Decay constants with Wilson fermions at β=6.0

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We present results of a high statistics study of f_π, f_K, f_D, f_{D_s}, and f_V^{-1} in the quenched approximation using Wilson fermions at β=6.0 on $32^3\times64$ lattices. We find that the various sources of systematic errors (due to setting the quark masses, renormalization constant, and lattice scale) are now larger than the statistical errors. Our best estimates, without extrapolation to the continuum limit, are f_π=134(4) MeV, f_K=159(3) MeV, f_D=229(7) MeV, f_{D_s}=260(4) MeV, and $f_V^{-1}(m_\rho)$=0.33(1), where only statistical errors have been shown. We discuss the extrapolation to the continuum limit by combining our data with those from other collaborations. [S0556-2821(96)04913-2]

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I. INTRODUCTION

Phenomenologically, f_D, f_{D_s}, f_B, and f_{B_s} are essential ingredients needed to determine the less well known elements of the Cabibbo-Kobayashi-Maskawa mixing matrix. As these heavy-light decay constants are at best very poorly measured, there has been a large effort by many lattice groups to estimate them from numerical simulations. Decay constants are among the most precise quantities that one can calculate on the lattice and a recent review has been presented by Allton [1]. In this paper we present results for f_π, f_K, f_K/f_π, f_D, f_D/f_π, f_{D_s}, f_{D_s}/f_D, and vector decay constant f_V^{-1} from simulations done on 170 $32^3\times64$ quenched lattices at β=6.0 using Wilson fermions. We emphasize that extrapolations to the continuum limit, incorporating results from other collaborations, are not very reliable as the combined data do not show an unambiguous pattern of $O(a)$ corrections.

Preliminary results from a subset of 100 lattices were presented in Ref. [2]. The raw lattice results have not changed significantly since then; however, we now present a more detailed analysis of the systematic errors. We estimate the uncertainty in the results due to extrapolation of the lattice data to physical values of the quark masses, the renormalization constants for the lattice currents, and the extraction of the lattice scale. We find that these various systematic errors are now much larger than the statistical errors. Finite size errors, if present, are smaller than the statistical errors. Our best estimates are now given in the scheme called TAD1 to evaluate the renormalization constants for the axial vector and vector currents.

The details of the lattices and the calculation of the spectrum are given in a companion paper [3]. In Sec. II we briefly summarize the lattice parameters, and in Sec. III we describe the lattice methodology and the consistency checks made to extract the decay constants using estimates from different types of fits and interpolating operators. The choice of renormalization constants for the axial vector and vector currents, Z_A and Z_V, is discussed in Sec. IV, the lattice scale in Sec. V, and the quark masses in Sec. VI. In Sec. VII we compare the data with predictions of quenched chiral perturbation theory. The extrapolation of the data to physical values of quark masses is discussed in Sec. VIII, and our best estimates at β=6.0 are summarized in Sec. IX. In Sec. X we compare our data with those from other collaborations (GF11 [4], JLQCD [5], and APE [6]; the MILC data presented in [7] are preliminary and, therefore, not included in this analysis) and extrapolate the combined data to the continuum limit. Finally, we present our conclusions in Sec. XI.

II. LATTICE PARAMETERS

The details of the 170 $32^3\times64$ gauge lattices used in this analysis are given in [3]. We refer the interested reader to it for further details of the signal in the two-point correlation functions and on the extraction of the spectrum. In this paper we concentrate on the analysis of systematic errors in decay constants associated with fixing the quark masses $\overline{m}=(m_u+m_d)/2$, m_s and m_c, the renormalization constants Z_A and Z_V, and the extrapolation to physical masses and the continuum limit.

