Quark matter in a strong magnetic field

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The effect of a strong magnetic field on the stability and gross properties of bulk as well as quasibulk quark matter is investigated using the conventional MIT bag model. Both the Landau diamagnetism and the paramagnetism of quark matter are studied. How the quark hadron phase transition is affected by the presence of a strong magnetic field is also investigated. The equation of state of strange quark matter changes significantly in a strong magnetic field. It is also shown that the thermal nucleation of quark bubbles in a compact metastable state of neutron matter is completely forbidden in the presence of a strong magnetic field. [S0556-2821(96)05712-8]

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I. INTRODUCTION

It is generally believed that if the density of neutron matter at the core of a neutron star exceeds a few times normal nuclear matter density a deconfining transition to quark matter may take place. Whatever the order of this transition is, it will convert a normal neutron star to a hybrid star with an infinite cluster of quark matter at the core and a crust of neutron matter. If the speculation of Witten [1] that a flavor symmetric strange quark matter (SQM) is the true ground state of matter at zero temperature and pressure is correct, then there is a possibility that the whole neutron star will be converted to a star of SQM, in which strange quarks are produced through the weak processes [2] and ultimately give rise to a dynamical chemical equilibrium among the constituents. The stability of such bulk and quasibulk objects has been thoroughly studied using the MIT bag model [3–6] as well as other confinement models; e.g., we have studied them using the dynamical density dependent quark mass [2,7–9] (D3QM) model of confinement, both at zero and finite temperatures. In this context we should mention that the results of the MIT bag model do not agree with the recent lattice Monte Carlo results. The latter predict a weak first-order or a second-order QCD phase transition, whereas according to the MIT bag model it is a strong first-order transition. On the other hand, we have seen that the predictions of the (D3QM) model more or less agree with the recent lattice Monte Carlo results. Since we do not know how to incorporate the effect of a strong magnetic field in lattice gauge calculations, we shall concentrate only on the MIT bag and D3QM models. To see the effect of a strong magnetic field on the second order quark-hadron phase transition we shall assume a metal-insulator-type transition in the presence of a strong magnetic field. The stability and gross properties of hybrid and strange stars have also been investigated [10–12]. Although the gross properties of such self-gravitating objects do not differ significantly from a pure neutron star [except in the mass-radio (MR) diagram], the presence of quark phase at the core may cause rapid cooling of the star and can also produce significant γ-ray bursts. The other important aspect is the superconducting nature of the quark matter core. Since quarks carry electric charges, unlike neutron matter which exhibits superfluidity in a neutron star, this particular phase as a whole behaves like a superconducting fluid (possibly type I—of course if there are very few protons present in neutron matter, they may form Cooper pairs and give rise to superconducting zones). As a result in such objects the magnetic field lines mainly pass through the crust region (if any), the consequence of which is the rapid ohmic decay of the magnetic field. Therefore the surface magnetic field for such objects will be low enough compared to the corresponding pure neutron star.

The other interesting part of such objects that must be studied is the time of quark matter nucleation. It is still not clear whether such a transition takes place immediately after the supernova explosion or much later [13]. If it occurs almost simultaneously with the explosion (if at all), then what is the effect of the strong magnetic field present in the neutron star on the (first or second order) quark-hadron phase transition? It is also interesting to study the effect of a strong magnetic field on the equation of state of quark matter.

From the observed features in the spectra of a pulsating accreting neutron star in a binary system, the strength of the surface magnetic field of a neutron star is found to be $\sim 10^{12}$ G. In the interior of a newly born neutron star, it probably reaches $\sim 10^{18}$ G [14]. It is therefore advisable to study the effect of a strong magnetic field on SQM, which we expect to be present at the core of a compact neutron star. As is well known, the energy of a charged particle changes significantly in the quantum limit if the magnetic field strength is equal to or greater than some critical value $B_m^{(c)} = m_i^2 c^3/(q_i \hbar)$ in Gauss, where m_i and q_i are, respectively, the mass and charge (absolute value) of the particle (e.g., $q_i = 2e/3$ for the u quark, $e/3$ for d and s quarks, and e for electron—here $e = |e|$ is the absolute value of the electronic charge), $\hbar$ and c are, respectively, the reduced Planck constant and velocity of light, both of which (along with the Boltzmann constant k_B) are taken to be unity in our choice of units. For an electron of mass 0.5 MeV, the strength of this critical field as mentioned above is $B_m^{(c)(e)} \approx 4.4 \times 10^{13}$ G, whereas for a light quark

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of current mass 5 MeV, this particular value becomes $=10^2\times B_m^{(c)(e)}$; on the other hand, for an s quark of current mass 150 MeV, it is $\sim 10^{20}$ G, which is too high to realize at the core of a neutron star. Therefore the quantum-mechanical effect of a neutron star magnetic field on an s quark may be ignored. The critical magnetic field as defined above is the typical strength at which the cyclotron lines begin to occur, and in this limit the cyclotron quantum is of the order of or greater than the corresponding rest energy. This is also equivalent to the requirement that the de Broglie wavelength is of the order of or greater than the Larmor radius of the particle in the magnetic field.

In recent years much interesting work has been done on the properties of dense astrophysical and cosmological matter in the presence of a strong magnetic field [15–17]. A study on how the primordial magnetic field affects the expansion rate of the Universe and synthesis of light elements in the early Universe a microsecond after the big bang has recently been done by Schramm *et al.* [18] (see also Ref. [19]). In a series of publications we have also reported the work done by us on the effect of the strong magnetic field on SQM using both the MIT bag model as well as the D3QM model of confinement [20–23]. We have also done some work on the magnetostatics of superconducting quark stars and also on the stability and gross properties of strange stars in the presence of a strong magnetic field [24].

The aim of the present paper is to investigate the stability and some gross properties of quark matter in the presence of a strong magnetic field. In Sec. II we study the thermodynamic properties of bulk SQM in the presence of a strong magnetic field. In Sec. III the thermal nucleation of stable quark matter bubbles in compact metastable neutron matter at the core of a neutron star in the presence of a strong magnetic field is investigated. In Sec. IV the properties of coexisting bulk phases in the presence of a strong magnetic field is studied. Assuming the possibility of a metal-insulator type of second order quark-hadron phase transition at the neutron star core at zero temperature, we investigate the effect of a strong magnetic field on such a transition in Sec. V. In Sec. VI, the paramagnetic properties of SQM are discussed in detail. Section VII contains conclusions of this work.

II. BULK SQM IN STRONG MAGNETIC FIELDS

To study the deconfining transition at the core and the stability of SQM in the presence of a strong magnetic field, we consider the conventional MIT bag model. The limitation of this model has already been discussed in the Introduction. For the sake of simplicity we are assuming that quarks are moving freely within the system and as usual the current masses of both u and d quarks are extremely low (in our actual calculation we have taken current mass for both of them to be 5 MeV, whereas for s quark, the current mass is taken to be 150 MeV).

For a constant magnetic field along the z axis ($\vec{B}_m=B_{m,(z)}=B_m=$const), the single particle energy eigenvalue is given by [25]

$$\epsilon_{k,n,s}^{(i)}=[k^2+m_i^2+q_iB_m(2n+s+1)]^{1/2}, \tag{2.1}$$

where $n=0,1,2,\ldots,$ are the principal quantum numbers for allowed Landau levels, $s=\pm1$ refers to spin up $(+)$ or down $(-)$ states, and k is the component of particle momentum along the direction of the external magnetic field. Setting $2\nu=2n+s+1$, where $\nu=0,1,2,\ldots$, we can rewrite the single particle energy eigenvalue in the form

$$\epsilon_\nu^{(i)}=(k^2+m_i^2+q_iB_m2\nu)^{1/2}. \tag{2.2}$$

Now it is very easy to show that the $\nu=0$ state is singly degenerate while all other states with $\nu\neq0$ are doubly degenerate.

