Coleman-Weinberg SO(10) grand unified theories as inflationary models

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The flat-space limit of the one-loop effective potential for SO(10) GUT's in spatially flat Friedmann-Robertson-Walker cosmologies is applied to study the dynamics of the early universe. The numerical integration of the corresponding field equations shows that, for such grand unified theories, a sufficiently long inflationary stage is achieved for suitable choices of the initial conditions. However, a severe fine-tuning of these initial conditions is necessary to obtain a large e-fold number. In the direction with residual symmetry $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$, one eventually finds parametric resonance for suitable choices of the free parameters of the classical potential. This phenomenon leads in turn to the end of inflation. [S0556-2821(96)04414-1]

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I. INTRODUCTION

Semiclassical effects in quantum field theory are at the very heart of many exciting developments in modern theoretical physics, e.g., radiative corrections to the Casimir force [1], trace anomalies [2], one-loop effective action [3,4], symmetry breaking in the early universe [5–7]. In the latter class of phenomena, the one-loop effective potential for grand unified theories (GUT's) in curved backgrounds is used in the field equations to determine the symmetry-breaking pattern which is relevant for the familiar low-energy phenomenology of particle physics [6,7]. In particular, in a previous paper by the first two authors, the one-loop effective potential for SO(10) GUT's in de Sitter space was derived for the first time [8]. The main motivation of our work was the analysis of spontaneous symmetry breaking in the early universe when GUT's in good agreement with low-energy phenomenology are studied. Among the nonsupersymmetric GUT's, the unified models based on SO(10) are still viable, and from the phenomenological point of view they turn out to be the most appealing ones for the following reasons [9–13].

(i) SO(10) models make it possible to obtain masses (M_X) of the leptoquarks mediating proton decay which are higher than the ones found within the minimal SU(5) model. This happens by virtue of an intermediate symmetry group in between SO(10) and $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. By fixing, in fact, at M_W the values of the coupling constants and $\sin^2 \theta_W$, and using the renormalization-group equations one gets, due to the above intermediate symmetry, a large value for the energy scale ($\approx M_X$) at which the SO(10) gauge group unifies interactions. Thus, for these models one recovers agreement with the experimental lower limit of 9.0×10^{32} yr for proton decay [10].

(ii) In SO(10) the exact cancellation of chiral anomalies is obtained.

(iii) The only SO(2n) group which contains a 16-dimensional representation is indeed SO(10). This makes it possible to accommodate fermions and antifermions of each generation in a single 16-dimensional complex representation.

(iv) On requiring that the GUT group should contain $SU(2)_L$ and $B - bL$ as local symmetries, while no exotic particles or mirror fermions should occur, one finds that SO(10) is unique and has $B - L$ as generator (hence $b = 1$).

(v) SO(10) models predict masses for τ and μ neutrinos of the order of magnitude relevant for cosmology and solar-neutrino astrophysics (including the effects of renormalization-group equations [12]).

(vi) SO(10) has also the property to be the intermediate symmetry of E_6, which is the low-energy limit of string theory [14].

On considering the particular mass matrix relevant for the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry-breaking direction, our main result was the numerical proof that, even when curvature effects are no longer negligible, the early universe can only reach the $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ absolute minimum [8].

Of course, supersymmetric GUT models might also be studied, and they are indeed receiving careful consideration in the current literature [15–20]. For example, unlike the minimal SU(5) model, in the minimal supersymmetric SU(5) model the three running coupling constants meet at a single unification point. This leads in turn to constraints on the new physics which may appear at energy scales between M_Z and the unification scale [15]. However, on considering SO(10) models and their supersymmetric version, it seems that only in the former the μ-neutrino mass is close to that particular value which could solve the solar-neutrino problem [21]. Thus, because nonsupersymmetric SO(10) appears in agreement with the whole set of observational data, we found it appropriate to complete our previous investigations of SO(10) models, before considering their supersymmetric versions.

A naturally occurring question is whether the analytic formulas obtained in Ref. [8] can be used to gain a better understanding of other phenomena occurring in the early universe. Indeed, it is well known that, according to the cosmological standard model based on Einstein's general relativity, the early universe is spatially homogeneous and

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isotropic [22,23], and gravity couples to the energy-momentum tensor of matter. Under the previous assumptions of symmetry and homogeneity (cf. Ref. [24]), the space-time metric can be locally cast in the form [22]

$$g = -dt \otimes dt + a^2(t)[d\chi \otimes d\chi + f^2(\chi)(d\theta_1 \otimes d\theta_1$$
$$+ \sin^2\theta_1 d\theta_2 \otimes d\theta_2)], \qquad (1.1)$$

where $a(t)$ is the cosmic scale factor, $\theta_1 \in]0,\pi[$, $\theta_2 \in]0,2\pi]$ and, denoting by k the constant curvature of the three-dimensional spatial sections, $f(\chi) = \sin\chi, \chi$ or $\sinh\chi$ if $k = 1, 0, -1$, respectively. Here we focus on spatially flat Friedmann-Robertson-Walker (FRW) cosmologies, for which $k = 0$.

The aim of the present work has been therefore to study the relevance of SO(10) GUT models for gravitational physics within the more general framework of FRW cosmologies. In this case, the analysis performed in Ref. [25] enables one to use, at least for typical GUT-inflationary models characterized by a phase-transition temperature $\leq 10^{16}$ GeV, the flat-space limit of the Coleman-Weinberg one-loop effective potential [5]. What happens is that the one-loop field equations in FRW cosmologies are an involved set of integro-differential equations [25]. In scalar electrodynamics, the nonlocal correction to the Coleman-Weinberg effective potential results from the coupling of a scalar field ϕ_c to the gravitational background and may lead to dissipative effects in the early universe. More precisely, nonlocal terms in the field equations may be approximated very well by a function of the form [25]

$$-A(\widetilde{\tau})\left(H\phi_c + \frac{d\phi_c}{d\widetilde{\tau}}\right)\frac{d\phi_c}{d\widetilde{\tau}},$$

where A is a positive-definite function of the conformal time $\widetilde{\tau}$. The first term in parentheses leads to a further quantum correction to the energy density of the scalar field (for slowly varying A), while the second term is purely dissipative [25]. However, nonlocal terms *do not* modify the pattern of minima found in static de Sitter space. Thus, for numerical purposes, even in the non-Abelian case, curvature effects on the one-loop potential are very small, and can be neglected at the GUT energy scale. This is the basic approximation on which the present paper relies.

