Moduli effects on neutrino oscillations

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We point out the possibility of detecting low-energy signals of moduli in superstring theory through neutrino oscillations. The idea is based on the characteristics that the couplings of moduli are different from matter to matter. We estimate the oscillation probability both in the base line and solar neutrino oscillations. In both cases, when there is at least one modulus of which the mass is less than or equal to 10^{-19} GeV, the interaction of the modulus significantly changes the conversion probability from one neutrino flavor to another. [S0556-2821(96)03511-4]

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I. INTRODUCTION

Recently data from the CERN e^+e^- collider LEP [1] suggest evidence of grand unified theories (GUT's) such as SU(5), SO(10), flipped SU(5), and so on. Furthermore, the data fit better if supersymmetry is included. On the theoretical side, to solve the gauge hierarchy problem the idea of supersymmetry (SUSY) is very persuasive. However, a SUSY GUT does not contain the interaction of gravity. At present it is conceived that superstring theory alone may include all interactions consistently in the theory. Phenomenologically, the heterotic superstring theory [2] is the most attractive. There are several ways of compactification, and after that very many vacua are produced [3]. They are parametrized, in general, by moduli [4] which are singlet superfields under the gauge group of the standard model, $\mathrm{SU}(3)_C\times \mathrm{SU}(2)_L\times \mathrm{U}(1)_Y$. For example, some of them describe the size and shape of compactified space. Although their vacuum expectation values (VEV's) are supposed to be of the order of Planck scale, masses of moduli are not known. Their interactions with matter are also model dependent. Even the number of moduli depends on the structure of the vacuum under consideration. The number of Kähler structure moduli, T_i, is given by the Hodge number $h^{(1,1)}$ and that of complex structure moduli, U_m, is $h^{(2,1)}$. They are, in general, large numbers in (2,2) Calabi-Yau manifolds. In (0,2) orbifolds, however, the former is at most 9 and the latter is at most 3 [there are other types of moduli, too: (0,2) untwisted moduli (Wilson lines) and twisted moduli]. In any case, there exist moduli. Both T_i and U_m behave similarly as particles. Since moduli have a very important role in superstring theory, it is very helpful to detect the moduli.

In this paper we would like to point out that moduli may give low-energy signals which could be tested in neutrino oscillation experiments without depending on a particular compactification scheme. Moduli generically couple to ordinary matter with nonrenormalizable interactions. Such couplings are expressed effectively in the superpotential as (in the lowest dimension)

$$P_{\text{nonren}}=\frac{c^I_{ijk}}{M_S}\varphi_i\varphi_j\varphi_k M_I \quad (I=1,2,3,\ldots), \tag{1}$$

where $\varphi_{i,j,k}$ are matter superfields, M_I are moduli superfields, and M_S is the string scale ($\sim 10^{18}$ GeV). c_{ijk} may contain a product of VEV's of many scalar fields [5]. Such terms at low energies induce Yukawa-type couplings between the ordinary matter and (real) scalar fields or pseudoscalar fields, i.e., moduli:

$$\mathcal{L}_Y=\frac{\langle H_2\rangle}{M_S}h^{(\nu)}_{ij}\overline{\nu^i_R}\nu^j_L M_I+\frac{\langle H_2\rangle}{M_S}h^{(u)}_{ij}\overline{u^i_R}u^j_L M_I+\frac{\langle H_1\rangle}{M_S}h^{(d)}_{ij}\overline{d^i_R}d^j_L M_I+\frac{\langle H_1\rangle}{M_S}h^{(l)}_{ij}\overline{l^i_R}l^j_L M_I+\text{H.c.}, \tag{2}$$

where i and j are generation indices ($i=1,2,3$), $\langle H_{1,2}\rangle$ are the vacuum expectation values of the Higgs doublets, and γ matrices are dropped. While the dilaton S interacts with ordinary matter universally[1] like a graviton, moduli interact (or noninteract) with various coupling constants. Moduli interact with ordinary matter as a coherent attractive or repulsive force. Since the interaction strength is comparable to that of gravity force, this behaves as a kind of fifth force if the mass of the exchanged particle is small enough [6,7]. The

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[1]This is the case at the string tree level. At the loop level, this universality is lost. And so the dilaton may take part in the neutrino oscillation, too.

potential of moduli is considered flat perturbatively to all orders. When spontaneous breaking of SUSY occurs, most or all moduli may get mass by nonperturbative effects. Therefore their masses are expected to be of the order of the gravitino mass. Namely, it would be as heavy as other scalar sparticles. But a few may have very tiny mass or massless after SUSY breaking. There are several arguments which support it.

(1) For real M_I the moduli mass μ may be induced by radiative corrections ($\mu \simeq 10^{-18}$ GeV) [6], or there may be a special cancellation in the mass equation. In Ref. [8], it is estimated that μ can be about $m_{3/2}^2/\mathrm{Re}M_I$, where $m_{3/2}$ is the gravitino mass.

(2) For imaginary M_I, in Ref. [7] it was argued that μ can be 2×10^{-24} GeV. However, in Ref. [8] it is said that they are massless. In Ref. [9], on the other hand, they are said to gain huge mass of the order of the SUSY-breaking scale.

We do not go into details of the models here and want to discuss the model-independent way as much as possible. We regard a mass of a modulus (especially a tiny one) as a free parameter and its interaction strength as parameters f_{ij}, and explore the possibility of finding the effects of moduli in terrestrial experiments, not in cosmology.