The general expression for the thermodynamic potential of the system is given by

$$\Omega=-T\ \ln Z=\sum_i\ [\Omega_{i,V}(T,\mu_i)V+\Omega_{i,S}(T,\mu_i)S+\Omega_{i,C}(T,\mu_i)C]+BV, \tag{2.3}$$

where the sum is over u,d,s quarks and electron (e).

The first term on the second line of Eq. (2.3) is the volume contribution, whose explicit form is given by [26]

$$\Omega_{i,V}(T,\mu_i)=-\frac{Tg_i}{(2\pi)^3}\int d^3k\ \ln\{1+\exp[\beta(\mu_i-\epsilon_i)]\}, \tag{2.4}$$

where g_i is the degeneracy of the ith species (equaling six for quarks and two for electron) and V is the volume occupied by the system.

The second term comes from the surface contribution and is given by [27]

$$\Omega_{i,S}(T,\mu_i)=\frac{g_iT}{64\pi^2}\int\ \frac{d^3k}{|\vec{k}|}\left[1-\frac{2}{\pi}\arctan\left(\frac{k}{m_i}\right)\right]\times\ln\{1+\exp[\beta(\mu_i-\epsilon_i)]\}=\sigma \tag{2.5}$$

and S is the area of the surface enclosing the volume V and σ is the surface energy per unit area or the surface tension of quark matter.

The last term corresponds to curvature correction, which becomes zero for a plane interface separating two phases. The explicit form for the curvature term is given by [28]

$$\Omega_{i,C}(T,\mu_i)=\frac{Tg_i}{48\pi^3}\int\ \frac{d^3k}{|\vec{k}|^2}\ln\{1+\exp[\beta(\mu_i-\epsilon_i)]\}\times\left\{1-\frac{3}{2}\frac{k}{m_i}\left[\frac{\pi}{2}-\arctan\left(\frac{k}{m_i}\right)\right]\right\} \tag{2.6}$$

where C is the length of the line element drawn on the surface S.

At finite $T\sim\mu$, one has to consider the presence of antiquarks and positrons in the system. We assume that for antiparticles $\overline{\mu_i}=-\mu_i$; i.e., they are also in chemical equilibrium with respect to annihilation processes. We would like to see the necessary changes for these expressions that has to be incorporated if a strong external magnetic field is present in the system. With these modified forms of thermodynamic potentials, we shall investigate the magnetism arising from a quantization of orbital motion of charged particles in the presence of a strong magnetic field, known as Landau diamagnetism. It is well known that in the presence of a magnetic field along the z axis, the path of the charged particle will be a regular helix whose axis lies along the z axis and whose projection on the x-y plane is a circle. If the magnetic field is uniform, both the linear velocity along the field direction and the angular velocity in the x-y plane will be constant, the latter arising from the constant Lorentz force experienced by the particle. Quantum mechanically the energy associated with the circular motion in the x-y plane is quantized in units of $2q_iB_m$. The energy associated with the linear motion along the z axis is also quantized, but in view of the smallness of the energy intervals, they may be taken as continuous variables. We thus have Eqs. (2.1) or (2.2) as single particle energy eigenvalues. These magnetized energy levels are degenerate because they result from an almost continuous set of zero field levels. All these levels for which the values of the quantity $k_x^2+k_y^2$ lies between $2q_iB_m\nu$ and $2q_iB_m(\nu+1)$ now coalesce together into a single level characterized by the quantum number ν. The number of these levels is given by

$$\frac{S}{(2\pi)^2}\int\int dk_x dk_y=\frac{Sq_iB_m}{2\pi}, \tag{2.7}$$

where S is the area of the orbit in the x-y plane. This expression is independent of ν. Then in the integral of the form $\int d^3k\, f(k)$, we can replace $\int\int dk_x dk_y$ with the expression given above, whereas the limit of k_z, which is a continuous variable, ranges from $-\infty$ to $+\infty$. Then we can rewrite Eqs. (2.4)–(2.6) in the presence of a strong magnetic field in the form

$$\Omega_{i,V}=-T\,\frac{q_ig_iB_m}{2\pi^2}\sum_{\nu=0}^{\infty}\int_0^{\infty}dk_z\ln\{1+\exp[\beta(\mu_i-\epsilon_i^{(\nu)})]\}, \tag{2.8}$$

$$\Omega_{i,S}=T\,\frac{q_ig_iB_m}{16\pi}\sum_{\nu=0}^{\infty}\int_0^{\infty}\frac{dk_z}{\sqrt{(k_z^2+k_{\perp,(i)}^2)}}\times\ln\{1+\exp[\beta(\mu_i-\epsilon_i^{(\nu)})]\}\left[1-\frac{2}{\pi}\arctan\left(\frac{k}{m_i}\right)\right], \tag{2.9}$$

and

$$\Omega_{i,C}=T\,\frac{q_ig_iB_m}{12\pi^2}\sum_{\nu=0}^{\infty}\int_0^{\infty}\frac{dk_z}{(k_z^2+k_{\perp,(i)}^2)}\times\ln\{1+\exp[\beta(\mu_i-\epsilon_i^{(\nu)})]\}\times\left\{1-\frac{3}{2}\frac{k}{m_i}\left[\frac{\pi}{2}-\arctan\left(\frac{k}{m_i}\right)\right]\right\}, \tag{2.10}$$

where $k_\perp^{(i)}=2\nu q_iB_m$.

Since the surface and curvature terms play significant roles during quark bubble nucleation in dense neutron matter, and are not at all important for a bulk quark matter system, we shall study their dependence on a strong magnetic field in the next section, when we discuss thermal nucleation of quark droplets in the presence of a strong magnetic field. In this section we shall consider only the bulk term.

We shall use Eq. (2.4) for s and $\bar{s}$ and Eq. (2.8) for u, d, and e and their antiparticles. In Eq. (2.4), $\epsilon_i=(k^2+m_i^2)^{1/2}$, whereas in Eq. (2.8), it is given by Eq. (2.2).

From the well-known thermodynamic relations [26], the expression for the kinetic pressure of the system is given by

$$P=-\sum_i\Omega_{i,V}, \tag{2.11}$$

the number density of the ith species is given by

$$n_i=-\left(\frac{\partial\Omega_{i,V}}{\partial\mu_i}\right)_T, \tag{2.12}$$

and finally the expression for the corresponding free-energy density is given by

$$U_i=\Omega_{i,V}+\mu_in_i-T\left(\frac{\partial\Omega_{i,V}}{\partial T}\right)_{\mu_i}. \tag{2.13}$$

The last term in Eq. (2.13) comes from the nonzero entropy of the system, which is zero for $T=0$. Then the total energy density of the confined SQM is given by

$$U=\sum_i U_i+B. \tag{2.14}$$

To evaluate all of these thermodynamic quantities as defined above for a given temperature and bag parameter, we need chemical potentials for all the constituents present in the system. Assuming the condition of β equilibrium, we have

$$\mu_d=\mu_s=\mu\quad\text{(say)} \tag{2.15}$$

and

$$\mu_u=\mu-\mu_e, \tag{2.16}$$

whereas the charge neutrality condition gives

$$2n_u-n_d-n_s-3n_e=0. \tag{2.17}$$

Again the baryon number density of the system is given by

$$n_B=\tfrac{1}{3}(n_u+n_d+n_s), \tag{2.18}$$

which is considered as a constant parameter.

We have solved these equations numerically to obtain chemical potentials for all the species for various values of B_m (including zero) and then evaluated the thermodynamic quantities of the SQM system.

Since for any nonzero temperature $T\gg0$ analytical expressions for the thermodynamic quantities cannot be obtained, we shall give zero temperature and for the sake of illustration low temperature analytical expressions, whereas in actual calculations for $T\neq0$, we shall use numerical solutions of the general expressions.