In Sec. II the scalar dynamics driving the inflationary phase is described. Section III, relying on Ref. [8], studies SO(10) GUT models with Higgs field in the 210-dimensional irreducible representation. In the SO(10) internal space, spherical coordinates for the Higgs field are used to obtain a convenient parametrization of the classical part of the potential. The flat-space limit of the one-loop effective potential is then studied in Sec. IV. Semiclassical field equations are studied in Sec. V, and their numerical solution is found in Sec. VI. The reheating process is then briefly described in Sec. VII, while the concluding remarks are presented in Sec. VIII.

II. INFLATION FROM SCALAR-FIELD DYNAMICS

In the inflationary models driven by scalar fields, the universe contains different species of matter, i.e., radiation and scalar particles (inflatons). The latter, characterized by a higher energy level with respect to the others, completely determine the dynamics of the universe, making all remaining contributions negligible. A theoretical scheme in which fundamental scalar particles naturally occur (Higgs field) and lead to symmetry breaking are the GUT's. In this case the energies involved are compatible with the levels required for an *efficient* inflation. Thus, we consider an inflationary model in which the inflaton coincides with the Higgs field of a GUT, corresponding to the highest energy scale of spontaneous symmetry breaking. In particular, we choose the most relevant ones which are the SO(10) GUT's with Higgs field belonging to the 210-dimensional irreducible representation (**210**) [8].

Within this framework (we denote hereafter by ϕ the fields corresponding to scalar particles), and assuming homogeneous field configurations $\phi = \phi(t)$, the *semiclassical* Lagrangian reads

$$\mathcal{L} = 6\dot{a}^2 a F(\phi) + 6\dot{a}a^2 F'(\phi)\dot{\phi} + a^3[\tfrac{1}{2}\dot{\phi}^2 - V^{(1)}(\phi)], \qquad (2.1)$$

where the overdot stands for d/dt, the prime for $\delta/\delta\phi$, and $F(\phi)$ denotes the arbitrary coupling of ϕ to the gravitational background with scalar curvature $R = 6[\ddot{a}/a + \dot{a}^2/a^2]$. In Eq. (2.1) the classical potential has been replaced by the one-loop effective potential $V^{(1)}(\phi)$; i.e., the gauge degrees of freedom have been integrated over. Note also that the first two terms in Eq. (2.1) yield the coupling $-a^3 F(\phi)R$, after integration by parts in the action.

Starting from Eq. (2.1) one gets the two semiclassical equations of motion:

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 + \left[\frac{\dot{a}}{a}\frac{F'}{F}\dot{\phi} + \frac{1}{2}\frac{F'}{F}\ddot{\phi} + \frac{1}{2}\frac{F''}{F}\dot{\phi}^2\right] - \frac{1}{8}\frac{\dot{\phi}^2}{F}$$
$$+ \frac{1}{4}\frac{V^{(1)}}{F} = 0, \qquad (2.2)$$

and

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + 6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right]F'(\phi) + \frac{\delta V^{(1)}}{\delta\phi} = 0. \qquad (2.3)$$

In general, for a nonminimally coupled real scalar field, $F(\phi) = (1/16\pi G) + (\xi/2)\phi^2$. In our case, in the light of the results obtained in Ref. [25], we neglect all contributions of the curvature to the scalar potential, and hence we can assume minimally coupled scalar fields (for which ξ vanishes).

III. SO(10) GUT MODELS WITH HIGGS FIELD IN THE 210 IRREDUCIBLE REPRESENTATION

In a gauge unifying theory such as SO(10), the Higgs field (fundamental scalar particles) belongs to one or more irreducible representations (irreps) of the gauge group, and its dynamics is ruled by a Higgs potential. These particles provide the correct residual symmetry for the model in the low-

energy limit, through a spontaneous symmetry-breaking mechanism. To study the inflation corresponding to the highest energy scale (M_X) of spontaneous symmetry breaking (SSB), we can consider the only contribution of the irreps, responsible for the SSB at that scale.

In the present case, we consider the most general renormalizable Higgs potential constructed by using a massless and minimally coupled irrep **210** only, which is obtained by the completely antisymmetrized product of four different **10**'s as

$$\Phi_{abcd} = N\mu_{[a} \otimes \nu_b \otimes \rho_c \otimes \sigma_{d]}, \tag{3.1}$$

where N is a normalization constant. The **210** irrep has four independent quartic invariants, i.e., $\|\phi\|^4$ and three nontrivial invariants; hence, the Higgs potential we are going to construct is a function of these [8]:

$$V(\phi) = V_0 + g_1\|(\phi\phi)_{\mathbf{45}}\|^2 + g_2\|(\phi\phi)_{\mathbf{210}}\|^2 + g_3\|(\phi\phi)_{\mathbf{1050}}\|^2$$
$$+ \lambda\|\phi\|^4, \tag{3.2}$$

where g_1, g_2, g_3, and λ are arbitrary coefficients, and V_0 is an arbitrary constant. We will see in due course what is the meaning of this constant and how it can be fixed. Unfortunately, in view of the technical difficulties to express Eq. (3.2) in terms of the 210 degrees of freedom, we restrict our analysis to the only directions invariant under the subgroup $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. This restriction, however, remains relevant for the aims of this paper, since it leads to the correct electroweak phenomenology at low energies for the model. The most general singlet ϕ_0 with respect to the group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ contained in the **210** representation is [8]

$$\frac{\phi_0}{\|\phi_0\|} = \frac{1}{\sqrt{6}}\sin\theta\sin\varphi(\phi_{1278} + \phi_{3478} + \phi_{5678} + \phi_{1290} + \phi_{3490}$$

$$+ \phi_{5690}) + \frac{1}{\sqrt{3}}\sin\theta\cos\varphi(\phi_{1234} + \phi_{3456} + \phi_{5612})$$

$$+ \cos\theta(\phi_{7890}), \tag{3.3}$$

where $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi]$. In particular, by varying θ and φ in their ranges, one gets the following residual-symmetry groups [8]:

$$\theta \in]0, \pi[\quad \text{and} \quad \varphi = 0, \pi \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R$$
$$\otimes U(1)_{B-L}, \tag{3.4a}$$

$$\theta = \frac{\pi}{2} \quad \text{and} \quad \varphi = 0, \pi \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R$$
$$\otimes U(1)_{B-L} \times D, \tag{3.4b}$$

$$\theta = 0, \pi \rightarrow SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R, \tag{3.4c}$$

$$\theta = \arctan(3) \quad \text{and} \quad \varphi = \arctan(\sqrt{2}) \quad \text{or} \tag{3.4d}$$

$$\theta = -\arctan(3) + \pi \quad \text{and} \quad \varphi = \arctan(\sqrt{2}) + \pi \rightarrow SU(5)$$
$$\otimes U(1), \tag{3.4e}$$

otherwise $\rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_{T_{3R}} \otimes U(1)_{B-L}$,
$$\tag{3.4f}$$

where T_{3R} is the z component of the $SU(2)_R$ group.