Section II has two subsections. In Sec. II A, taking the influence of moduli interaction into consideration, we obtain the oscillation probability. In Sec. II B we examine how the moduli interaction affects the planning experiments. In Sec. III, we estimate the moduli effect on the solar neutrino oscillation. In Sec. IV, we argue the problematic points and mention the prospect of future experiments.

II. MODULI EFFECTS

A. Oscillation probability

In this section we deal with the accelerator experiments and derive the ν_μ-ν_τ oscillation probability including the effect of moduli interaction. We assume that there is, for simplicity, at least one modulus which interacts with ν_τ and/or ν_μ and u or d quark (or electron). For example, $h_{33}^{(\nu)}\neq 0$, $h_{11}^{(u)}\neq 0$, and others can be zero in Eq. (2). Although the interaction strength is gravitational, it may be detectable in the neutrino oscillations when μ is very tiny. We take μ in the range of 10^{-22}–10^{-14} GeV.

We define the mass eigenstate as (ν_2^m,ν_3^m) and the flavor eigenstate as (ν_μ,ν_τ). The latter eigenstate is expressed by the former with a mixing angle θ as

$$\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}=U\begin{pmatrix}\nu_2^m\\ \nu_3^m\end{pmatrix},\quad U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}. \tag{3}$$

The neutrino interaction with matter through moduli which is derived from Eq. (2) can be replaced by the Yukawa potential as moduli interact coherently and we put its coupling constants as $f_{ij}G_M$. G_M is the common coupling constant of the modulus so that the maximum value among $|f_{ij}|$ is unity. The Hamiltonian of the mass eigenstate is changed to

$$H=\begin{pmatrix}p+\dfrac{m_2^2}{2p}-f'_{22}\phi & -f'_{23}\phi\\ -f'_{32}\phi & p+\dfrac{m_3^2}{2p}-f'_{33}\phi\end{pmatrix}, \tag{4}$$

where p is the momentum of a neutrino beam, and m_2 and m_3 are the masses of mass eigenstates. In Eq. (4), $f'_{ij}\phi$ represent the potentials induced by moduli interaction and

$$\begin{pmatrix}f'_{22} & f'_{23}\\ f'_{32} & f'_{33}\end{pmatrix}=U^{-1}\begin{pmatrix}f_{22} & f_{23}\\ f_{32} & f_{33}\end{pmatrix}U. \tag{5}$$

We can take $f_{23}=f_{32}$. Because of minus signs before $f'_{ij}\phi$, $\phi>0$ means that it is an attractive potential and $\phi<0$ means repulsive. At least in orbifold models [10] either the diagonal (f_{33} or f_{22}) or nondiagonal (f_{23}) part may be considered to vanish or to be very small. Let us consider the following simple two cases: (A) $\Delta f=1$ ($\Delta f\equiv f_{33}-f_{22}$), $f_{23}=0$; (B) $\Delta f=0$, $f_{23}=1$.

The flavor eigenstate obeys the Schrödinger-like matrix equation

$$i\frac{d}{dx}\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}=UHU^{-1}\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}. \tag{6}$$

It does not make any difference to the probability of the ν_μ-ν_τ transition if we subtract from UHU^{-1} any multiple of the unit matrix. We choose the Hamiltonian matrix traceless for the sake of convenience: namely,

$$i\frac{d}{dx}\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}-a & b\\ b & a\end{pmatrix}\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}, \tag{7}$$

where

$$a\equiv\frac{\Delta m^2}{4E}\cos 2\theta-\frac{\Delta f}{2}\phi, \tag{8}$$

$$b\equiv\frac{\Delta m^2}{4E}\sin 2\theta-f_{23}\phi, \tag{9}$$

and $\Delta m^2\equiv m_3^2-m_2^2$. The momentum p is replaced by the neutrino energy E hereafter. Solving this, we obtain the oscillation probability

$$P(\nu_\mu\to\nu_\tau)=\sin^2 2\theta_M\sin^2\left\{\left[\left(\frac{\Delta m^2}{4E}\right)^2+\left(\frac{\Delta f'\phi}{2}\right)^2-\frac{\Delta m^2}{4E}\Delta f'\phi+(f'_{23}\phi)^2\right]^{1/2}L\right\}. \tag{10}$$

It is rewritten as

$$P(\nu_\mu\to\nu_\tau)=\frac{b^2}{a^2+b^2}\sin^2(\sqrt{a^2+b^2}L), \tag{11}$$

where $\theta_M=\theta+\zeta$ (ζ is the mixing angle from eigenstate of H to mass eigenstate), $\tan 2\theta_M=b/a$, and so

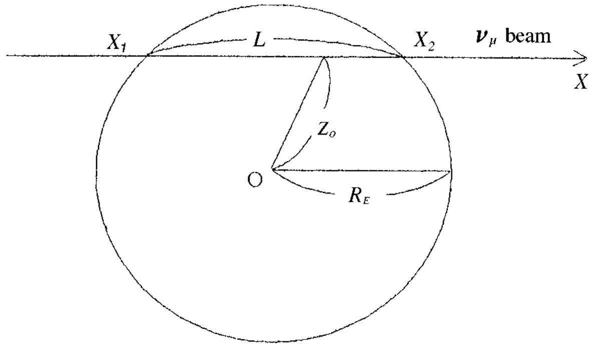

FIG. 1. Schematic view of base line neutrino oscillations. The neutrino beam is injected from the accelerator at the point x_1 and detected at the point x_2. The base line length L is the distance between x_1 and x_2.

$$\sin^2 2\theta_M = \frac{b^2}{a^2+b^2}. \tag{12}$$

L is the distance between an accelerator and a detector. The first term inside the brackets in Eq. (10) is due to the oscillation in the vacuum, and the last three terms are due to moduli interaction.

