Photonic decay widths of some heavy pseudoscalar mesons

H. Routh and V. P. Gautam
Theoretical Physics Department, Indian Association for the Cultivation of Science, Calcutta-700032, India
(Received 14 June 1995; revised manuscript received 17 November 1995)

The photonic decays of the heavy pseudoscalar mesons B_L, B_{sL}, and D_L are studied in the single loop quark model. The contribution to the two γ decays of the heavy pseudoscalar mesons are computed in the standard model as a function of the quark mass. It is interesting to note the equality of the order for the two γ decay amplitudes of B_L, B_{sL}, and D_L. [S0556-2821(96)06011-0]

PACS number(s): 13.20.Fc, 13.20.Jf, 13.40.Hq

INTRODUCTION

We are studying the exotic flavor-changing two γ decays of the heavy pseudoscalar (B and D) mesons. These are of second order in both the weak and the electromagnetic interactions.

The electromagnetic decay of the pseudoscalar meson has been studied by the baryon loop model and the constituent quark model with $n_f=2$–4 flavors [1]. The two γ decays of the pseudoscalar mesons such as π^0, η, η', η_b, η_c, and η_t are studied in the one loop quark model [2]. The one loop quark diagram contributions to the $K_L\to 2\gamma$ has been computed in the standard model as a function of the quark mass [3]. Lattice computation of the decay constants of B and D mesons have been carried out by Bernard *et al.* [4] who obtained the following estimates from the numerical interpolation of the static and the intermediate mass results as

$$f_B=187\pm 34\pm 15\ \text{MeV},$$

$$f_{B_s}=207\pm 34\pm 22\ \text{MeV},$$

$$f_D=208\pm 35\pm 12\ \text{MeV}.$$

In our calculations we use the central values of these decay constants. Here we present detailed quark model calculations of the effective $B_L\to 2\gamma$, $B_{sL}\to 2\gamma$, and $D_L\to 2\gamma$ transitions.

THEORY

All the external momenta are assumed to be small compared to M_W. For the invariance of CPT, gauge and also for the angular momentum conservations, the amplitude of the radiative flavor-changing two γ decay of heavy pseudoscalar meson (P_L) must be proportional to $\varepsilon_1^\mu\varepsilon_2^\nu q_1^\alpha q_2^\beta$, where $\varepsilon_{1,2}$ and $q_{1,2}$ are the polarization and the momentum vectors of the two photons. The amplitude of the quark model calculation is given by [3]

$$A(P_L\to 2\gamma)=i\frac{G_F\alpha}{\pi}f_p\varepsilon_{\mu\nu\alpha\beta}\varepsilon_1^\mu\varepsilon_2^\nu q_1^\alpha q_2^\beta\sum_{j=u,c,t}\mathrm{Re}(\lambda_{jm}\lambda_{jn}^*)\times[A_j^{(i)}+A_j^{(r)}],\tag{1}$$

where $m=d,\ n=b$ for B_L, $m=s,\ n=b$ for B_{sL}, $m=c,\ n=u$ for D_L. Also

$$A_j^{(i)}=(Q+1)^2\left\{2+4(x_j/x_p)\int_0^1(dy/y)\times\ln[1-y(1-y)(x_j/x_p)]\right\}\tag{2}$$

is the one particle irreducible contribution of the jth quark and

$$A_j^{(r)}=\xi\left[Q^2\left(-1+\frac{1-5x_j-2x_j^2}{(1-x_j)^3}-\frac{6x_j^2\ln x_j}{(1-x_j)^4}\right)+Q\left(-3+\frac{3-9x_j}{(1-x_j)^2}-\frac{6x_j\ln x_j}{(1-x_j)^3}\right)\right]\tag{3}$$

is the one particle reducible contribution where $Q=-1/3$ for the d, s, and b and $Q=2/3$ for the u, c, and t quarks. The term $A_j^{(r)}$ is simply related to the one-loop effective γP coupling [5]. The schematic diagram for the two γ decay of P^0 and $\overline{P^0}$ is shown in Fig. 1. Further $x_j=(m_j/M_W)^2$, $x_p=(m_p/M_W)^2$ and λ_{ij} is the charged current mixing matrix for six quarks [6]. The parameter ξ is given by