Inserting Eq. (3.3) into Eq. (3.2) one finds the tree-level potential

$$V(\phi_0) = V_0 + \left(\frac{\alpha}{8}f_\alpha + \frac{\gamma}{4}f_\gamma + \frac{\delta}{9}f_\delta + (\lambda - \delta)\right)\|\phi_0\|^4, \tag{3.5}$$

where [8]

$$\alpha \equiv \frac{4}{945}(-108g_1 + 28g_2 + 140g_3), \tag{3.6}$$

$$\gamma \equiv \frac{8}{35}g_1, \tag{3.7}$$

$$\delta \equiv -\frac{1}{10}g_3, \tag{3.8}$$

$$f_\alpha \equiv \sin^4\theta + \sin^2\theta\sin^2\varphi[2\sin\theta\cos\varphi + \sqrt{3}\cos\theta]^2$$
$$+ \frac{3}{4}\sin^4\theta\sin^4\varphi, \tag{3.9}$$

$$f_\gamma \equiv \sin^2\theta\left[\cos\theta\cos\varphi + \frac{1}{\sqrt{3}}\sin\theta\sin^2\varphi\right]^2$$

$$+ \sin^4\theta\sin^2\varphi\cos^2\varphi + f_\alpha, \tag{3.10}$$

$$f_\delta \equiv [2\sin^2\theta\cos^2\varphi - \frac{1}{2}\sin^2\theta\sin^2\varphi - 3\cos^2\theta]^2 + 30f_\gamma - 25f_\alpha. \tag{3.11}$$

Since in the following analysis δ is always negative and α may take negative values, the tree-level potential (3.5) is unbounded from below, unless we impose the restriction [8]

$$\lambda \geq \frac{|\alpha|}{8}(f_\alpha)_{max} + \frac{|\delta|}{9}(f_\delta)_{max}. \tag{3.12}$$

Note also that contributions proportional to a cubic term in the potential are set to zero, since we are assuming $\phi \rightarrow -\phi$ invariance of our model.

IV. ONE-LOOP EFFECTIVE POTENTIAL

As shown by Coleman and Weinberg [5], the effects of quantum fluctuations on the scalar potential, due to both gauge bosons and self-interactions, may account for the spontaneous symmetry breaking occurring in gauge theories without assuming from the beginning the presence of negative quadratic terms for the Higgs field. These phenomena are best tackled in terms of the one-loop effective potential [5–8,25,26]. This provides, in our case, the appropriate tool for studying the phase transition due to scalar dynamics.

The one-loop effective potential $V^{(1)}$ corresponding to the classical expression (3.5) in the flat-space limit is obtained in Ref. [8]. On defining

$$h_1 \equiv \cos^2\theta + \sin^2\theta\left[\tfrac{1}{2}\sin^2\varphi + \tfrac{2}{3}\cos^2\varphi\right]$$

$$+ \sqrt{\tfrac{2}{3}}\sin\theta\sin\varphi\left[\cos\theta + \tfrac{2}{\sqrt{3}}\sin\theta\cos\varphi\right], \tag{4.1}$$

$$h_2 \equiv \cos^2\theta + \sin^2\theta\left[\tfrac{1}{2}\sin^2\varphi + \tfrac{2}{3}\cos^2\varphi\right] - \sqrt{\tfrac{2}{3}}\sin\theta\cos\theta\sin\varphi, \tag{4.2}$$

$$h_3 \equiv \tfrac{1}{2}\sin^2\theta\sin^2\varphi, \tag{4.3}$$

$$h_4 \equiv \tfrac{2}{3}\sin^2\theta, \tag{4.4}$$

$$h_5^2 \equiv \frac{3}{2}h_1^2 + \frac{3}{2}h_2^2 + \frac{h_3^2}{4} + \frac{3}{4}h_4^2, \tag{4.5}$$

one gets

$$V^{(1)}(\phi_0) \sim V_0 + \left(\frac{\alpha}{8}f_\alpha + \frac{\gamma}{4}f_\gamma + \frac{\delta}{9}f_\delta + (\lambda - \delta)\right)\|\phi_0\|^4$$

$$- \frac{3\mathcal{G}^4}{8\pi^2}\|\phi_0\|^4\left[\frac{3}{4}h_1^2[3 - \ln(h_1^2)]\right.$$

$$+ \frac{3}{4}h_2^2[3 - \ln(h_2^2)] + \frac{h_3^2}{8}[3 - \ln(h_3^2)]$$

$$+ \left.\frac{3}{8}h_4^2[3 - \ln(h_4^2)] - h_5^2\ln\left(\frac{\mathcal{G}^2\|\phi_0\|^2}{\mu^2}\right)\right], \tag{4.6}$$

where $\mathcal{G}$ is the gauge coupling constant and μ is the renormalization mass.

V. SEMICLASSICAL FIELD EQUATIONS

After defining the dimensionless time $\tau \equiv \mu t/\mathcal{G}$ the Higgs field ϕ_0 can be seen, in the SO(10)-space, as the position vector in R^3, given in spherical coordinates as $\widetilde{\phi}_0 \equiv (\mathcal{G}/\mu)\phi_0 = y\hat{e}_y$. Thus, by taking its derivative with respect to τ one has

$$\frac{d}{d\tau}\widetilde{\phi}_0 = \frac{dy}{d\tau}\hat{e}_y + y\frac{d\theta}{d\tau}\hat{e}_\theta + y\sin\theta\frac{d\varphi}{d\tau}\hat{e}_\varphi. \tag{5.1}$$

With our notation, it is convenient to introduce a dimensionless expression for Eq. (4.6) obtained multiplying it by $\mathcal{G}^4/\mu^4$: i.e.,

$$\widetilde{V}^{(1)}(y,\theta,\varphi) \sim \widetilde{V}_0 + \left(\frac{\alpha}{8}f_\alpha + \frac{\gamma}{4}f_\gamma + \frac{\delta}{9}f_\delta + (\lambda - \delta)\right)y^4$$

$$- \frac{3\mathcal{G}^4}{8\pi^2}y^4\left[\frac{3}{4}h_1^2[3 - \ln(h_1^2)]\right.$$

$$+ \frac{3}{4}h_2^2[3 - \ln(h_2^2)] + \frac{h_3^2}{8}[3 - \ln(h_3^2)]$$

$$+ \left.\frac{3}{8}h_4^2[3 - \ln(h_4^2)] - h_5^2\ln(y^2)\right]. \tag{5.2}$$

As far as the constant $\widetilde{V}_0$ is concerned, it has to be fixed by requiring that a vanishing potential energy should correspond to the absolute minimum for $\widetilde{V}^{(1)}$. By denoting with y_m, θ_m, and φ_m the absolute-minimum coordinates, we have $\widetilde{V}^{(1)}(y_m,\theta_m,\varphi_m) = 0$. Moreover, y_m makes it possible to determine also μ, bearing in mind that the spontaneous symmetry-breaking scale is M_X, which is fixed for the particular model by the low-energy predictions, and then $\mu = M_X/y_m$.