To calculate decay constants we used the Wuppertal source quark propagators at five values of quark mass given by κ=0.135 (C), 0.153 (S), 0.155 (U_1), 0.1558 (U_2), and 0.1563 (U_3). These quarks correspond to pseudoscalar mesons of mass 2835, 983, 690, 545, and 431 MeV, respectively, where we have used $1/a$=2.33 GeV for the lattice scale. We construct two types of correlation functions, smeared-local ($\Gamma_{\rm SL}$) and smeared-smeared ($\Gamma_{\rm SS}$) functions, which are combined in different ways to extract the decay constants as discussed below. The three light quarks allow us to extrapolate the data to the physical isospin-symmetric light quark mass $\overline{m}$, while the C and S κ values are selected to be close to the physical charm and strange quark masses. The physical value of strange quark lies between S and U_1, and we use these two points to interpolate to it. In most cases we find that extrapolation to $\overline{m}$ can be done using the six combinations of light quarks U_1U_1, U_1U_2, U_1U_3, U_2U_2, U_2U_3, and U_3U_3. For brevity we will denote this combination by $\{U_iU_j\}$ and the three degenerate cases by $\{U_iU_i\}$.

III. LATTICE METHOD FOR CALCULATING f_P AND f_V

The lattice definition of the pseudoscalar decay constant f_P, using the convention that the experimental value is f_π=131 MeV, is [8]

TABLE I. Data, in lattice units, for the pseudoscalar decay constant f_P for the six different ways of combining the SL and SS correlators described in the text. The renormalization scheme used to generate this data is TAD1 as described in Table V, and the meson mass used in the analysis is taken to be the pole mass.

	f_π^a	f_π^b	f_π^c	f_π^d	f_π^e	f_π^f
$ChCh$	0.198(2)	0.198(3)	0.197(2)	0.197(2)	0.197(3)	0.197(3)
$ChSt$	0.129(2)	0.129(2)	0.129(2)	0.128(2)	0.129(2)	0.128(2)
ChU_1	0.115(2)	0.116(2)	0.116(2)	0.115(2)	0.116(2)	0.115(2)
ChU_2	0.110(2)	0.110(3)	0.111(2)	0.110(2)	0.111(2)	0.110(2)
ChU_3	0.107(3)	0.108(3)	0.109(2)	0.108(2)	0.109(2)	0.108(2)
$StSt$	0.093(1)	0.093(1)	0.093(1)	0.093(1)	0.094(2)	0.093(2)
StU_1	0.084(1)	0.084(1)	0.085(1)	0.084(1)	0.085(2)	0.084(1)
StU_2	0.081(1)	0.080(1)	0.081(1)	0.081(1)	0.081(2)	0.080(2)
StU_3	0.078(1)	0.078(1)	0.079(1)	0.078(1)	0.078(2)	0.077(2)
U_1U_1	0.076(1)	0.076(1)	0.077(1)	0.076(1)	0.076(2)	0.076(2)
U_1U_2	0.073(1)	0.072(1)	0.073(1)	0.073(1)	0.073(2)	0.072(2)
U_1U_3	0.070(1)	0.070(1)	0.071(1)	0.071(1)	0.070(2)	0.070(2)
U_2U_2	0.069(1)	0.069(1)	0.070(1)	0.069(1)	0.069(2)	0.069(2)
U_2U_3	0.067(1)	0.066(1)	0.068(1)	0.067(1)	0.066(2)	0.066(2)
U_3U_3	0.064(1)	0.064(1)	0.066(1)	0.065(1)	0.064(3)	0.064(2)

$$f_\pi = \frac{Z_A \langle 0 | A_4^{\mathrm{local}} | \pi(\vec{p}) \rangle}{E_\pi(\vec{p})}, \tag{3.1}$$

where Z_A is the axial vector current renormalization constant connecting the lattice scheme to the continuum modified minimal subtraction scheme ($\overline{\mathrm{MS}}$). In order to extract f_π we study, in addition to the two-point correlation functions Γ, two kinds of ratios of correlators:

$$R_1(t) = \frac{\Gamma_{\mathrm{SL}}(t)}{\Gamma_{\mathrm{SS}(t)}} \overset{t\to\infty}{\sim} \frac{\langle 0 | A_4^{\mathrm{local}} | \pi \rangle}{\langle 0 | A_4^{\mathrm{smeared}} | \pi \rangle},$$

$$R_2(t) = \frac{\Gamma_{\mathrm{SL}}(t)\Gamma_{\mathrm{SL}}(t)}{\Gamma_{\mathrm{SS}}(t)} \overset{t\to\infty}{\sim} \frac{|\langle 0 | A_4^{\mathrm{local}} | \pi \rangle|^2}{2M_\pi} e^{-M_\pi t}. \tag{3.2}$$