Now for $T=0$, we have the number density of the component i $(i=u,\,d,$ or $e)$:

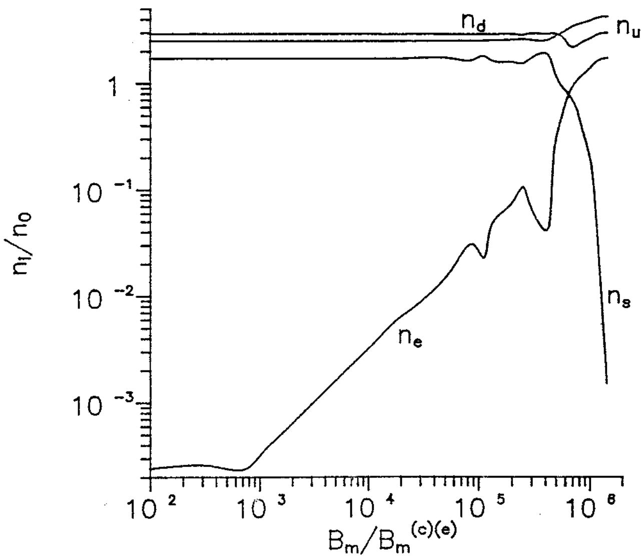

FIG. 1. The variation of u, d, s quarks and electron number density with the magnetic field intensity B_m, $n_B=2.5n_0$.

$$n_i=n_i^*\left[\zeta_i^{1/2}+2\sum_{\nu=1}^{[\nu_{\max}^{(i)}]}(\zeta_i-\nu)^{1/2}\right] \tag{2.19}$$

where

$$n_i^*=\frac{q_iB_m}{2^{1/2}\pi^2}\,g_i \tag{2.20}$$

$$\zeta_i=\frac{\mu_i^2-m_i^2}{2q_iB_m} \tag{2.21}$$

and $[\nu_{\max}^{(i)}]$ is the greatest integer not exceeding ζ_i. The ν limit becomes finite for the $T\to 0$ limit only, when the maximum available energy of a particle is approximately equal to its Fermi energy. On the other hand, for s quark, the expression for number density is given by the usual expression

$$n_s=\frac{(\mu_s^2-m_s^2)^{3/2}}{\pi^2}. \tag{2.22}$$

In Fig. 1 we have shown the variation of number of densities for u, d, s quarks and the electron with the magnetic field intensity B_m. At low enough magnetic field strength, all the number densities remain almost constant and are equal to their zero field values. For relatively higher field strengths $>10^3\times B_m^{(c)(e)}$ G, electron density increases with B_m. The s-quark density decreases with B_m at relatively higher values than that of the electron and becomes negligibly small for extremely large field strength $>10^5\times B_m^{(c)(e)}$ G. On the other hand, u-quark density becomes almost two times the zero field value at this extremely large field strength, whereas the d-quark density remains almost constant within this range of magnetic field. All the number densities, however, show oscillating behavior with the increase of magnetic field intensity. The number densities show oscillating nature, as the consecutive Landau levels for different constituents pass through the corresponding Fermi levels. The electron density increases to $\sim 10^4$ times the zero field value at extremely large magnetic field. One can very easily check that these oscillating and increasing and/or decreasing natures of number densities are consistent with the charge neutrality and constancy of baryon number density conditions. The physical meaning of change in s-quark density and electronic density is the shift of the β-equilibrium point in the presence of a strong magnetic field. As a cross check one should try to see how the rates of weak processes in which s quark and/or electron are created or annihilated, and change in the presence of a strong magnetic field (a detailed analysis on the effect of a strong magnetic field on weak and electromagnetic processes in SQM will be reported elsewhere [29]).

The expression for kinetic pressure and free-energy density for the ith species are given by ($i=u$, d, and e)

$$P_i=-\Omega_{i,V}=\frac{q_ig_iB_m}{2\pi^2}\sum_{\nu=0}^{[\nu_{\max}^{(i)}]}\left\{\frac{1}{2}\,\mu_i(\mu_i^2-M_\nu^{(i)2})^{1/2} -\frac{1}{2}\,M_\nu^{(i)2}\ln\left[\frac{\mu_i+(\mu_i^2-M_\nu^{(i)2})^{1/2}}{M_\nu^{(i)}}\right]\right\} \tag{2.23}$$

and

$$\epsilon_i=\Omega_{i,V}+\mu_in_i=\frac{q_ig_iB_m}{2\pi^2}\sum_{\nu=0}^{[\nu_{\max}]}\left\{\frac{1}{2}\,\mu_i(\mu_i^2-M_\nu^{(i)2})^{1/2} +\frac{1}{2}\,M_\nu^{(i)2}\ln\left[\frac{\mu_i+(\mu_i^2-M_\nu^{(i)2})^{1/2}}{M_\nu^{(i)}}\right]\right\}, \tag{2.24}$$

where $M_\nu^{(i)}=(m_i^2+2\nu q_iB_m)^{1/2}$. For s quark, they are given by the usual expressions

$$P_s=\frac{1}{8\pi^2}\left\{2\mu_s(\mu_s^2-m_s^2)^{3/2}-3m_s^2\mu_s(\mu_s^2-m_s^2)^{1/2} +3m_s^4\ln\left[\frac{\mu_s+(\mu_s^2-m_s^2)^{1/2}}{m_s}\right]\right\} \tag{2.25}$$

and

$$\epsilon_s=\frac{3}{8\pi^2}\left\{2\mu_s^3(\mu_s^2-m_s^2)^{1/2}-m_s^2\mu_s(\mu_s^2-m_s^2)^{1/2} -m_s^4\ln\left[\frac{\mu_s+(\mu_s^2-m_s^2)^{1/2}}{m_s}\right]\right\}. \tag{2.26}$$

These two expressions are without the bag pressure or vacuum energy density. Here we have also not taken magnetic pressure and magnetic energy density into account. They are given by the usual expressions, $\pm B_m^2/8\pi$, where the plus sign is for magnetic energy density and the minus sign is for magnetic pressure. Now for $B_m=10^{15}$ G, the magnetic energy density is $\epsilon_{15}=32.283$ MeV^4, which is too small compared with the free energy density $\sim 10^6$ MeV^4, whereas for $B_m\sim 10^{18}$ G, it is given by $\epsilon_{18}=10^6\times\epsilon_{15}$. The same conclusion is also valid for the magnetic pressure of the system. Therefore, magnetic energy density as well as the magnetic pressure will be comparable to the particle free-energy den-

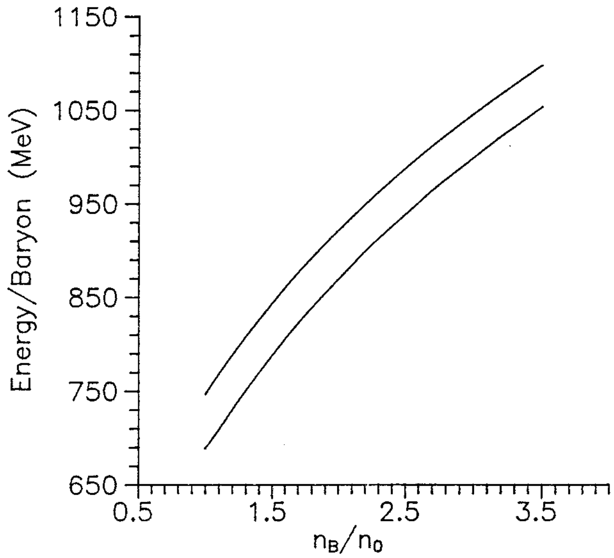

FIG. 2. Plot of energy per baryon against the baryon number density for $B_m=0$ (upper curve) and $B_m=10^3\times B_m^{(c)(e)}$ (lower curve).