In this scheme, the semiclassical equations for $\widetilde{\phi}_0$ [27] take the form (from now on, the dot denotes differentiation with respect to τ)

$$\ddot{y} = y[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2] - 3K\dot{y}\sqrt{\widetilde{\rho}_\phi} - \frac{\partial}{\partial y}\widetilde{V}^{(1)}(\widetilde{\phi}), \tag{5.3}$$

$$\ddot{\theta} = -2\frac{\dot{y}}{y}\dot{\theta} + \sin\theta\cos\theta\dot{\varphi}^2 - 3K\dot{\theta}\sqrt{\widetilde{\rho}_\phi} - \frac{1}{y^2}\frac{\partial}{\partial\theta}\widetilde{V}^{(1)}(\widetilde{\phi}), \tag{5.4}$$

$$\ddot{\varphi} = -2\frac{\dot{y}}{y}\dot{\varphi} - 2\frac{\cos\theta}{\sin\theta}\dot{\varphi} - 3K\dot{\varphi}\sqrt{\widetilde{\rho}_\phi} - \frac{1}{y^2\sin^2\theta}\frac{\partial}{\partial\varphi}\widetilde{V}^{(1)}(\widetilde{\phi}). \tag{5.5}$$

With our notation, $K \equiv \sqrt{8\pi G\mu^2/3\mathcal{G}^2}$, and

$$\widetilde{\rho}_\phi \equiv \tfrac{1}{2}[\dot{y}^2 + y^2(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2)] + V^{(1)}(\phi). \tag{5.6}$$

Inflation eventually ends when, by virtue of decay mechanisms, the energy stored in the scalar configuration is released to lighter degrees of freedom (radiation). This is the so-called reheating process, and the knowledge of the value of τ, say τ_f, for which this actually happens, enables one to determine the total e-fold number $N(\tau_f)$ of the inflationary model by solving the differential equation

$$\dot{N}(\tau) \equiv \frac{d}{d\tau}\ln\left(\frac{a(\tau)}{a(0)}\right) = K\sqrt{\widetilde{\rho}_\phi}. \tag{5.7}$$

A brief description of the reheating mechanism for our particular model is presented in Sec. VII.

VI. NUMERICAL ANALYSIS

A necessary requirement to perform a numerical analysis of the differential equations obtained in Sec. V, is a good knowledge of the absolute minimum of the potential (5.2). As already stated in Sec. III, if the inequality (3.12) is satisfied, the effective potential is bounded from below, but unfortunately, this does not ensure that quantum corrections to the tree-level potential will make it possible to fix the value of the field at the minimum, y_m, so that it is sensibly different from zero. Thus, one has to choose among the values of α, γ, δ, and λ satisfying Eq. (3.12), the ones for which a symmetry breaking actually occurs [8].

In Table I, we report for two choices of the free parameters of the classical potential (3.5) the corresponding values of θ_m, φ_m, y_m and hence the values of μ and $\widetilde{V}_0$. The parameters α, γ, δ and λ have been chosen in such a way that small couplings are achieved, and the minimum of y is of order 1.

TABLE I. Values of θ_m, φ_m, y_m, μ, and $\widetilde{V}_0$ for two choices of the free parameters of Eq. (3.5).

α	γ	δ	λ	θ_m	φ_m	y_m	μ (10^{16} GeV)	$\widetilde{V}_0$
$-3.\times 10^{-2}$	$5.\times 10^{-5}$	$-5.\times 10^{-6}$	$1.\times 10^{-2}$	1.31	0.94	1.68	0.34	2.49×10^{-2}
$-1.\times 10^{-4}$	$5.\times 10^{-7}$	$-5.\times 10^{-8}$	3.39×10^{-5}	1.70	1.35	2.09	0.27	3.50×10^{-2}

In Fig. 1, we plot the solution for y (obtained by means of the NAG Library routine D02CAF) corresponding to the initial conditions relevant for the $SU(4)_{PS}\otimes SU(2)_L\otimes SU(2)_R$ symmetry-breaking direction [see Eq. (3.4c)], i.e., $\theta(0)=0, y(0)=10^{-5}, \dot\theta(0)=0, \dot y(0)=4.\times 10^{-8}$. Note that a vanishing value of $\theta(0)$ and $\dot\theta(0)$ is the one resulting from our particular choice of residual-symmetry direction, along which the inflationary dynamics evolves. Moreover, a very small value of $y(0)$, which approaches zero, reflects our choice to start from a singlet state which has a complete SO(10) symmetry [hence, $y(0)=0$]. Of course, a $y(0)$ value which differs from zero is only taken for numerical convenience, but our results are essentially independent of $y(0)$, providing this is very small. For numerical purposes, it has been thus convenient to reexpress Eqs. (5.3)–(5.5) in terms of the variables defined in Eqs. (A2)–(A4) of the Appendix. However, we keep using the y, θ, φ parametrization in our paper, since it makes it easier to describe symmetry breaking. What happens is that y characterizes the dynamics which is independent of the symmetry-breaking direction, whereas θ and φ define the various symmetry-breaking directions. After a slow-roll phase which is not shown in Fig. 1

for the sake of clarity, y starts increasing until it reaches a region where it oscillates in the neighborhood of a relative minimum. The corresponding value of θ remains equal to 0. Interestingly, the time necessary to reach the region of rapid oscillation for y is of order $4.\times 10^{-35}$ sec, and the corresponding e-fold number [see Eq. (5.7)] is $\simeq 100$. However, such a large e-fold number is only achieved with the help of a severe fine-tuning of the $\dot y(0)$ value. This is indeed a peculiar property of Coleman-Weinberg potentials, which have a relative maximum at $y=0$. Further details about the oscillating phase and the end of inflation will be given in Sec. VII.

In Fig. 2, y is plotted against τ when the choice of α, γ, δ, and λ described on the second line of numerical values of Table I is made. In such a case, the following initial conditions are chosen: $\theta(0)=0$, $y(0)=10^{-5}$, $\varphi(0)=\pi/4$, $\dot\theta(0)=\sqrt{2}\times 10^{-3}$, $\dot y(0)=3.\times 10^{-8}$, $\dot\varphi(0)=0$. In this case, however, y oscillates in a neighborhood of the *absolute* minimum (cf. Fig. 1). The same happens for θ and φ, whose evolution is not shown for brevity. Moreover, a fine-tuning of the initial conditions leads again to an e-fold number of order 100, and the duration of the inflationary stage is the same as in Fig. 1.