Next we evaluate ϕ in base line neutrino experiments. In a relativistic case ϕ is represented as the product of the energy of a neutrino beam and the potential per unit mass due to moduli interaction with matter [11]. For an attractive force we get

$$\phi = EV,$$

$$V = G_M \frac{M}{r} \exp(-\mu r). \tag{13}$$

Here M is the mass of the matter which interacts with the neutrino by interachanging moduli. ϕ changes its sign for a repulsive force. There may be a case that $\Delta f = -1$ and $f_{23}=0$. In this case the attractive force gives the same results as those of the repulsive one in case (A) when both coupling constants are equal to each other. So we will not discuss the case of $\Delta f = -1$ and $f_{23}=0$. To estimate V, we consider the following two cases.

(1) The contribution to V from the whole Earth is added up. We assume the density ρ to be constant. Then

$$\phi_{\text{global}} = \frac{2\pi G_M \rho E}{\mu^2} \left\{ 2 - \frac{R_E + \mu^{-1}}{z_0} \times [e^{-\mu(R_E - z_0)} - e^{-\mu(R_E + z_0)}] \right\}, \tag{14}$$

where R_E denotes the radius of the Earth. z_0 is the average distance between the neutrino trajectory and the center of the Earth (see Fig. 1). We can put $z_0 \simeq R_E - L^2/12R_E$. Since in the planning base line experiments $L \ll 2R_E$ and the main contribution to V comes from the parts near the neutrino trajectory, we put ρ to be the density of the surface layer of the Earth: $\rho = \rho_{\text{sur}} = 2.76$ [g cm^{-3}].

(2) μ^{-1} is the scale of the region where moduli interaction is effective. Consequently, the sphere within the radius μ^{-1} is sufficient for the estimation of V. We obtain

$$\phi_{\text{local}} = \frac{4\pi G_M \rho_{\text{sur}} E}{\mu^2} \left(1 - \frac{2}{e} \right). \tag{15}$$

The value of ϕ given by Eq. (14) is almost tantamount to that of Eq. (15) because of the exponential damping appearing in Eq. (13). Hence we use Eq. (14) hereafter.

As is obviously seen from Eq. (14) or (15), the potential ϕ is proportional to $\mu^{-2}E$; that is to say, the smaller μ is, the larger the effect of moduli is. The effect of the moduli is enhanced by E^2 relative to the vacuum oscillation part $\Delta m^2/4E$. In case (A) a very large ϕ, i.e., $|\Delta f \phi/2| \gg (\Delta m^2/4E) \cos\theta$ and so $a^2 \gg b^2$, leads to a very small $P(\nu_\mu \to \nu_\tau)$. On the other hand, in case (B) a large ϕ means that $b^2 \gg a^2$ and the magnitude $\sin^2 2\theta_M = b^2/(a^2+b^2)$ approaches 1. Then the oscillation length defined by $l = \pi/\sqrt{a^2+b^2}$ is much smaller than the oscillation length in the vacuum, $l_v = 4\pi E/\Delta m^2$. Therefore, for $L \simeq l_v$ the probability is averaged to be a half of the magnitude. Contrary to these, when $|\phi| \ll (\Delta m^2/4E)\cos\theta$ (or $\sin\theta$), the effect cannot be seen.

It is noted that a resonance similar to the solar neutrino oscillation occurs under a certain condition in case (A). When

$$\frac{\Delta m^2}{4E} \cos 2\theta = \frac{\Delta f}{2} \phi, \tag{16}$$

then $a=0$, and the magnitude is unity. Usually, Δm^2 is considered to be positive, and so the resonance occurs in the attractive (repulsive) force for positive (negative) Δf. On the contrary, if $(\Delta m^2/4E)\sin 2\theta \simeq f_{23}\phi$ in case (B) with the attractive force, then $b \simeq 0$ and $P(\nu_\mu \to \nu_\tau)$ is strongly suppressed.

B. Oscillations on long and short base lines

In this section we discuss long and short base line neutrino oscillations. In the planning experiments the muon neutrino (ν_μ) beam with energy E (of the order of 1 GeV to a few 10 GeV) propagates along the trajectory.

We now evaluate the oscillation probability. The force induced by the interaction of moduli with very tiny mass behaves like a fifth force, which many experiments have tested and given limitations to. Restrictions on the coupling constant G_5 as a function of the range λ have been given. First fixing the value of μ where $\mu = \lambda^{-1}$, we take G_M in Eq. (13) at the maximum value of allowable G_5. Denoting $\alpha = G_M/G_N$, where G_N is the gravitational constant, we impose restrictions for the attractive force from Ref. [12]; for example, $(2.0\times10^{-22}, 3.0\times10^{-6})$, $(2.0\times10^{-20}, 1.6\times10^{-4})$, $(2.0\times10^{-18}, 5.0\times10^{-4})$, in terms of ($\mu$ [GeV], α) (see Table I). Similarly, for the repulsive force the restrictions are found in Ref. [13] (see Table III).

We will comment on Eq. (10) here. The quantity in the braces can be written as

TABLE I. Numerical values of $\sin^2 2\theta_M$ and πl^{-1} in units of $(\mathrm{km})^{-1}$ in the short base line experiment (CHORUS) for the attractive force. For the oscillation in the vacuum, $\sin^2 2\theta_M = 4.0\times10^{-4}$, $\pi l^{-1} = 4.2$.