In the case of R_1 we have to extract $\langle 0 | A_4^{\mathrm{smeared}} | \pi \rangle$ separately from the Γ_{SS} correlator. For each of the two ratios R_1 and R_2 the smeared source J used to create the pion can be either π or A_4. This gives four ways of calculating f_π, which we label as f_π^a (using the ratio R_1 with $J=\pi$), f_π^b (using the ratio R_1 with $J=A_4$), f_π^c (using the ratio R_2 with $J=\pi$), and f_π^d (using the ratio R_2 with $J=A_4$). The fifth way, f_π^e, consists of combining the mass and amplitude of the two-point correlation functions $\langle A_4 P \rangle_{\mathrm{LS}}$ and $\langle PP \rangle_{\mathrm{SS}}$, and the sixth way, f_π^f, uses $\langle A_4 A_4 \rangle_{\mathrm{LS}}$ and $\langle A_4 A_4 \rangle_{\mathrm{SS}}$.

The lattice results for mesons at $\vec{p}=0$ for the different combinations of quarks are given in Table I using the renormalization scheme Z_{TAD1} defined in Table V. All errors are estimated by a single elimination jackknife procedure. The results from the six ways of combining the two correlators are mutually consistent. Since the different methods use the same correlators, the data are highly correlated; however, consistent results do indicate that fits have been made to the lowest state in each of these correlators and reassure us of the statistical quality of the data.

The results for f_π using correlators at nonzero momentum are given in Table II. The data show that in almost all cases the results are consistent within 2σ. The most noticeable differences are in the $\vec{p}=(2,0,0)$ values for lighter quarks. The signal in these channels is not very good, and it is likely that in these cases there exists contamination from excited states over the range of time slices to which fits have been made. We regard the overall consistency of the data as another successful check of the lattice methodology. Henceforth we shall restrict the analysis to the $\vec{p}=(0,0,0)$ case as it has the best signal.

The dimensionless vector decay constants are defined as

$$Z_V \langle 0 | V_\mu^{\mathrm{local}} | V \rangle = \frac{\epsilon_\mu M_V^2}{f_V}, \tag{3.3}$$

where V_μ is the vector current and $|V\rangle$ is the lowest 1^- state with mass M_V. The experimental quantities are related to f_V^{-1} by

$$f_\rho^{-1} = \frac{1}{\sqrt{2}} f_V^{-1}(M_\rho) = 0.199(5),$$

$$f_\phi^{-1} = -\tfrac{1}{3} f_V^{-1}(M_\phi) = -0.078(1),$$

$$f_{J/\psi}^{-1} = \tfrac{2}{3} f_V^{-1}(M_{J/\psi}) = 0.087(3), \tag{3.4}$$

where the values are calculated using the rate $\Gamma(V \to e^+e^-)$ given by the Particle Data Group (PDG) [9]. We extract the relevant matrix element in the same two ways as described in Eq. (3.2) for f_P. To study discretization errors we study three lattice transcriptions of the vector current (local, extended, and conserved):

$$V_\mu^L(x) = \overline{\psi(x)} \gamma_\mu \psi(x), \tag{3.5}$$

$$V_\mu^E(x) = \overline{\psi(x)} \gamma_\mu U_\mu(x) \psi(x+\hat{\mu}) + \overline{\psi(x+\hat{\mu})} \gamma_\mu U_\mu^\dagger(x) \psi(x),$$

TABLE II. Data, in lattice units, for the pseudoscalar decay constant f_P, averaged over the six different ways of combining the SL and SS correlators, measured at different momenta. The renormalization scheme is TAD1 as described in Table V, and the meson mass used in the analysis is taken to be the pole mass.