The cases described in Figs. 1 and 2 are indeed also relevant for the analysis of the reheating stage of the early
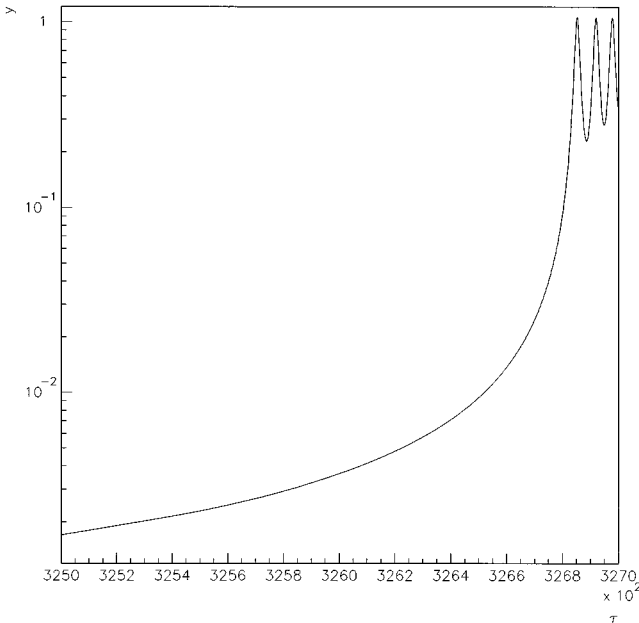

FIG. 1. The solution of Eq. (5.3) is plotted against τ, for the first set of values of $\alpha, \gamma, \delta, \lambda$ shown in Table I. After a slow-roll phase which is not shown, y starts increasing until it reaches a region where it oscillates in the neighborhood of a relative minimum.
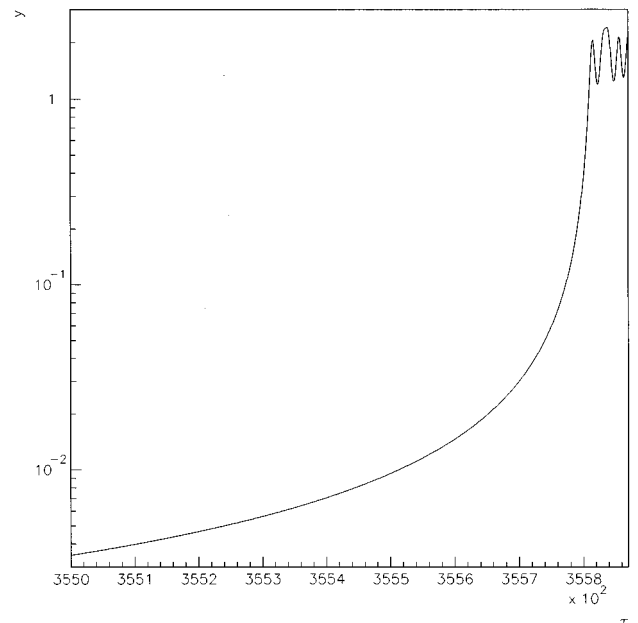

FIG. 2. The solution of Eq. (5.3) is plotted against τ, for the second set of values of $\alpha, \gamma, \delta, \lambda$ shown in Table I. Unlike Fig. 1, y oscillates in a neighborhood of the absolute maximum.

universe, as shown in Sec. VII, where a brief outline of such a phenomenon is presented.

VII. REHEATING

When the energy stored in the scalar-field configuration is released to the relativistic degrees of freedom (hereafter ρ_R denotes the corresponding energy density) and begins to dominate the total energy of the universe, the evolution undergoes the so-called reheating phase. Such a phase, which is the necessary last stage of the inflationary dynamics, is as important as the exponential expansion itself, since it ensures the end of the exponential growth and the reheating of the universe. In fact, if the radiation energy dominates the total energy of the system, through the equation of state, the energy density and the pressure p_R satisfy the condition $\rho_R + 3p_R = 4\rho_R > 0$, which implies $\ddot{a} < 0$.

According to the recent models of reheating, this process can be divided into three stages [28–30]. In the first stage, the scalar field ϕ decays into massive bosons via parametric resonance. The second stage consists in the decay of previously produced particles, and the last stage leads to thermalization (for our purposes, only the first two stages will be considered).

As far as the parametric resonance is concerned, it can be studied starting from the equation for quantum fluctuations of a scalar field ω, quadratically coupled to ϕ. As shown in Ref. [28], on setting $z = m_\phi t$, such an equation may be cast in the form

$$\omega_k'' + [A(k) - 2q\cos(2z)]\omega_k = 0, \qquad (7.1)$$

where the prime denotes differentiation with respect to z. Moreover, $A(k) = k^2/m_\phi^2 a^2 + 2q$ ($\vec{k}/a$ being the physical momentum), and $q = g^2 \Phi^2/4m_\phi^2$. In the model considered in Refs. [28–30], Φ is the amplitude of oscillations of the field ϕ, and g is a small coupling constant. Interestingly, an exponential instability of the solutions exists and it can be interpreted as a rich particle production. This phenomenon is best tackled by studying the stability-instability chart of the Mathieu equation [28–30].

In particular, the field ω may be given by the fluctuations of the scalar field ϕ itself. In such a case, one has to start from Eq. (A9) of the Appendix, where the effect of $3H d\delta z/dt$ is neglected, following Ref. [28]. Parametric resonance is only achieved if $\widetilde{M}(z_1^0, z_2^0, z_3^0)$ therein is positive for some values of $\alpha, \gamma, \delta, \lambda$, in the neighborhood of which the scalar field starts oscillating. In Figs. 3 and 4, we plot the function $\widetilde{M}$ against θ and φ, for the values of α, γ, δ, and λ reported on the first and second line of numerical values of Table I, respectively. In each figure, the closed curve represents those values of θ and φ for which $\widetilde{M}$ vanishes. Within the region bounded by such a curve, $\widetilde{M}$ takes negative values, whereas it is positive outside.

Interestingly, in Fig. 3, θ_m and φ_m lie in the region of parametric resonance, i.e., outside the region where $\widetilde{M} < 0$, while the converse holds in Fig. 4. Of course, since Fig. 1 corresponds to a case when θ remains equal to zero, para-

metric resonance is indeed achieved because, for this particular choice of parameters, $\widetilde{M}(y, \theta = 0, \varphi)$ is positive, as shown in Fig. 3.