			Case (A)		Case (B)	
λ (m)	μ (GeV)	α	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}
1×10^{5}	2.0×10^{-21}	3.0×10^{-5}	9.4×10^{-8}	2.8×10^{2}	1.0	5.6×10^{2}
5×10^{4}	3.9×10^{-21}	5.7×10^{-5}	4.2×10^{-7}	1.3×10^{2}	1.0	2.7×10^{2}
2×10^{4}	9.9×10^{-21}	1.6×10^{-4}	2.3×10^{-6}	5.6×10^{1}	1.0	1.2×10^{2}
1×10^{4}	2.0×10^{-20}	1.6×10^{-4}	6.0×10^{-5}	1.1×10^{1}	9.8×10^{-1}	3.0×10^{1}
5×10^{3}	3.9×10^{-20}	1.6×10^{-4}	3.6×10^{-2}	4.5×10^{-1}	7.6×10^{-1}	8.6
2×10^{3}	9.9×10^{-20}	1.6×10^{-4}	5.5×10^{-4}	3.6	6.6×10^{-2}	4.4
1×10^{3}	2.0×10^{-19}	1.6×10^{-4}	4.3×10^{-4}	4.1	2.7×10^{-3}	4.2
5×10^{2}	3.9×10^{-19}	1.9×10^{-4}	4.1×10^{-4}	4.2	1.7×10^{-6}	4.2
2×10^{2}	9.9×10^{-19}	2.6×10^{-4}	4.0×10^{-4}	4.2	2.4×10^{-4}	4.2
1×10^{2}	2.0×10^{-18}	5.0×10^{-4}	4.0×10^{-4}	4.2	3.2×10^{-4}	4.2
5×10^{1}	3.9×10^{-18}	1.8×10^{-4}	4.0×10^{-4}	4.2	3.9×10^{-4}	4.2
2×10^{1}	9.9×10^{-18}	1.6×10^{-3}	4.0×10^{-4}	4.2	3.9×10^{-4}	4.2

$$\left(\frac{\Delta m^2}{4E} - \frac{\Delta f}{2}\phi\right)L, \tag{17}$$

for $f_{23}=0$ and $\cos 2\theta = 1$. In order to estimate the moduli effect roughly, we compare the value due to ϕ with the vacuum part in the following way. As is obvious,

$$\frac{\Delta m^2}{4E}L = 1.27\frac{(\Delta m^2/\mathrm{eV}^2)}{(E/\mathrm{GeV})}\left(\frac{L}{\mathrm{km}}\right). \tag{18}$$

For ϕ, using Eq. (14),

$$\frac{1}{2}\phi L \simeq \frac{\pi G_M \rho E L}{\mu^2} = 1.23\left(\frac{\alpha}{10^{-4}}\right)\left(\frac{\mu}{10^{-20}\ \mathrm{GeV}}\right)^{-2}\left(\frac{E}{\mathrm{GeV}}\right) \times\left(\frac{L}{\mathrm{km}}\right). \tag{19}$$

When all physical quantities are the same in the denoted units, both values of Eqs. (18) and (19) are almost the same and close to $\pi/2$. The above two equations are also useful to calculate a and b given by Eqs. (8) and (9).

Let us consider two versions of Δm^2 and θ. First, if ν_τ is regarded as a candidate of dark matter, then Δm^2 is expected to be about 100 eV^2 [14] in which the mixing angle is supposed to be very small ($\theta \simeq 1.0\times10^{-2}$). Second, according to Kamiokande atmospheric neutrino data, $\Delta m^2 \simeq 10^{-2}$ eV^2 [15] and the maximal mixing ($\theta \simeq \pi/4$) is suggested.

The short base line experiments, such as (i) CHORUS ($E=10$ GeV, $L=0.8$ km [16], expect the former version. We take $\theta = 1.0\times10^{-2}$ and $\Delta m^2 = 100$ eV^2 for this experiment. The long base line experiments, such as (ii) KEK $\to$Kamioka ($E=1.4$ GeV, $L=250$ km [17], (iii) Fermilab $\to$SOUDAN2 ($E=10$ GeV, $L=800$ km) [18], expect the latter version. So we fix $\theta=\pi/4$ and $\Delta m^2 = 10^{-2}$ eV^{-2}.

Our results are shown in Tables I–IV. The first column shows the values of λ, the second the values of μ, and the

TABLE II. Same quantities as in Table I in long base line experiments (KEK and Fermilab), for the attractive case. The values of λ and α are the same as those of the corresponding μ in Table I. For the oscillation in the vacuum, $\sin^2 2\theta_M = 1.0$, $(\pi l^{-1})_{\mathrm{KEK}} = 9.1\times10^{-3}$, $(\pi l^{-1})_{\mathrm{Fermilab}} = 1.3\times10^{-3}$.