	$\vec{p}$=(0,0,0)	$\vec{p}$=(1,0,0)	$\vec{p}$=(1,1,0)	$\vec{p}$=(1,1,1)	$\vec{p}$=(2,0,0)
$ChCh$	0.198(2)	0.198(2)	0.203(2)	0.200(3)	0.201(3)
$ChSt$	0.129(2)	0.129(2)	0.131(2)	0.129(2)	0.129(2)
ChU_1	0.116(2)	0.116(2)	0.118(2)	0.115(2)	0.115(2)
ChU_2	0.111(2)	0.111(2)	0.112(2)	0.110(2)	0.110(3)
ChU_3	0.108(2)	0.108(3)	0.110(3)	0.107(3)	0.108(3)
$StSt$	0.093(1)	0.094(1)	0.095(2)	0.096(2)	0.099(2)
StU_1	0.084(1)	0.085(1)	0.086(2)	0.087(2)	0.090(2)
StU_2	0.080(1)	0.081(2)	0.082(2)	0.083(2)	0.086(2)
StU_3	0.078(1)	0.079(2)	0.080(3)	0.080(2)	0.083(2)
U_1U_1	0.076(1)	0.077(2)	0.078(2)	0.079(2)	0.083(3)
U_1U_2	0.073(1)	0.073(2)	0.074(2)	0.075(3)	0.080(3)
U_1U_3	0.070(1)	0.071(2)	0.072(2)	0.072(3)	0.078(4)
U_2U_2	0.069(1)	0.070(2)	0.071(2)	0.071(3)	0.077(4)
U_2U_3	0.067(1)	0.068(2)	0.069(3)	0.068(3)	0.075(5)
U_3U_3	0.064(1)	0.066(2)	0.067(3)	0.064(4)	0.074(5)

$$V^C_\mu(x) = \overline{\psi(x)}(\gamma_\mu - r)U_\mu(x)\psi(x+\hat{\mu}) + \overline{\psi(x+\hat{\mu})}(\gamma_\mu + r)U^\dagger_\mu(x)\psi(x),$$

where for degenerate quarks the last form is the conserved current. In Tables III and IV we show the lattice data for the 15 mass combinations as a function of the different methods or currents, and versus the renormalization schemes for the local current. Overall, the data show that the two methods in Eq. (3.2) give consistent results for all three currents. The results from the local and extended vector currents also agree, while those from the conserved current are ≈10% smaller. These points will be discussed in more detail later.

In order to extract results that can be compared with experiments, we analyze the data in terms of the five sources of systematic errors discussed below.

IV. RENORMALIZATION CONSTANTS Z_A AND Z_V

Reliable calculations of decay constants depend on our ability to calculate the renormalization constants Z_A and Z_V linking the lattice and continuum regularization schemes. In our analysis we use one-loop matching with the tadpole subtraction scheme of Lepage and Mackenzie. An outline of the scheme, which includes picking a good definition of the lattice α_s and the scale q^* at which to evaluate it is as follows. Lepage and Mackenzie show that α_v (to be defined below) is

TABLE III. Lattice data for the vector decay constant f_V^{-1} for the two different ways of combining the SL and SS correlators, and for the three different lattice vector currents described in the text. The renormalization scheme in all cases is TAD1 as described in Table V, and the meson mass used in the analysis is taken to be the pole mass.

	f^a_ρ Loc.	f^b_ρ Loc.	f^a_ρ Ext.	f^b_ρ Ext.	f^a_ρ Con.	f^b_ρ Con.
$ChCh$	0.186(02)	0.186(03)	0.184(03)	0.185(02)	0.168(03)	0.169(02)
$ChSt$	0.184(03)	0.186(03)	0.190(04)	0.191(03)	0.171(03)	0.172(03)
ChU_1	0.174(03)	0.176(03)	0.182(04)	0.182(03)	0.162(04)	0.164(03)
ChU_2	0.170(04)	0.172(04)	0.177(05)	0.178(04)	0.157(05)	0.159(03)
ChU_3	0.166(04)	0.170(05)	0.175(07)	0.176(05)	0.154(06)	0.157(04)
$StSt$	0.291(04)	0.293(05)	0.303(06)	0.308(05)	0.268(05)	0.274(04)
StU_1	0.300(04)	0.301(06)	0.312(07)	0.318(06)	0.273(06)	0.282(04)
StU_2	0.302(04)	0.299(09)	0.314(08)	0.319(06)	0.273(07)	0.282(05)
StU_3	0.303(05)	0.297(10)	0.313(11)	0.318(08)	0.271(09)	0.281(06)
U_1U_1	0.316(05)	0.314(05)	0.329(10)	0.334(05)	0.285(09)	0.296(04)
U_1U_2	0.320(05)	0.317(06)	0.334(13)	0.338(06)	0.288(11)	0.299(05)
U_1U_3	0.322(06)	0.317(06)	0.333(16)	0.334(09)	0.288(14)	0.300(05)
U_2U_2	0.325(06)	0.320(07)	0.338(17)	0.337(10)	0.292(15)	0.303(06)
U_2U_3	0.326(07)	0.319(08)	0.334(23)	0.334(13)	0.292(20)	0.303(07)
U_3U_3	0.326(07)	0.316(10)	0.326(31)	0.327(16)	0.291(27)	0.302(08)