Thus, our analysis shows that the inflationary dynamics with residual symmetry $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ [see Eq. (3.4c)] leads to slow roll and also to parametric resonance. These properties add evidence in favor of SO(10) GUT models being physically relevant, although one should bear in mind that fine-tuning problems remain. However, the case described by Fig. 4 is not, by itself, ruled out. One has instead to consider a scalar field ω which is *not* given by the fluctuations of ϕ itself [28–30].

We now briefly consider the second stage of the reheating process. Once that the quantum fluctuations of the **210** irrep have been produced via parametric resonance, they have to decay into relativistic degrees of freedom, whose mass is negligible with respect to the M_X scale. Note that the decay of the **210** representation into a fermion-antifermion pair cannot occur, since it makes it necessary to introduce a Yukawa coupling of the **210** to fermions, which would lead in turn to an undesirable mass term for fermions of order M_X. On similar ground, any gauge boson coupled to **210** would acquire mass at the scale M_X, and hence such a coupling cannot lead to decays into relativistic degrees of freedom. One is thus left with decays into other Higgs particles of smaller mass. Indeed, the presence of lighter Higgs fields in GUT's is particularly evident in SO(10), where the spontaneous symmetry breaking which leads to the standard electroweak model occurs in two steps at different mass scales: i.e.,

$$\text{SO}(10) \xrightarrow{M_X} G \xrightarrow{M_R} \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y, \qquad (7.2)$$

where M_X is of order 10^{15}–10^{16} GeV to be compatible with the lower limit on proton decay, G is one of the intermediate symmetry groups appearing in Eqs. (3.4a)–(3.4f), and the scale M_R is the one relevant for neutrino physics. Note that every breaking phase in Eq. (7.2) is mediated by a different scalar field belonging to a irrep of SO(10). A realistic model can be constructed, for example, by considering in addition to the **210** (ϕ), the scalar fields (ψ, $\overline{\psi}$) belonging to the representation $126 \oplus \overline{126}$, and two 10-dimensional irreducible representations (χ) which mediate the electroweak symmetry breaking [31]:

$$\text{SO}(10) \xrightarrow{<\phi>} G \xrightarrow{<\psi>} \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$$

$$\xrightarrow{<\chi>} \text{SU}(3)_C \otimes \text{U}(1)_Q. \qquad (7.3)$$

In this model, to obtain the above symmetry-breaking pattern, one has to replace the simple tree-level potential of Eq. (3.2) by a much more complicated expression $V(\phi, \psi, \overline{\psi}, \chi)$ [see Eqs. (7)–(11) of Ref. [31]]. Still, as far as the inflation at the highest energy scale is concerned, only the terms of the potential containing uniquely the **210** representation with the associated scalars are relevant. This is why we neglected in Eq. (3.2) the contributions resulting from the other representations. The other terms, however, are important when the second stage of reheating is considered, since they can me-
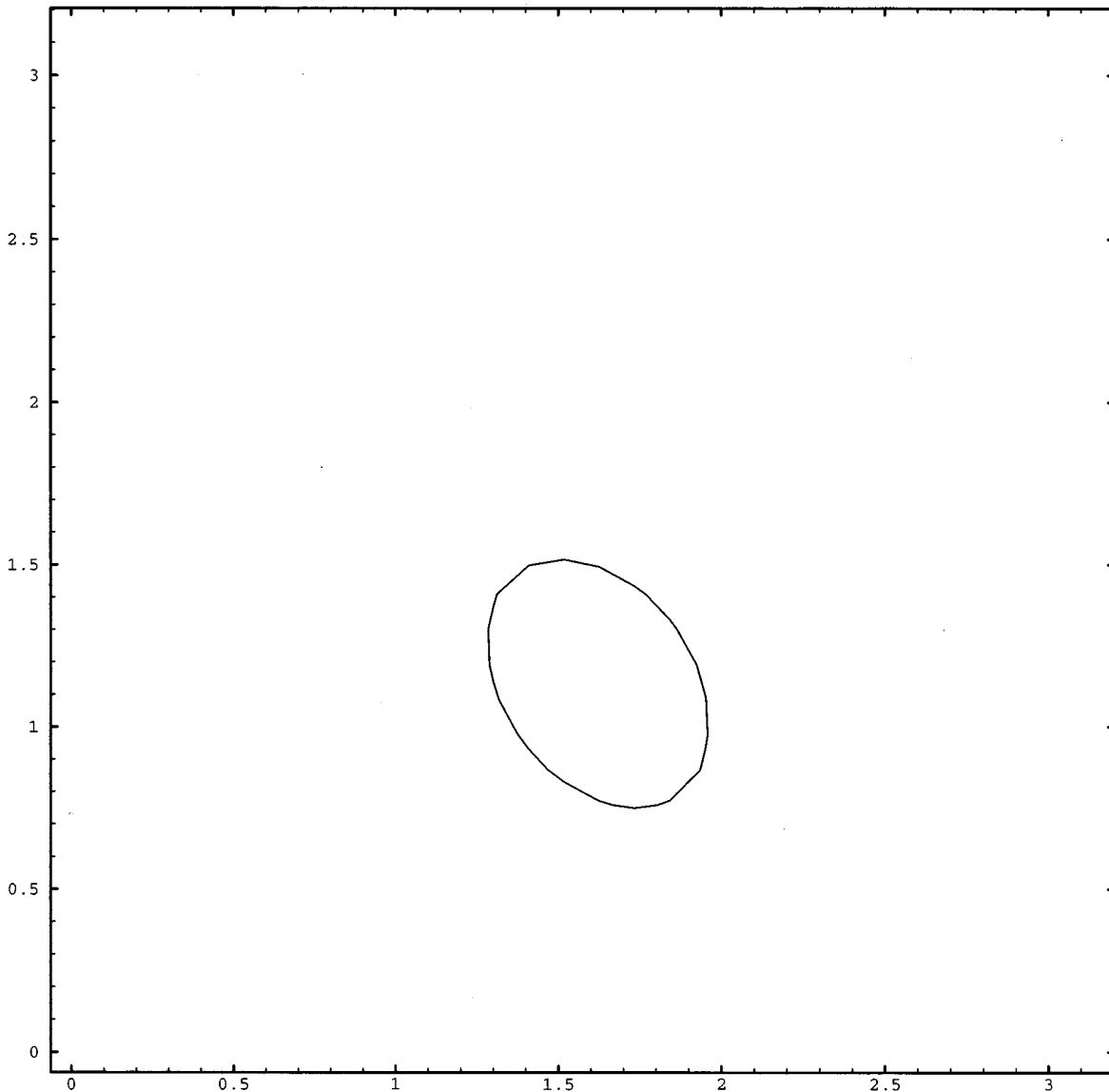
FIG. 3. The function defined in Eq. (A10), divided by y^2, is plotted against θ and φ, for the first set of values of $\alpha, \gamma, \delta, \lambda$ given in Table I. Note that θ_m and φ_m lie in the region of parametric resonance, i.e., outside the region where $\widetilde{M}$ is negative.

diate the decay of the massive Higgs particles in the lighter ψ, $\overline{\psi}$, and χ. The interacting terms between different scalar representations are the ones actually relevant in that they do not provide mass for light particles, while decay processes are allowed. Such interacting terms occur in quartic form, involve the whole set of representations of SO(10), and, hence, being linear in ϕ, do not provide mass for the lighter Higgs particles [Eq. (11) of Ref. [31]].