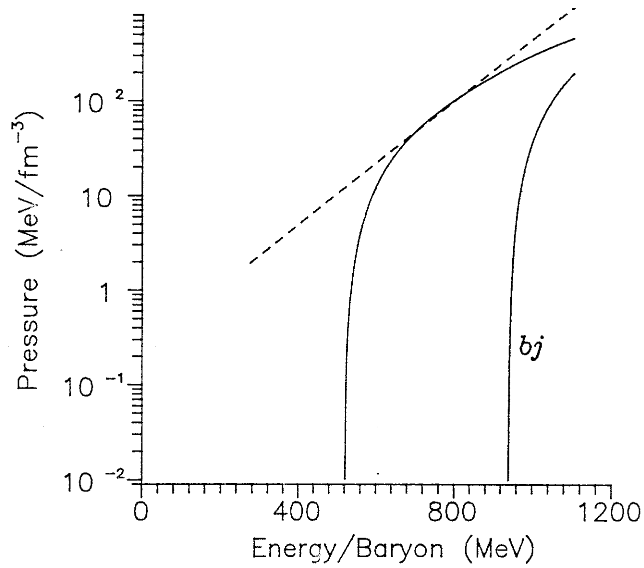

FIG. 3. Pressure vs energy per baryon curves (equation of states) for $T=0$. The dotted curve is for $B_m=0$ and solid one is for $B_m=10^3\times B_m^{(c)(e)}$. The curve indicated by bj is for B-J equation of state.

sity and the kinetic pressure, respectively, if and only if the magnetic field intensity is too high. For $B_m\geqslant 10^{20}$ G, the magnetic energy density as well as the magnetic pressure play the main roles. This magnetic field strength is, however, too high to realize at the core of neutron stars or any other stellar objects of our interest.

To study the stability of bulk SQM in the presence of a strong magnetic field we shall use the stability condition $\Sigma_i P_i - B = 0$, where $i=u,\ d,\ s$, and e, and solve the Eqs. (2.15)–(2.18) numerically for $T=0$. Since $\nu_{\max}^{(i)}$ is a function of μ_i, we have solved these equations self-consistently for $B_m\neq 0$. In Fig. 2 we have plotted energy per baryon of SQM against the baryon number density. The upper curve is for $B_m=0$, whereas the lower one is for $B_m=10^3\times B_m^{(c)(e)}$. This figure shows clearly that the presence of a strong magnetic field makes SQM energetically more stable.

We shall now write out for the sake of completeness the low-temperature analytical expressions for thermodynamic parameters. The number density is given by

$$n_i = n_i^{(0)} - \frac{q_i g_i B_m}{12} T^2 \sum_{\nu=0}^{[\nu_{\max}^{(i)}]} (\mu_i^2 - M_\nu^{(i)2})^{1/2} \frac{M_\nu^{(i)2}}{(\mu_i^2 - M_\nu^{(i)2})^2} \tag{2.27}$$

for $i=u,\ d$, and e, and as before, for s quark it is given by

$$n_s = n_s^{(0)} + \frac{T^2}{6} \frac{2\mu_s^2 - m_s^2}{(\mu_s^2 - m_s^2)^{1/2}}. \tag{2.28}$$

The expression for pressure for $u,\ d$, and e is given by the general expression

$$P_i = P_i^{(0)} + \frac{q_i g_i B_m}{12} T^2 \sum_{\nu=0}^{[\nu_{\max}^{(i)}]} \frac{\mu_i}{(\mu_i^2 - M_\nu^{(i)2})^2} \tag{2.29}$$

and for s quark it is given by

$$P_s = P_s^{(0)} + \frac{T^2}{2} \mu_s (\mu_s^2 - m_s^2)^{1/2}. \tag{2.30}$$

In all these equations, the superscript zero refers to zero temperature cases. In this particular low-temperature case the free-energy density for the ith species is given by

$$\epsilon_i = -P_i + \mu_i n_i + T s_i \tag{2.31}$$

where $i=u,\ d,\ s$, or e. Here the last term comes from the nonzero entropy of the system and is given by

$$T s_i = -T\left(\frac{\partial \Omega_{i,V}}{\partial T}\right)_{\mu_i} = \frac{q_i g_i B_m}{6} T^2 \sum_{\nu=0}^{[\nu_{\max}^{(i)}]} \frac{\mu_i}{(\mu_i^2 - M_\nu^{(i)2})^{1/2}} \tag{2.32}$$

for $i=u,\ d$, and e, whereas for s quark it is given by

$$T s_s = -T\left(\frac{\partial \Omega_{s,V}}{\partial T}\right)_{\mu_s} = T^2 \mu_s (\mu_s^2 - m_s^2)^{1/2}. \tag{2.33}$$

In Figs. 3 and 4 we have shown the equation of states for SQM at $T=0$ and 40 MeV respectively. In both these figures the dotted curves are for $B_m=0$, whereas the solid ones are for $B_m=10^3\times B_m^{(c)(e)}$. For the sake of comparison we have plotted Bethe-Johnson (B-J) equation of state [14] in Fig. 3, indicated by the symbol bj. These two figures show that in the presence of a strong magnetic field the equation of state of SQM changes significantly. In both these figures we have taken $B^{1/4}=0$, which is a constant scaling factor. Figure 5 shows the variation of energy per baryon of stable SQM with the magnetic field strength for $n_B=2.5n_0$ and $T=0$. This figure shows that for low B_m, the energy per baryon remains almost constant and rises sharply as B_m exceeds $10^4\times B_m^{(c)(e)}$, and the matter becomes energetically unstable. In Fig. 6 we have plotted the variation of kinetic pressure of

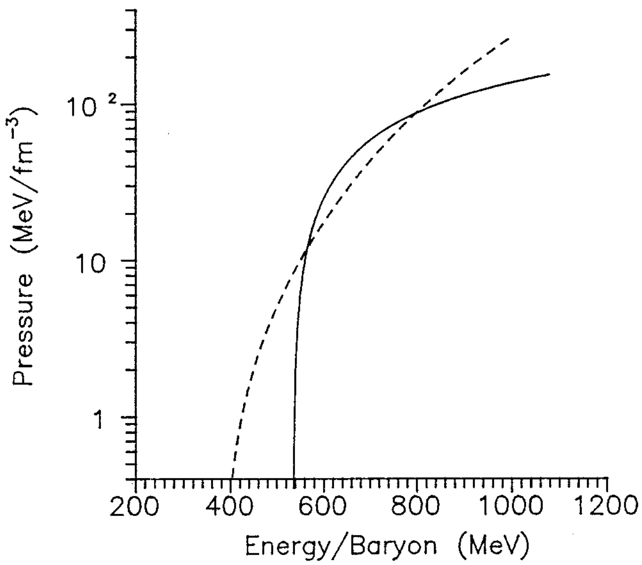

FIG. 4. Pressure vs energy per baryon curves (equation of states) for $T=40$ MeV. The dotted curve is for $B_m=0$ and the solid one is for $B_m=10^3\times B_m^{(c)(e)}$.

SQM with the magnetic field intensity for $B^{1/4}=0$ and for the same value of n_B as before. This figure shows that for relatively low values of magnetic field strength the kinetic pressure initially remains almost independent of magnetic field strength and then decreases sharply as B_m exceeds $10^4\times B_m^{(c)(e)}$ and finally becomes negative for extremely large values for B_m. Therefore for extremely large magnetic field strength, the system becomes energetically as well as mechanically unstable (these two conclusions are valid if the magnetic field is confined within the quark matter region only, which is possibly not true).