TABLE IV. Lattice data for the vector decay constant f_V^{-1} as a function of the different renormalization schemes given in Table V. The results are for the local current, and the meson mass used in the analysis is taken to be the pole mass.

	$Z_{\mathrm{TAD}a}$	Z_{TAD1}	Z_{TAD2}	$Z_{\mathrm{TAD}\pi}$	$Z_{\mathrm{TAD}U_0}$	Z_{TFG11}	Z_{BST_π}
$ChCh$	0.184(2)	0.186(2)	0.193(2)	0.196(3)	0.173(2)	0.188(2)	0.119(2)
$ChSt$	0.183(3)	0.185(3)	0.192(3)	0.195(3)	0.172(3)	0.187(3)	0.144(2)
ChU_1	0.173(3)	0.175(3)	0.182(3)	0.185(3)	0.163(3)	0.177(3)	0.140(2)
ChU_2	0.169(4)	0.171(4)	0.177(4)	0.180(4)	0.158(3)	0.172(4)	0.138(3)
ChU_3	0.166(4)	0.168(5)	0.174(5)	0.177(5)	0.156(4)	0.169(5)	0.136(4)
$StSt$	0.289(4)	0.292(4)	0.303(4)	0.308(4)	0.271(4)	0.295(4)	0.278(4)
StU_1	0.297(5)	0.301(5)	0.312(5)	0.317(5)	0.279(4)	0.304(5)	0.294(5)
StU_2	0.298(6)	0.301(6)	0.312(6)	0.317(6)	0.280(6)	0.304(6)	0.297(6)
StU_3	0.297(7)	0.300(7)	0.311(7)	0.317(7)	0.279(6)	0.303(7)	0.298(7)
U_1U_1	0.312(5)	0.315(5)	0.327(5)	0.333(5)	0.293(4)	0.318(5)	0.316(5)
U_1U_2	0.315(5)	0.319(5)	0.331(5)	0.336(5)	0.296(5)	0.322(5)	0.322(5)
U_1U_3	0.316(6)	0.319(6)	0.331(6)	0.337(6)	0.297(5)	0.322(6)	0.325(6)
U_2U_2	0.319(6)	0.322(6)	0.335(6)	0.340(6)	0.300(6)	0.325(6)	0.329(6)
U_2U_3	0.319(7)	0.322(7)	0.335(7)	0.340(7)	0.299(6)	0.325(7)	0.331(7)
U_3U_3	0.318(8)	0.321(8)	0.334(8)	0.339(8)	0.299(7)	0.324(8)	0.332(8)

a better expansion parameter than the bare lattice coupling. To pick the value of q^* we need to know the ''mean'' momentum flow relevant to a given matrix element. Again it has been pointed out by Lepage and Mackenzie that q^*, estimated by calculating the mean momentum in the loop integrals, is dominated by tadpole diagrams which are lattice artifacts. If the tadpoles are not removed, then this scale is typically π/a. They have proposed a mean-field-improved version of the lattice theory which removes the contribution of tadpoles. The effect of this is threefold. One, it typically changes q^* to $1/a$; i.e., the matching scale becomes more infrared if the tadpole diagram is removed. Second, the renormalization of the quark field changes from $\sqrt{2\kappa}\rightarrow\sqrt{8\kappa_c}\sqrt{1-3\kappa/4\kappa_c}$. Finally, the perturbative expression for $8\kappa_c$ is combined with the coefficient of α_v in the one-loop matching relations to remove the tadpole contribution.