$$\xi=\frac{m_P^2}{16}\left\langle P_L\left|\frac{1}{q_1\cdot p_2}+\frac{1}{q_2\cdot p_2}+\frac{1}{q_1\cdot p_2}+\frac{1}{q_2\cdot p_1}\right|P_L\right\rangle,$$

where $p_{1,2}$ are the momenta of the valence quarks inside the pseudoscalar mesons. In [7] it is called the charge radius of the pseudoscalar meson which can be understood from Fig. 2. Now it has been argued [8–10] that if the entire CP violation in the K^0-$\overline{K^0}$ sector comes from δ, then

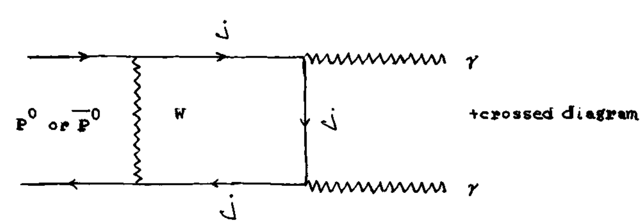

FIG. 1. The schematic diagram for the two γ decay of P^0 and $\overline{P_0}$; here $P^0=B^0$, B_s^0, and D^0. Also $j=u,c,t$ for B^0 and B_s^0 and d,s,b for D^0.

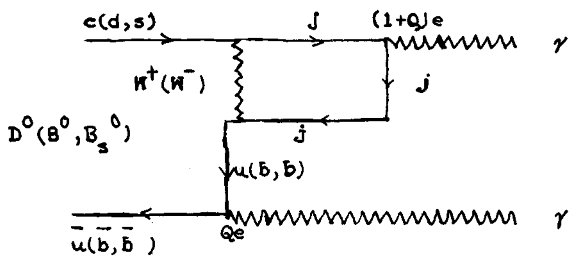

FIG. 2. The diagram for the 2γ decay showing the possibility of the vertex as appears for the charge radii of P^0; here $P^0=B^0$ and B_s^0 and D^0. Also $j=u,c,t$ for B^0 and B_s^0 and d,s,b for D^0.

$$s_2 s_3 \sin\delta = 10^{-3}, \tag{4a}$$

which is consistent with what is needed for ε in K decays.

The decay width of the heavy pseudoscalar mesons is given by

$$\Gamma(P_L \to 2\gamma) = (m_{P_L}^3/64\pi)|A(P_L \to 2\gamma)|^2. \tag{4b}$$

The neutral heavy mesons (B_L, B_{sL}, and D_L) exist in the two CP eigenstates. The mass eigenstates are the mixture of these states. The mixing may be understood by the box diagram as shown in Fig. 3. The two mass eigenstates can be written as

$$|P_L\rangle = a|P^0\rangle + b|\overline{P^0}\rangle,$$

$$|P_S\rangle = a|P^0\rangle - b|P^0\rangle, \quad \text{where } a^2+b^2=1.$$

Here L and S stand for mixed CP states.

CALCULATIONS

A. Decay width of $B_L \to 2\gamma$

For simplicity we assume the mass of the u quark to be zero. We define a portion of the right-hand side of Eq. (1) by E_1 in the following way:

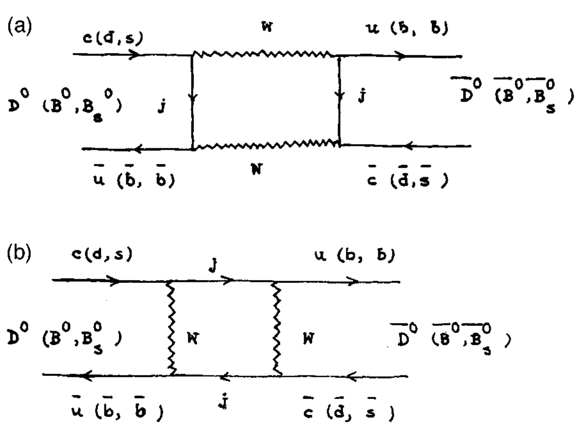

FIG. 3. The diagrams (a) and (b) for the conversion of $P^0 \leftrightarrow \overline{P^0}$; here $P^0=B^0$, B_s^0, and D^0. Also $j=u,c,t$ for B^0 and B_s^0 d,s,b for D^0.