III. NUCLEATION OF QUARK BUBBLE IN COMPACT NEUTRON MATTER

In this section we shall investigate the first order quark-hadron phase transition initiated by the nucleation of droplets

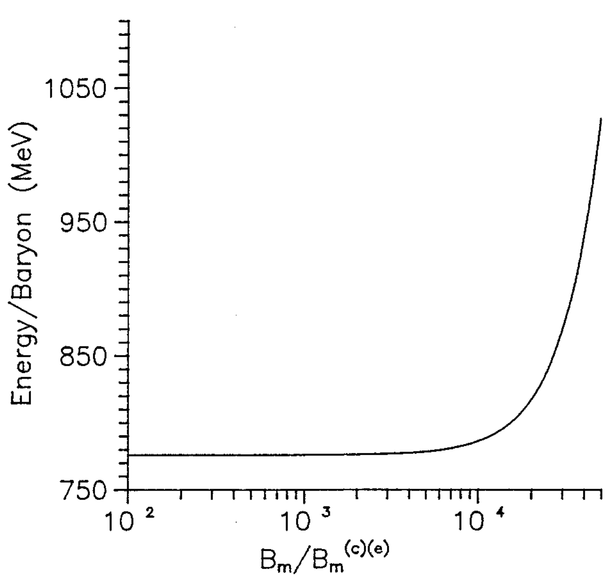

FIG. 5. The variation of energy per baryon for stable SQM with the magnetic field strength for $n_B=2.5n_0$ and $T=0$.

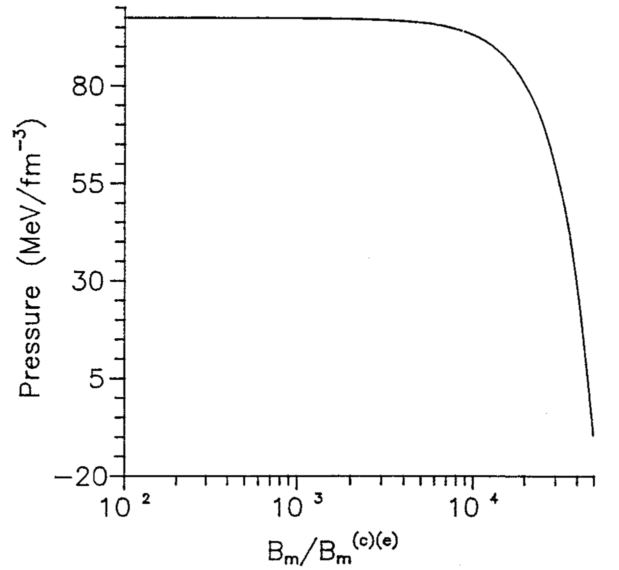

FIG. 6. The variation of pressure with the magnetic field strength for $n_B=2.5n_0$, $T=0$, and $B^{1/4}=0$.

of quark matter in the presence of a strong magnetic field. In particular we shall study the effect of a strong magnetic field on the nucleation of quark droplets in dense neutron matter. Since the surface and curvature energies of the quark bubble play a crucial role in droplet nucleation we would like to see how these two quantities are modified by the presence of a strong magnetic field.

The nucleation rate of stable quark matter bubble in metastable neutron matter per unit volume due to thermal fluctuation is given by [30]

$$I=I_0\exp(-W_m/T)\approx T^4\exp(-W_m/T), \qquad (3.1)$$

where W_m is the minimum thermodynamic work to be done to create a critical quark droplet and is given by [31]

$$W_m=\frac{4}{3}\pi\frac{\sigma^3}{(\Delta P)^2}[2+2(1+b)^{3/2}+3b], \qquad (3.2)$$

where $\sigma=\sigma_q+\sigma_n$ is the surface tension and $\Delta P=P_q-P_n$ is the pressure difference. Here $\Delta P=P_q-P_n$ is a positive quantity and both σ_q and σ_n are also positive, $b=2\gamma(\Delta P)/\sigma^2$, and $\gamma=\gamma_q-\gamma_n$ stands for curvature energy density. Here phase ''q'' is a droplet of quark matter and phase ''n'' stands for the metastable neutron matter. In Eq. (3.1), the preexponential factor is chosen to be T^4, where T is the temperature of the metastable medium (for an exact expression see Ref. [32]). In Eq. (3.2) we have assumed that the two phases are in chemical equilibrium ($\Delta\mu=0$). Since the bubble nucleation time $\tau_{\rm bubble}\approx10^{-23}$ sec$\approx\tau_{\rm strong}$, the strong interaction time scale, the creation of strange quarks through weak processes within the quark droplets may be ignored. Therefore the constituents of quark droplets are only u and d quarks. Since the temperature ($\sim$4–5 MeV) $\ll$ quark chemical potential ($\sim$300 MeV), the presence of antiquarks can also be ignored. On the other hand, if we assume the presence of hyperons at the core of a neutron star (which is believed to be true), then immediately after the phase transition to quark matter, s quarks will also be present in the quark bubble. However, the quantum mechanical effect of strong magnetics on the s-quark part of the strangelet can be ignored as long as the magnetic field strength is $\leqslant 10^{20}$ G, which has already been discussed. Therefore in the surface energy term of the strangelet a finite contribution will come from the s-quark part and is given by Eq. (2.5). On the other hand the surface tension or the surface energy per unit area for u and d quarks are given by Eq. (2.9). It is obvious from this equation that the surface energy diverges logarithmically in the infrared limit (i.e., $k_z \to 0$) for $\nu=0$, i.e., for the ground-state Landau level. To show this more explicitly, we shall make a zero temperature approximation. In this case the upper limit of the ν sum can be obtained from the condition

$$k_z^2=\mu_i^2-m_i^2-2\nu q_i B_m \geqslant 0. \tag{3.3}$$

This gives

$$\nu \leqslant \frac{\mu_i^2-m_i^2}{2q_iB_m}=[\nu_{\rm max}^{(i)}] \quad \text{(say)}, \tag{3.4}$$

which is the largest integer not exceeding $(\mu_i^2-m_i^2)/(2q_iB_m)$. Therefore as has been mentioned earlier the upper limit $[\nu_{\rm max}^{(i)}]$ cannot be the same for u and d quarks. To visualize the effect of a magnetic field on the nucleation rate of a quark droplet, we shall first rewrite Eq. (2.9) in the form

$$\sigma=\frac{TB_m}{16\pi}\sum_{i=u,d} g_i q_i \sum_{\nu=0}^{\infty}\int_0^{\infty}\frac{dk_z}{\sqrt{k_z^2+k_{\perp(i)}^2}} \times \ln\left[1+\exp\left(-\frac{\epsilon_i^{(\nu)}-\mu_i}{T}\right)\right]G, \tag{3.5}$$

where

$$G=\left[1-\frac{2}{\pi}\arctan\left(\frac{k}{m_i}\right)\right]$$

for the MIT bag model and is equal to unity for the D3QM model. Now we shall evaluate integral (3.5) by parts for $G=1$ and take $T\to 0$ limit, then

$$\sigma(G=1)=\frac{B_m}{16\pi}\sum_{i=u,d} g_i q_i\left[\sum_{\nu=0}^{\nu_{\rm max}^{(i)}}\int_0^{k_{F_i}}\frac{k_z dk_z}{\sqrt{k_z^2+m_i^2+k_{\perp,(i)}^2}} \times \ln\left(k_z+\sqrt{k_z^2+k_{\perp,(i)}^2}\right) - \sum_{\nu=0}^{\nu_{\rm max}} \ln(k_{\perp,(i)})\left(\mu_i-\sqrt{k_{\perp,(i)}^2+m_i^2}\right)\right]. \tag{3.6}$$

It is easy to check that Eq. (3.6) is also a part of σ, given by Eq. (3.5) for which $G\neq 1$. Since $\mu_i > \sqrt{k_{\perp,(i)}^2+m_i^2}$, therefore, for $\nu=0$ the second term of Eq. (3.6) becomes $+\infty$. The other part of the integral (3.6) has been evaluated numerically and found to be a finite number. Similarly the integral (3.5) with the second part of G (which is $\neq 1$) has also been obtained numerically and is found to be finite. Therefore the diverging property of $\sigma(G=1)$ for $\nu=0$ is also true for the whole surface energy area, given by Eq. (3.5). The term $\nu=0$ corresponds to the lowest Landau level. If the magnetic field is too strong only this level will be populated, which is also obvious from Eq. (3.4). Although we have assumed here $T\to 0$, this important conclusion is equally valid for any finite T. Since both P_n and P_q are finite even in the presence of a strong magnetic field and T is also finite, we have from Eq. (3.1) $I=0$, which means the thermal nucleation rate of droplet formation becomes zero; i.e., there cannot be a single quark droplet formation from metastable neutron matter due to thermal fluctuation at the core if the magnetic field is of the order of or greater than the corresponding critical value. Therefore the formation of a quark droplet at the magnetized neutron star core will be controlled by some other mechanism, namely, the quantum effects, triggered by strangelet capture, etc.

