}^2}{16\hat{M}^4}. \tag{25}$$

Here, primes denote derivatives with respect to the dimensionless time variable $\tau \equiv M_P t$ and

$$\hat{M} \equiv \frac{M}{M_P} \simeq \frac{1}{12.83}, \quad \hat{\mu} \equiv \frac{\mu}{M_P} \simeq \frac{1}{13737}. \tag{26}$$

We have conducted numerical integration of Eqs. (22) and (23) for an extensive set of initial values of the fields in the ranges $0.01 \leqslant \hat{\sigma} \leqslant 1.2$ and $0.01 \leqslant \hat{\chi} \leqslant 0.5$ and with vanishing initial velocities. The integration of these two coupled equations was performed by implementing a variant of the Bulrish-Stoer [4] variable step method in a Fortran program run mainly on a number of workstations. As a general rule, the initial step for the dimensionless time variable τ was chosen to be ten while the sought accuracy was put to 10^{-12}. This choice was found to ensure reasonable stability in the cases where $\hat{\sigma}$ settles down to a constant nonzero value relatively early. In the opposite cases as well as near some transition regions, the initial step was decreased to five or less, while the sought accuracy was increased by two or three orders of magnitude depending on the case.

The results of our search are summarized in Figs. 1 and 2. Each point shown on the $\hat{\sigma}$-$\hat{\chi}$ plane corresponds to a given set of initial conditions and depicts a definite evolution pattern for the $\hat{\sigma}$-$\hat{\chi}$ system according to the symbol attributes being used. We have used filled circles to specify the evolution pattern where both fields oscillate and fall rapidly to the supersymmetric minima in Eq. (5) without producing any appreciable amount of inflation. Open triangles correspond to the case where $\hat{\sigma}$ starts off at relatively large values ($\hat{\sigma} > \hat{\chi}$) and decreases slowly, tending asymptotically to a constant value $\hat{\sigma} \geqslant \hat{\sigma}_H \approx 0.0225$. The field $\hat{\chi}$ oscillates and relaxes at the bottom of the valley in Eq. (6). The system

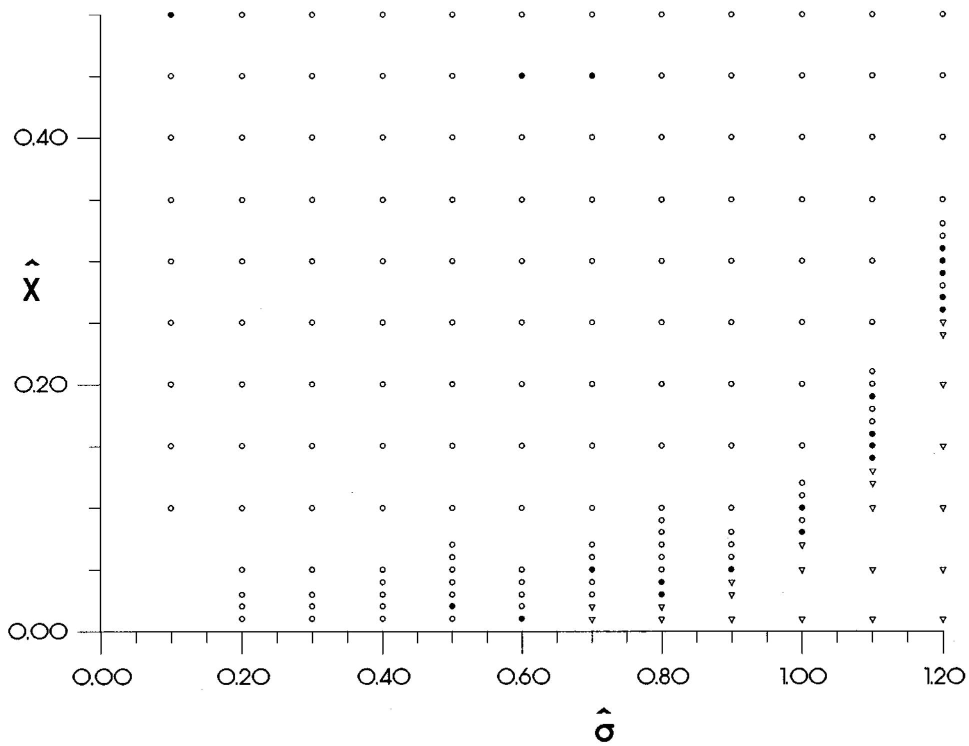

FIG. 1. Evolution patterns for the $\hat{\sigma}$-$\hat{\chi}$ system. Filled circles represent points that do not lead to inflation. Open triangles give adequate inflation having only the $\hat{\chi}$ field oscillating. Open circles give adequate inflation with both fields oscillating initially.

then evolves through the valley of minima in Eq. (6) giving an adequate amount of inflation. Finally, open circles correspond to the pattern where both fields start oscillating at the beginning. Then, $\hat{\sigma}$ settles down at large values and the system subsequently follows the valley of minima in Eq. (6) as in the preceding case.