	Case (A)				Case (B)			
	KEK		Fermilab		KEK		Fermilab	
μ (GeV)	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}
2.0×10^{-21}	4.7×10^{-7}	1.3×10^{1}	1.6×10^{-10}	1.0×10^{2}	1.0	2.6×10^{1}	1.0	2.0×10^{2}
3.9×10^{-21}	2.0×10^{-6}	6.3	6.0×10^{-10}	5.1×10^{1}	1.0	1.3×10^{1}	1.0	1.0×10^{2}
9.9×10^{-21}	9.5×10^{-6}	2.9	2.1×10^{-9}	2.7×10^{1}	1.0	5.9	1.0	5.4×10^{1}
2.0×10^{-20}	1.4×10^{-4}	7.6×10^{-1}	2.6×10^{-8}	7.9	1.0	1.5	1.0	1.6×10^{1}
3.9×10^{-20}	2.0×10^{-3}	2.0×10^{-1}	3.1×10^{-7}	2.3	1.0	4.0×10^{-1}	1.0	4.6
9.9×10^{-20}	5.4×10^{-2}	3.9×10^{-2}	1.0×10^{-5}	4.0×10^{-1}	1.0	6.6×10^{-2}	1.0	8.0×10^{-1}
2.0×10^{-19}	4.0×10^{-1}	1.4×10^{-2}	1.6×10^{-4}	1.0×10^{-1}	1.0	1.3×10^{-2}	1.0	2.0×10^{-1}
3.9×10^{-19}	8.5×10^{-1}	9.8×10^{-3}	1.8×10^{-3}	3.0×10^{-2}	1.0	1.5×10^{-3}	1.0	5.9×10^{-2}
9.9×10^{-19}	9.9×10^{-1}	9.1×10^{-3}	3.6×10^{-2}	6.7×10^{-3}	1.0	7.2×10^{-3}	1.0	1.2×10^{-2}
2.0×10^{-18}	1.0	9.1×10^{-3}	1.4×10^{-1}	3.4×10^{-3}	1.0	8.2×10^{-3}	1.0	5.0×10^{-3}
3.9×10^{-18}	1.0	9.1×10^{-3}	9.5×10^{-1}	1.3×10^{-3}	1.0	9.0×10^{-3}	1.0	7.0×10^{-4}
9.9×10^{-18}	1.0	9.1×10^{-3}	9.1×10^{-1}	1.3×10^{-3}	1.0	8.9×10^{-3}	1.0	4.6×10^{-4}

TABLE III. Same quantities as in Table I in the CHORUS experiment for the repulsive force. For the oscillation in the vacuum, $\sin^2 2\theta_M = 4.0\times10^{-4}$, $\pi l^{-1} = 4.2$.

			Case (A)		Case (B)	
λ (m)	μ (GeV)	α	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}
1×10^5	2.0×10^{-21}	3.6×10^{-4}	6.3×10^{-10}	3.4×10^3	1.0	6.7×10^3
5×10^4	3.9×10^{-21}	3.6×10^{-4}	9.9×10^{-9}	8.5×10^2	1.0	1.7×10^3
2×10^4	9.9×10^{-21}	5.9×10^{-3}	1.4×10^{-9}	2.2×10^3	1.0	4.5×10^3
1×10^4	2.0×10^{-20}	1.3×10^{-3}	4.8×10^{-7}	1.3×10^2	1.0	2.5×10^2
5×10^3	3.9×10^{-20}	1.9×10^{-3}	3.0×10^{-6}	4.9×10^1	1.0	9.0×10^1
2×10^3	9.9×10^{-20}	5.2×10^{-3}	1.2×10^{-5}	2.4×10^1	9.9×10^{-1}	4.0×10^1
1×10^3	2.0×10^{-19}	1.0×10^{-2}	3.8×10^{-5}	1.4×10^1	9.5×10^{-1}	1.9×10^1
5×10^2	3.9×10^{-19}	8.9×10^{-3}	1.8×10^{-4}	6.3	5.1×10^{-1}	6.0
2×10^2	9.9×10^{-19}	7.7×10^{-3}	3.5×10^{-4}	4.5	2.4×10^{-2}	4.3
1×10^2	2.0×10^{-18}	7.2×10^{-3}	3.9×10^{-4}	4.3	2.7×10^{-3}	4.2
5×10^1	3.9×10^{-18}	7.1×10^{-3}	4.0×10^{-4}	4.2	7.8×10^{-4}	4.2
2×10^1	9.9×10^{-18}	7.1×10^{-3}	4.0×10^{-4}	4.2	4.5×10^{-4}	4.2

third is assigned to α's in Tables I and III. When the values of λ are fixed, the probability can be calculated from Eq. (11). Here we evaluate the following two quantities involved in the formula of the probability:

$$\sin^2 2\theta_M = \frac{b^2}{a^2+b^2}, \tag{20}$$

$$\frac{\pi}{l} = \sqrt{a^2+b^2}. \tag{21}$$

$\sin^2 2\theta_M$ represents the magnitude of probability. l is the oscillation length. So $P(\nu_\mu \to \nu_\tau) = 0$ when $L = l$. We list these values in each table. The results for the attractive force are listed in Tables I and II, and those for the repulsive force in Tables III and IV.

Table I represents the estimation for the CHORUS experiment. In case (A), $\sin^2 2\theta_M$ is reduced to the comparatively lower values in the whole range of μ because b is small and constant. When μ is small, the probability changes rapidly on account of large values of πl^{-1}. When μ is larger than about 4×10^{-19} GeV, no moduli effects can be seen: $P(\nu_\mu \to \nu_\tau)$ shows no difference from the oscillation in the vacuum. A particular value of μ causes a phenomenon like a resonance which gives the largest value to $\sin^2 2\theta_M$. We will discuss this phenomenon in more detail later. In case (B), for a small value of μ, $\sin^2 2\theta_M$ is nearly unity, but l is very small. Therefore, the probability is supposed to be averaged to one-half and this may be observable. When μ is heavier, $\sin^2 2\theta_M$ is smaller and the neutrino oscillates more slowly to make little difference than that in the vacuum. In this case, however, an incident which we may call ''antiresonance'' occurs when μ takes the value such as b vanishes; namely, the moduli effect cancels the oscillation in the vacuum.