Following Ref. [28], we shall now investigate the effect of a strong magnetic field on the quark curvature term that also plays an equally important role as surface tension during thermal nucleation. As before, we can rewrite Eq. (2.10) in the form

$$\gamma=\frac{T}{12\pi^2}\sum_{i=u,d} g_i q_i \sum_{\nu=0}^{\infty}\int_0^{\infty}\frac{dk_z}{k_z^2+k_{\perp,(i)}^2} \times \ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\}G, \tag{3.7}$$

where

$$G=1-\frac{3}{2}\frac{k}{m_i}\left[\frac{\pi}{2}-\arctan\left(\frac{k}{m_i}\right)\right]$$

for the MIT bag model and equal to unity for the D3QM model.

Now we shall evaluate Eq. (3.7) term by term. Consider

$$I_1=\int_0^{\infty}\frac{dk_z}{k_z^2+k_{\perp,(i)}^2}\ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\}. \tag{3.8}$$

Obviously, this integral diverges in the infrared limit for $\nu=0$. Here the divergence is $1/k_z$ type. To see it more explicitly, let us integrate Eq. (3.8) by parts:

$$I_1=\frac{1}{Tk_{\perp,(i)}}\int_0^{\infty}\arctan\left(\frac{k_z}{k_{\perp,(i)}}\right)\frac{k_z dk_z}{(k_z^2+k_{\perp,(i)}^2+m_i^2)^{1/2}} \times \frac{1}{\exp[\beta(\epsilon_i-\mu_i)]+1}, \tag{3.9}$$

which diverges for $\nu=0$, but the divergence is not logarithmic ($I_1\sim 1/\nu$ as $\nu\to 0$).

Next consider the second term

$$I_2=-\frac{3\pi}{4m_i}\int_0^{\infty}\frac{dk_z}{(k_z^2+k_{\perp,(i)}^2)^{1/2}}\ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\}, \tag{3.10}$$

which has logarithmic divergence in the infrared limit for $\nu=0$. Integrating by parts, we have

$$I_2=\frac{3\pi}{4m_i}\left\{(\ln(k_{\perp,(i)})\ln\{1+\exp[-\beta(\epsilon_i-\mu_i)]\})\right.$$
$$-\frac{1}{T}\int_0^\infty \ln[k_z+(k_z^2+k_{\perp,(i)}^2)^{1/2}]\,\frac{k_z dk_z}{(k_z^2+k_{\perp,(i)}^2+m_i^2)^{1/2}}$$
$$\left.\times\left[\frac{1}{\exp[\beta(\epsilon_i-\mu_i)]+1}\right]\right\}. \tag{3.11}$$

For $\nu=0$, the first term diverges logarithmically, but unlike the second term of Eq. (3.6), it becomes $-\infty$. The second term of Eq. (3.11) and the last term of Eq. (3.7) have been checked numerically and they remain finite for all values of ν in the infrared as well as ultraviolet limits. The divergences of the first two terms of this equation cannot cancel each other. The first divergence is much faster as $\nu\to 0$ than the second one; therefore, the overall divergence of the curvature term (3.7) remains positive as $\nu\to 0$. From Eq. (3.2) it is obvious that if somehow the quantity b becomes finite (even zero), still the effect of σ makes W_m infinitely large. Therefore, in general, both the surface tension as well as curvature terms of a quark matter bubble diverge in the presence of a strong magnetic field. Physically it means that an infinite amount of work has to be done to create a critical quark bubble. As a consequence, thermal nucleation of such a bubble in metastable neutron matter will be completely forbidden. It can only occur via some nonthermal mechanisms. Therefore, in the future, if we get some direct or indirect experimental evidences for quark core and if the above conclusion is assumed to be correct, either the magnetic field of such objects will be extremely low ($<B_m^{(c)}$) from its time of birth (if quark matter is assumed to be produced immediately after supernova explosion), or it must be produced much later, when the neutron star magnetic field has decayed to a value $B_m<B_m^{(c)}$ or the quark droplets are produced through some nonthermal means. Since all these divergences are appearing in the infrared limit for $\nu=0$, one can in principle remove them by introducing a lower cutoff for k_z. Of course, in that case one has to think about the physical meaning of this lower cutoff of longitudinal momentum.

IV. COEXISTING BULK PHASES

From the discussion in the previous section, we have seen that it is impossible to have a mixed phase of dense neutron matter and droplets of quark matter in the presence of a strong magnetic field of strength greater than the corresponding critical value. In this section we would like to see the effect of a strong magnetic field on the coexisting bulk phases.

Assuming for the sake of simplicity that only neutrons present in the hadronic phase obey the free hadronic gas equation of state, the conditions to be satisfied by the coexisting phases are

$$\mu_n=\mu_u+2\mu_d, \tag{4.1}$$

$$P_n=P_q, \tag{4.2}$$

and

$$T_n=T_q=T_c. \tag{4.3}$$

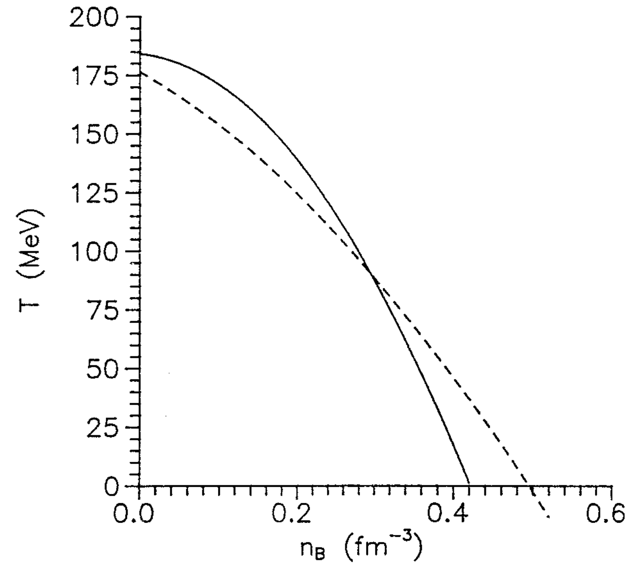

FIG. 7. Phase diagram for a bulk coexisting QGP and hadronic phases for $B_m=0$ (dotted curve) and $B_m=10^3\times B_m^{(c)(e)}$ (solid curve).

We also assume that the condition of β equilibrium is satisfied in the bulk quark matter phase.

In Fig. 7 we show the phase diagram (n_B vs T diagram) for such a mixed phase. The dotted curve is for $B_m=0$, whereas the solid curve is for $B_m=10^3\times B_m^{(c)(e)}$. This figure shows that the phase diagram changes significantly in presence of a strong magnetic field.

This observation is especially important during quark-hadron phase transition in the early Universe (where $n_B\approx 0$) if it would have taken place in the presence of a strong magnetic field of our interest (for a detailed discussion, see the in-press paper of Ref. [23], where the effect of a strong magnetic field on the baryon number inhomogeneity during cosmic QCD phase transition is also discussed).