The evolution pattern represented by open triangles includes the pattern found in the limiting case $\hat{\sigma} \gg 1 \gg \hat{\chi}$ analyzed earlier by means of semianalytic arguments. In general, however, $\hat{\sigma}$ does not remain frozen as in the limiting case, but its variation over a large period of time is small. The open circles area being the least expected deserves further

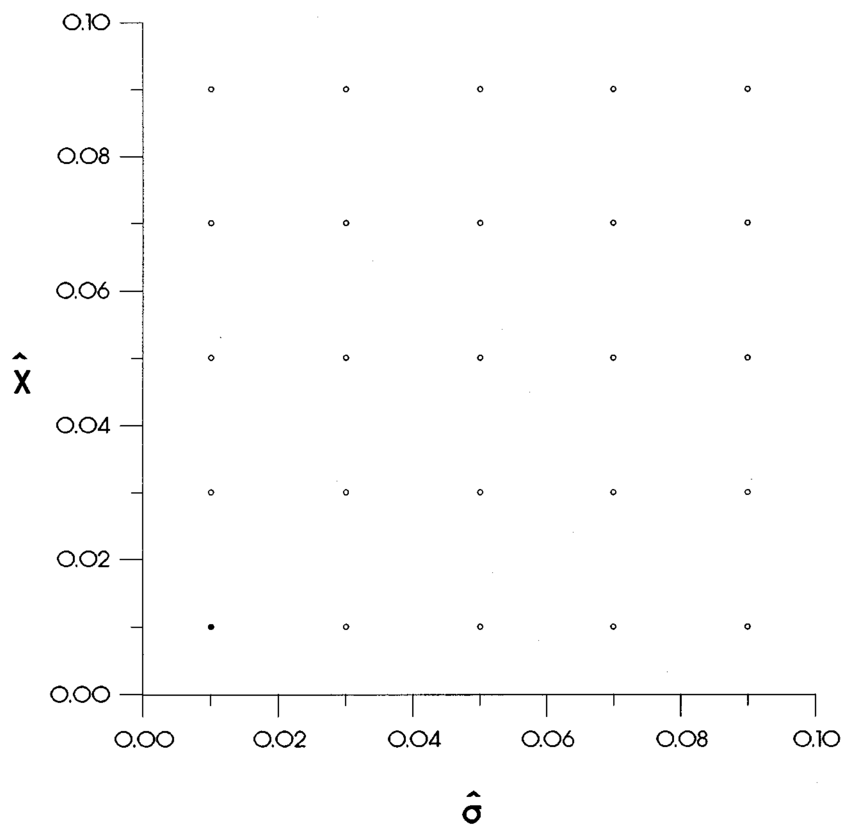

FIG. 2. Same as in Fig. 1. The initial values are now restricted to lie near the beginning of the axes.

attention. Here, although $\hat{\sigma}$ starts at moderate or small values, it appears to increase in amplitude absorbing energy from the fast oscillating field $\hat{\chi}$ and creates conditions that eventually lead to evolution of the open triangles type. It is a beautiful example of large energy transfer between two strongly coupled nonlinear oscillators.

The points depicted in Figs. 1 and 2 follow a remarkably regular pattern, although there is some intermingling. This intermingling persists even when the sought accuracy is increased to a maximum and the step is lowered to a minimum, and thus we have to assume that it really exists. It would, of course, be very desirable to get three distinct regions separated by critical lines but we have no reason to *a priori* exclude the possibility of intermingling.

The problem to be addressed next concerns the ''naturality'' of initial conditions. A closer look at Fig. 1 reveals that the open triangle area, although leading to successful inflation, cannot be considered as being completely ''natural'' since it requires relatively large differences between the initial values of the fields. The open circles region appears to be significantly better since the initial values can be of the same order of magnitude. Considering that the initial kinetic energies for both fields are taken to be zero, the initial energy density equals the potential energy density given in Eq. (25). It turns out that, when $\hat{\chi}$ and $\hat{\sigma}$ are of the same order of magnitude and approximately equal to a few tenths, we get acceptable initial energy densities. Thus, part of the open circles area corresponds to ''natural'' initial conditions that lead to successful inflation. Inclusion of supergravity will certainly invalidate the preceding analysis, except for an area where all initial field values are much smaller than the Planck scale. This area is shown in Fig. 2 and a closer inspection shows that all but one of the points fulfil all the conditions for adequate inflation. In summary, taking into account all the previously stated results, we feel confident to conclude that the smooth hybrid inflationary model appears to be ''natural'' and satisfactory.

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