$$E_1 = \sum_{j=u,c,t} \mathrm{Re}(\lambda_{jd}\lambda_{jb}^*)[A_j^{(i)} + A_j^{(r)}]$$

$$= -s_1[\tfrac{8}{9}c_1 s_3 - (16x_c/9x_{B_L})c_2(c_1c_2s_3 + s_2c_3\cos\delta_1)U_1 - \xi_{B_L}Rs_2(c_1s_2s_3 - c_2c_3\cos\delta_1)],$$

where

$$|U_1| = \int_0^1 (dy/y)\ln[1 - y(1-y)x_{B_L}/x_c] = 7.657,$$

where we take $m_{B_L} = 5279$ MeV [11],

$$R = \frac{7x_t - 5x_t^2 - 8x_t^3}{9(1-x_t)^3} + \frac{2x_t^2(2-x_t)\ln x_t}{3(1-x_t)^4} = 0.52,$$

where we take $m_t = 174$ GeV [12],

$$\xi_{B_L} = (m_{B_L}^2/m_{\pi^0}^2)\xi_{\pi^0} = 87.90,$$

with $\xi_{\pi^0} = 5.9\times 10^{-2}c_1$ [3] and $\delta = 0.7°$ using Eq. (4a). Therefore, the amplitude for the two γ decay of B_L is given by

$$|A(B_L \to 2\gamma)| = \frac{G_F\alpha}{(2)^{1/2}\pi} f_B |E_1| = 1.19\times 10^{-11}\ \mathrm{MeV}^{-1}.$$

Here we have taken $f_B = 187$ MeV [4]. The factor $(2)^{1/2}$ comes due to normalization of the decay constant. Hence the decay width is given by

$$\Gamma(B_L \to 2\gamma) = \frac{m_{B_L}^3}{64\pi}|A(B_L - 2\gamma)|^2 = 1.04\times 10^{-13}\ \mathrm{MeV}.$$

B. Decay width of $B_{sL} \to 2\gamma$

Here also we take mass of the u quark as zero and thus $x_u = 0$. We define E_2 as

$$E_2 = \sum_{j=u,c,t} \mathrm{Re}(\lambda_{js}\lambda_{jb}^*)[A_j^{(i)} + A_j^{(r)}]$$

$$= \tfrac{8}{9}s_1^2 s_3 c_3 + \frac{16x_c}{9x_{B_{sL}}}(c_1^2c_2^2c_3s_3 + c_1c_2s_2c_3^2\cos\delta_2 - c_1c_2s_2s_3^2\cos\delta_2 - s_2^2s_3c_3)U_2 + \xi_{B_{sL}}R(c_1^2s_2^2c_3s_3 + c_1c_2s_2s_3^2\cos\delta_2 - c_1c_2s_2c_3^2\cos\delta_2 - c_2^2c_3s_3),$$

where

$$|U_2| = \int_0^1 (dy/y)\ln\left[1 - y(1-y)\frac{x_{B_{sL}}}{x_c}\right] = 7.746,$$

$$R = \frac{7x_t - 5x_t^2 - 8x_t^3}{9(1-x_t)3} + \frac{2x_t^2(2-3x_t)\ln x_t}{3(1-x_t)^4} = 0.52,$$

$\delta = 0.7°$ using Eq. (4a), $\zeta_{B_{sL}} = (m_{B_{sL}}/m_{\pi^0})^2\xi_{\pi^0} = 91.60$, where we take $m_{B_{sL}} = 5375$ MeV [11].