V. METAL-INSULATOR TYPE OF TRANSITION

Since the order of quark-hadron transition is still not known exactly, in this section we shall consider a metal-insulator type of second-order quark-hadron phase transition at zero temperature in the presence of a strong magnetic field. As the density of neutron matter at the core of a neutron star increases, the separation between two neutrons decreases and at some critical value, when the separation becomes ~0.4 fm (the hard core radius), neutrons virtually lose their individuality and quarks can percolate out, giving rise to a color metal. This transition is more or less identical with the metal-insulator transition observed in condensed matter physics, which takes place at high pressure. Here neutrons, which are colorless objects, play the role of neutral atoms of the insulator placed at the lattice points and orbital bound electrons may be compared with the confined quarks.

Unlike a first-order transition, where the two phases are in equilibrium at the critical point, here they are indistinguishable. The conditions that must be satisfied in this case at the critical point are

$$\epsilon_n=\epsilon_q, \tag{5.1}$$

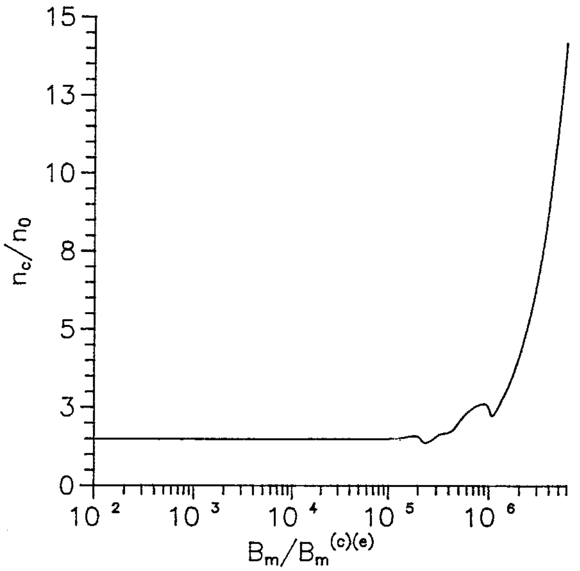

FIG. 8. The variation of n_c with the magnetic field intensity B_m.

$$\mu_n = 2\mu_d + \mu_u, \tag{5.2}$$

and of course

$$P_n = P_q. \tag{5.3}$$

Here we have assumed the presence of u and d quarks only, the numerical solution of Eqs. (5.1)–(5.3) give critical baryon number density $n_c = 1.49n_0$ and $B^{1/4} = 146.52$ MeV for $B_m = 0$. In the presence of a strong magnetic field, both the critical baryon number density and the corresponding bag pressure depend on the strength of the magnetic field. In Fig. 8 we show the variation of n_c with the magnetic field intensity B_m and in Fig. 9 the same type of variation is shown for $B^{1/4}$. These figures show that for low magnetic field strength, both the critical density and bag pressure are almost independent of B_m and very close to their zero magnetic field values. But for large magnetic field strength $> 10^4 \times B_m^{(c)(e)}$ G, both these quantities increase sharply. These figures also show that for extremely large magnetic field strength, the critical density becomes too high to achieve at the core of a neutron star. This implies that for an extremely strong magnetic field at the neutron star core, both the first order transition through the nucleation of quark bubbles and a second order metal-insulator type of quark hadron transition are impossible.

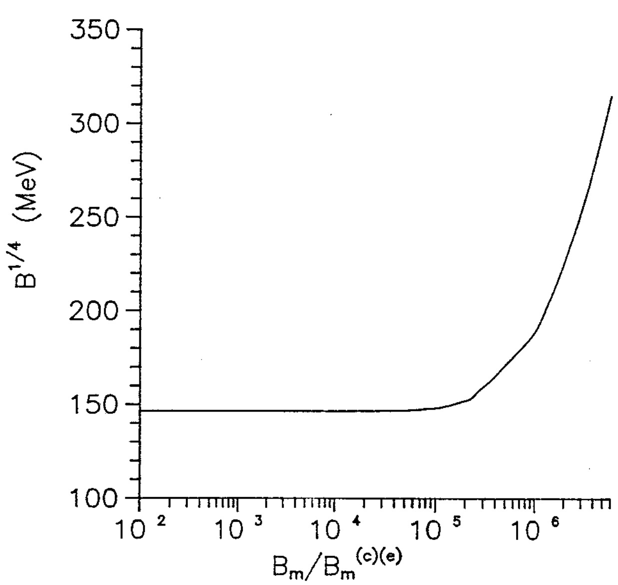

FIG. 9. The variation of $B^{1/4}$ with the magnetic field intensity B_m.

VI. PAULI PARAMAGNETISM OF QUARK MATTER

As mentioned in the Introduction, the magnetism of quark gas at $T = 0$ consists of two parts: a paramagnetic part due to the intrinsic spin magnetic moment, known as Pauli paramagnetism, and a diamagnetic part called Landau diamagnetism, arising from the quantization of the orbital motion of the quarks in the strong magnetic field. The second part has already been discussed. In this section we shall consider the Pauli paramagnetism of quark matter.

It is known that the quarks have nonzero magnetic dipole moment, given by $\eta_u = 1.852\eta_N$, $\eta_d = -0.972\eta_N$ and $\eta_s = -0.581\eta_N$, where $\eta_N = e/2m_p = 3.152\times 10^{-14}$ MeV/Tesla, the nuclear magneton. For the electron, it is given by $\eta_e = 1.00116\eta_B$, where $\eta_B = e/2m_e = 5.788\times 10^{-11}$ MeV/Tesla, the Bohr magneton.

Therefore in the presence of a magnetic field of strength B_m, the magnetic potential energy of the ith component is $-\eta_i B_m$, where i stands for u, d, or s quarks or electron. Then the corresponding number density is given by [33]

$$n_i = \frac{g_i}{2\pi^2}\left(\int_{\epsilon=m_i}^{\epsilon_i(B_m)} k^2 dk + \int_{\epsilon=m_i+2\eta_i B_m}^{\epsilon_i(B_m)} k^2 dk\right) \tag{6.1}$$

which gives, after some simple algebraic manipulation,

$$n_i = \frac{2\eta_i B_m(2\eta_i B_m + 2m_i)}{6\pi^2}(2x_i^{3/2} - 1), \tag{6.2}$$

where

$$x_i = \frac{\epsilon_i^2(B_m) - m_i^2}{2\eta_i B_m(2\eta_i B_m + 2m_i)} \tag{6.3}$$

and ϵ_i is the single particle energy. From Eq. (6.3) one can write

$$\epsilon_i^2(B_m) = m_i^2 + x_i(R_{3/2})^{-2/3}(\epsilon_{i,0}^2 - m_i^2), \tag{6.4}$$

where $\epsilon_{i,0}$ is the single particle energy in absence of a magnetic field, given by

$$\epsilon_{i,0} = m_i^2 + (3\pi^2 n_i)^{2/3} \tag{6.5}$$

and

$$R_{3/2}(x_i) = \tfrac{1}{2}(2x^{3/2} - 1). \tag{6.6}$$

Now the total particle number density for the ith component can be decomposed into two parts, with the magnetic moment along the direction of the magnetic field and opposite to it, and we have

$$n_i = n_i^{(\uparrow)}(B_m) + n_i^{(\downarrow)}(B_m), \tag{6.7}$$

where

$$n_i^{(\uparrow)}(B_m) = \frac{n_i}{2}x_i^{3/2}[R_{3/2}(x_i)]^{-1}, \tag{6.8}$$

the number density for the ith component with the direction of magnetic dipole moment along the direction of external magnetic field, and

$$n_i^{(\downarrow)}(B_m)=\frac{n_i}{2}\,x_i^{3/2}[R_{3/2}(x_i)]^{-1}(1-x_i^{-3/2}), \tag{6.9}$$

the same number density as above, but where the direction of the magnetic dipole moment is now opposite to the direction of the external field. In the absence of a magnetic field, or for $B_m\to 0$, the quantity $x_i=\infty$, which gives

$$\epsilon_i=\epsilon_{i,0}, \quad n_i^{(\uparrow)}=n_i^{(\downarrow)}. \tag{6.10}$$