Our calculations on KEK and Fermilab-experiments are listed in Table II for the attractive case. Here we take the angle $\theta = \pi/4$, and so the first cosine term in Eq. (8) is zero. As seen in case (A) of KEK, only in a narrow range of μ,

TABLE IV. Same quantities as in Table I in the KEK and Fermilab experiments for the repulsive force. The values of λ and α are the same as those of the corresponding μ as in Table III. For the oscillation in the vacuum, $\sin^2 2\theta_M = 1.0$, $(\pi l^{-1})_{\mathrm{KEK}} = 9.1\times10^{-3}$, and $(\pi l^{-1})_{\mathrm{Fermilab}} = 1.3\times10^{-3}$.

	Case (A)				Case (B)			
	KEK		Fermilab		KEK		Fermilab	
μ (GeV)	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}	$\sin^2 2\theta_M$	πl^{-1}
2.0×10^{-21}	3.3×10^{-9}	1.6×10^2	1.1×10^{-12}	1.2×10^3	1.0	3.2×10^2	1.0	2.4×10^3
3.9×10^{-21}	5.1×10^{-8}	4.0×10^1	1.5×10^{-11}	3.3×10^2	1.0	8.0×10^1	1.0	6.5×10^2
9.9×10^{-21}	7.0×10^{-9}	1.1×10^2	1.6×10^{-12}	1.0×10^3	1.0	2.2×10^2	1.0	2.0×10^3
2.0×10^{-20}	2.2×10^{-6}	6.2	3.9×10^{-10}	6.4×10^1	1.0	1.2×10^1	1.0	1.3×10^2
3.9×10^{-20}	1.4×10^{-5}	2.4	2.2×10^{-9}	2.7×10^1	1.0	4.8	1.0	5.4×10^1
9.9×10^{-20}	5.4×10^{-5}	1.2	9.5×10^{-9}	1.3×10^1	1.0	2.5	1.0	2.6×10^1
2.0×10^{-19}	1.7×10^{-4}	6.9×10^{-1}	4.0×10^{-8}	6.3	1.0	1.4	1.0	1.3×10^1
3.9×10^{-19}	2.6×10^{-3}	1.8×10^{-1}	8.1×10^{-7}	1.4	1.0	3.6×10^{-1}	1.0	2.8
9.9×10^{-19}	1.0×10^{-1}	2.8×10^{-2}	4.2×10^{-5}	1.9×10^{-1}	1.0	6.3×10^{-2}	1.0	3.9×10^{-1}
2.0×10^{-18}	6.7×10^{-1}	1.1×10^{-2}	7.8×10^{-4}	4.5×10^{-2}	1.0	2.2×10^{-2}	1.0	9.2×10^{-2}
3.9×10^{-18}	9.7×10^{-1}	9.2×10^{-3}	1.3×10^{-2}	1.1×10^{-2}	1.0	1.2×10^{-2}	1.0	2.4×10^{-2}
9.9×10^{-18}	1.0	9.0×10^{-3}	3.3×10^{-1}	2.2×10^{-3}	1.0	9.6×10^{-3}	1.0	4.9×10^{-3}

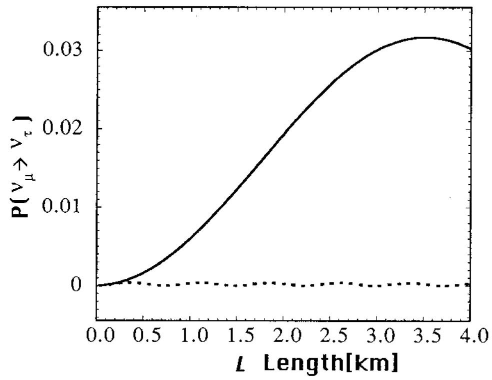

FIG. 2. ν_μ-ν_τ oscillation probability as a function of the length. The solid line is drawn by taking numerical values in case (A) of Table I with $\mu = 3.94 \times 10^{-20}$ [GeV]. The dotted line represents the probability in the vacuum.

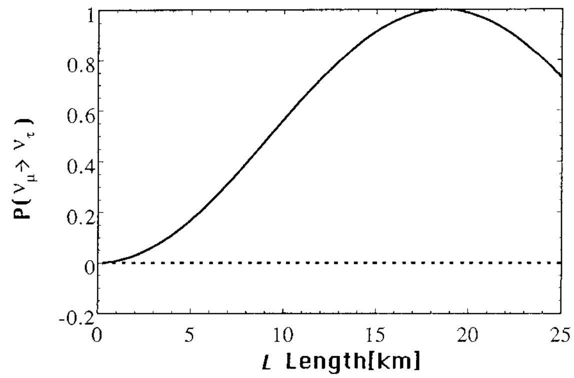

FIG. 3. Same probability as in Fig. 2. The solid line means the same as in Fig. 2, but with $\mu = 3.74 \times 10^{-20}$ [GeV] and shows a resonance behavior. The dotted line is the same as in Fig. 2.

10^{-19} GeV $\lesssim \mu \lesssim 10^{-18}$ GeV, the effect of moduli may be detectable by taking small l into account. For μ smaller than 10^{-19} GeV, $\sin^2 2\theta_M$ is less than 10^{-2}, which is so small that the conversion of ν_μ to ν_τ cannot be detected in long base line experiments. For μ larger than 10^{-18} GeV, the effect is too small to discriminate it from oscillations in the vacuum. In case (A) of Fermilab, the range of μ where the effect may be observable shifts to a range around several times 10^{-18} GeV. In case (B), $\sin^2 2\theta_M$ is unity for any μ because of the maximal mixing angle $\theta = \pi/4$. The effect may only be seen in a small l.