Therefore, the amplitude for the two γ decay of B_{sL} is given by

$$|A(B_{sL}\to 2\gamma)|=\frac{G_F\alpha}{(2)^{1/2}\pi}f_{B_s}|E_2|=9.04\times 10^{-11}\ \text{MeV}^{-1}.$$

Here we take $f_{B_s}=207$ MeV [4]. The factor $(2)^{1/2}$ comes due to normalization of the decay constant. Hence the decay width is given by

$$\Gamma(B_{sL}\to 2\gamma)=\frac{m_{B_{sL}}^3}{64\pi}|A(B_{sL}\to 2\gamma)|^2=6.32\times 10^{-12}\ \text{MeV}.$$

C. Decay width of $D_L\to 2\gamma$

For simplicity we assume the mass of d quark as zero thus $x_d=0$. We define E_3 as

$$E_3=\sum_{j=d,s,b}\text{Re}(\lambda_{jc}\lambda_{ju}^*)[A_j^{(i)}+A_j^{(r)}]$$

$$=s_1[\tfrac{50}{9}c_1c_2-(100x_s/9x_{D_L})c_3(c_1c_2c_3-s_2s_3\cos\delta_3)U_3$$

$$-\xi_{D_L}R's_3(c_1c_2s_3+s_2c_3\cos\delta_3)],$$

where

$$|U_3|=\int_0^1(dy/y)\ln[1-y(1-y)x_D/x_s]=7.983,$$

$$R'=\frac{2(11x_b^3-10x_b^2-13x_b)}{9(1-x_b)^3}-\frac{4x_b^2(5-3x_b)\ln x_b}{3(1-x_b)^4}$$

$$=0.0087,$$

$\delta=0.7°$, using Eq. (4a) and

$$\xi_{D_L}=(m_{D_L}/m_{\pi^0})^2\xi_{\pi^0}=10.94.$$

Therefore, the amplitude for the two γ decay of D_L is given by

$$|A(D_L\to 2\gamma)|=\frac{G_F\alpha}{(2)^{1/2}\pi}f_D|E_3|=0.88\times 10^{-11}\ \text{MeV}.$$

Here we have taken $f_D=208$ MeV. The factor $(2)^{1/2}$ comes due to normalization of the decay constant. Therefore, the decay width is given by

$$\Gamma(D_L\to 2\gamma)=\frac{m_{D_L}^3}{64\pi}|A(D_L\to 2\gamma)|^2$$

$$=2.50\times 10^{-15}\ \text{MeV}.$$

RESULTS AND DISCUSSION

From our results it is observed that the two γ decay widths of B_L, B_{sL}, and D_L are not too sensitive to the variation in the (quark level) CP-violating parameter δ even up to a few degrees. From our calculation it is also interesting to note the equality of the order for the two γ decay amplitudes of B_L, B_{sL}, and D_L. However, their decay widths increase gradually from D_L to B_L (B_{sL}) due to the increase of their mass. Now here we like to mention that at the hadronic level the CP violation manifestation is in terms of ε while at the quark level its measure is the CKM phase δ.

[1] L. I. Ametller, C. Ayala, and A. Bramon, Phys. Rev. D **29**, 916 (1984).
[2] S. Basu, V. P. Gautam, and L. Chatterjee, J. Phys. G **13**, L13 (1987).
[3] E. Ma and A. Pramudita, Phys. Rev. D **24**, 2476 (1981).
[4] C. W. Bernard, J. N. Labrenz, and A. Soni, Phys. Rev. D **49**, 2536 (1994).
[5] E. Ma and A. Pramudita, Phys. Rev. D **24**, 1410 (1981).
[6] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[7] X.-G. He, S. Pakvasa, E. A. Paschos, and Y. L.Wu, Phys. Rev. Lett. **64**, 1003 (1990).
[8] J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B109**, 213 (1976).
[9] J. Ellis, M. K. Gaillard, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B131**, 285 (1977).
[10] A. Ali and Z. Z. Aydin, Nucl. Phys. **B148**, 165 (1979).
[11] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[12] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **50**, 2966 (1994).