The second relation implies equal occupation probability for up and down spin states. On the other hand for $x_i=1$, we have

$$n_i^{(\uparrow)}=n_i, \quad \text{and } n_i^{(\downarrow)}=0, \tag{6.11}$$

which is the complete saturation of the spin states along the direction of the magnetic field. In this condition

$$\epsilon_i^2=m_i^2+2^{2/3}(\epsilon_{i,0}^2-m_i^2). \tag{6.12}$$

The threshold magnetic field strength for complete saturation of the ith component can be obtained by putting $x_i=1$ in Eq. (6.3), which gives

$$B_m^{(\mathrm{th})(i)}=\frac{\left[2^{2/3}\left(\frac{6\pi^2 n_i}{g_i}\right)^{2/3}+m_i^2\right]^{1/2}-m_i}{2\eta_i}. \tag{6.13}$$

For $n_i=2.5n_B$, the threshold values for u, d, and s quarks are 3.4×10^{19} G, 6.46×10^{19} G, and 7.59×10^{19} G, respectively, which are of course not too low.

The energy density for such a Pauli paramagnetic system is given by

$$U=\sum_i U_i, \tag{6.14}$$

where

$$U_i(B_m)=\frac{g_i}{2\pi^2}\left(\int_{\epsilon=m_i}^{\epsilon_i(B_m)}\epsilon k^2dk+\int_{\epsilon=m_i+2\eta_iB_m}^{\epsilon_i(B_m)}\epsilon k^2 dk\right), \tag{6.15}$$

which after some straightforward algebraic manipulation becomes

$$U_i(B_m)=\frac{g_i}{16\pi^2}\left(2(\epsilon_i^2-m_i^2)^{1/2}(2\epsilon_i^2-m_i^2)\epsilon_i -2m_i^4\ln\left[\frac{\epsilon_i+(\epsilon_i^2-m_i^2)^{1/2}}{m_i}\right] -[y_i(y_i+2m_i)](m_i^3+5m_i^2y_i+6m_iy_i^2+2y_i^3) +m_i^4\ln\left\{1+\frac{y_i+[y_i(2m_i+y_i)]^{1/2}}{m_i}\right\}\right), \tag{6.16}$$

where $y_i=2n_iB_m$.

The kinetic pressure is given by

$$P=\sum_i P_i, \tag{6.17}$$

where

$$P_i(B_m)=\frac{g_i}{6\pi^2}\left[\int_{\epsilon=m_i}^{\epsilon(B_m)}\frac{k^4dk}{(k^2+m_i^2)^{1/2}} +\int_{\epsilon=m_i+2\eta_i}^{\epsilon(B_m)}\frac{k^4dk}{(k^2+m_i^2)^{1/2}}\right], \tag{6.18}$$

which, after some algebraic simplification reduces to

$$P_i(B_m)=\frac{g_i}{48\pi^2}\left(2(\epsilon_i^2-m_i^2)^{1/2}(2\epsilon_i^2-m_i^2)\epsilon_i +6m_i^4\ln\left[\frac{\epsilon_i+(\epsilon_i^2-m_i^2)^{1/2}}{m_i}\right] -[y_i(y_i+2m_i)](-3m_i^3+m_i^2y_i+6m_iy_i^2+2y_i^3) -3m_i^4\ln\left\{1+\frac{y_i+[y_i(2m_i+y_i)]^{1/2}}{m_i}\right\}\right). \tag{6.19}$$

In the field free case ($B_m=0$), $y_i=0$, then both the expressions for energy density and pressure reduce to the corresponding zero field expressions, with ϵ_i replaced by $\epsilon_i^{(0)}$. On the other hand, for the saturation condition, $x_i=1$ and $y_i=\epsilon_i-m_i$, we have

$$U_i(x_i=1)=\frac{g_i}{16\pi^2}\left\{2(\epsilon_i^2-m_i^2)^{1/2}(2\epsilon_i^2-m_i^2)\epsilon_i -2m_i^4\ln\left[\frac{\epsilon_i+(\epsilon_i^2-m_i^2)^{1/2}}{m_i}\right]\right\}, \tag{6.20}$$

where in this expression ϵ_i is given by Eq. (6.12). Similarly the expression for pressure is given by

$$P_i(x_i=1)=\frac{g_i}{48\pi^2}\left\{2(\epsilon_i^2-m_i^2)^{1/2}(2\epsilon_i^2-m_i^2)\epsilon_i +6m_i^4\ln\left[\frac{\epsilon_i+(\epsilon_i^2-m_i^2)^{1/2}}{m_i}\right]\right\}. \tag{6.21}$$

The bulk interaction energy per unit volume is given by

$$U_{in}=\sum_i U_{in}^{(i)} \tag{6.22}$$

where

$$U_{in}^{(i)}=\eta_iB_m[n_i^{(\uparrow)}(B_m)-n_i^{(\downarrow)}(B_m)]. \tag{6.23}$$

Substituting the values of $n_i^{(\uparrow)}$ and $n_i^{(\downarrow)}$, and after some simple algebraic manipulation, we have

$$U_{in}^{(i)}=n_i\eta_iB_m\left[1-\frac{x_i^{3/2}}{R_{3/2}(x_i)}\right], \tag{6.24}$$

which becomes zero for $B_m=0$ and at the saturation limit,

$$U_{in}^{(i)}(x_i=1)=n_i\eta_iB_m. \tag{6.25}$$

Finally, the chemical potential for the species i in presence of a magnetic field B_m is given by

$$\mu_i = \epsilon_i - n_i B_m . \tag{6.26}$$

Substituting the values for ϵ_i and $\eta_i B_m$, we have, after some simple algebra,

$$\mu_i = \{m_i^2 + x_i [R_{3/2}]^{-2/3} (\epsilon_{(i,0)}^2 - m_i^2)\}^{1/2} - \frac{1}{2}\left\{\left[\frac{\epsilon_i^2 - m_i^2(1-x_i^2)}{x_i}\right]^{1/2} - m_i\right\}, \tag{6.27}$$

which becomes $\epsilon_{i,0}$ for $y_i=0$, or equivalently for $B_m=0$ or $x_i \to \infty$. On the other hand in the saturation limit, we have

$$\mu_i(x_i=1) = \tfrac{1}{2}[\epsilon_i(x_i=1) + m_i] \tag{6.28}$$

VII. CONCLUSIONS AND DISCUSSIONS

From the theoretical investigations of SQM in the presence of a strong magnetic field we can make the following conclusions.

(i) In the presence of a strong magnetic field of strength of the order of greater than some critical value as discussed in the main text, the SQM becomes energetically more stable as long as $B_m \leqslant 10^{20}$ G. Above this value, since the magnetic energy density and also the magnetic pressure play the main roles, the system becomes energetically as well as mechanically unstable (the pressure of the system becomes negative).

(ii) The equation of state of magnetized SQM changes significantly. The β-equilibrium point has also been shifted.

(iii) There cannot be any thermal nucleation of quark droplets in a metastable state of compact neutron matter.

(iv) If bulk QGP and hadronic matter can coexist in the presence of a strong magnetic field, then a significant change in the phase diagram has also been observed.

(v) If the quark-hadron phase transition at low temperature and high density is like a metal-insulator type of second-order transition, in the presence of an extremely strong magnetic field as mentioned above, the critical density at which such a transition may occur is found to be too large, which is almost impossible to achieve at the core of a neutron star.

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