Next we turn to the repulsive force. The numerical results of $\sin^2 2\theta_M$ and πl^{-1} are listed in Tables III and IV. In case (A) of Table III, the moduli effect makes the values of a large. Therefore, $\sin^2 2\theta_M$ is so small that the effect is hard to observe. On the other hand, in case (B) of the CHORUS experiment, Table III shows that large ϕ's with small μ's ($\lesssim 10^{-19}$ GeV) enhance $\sin^2 2\theta_M$ to be unity. On the long base line experiments (see Table IV), $\sin^2 2\theta_M = 1$ irrespective of the moduli effect. The effect may be seen only through the oscillation length.

We illustrate the oscillation probability as a function of the distance L [km] in case (A) for the CHORUS experiment with attractive force. In Fig. 2 we show the probability vs L near the resonance and the mass of the modulus is set at $\mu = 3.94 \times 10^{-20}$ [GeV]. The dotted line denotes the probability of the oscillation in the vacuum, and the solid line corresponds to the oscillation including the moduli effect. The former magnitude, the value of which is $\sin^2 2\theta_M = 4 \times 10^{-4}$, is much smaller than the latter and changes much more frequently with L. The exact resonance occurs at $\mu = 3.74 \times 10^{-20}$ [GeV] as shown in Fig. 3. The probability including the moduli effect increases more slowly than in Fig. 2 and reaches the maximum value around $L = 18$ [km]. If such a bump is found experimentally, the mass of the modulus will be determined.

It is noted that the values of μ in the discussion above must be changed if we take smaller values of α than the present ones which are upper limits in the experimental restrictions on the fifth force. However, as seen in Eq. (19), a smaller μ corresponding to a smaller α, which makes $\alpha\mu^{-2}$ invariant, gives a similar result on neglecting the α dependence on μ.

III. SOLAR NEUTRINO OSCILLATIONS

Now we will roughly examine to what degree the moduli interaction influences solar neutrino oscillations. We assume that ν_e's are generated in the region near the distance $R_{\rm min}$ from the center of the Sun. While they propagate along the R axis to the surface, they partly change into ν_μ. The final eigenstate of mass including the Mikheyev-Smirnov-Wolfenstein (MSW) effect [19,20] and also the moduli interaction is defined as $(\tilde{\nu}_1, \tilde{\nu}_2)$ by which the flavor eigenstate (ν_e, ν_μ) is written as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_s & \sin\theta_s \\ -\sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix}, \tag{22}$$

where θ_s is the sum of the mixing angles: One is from the eigenstate (ν_1^s, ν_2^s) of mass and the MSW effect to (ν_e, ν_μ) and the other from (ν_1^s, ν_2^s) to $(\tilde{\nu}_1, \tilde{\nu}_2)$.

We use again a traceless Hamiltonian for (ν_e, ν_μ):

$$i\frac{d}{dR}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -a_s(R) & b_s(R) \\ b_s(R) & a_s(R) \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \tag{23}$$

where

$$a_s(R) = \frac{\Delta m'^2}{4E}\cos 2\theta' - \frac{\sqrt{2}}{2} G_F N_e(R) - \frac{\Delta f_s}{2}\phi(R), \tag{24}$$

$$b_s(R) = \frac{\Delta m'^2}{4E}\sin 2\theta' - f_{12}\phi(R). \tag{25}$$

Here θ' is the mixing angle from the mass eigenstate with eigenvalues m_1 and m_2 to the flavor state, $\Delta m'^2 = m_2^2 - m_1^2$, and $\Delta f_s = f_{22} - f_{11}$. In Eq. (24), the term including Fermi's coupling constant G_F and the number density of electrons, $N_e(R)$, represents MSW effect. N_e strongly depends on R [21]:

$$N_e(R)=245N_A\exp\left(-10.54\frac{R}{R_\odot}\right)\ \mathrm{cm}^{-3}, \tag{26}$$

where N_A is Avogadro's number and $R_\odot$ is the radius of the Sun.

With respect to $\phi(R)$ in Eqs. (24) and (25), assuming $\lambda=\mu^{-1}\ll R_\odot$, we can use Eq. (15). By replacing ρ_{sur} with $\rho_\odot$ we get

$$\phi(R)=\frac{4\pi G_M\rho_\odot(R)E}{\mu^2}\left(1-\frac{2}{e}\right). \tag{27}$$

Then the density of the Sun, $\rho_\odot(R)$, is replaced using $N_e(R)$ as

$$\rho_\odot(R)=\frac{m_N N_e(R)}{Y_e}, \tag{28}$$

where m_N is the mass of a nucleon and Y_e is the electron number per nucleon: $Y_e\simeq 1$. Under the adiabatic approximation, Eq. (23) leads to the probability at the distance R, similarly to Eq. (11):

$$P(\nu_e\to\nu_\mu,R)=\frac{b_s(R)^2}{a_s(R)^2+b_s(R)^2}\times\sin^2\left\{\int_{R_{\min}}^{R}[a_s(R)^2+b_s(R)^2]^{1/2}dR\right\}. \tag{29}$$

The above equation reproduces the probability of MSW when $\phi(R)=0$. We do not discuss this probability in detail, but examine the effect of moduli qualitatively.