VIII. CONCLUDING REMARKS

Our paper has studied the relevance for inflationary cosmology of a nonsupersymmetric GUT which is consistent with the available data on nucleon lifetimes. This is the SO(10) model in the 210-dimensional irreducible representation [12].

Starting from a quartic tree-level potential in the case of minimal coupling, we have studied the flat-space limit of the semiclassical field equations, where the one-loop effective

potential contains the logarithmic terms which result from the Coleman-Weinberg method for the evaluation of radiative corrections [5]. Following Ref. [8], when the Higgs scalar field belongs to the 210-dimensional irreducible representation of SO(10) (this corresponds to the highest energy levels), attention has been restricted to the mass matrix relevant for the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry-breaking direction, to agree with the low-energy phenomenology of the standard model of particle physics.

The main result of our investigation is that the inflationary dynamics with residual symmetry $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ leads to a sufficiently long inflationary stage, and then a reheating process occurs via parametric resonance and subsequent decay of the particles produced previously. Such processes are three-body decays of **210** into the massless components of the **126**, $\overline{\textbf{126}}$, and **10** representations, and result from the mutual quartic coupling of all irreducible representations. The intermediate symmetry $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ has been extensively studied in
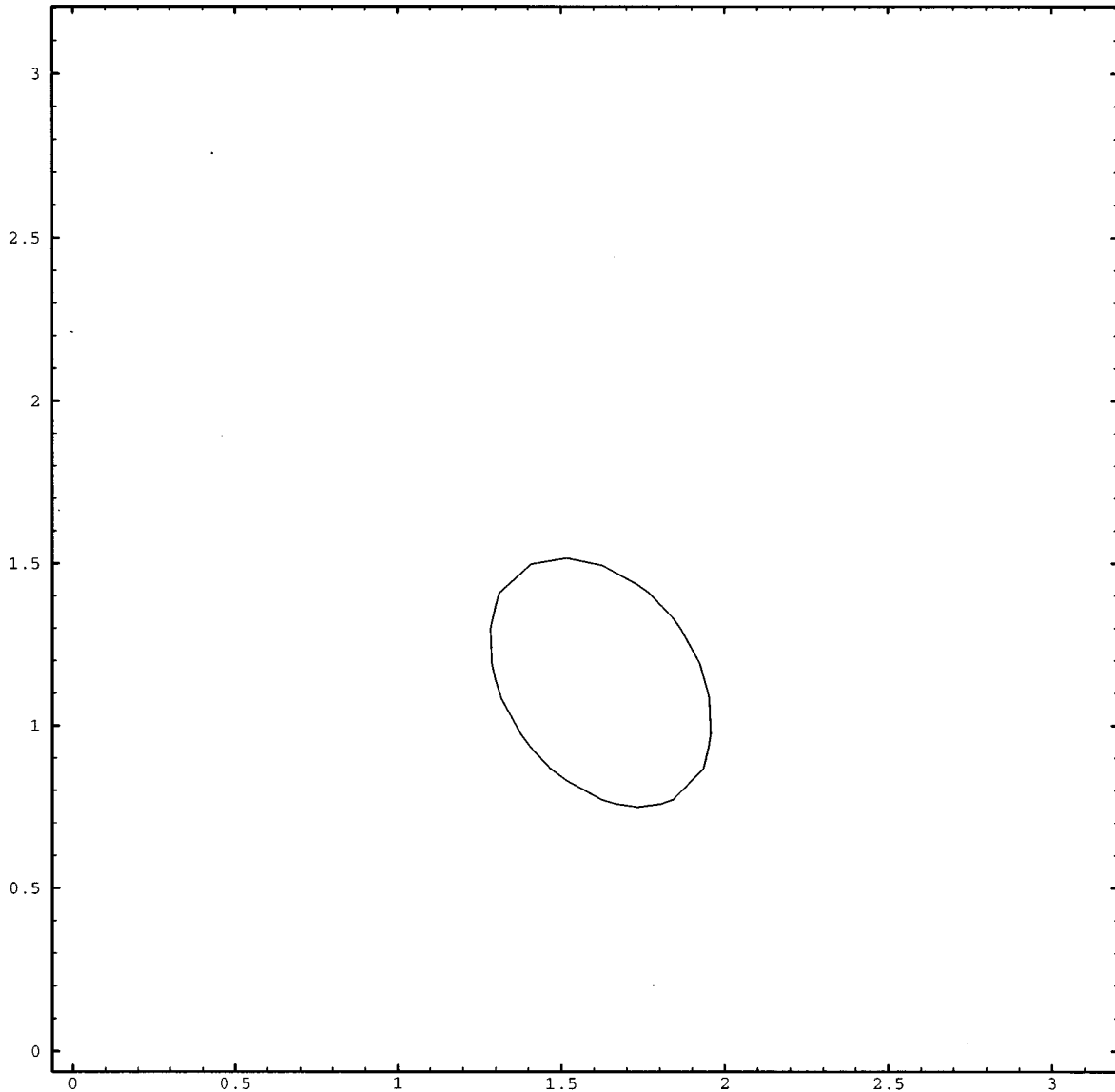
FIG. 4. The function defined in Eq. (A10), divided by y^2, is plotted against θ and φ, for the second set of values of $\alpha, \gamma, \delta, \lambda$ given in Table I. Unlike Fig. 3, θ_m and φ_m lie outside the region of parametric resonance.

the literature on particle physics, since it occurs in the most promising SO(10) models [9–13]. Thus, a natural and deep link between low-energy phenomenology, grand unification, inflationary cosmology, and physical processes in the very early universe seems to emerge from our work.