Let us consider case (A$'$), $\Delta f_s=1$, $f_{12}=0$, and case (B$'$), $\Delta f_s=0$, $f_{12}=1$, separately. In case (A$'$), both the G_F term and ϕ term are proportional to $N_e(R)$. We have

$$\left(\frac{\Delta f_s}{2}\right)\frac{\phi(R)}{N_e(R)}=1.04\times10^{-5}\left(\frac{\alpha}{10^{-4}}\right)\left(\frac{\mu}{10^{-20}\ \mathrm{GeV}}\right)^{-2}\times\left(\frac{E}{\mathrm{MeV}}\right)\frac{1}{\mathrm{GeV}^2}. \tag{30}$$

Equation (30) is equally matched with $(\sqrt{2}/2)G_F=8.25\times10^{-6}$ [GeV^{-2}]. Equation (30) reads that when $\mu\sim10^{-20}$ GeV, $\alpha\sim10^{-4}$, $E\sim1$ MeV, the ϕ term is comparable to the G_F term in Eq. (24) everywhere in the Sun. In addition to that, for $\Delta f_s\phi>0$ the resonance $(a_s=0)$ occurs at a smaller value of $N_e(R)$ than that when only the MSW mechanism works. Resonance never occurs when $\Delta f_s\phi<0$ and $|\Delta f_s\phi|>(G_F$ term).

In case (B$'$) both terms of the right-hand side in Eq. (25) are reexpressed as

$$\frac{\Delta m'^2}{4E}\sin2\theta'=1.27\times10^{-3}\left(\frac{\Delta m'^2}{10^{-6}\ \mathrm{eV}^2}\right)\left(\frac{E}{\mathrm{MeV}}\right)^{-1}\times\sin2\theta'\frac{1}{\mathrm{km}}, \tag{31}$$

$$f_{12}\phi(R)=\phi(R)=4.90\times10^{-4}\left(\frac{\alpha}{10^{-4}}\right)\left(\frac{\mu}{10^{-20}\ \mathrm{GeV}}\right)^{-2}\times\left(\frac{E}{\mathrm{MeV}}\right)\left(\frac{\rho_\odot(R)}{\mathrm{g\ cm}^{-3}}\right)\frac{1}{\mathrm{km}}. \tag{32}$$

Solar neutrino experiments suggest $\sin^2 2\theta'\simeq3\times10^{-2}$. Then for $\rho_\odot(R)\simeq0.5$ [g cm^{-3}] and for other typical values expressed in Eqs. (31) and (32), both values are almost the same. This means that in the case of the attractive force $[f_{12}\phi(R)>0]$ the probability is very small because of $b_s\simeq0$. For $|\phi(R)|\gg\Delta m'^2/4E$, the probability is suppressed in case (A$'$), but on the other hand increases in case (B$'$).

Next we will examine the argument of sine in Eq. (29) at the solar surface only in a simplified case. We compare the argument due to moduli terms alone with that in the vacuum. The integration with respect to R is taken from $R_{\min}=0.1R_\odot$ to $R_\odot$. The vacuum part in the argument can be obtained easily from Eq. (31) as

$$0.9\frac{\Delta m'^2}{4E}R_\odot=7.94\times10^2\left(\frac{\Delta m'^2}{10^{-6}\ \mathrm{eV}^2}\right)\left(\frac{E}{\mathrm{MeV}}\right)^{-1}. \tag{33}$$

On the moduli part, using Eq. (26), we get

$$\frac{1}{2}\int_{R_{\min}}^{R_\odot}\phi(R)dR=1.38\times10^3\left(\frac{\alpha}{10^{-4}}\right)\times\left(\frac{\mu}{10^{-20}\ \mathrm{GeV}}\right)^{-2}\left(\frac{E}{\mathrm{MeV}}\right). \tag{34}$$

Comparing both values in Eqs. (33) and (34), the moduli effect matches the oscillation in the vacuum if moduli mass is around 10^{-20} GeV.

IV. CONCLUDING REMARKS

In this paper we have pointed out new signals which would support the heterotic superstring theory in neutrino oscillation experiments. The theory always accommodates moduli. There are, however, too many candidates of the vacuum to determine the masses and the interactions of moduli. Here we assume that at least one modulus has a tiny mass such as $\mu\sim10^{-22}$–10^{-14} GeV. Its interaction is expected to depend on flavors and to affect base line and solar neutrino oscillations.

The oscillation probability of ν_μ-ν_τ in planning base line experiments and of ν_e-ν_μ in the Sun are numerically estimated. It is concluded that the effect of moduli is significant when its mass is less than or equal to 10^{-19} GeV under our choice of the values of parameters. Note that when the mixing angle θ of the mass eigenstate to the flavor one is very small, the maximum value of the oscillation probability is very small in the vacuum. In CHORUS experiments, the value is 4×10^{-4} (see Table I). However, taking the moduli effect into account, a particular value of μ makes the maximum value of the probability unity as in the situation of the solar neutrino oscillation when the MSW mechanism exists.

One of parameters is the ratio α of the moduli coupling constant to the gravitational constant. We took the values of α as maximum values satisfying restrictions from experiments on the fifth force. Such values of α are not ensured. As seen in Eq. (19), however, by decreasing the value of μ for a smaller value of α, we get a similar result. There are also ambiguities about the signs of the difference of coupling constants, $\Delta f(\Delta f_s)$.

Changing the neutrino energy E and/or the length of the base line, L, the effect of moduli varies. So if neutrino oscillation experiments are scrupulously performed with various conditions as well as solar ones, one may get a clue of the moduli as to the form of interaction with matter and to its mass. In the present paper, we have given the basic equations for the oscillation probability with which one can estimate the moduli effect for a given condition. Note that though we have mentioned only moduli, the present results can be extended to other objects. Namely, any particle is a candidate which has a tiny mass and its interaction depends on flavors and if its strength is adequate. A SUSY Majoron [22] might be one of them.

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