Of course, we have only studied some *particular* values of the parameters of the tree-level potential. Although particle physics and cosmology lead to restrictions on such parameters, our choices are by no means exhaustive. It now appears interesting to get a more quantitative understanding of the reheating process outlined in Sec. VII. Moreover, from the point of view of perturbative properties of quantum field theory, it is necessary to study in detail the nonlocal contributions to the semiclassical field equations. These arise already in scalar electrodynamics [25], by virtue of the coupling of the scalar field to the gravitational background, and are receiving careful consideration since they are related to dissipative and nondissipative phenomena in the early universe [25,32]. In particular, it appears desirable to extend our

analysis of semiclassical field equations to other FRW models, i.e., closed or open and hyperbolic. As a first step of this program, work is in progress by some of us on the semiclassical field equations of scalar electrodynamics in a closed FRW universe [33]. After completing such a calculation, it will be possible to extend it to the non-Abelian models relevant for grand unification.

Other interesting problems arise in finite-temperature calculations [34–38]. Temperature-induced symmetry-changing phase transitions in quantum field theory play indeed an important role in the investigation of physical phenomena in the very early universe. As shown in Ref. [36], in some of the improved one-loop calculations, difficulties result from the naive replacement of improved propagators in the one-loop effective potential. One needs instead a self-consistent loop expansion of the effective potential in terms of the full propagator, along the lines of the technique developed in Ref. [39]. The extension of these techniques to GUT models in FRW cosmologies is a highly nontrivial task which might

shed new light on inflationary cosmology, leading to a better understanding of the nature of phase transitions and of the behavior of the Higgs field [40,41]. On the other hand, the whole perturbative approach might fail in the neighborhood of a phase transition. Thus, a thorough understanding of nonperturbative properties of thermal field theory is necessary to investigate such issues. This goes well beyond the aims of our paper and the knowledge of the authors, and appears as one of the main open problems of inflationary cosmology. Last, it also appears worth applying the technique described in Ref. [42] to our semiclassical analysis of GUT models in cosmological backgrounds.

APPENDIX

To obtain the equation for the quantum fluctuations of the **210** irrep, we follow the notation of Ref. [8], for which the most general singlet ϕ_0 with respect to the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is written down in Eq. (3.3). Within this framework, the Lagrangian for scalar fields only, which are assumed to be spatially homogeneous, reads

$$\mathcal{L}_{\phi_0} = a^3 \left[\frac{1}{2}\left(\frac{dz_1}{dt}\right)^2 + \frac{1}{2}\left(\frac{dz_2}{dt}\right)^2 + \frac{1}{2}\left(\frac{dz_3}{dt}\right)^2 \right] + \sum_{l=1}^{3} g^{ij} z_{l,i} z_{l,j}$$
$$- V(z_1, z_2, z_3), \quad (A1)$$

where there is summation over the repeated indices i and j. We now take the one-loop expansion of Eq. (A1) by writing $z_i = z_i^0 + \delta z_i$, and we make the ansatz $\delta z_i = \delta z$ for simplicity. The three independent degrees of freedom of the scalar field turn out to be

$$z_1^0 = \|\phi_0\| \sin\theta \cos\varphi, \quad (A2)$$

$$z_2^0 = \|\phi_0\| \sin\theta \sin\varphi, \quad (A3)$$

$$z_3^0 = \|\phi_0\| \cos\theta. \quad (A4)$$

Thus, the tree-level potential (3.5) can be rewritten in terms of Eqs. (A2)–(A4) by pointing out that

$$\|\phi_0\|^2 = (z_1^0)^2 + (z_2^0)^2 + (z_3^0)^2, \quad (A5)$$

$$f_\alpha \|\phi_0\|^4 \equiv [(z_1^0)^2 + (z_2^0)^2]^2 + (z_2^0)^2 (2z_1^0 + \sqrt{3}z_3^0)^2 + \tfrac{3}{4}(z_2^0)^4, \quad (A6)$$

$$f_\gamma \|\phi_0\|^4 \equiv \left(z_1^0 z_3^0 + \frac{(z_2^0)^2}{\sqrt{3}} \right)^2 + (z_1^0 z_2^0)^2 + f_\alpha \|\phi_0\|^4, \quad (A7)$$

$$f_\delta \|\phi_0\|^4 \equiv (30 f_\gamma - 25 f_\alpha) \|\phi_0\|^4$$
$$+ \left(2(z_1^0)^2 - \frac{(z_2^0)^2}{2} - 3(z_3^0)^2 \right)^2. \quad (A8)$$

One thus gets the following equation for the quantum fluctuation:

$$\frac{d^2 \delta z}{dt^2} + 3H \frac{d \delta z}{dt} + \left[\frac{k^2}{a^2} + \widetilde{M}(z_1^0, z_2^0, z_3^0) \right] \delta z = 0, \quad (A9)$$

where $k \equiv \sqrt{\vec{k}^2}$, $\widetilde{M}(z_1^0, z_2^0, z_3^0)$ is defined as

$$\widetilde{M}(z_1^0, z_2^0, z_3^0) \equiv \tfrac{2}{3} [c_{110} z_1^0 z_2^0 + c_{101} z_1^0 z_3^0 + c_{011} z_2^0 z_3^0 + c_{200}(z_1^0)^2$$
$$+ c_{020}(z_2^0)^2 + c_{002}(z_3^0)^2], \quad (A10)$$

and the c_{abd} coefficients are given by

$$c_{110} \equiv (3 + \sqrt{3})\alpha + 7\left(1 + \frac{1}{\sqrt{3}} \right)\gamma + \frac{80}{9}(2 + \sqrt{3})\delta + 8\lambda, \quad (A11)$$

$$c_{101} \equiv \frac{\sqrt{3}}{2}\alpha + \left(1 + \frac{7}{2\sqrt{3}} \right)\gamma + \frac{40}{9}\sqrt{3}\delta + 8\lambda, \quad (A12)$$

$$c_{011} \equiv \left(\frac{3}{2} + \sqrt{3} \right)\alpha + \left(3 + \frac{7}{\sqrt{3}} \right)\gamma + \frac{80}{9}\sqrt{3}\delta + 8\lambda, \quad (A13)$$

$$c_{200} \equiv \tfrac{3}{2}\alpha + \tfrac{7}{2}\gamma + \tfrac{40}{9}\delta + 10\lambda, \quad (A14)$$

$$c_{020} \equiv \frac{1}{2}\left(\frac{39}{8} + \sqrt{3} \right)\alpha + \frac{1}{2}\left(\frac{45}{4} + \frac{7}{\sqrt{3}} \right)\gamma$$
$$+ \frac{20}{9}(5 + 2\sqrt{3})\delta + 10\lambda, \quad (A15)$$

$$c_{002} \equiv \tfrac{3}{8}\alpha + \gamma + 10\lambda. \quad (A16)$$

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