To get $\alpha_{\overline{\mathrm{MS}}}(q^*)$ we use the following Lepage-Mackenzie scheme. The coupling α_v is defined at scale $q=3.41/a$ to be

$$\alpha_v\left(\frac{3.41}{a}\right)[1-(1.19+0.025n_f)\alpha_v]=-\frac{3}{4\pi}\ln(\tfrac{1}{3}\ \mathrm{Tr\ Plaq}), \tag{4.1}$$

which is related to $\alpha_{\overline{\mathrm{MS}}}$ at scale $q=3.41/a$ by

$$\frac{1}{\alpha_{\overline{\mathrm{MS}}}}=\frac{1}{\alpha_v}+0.822. \tag{4.2}$$

We then run $\alpha_{\overline{\mathrm{MS}}}$ from q to q^* by integrating the two-loop β function. To translate the results from q^* to any other point, one uses the standard continuum running.

At the lowest order there are two equally good tadpole factors, U_0=plaquette$^{1/4}$ or $8\kappa_c$. To the accuracy of the mean-field improvement, one expects $8\kappa_c U_0=1$. Deviations from this relation (≈10% for the Wilson action at β=6.0) are a measure of possible residual errors. Writing the tadpole factor as $1-X\alpha_{\overline{\mathrm{MS}}}(q^*)$, we define a given Z factor to be

$$Z=1+\alpha_{\overline{\mathrm{MS}}}(q^*)\left(\frac{\gamma_0}{4\pi}\ln(q^*a)+(C-X)\right), \tag{4.3}$$

where C is the difference between the finite part of the continuum $\overline{\mathrm{MS}}$ and lattice one-loop result. Thus Z_A for the local operator in the tadpole-improved schemes is

$$\sqrt{Z_\psi^1 Z_\psi^2}Z_A^L=\sqrt{1-3\kappa_1/4\kappa_c}\sqrt{1-3\kappa_2/4\kappa_c}\times[1-\alpha_{\overline{\mathrm{MS}}}(q^*)(1.68-X)]. \tag{4.4}$$

In order to examine the dependence of the decay constants on Z and the renormalization of the quark field, we present our results for seven different commonly used schemes described in Table V. The schemes $Z_{\mathrm{TAD}a}$, Z_{TAD1}, Z_{TAD2}, $Z_{\mathrm{TAD}\pi}$, and Z_{TGF11} are all self-consistent to $O(\alpha_s)$. The scheme $Z_{\mathrm{TAD}U0}$ is *ad hoc* as we have replaced $8\kappa_c$ by U_0 in only one part. We shall quote, as our best estimates, results obtained in the Z_{TAD1} scheme and use the difference between it and $Z_{\mathrm{TAD}\pi}$ as an estimate of the systematic error due to turning q^*. Finally, an estimate of the residual perturbative errors is taken to be the difference between Z_{TAD1} and $Z_{\mathrm{TAD}U_0}$, and is given in column labeled Z_A in Table X. This, we believe, is an overestimate of the error we make by using the one-loop coefficient of α_v.

The renormalization of the local vector current Z_V^L proceeds in the same way as Z_A. In the case of both the extended and conserved currents, there is no tadpole contribution in C as it cancels between the wave-function renormalization and the vertex correction. Consequently, we use the nonperturbative value for $8\kappa_c$. The complete renormalization factors in the tadpole-improved schemes for relating the lattice results to the continuum are

$$\sqrt{Z_\psi^1 Z_\psi^2}Z_V^L=\sqrt{1-3\kappa_1/4\kappa_c}\sqrt{1-3\kappa_2/4\kappa_c}\times[1-\alpha_{\overline{\mathrm{MS}}}(q^*)(2.182-X)], \tag{4.5}$$

TABLE V. Different renormalization schemes used in the analysis. The two possible tadpole factors are $U_0=\text{plaq}^{1/4}=0.878$ and $1/8\kappa_c=0.795$. The one-loop perturbative expansions for these are $U_0=1-1.0492\alpha$ and $8\kappa_c=1+1.364\alpha$, respectively. The sixth scheme Z_{TGF11} is the one used by the GF11 Collaboration with a slightly different definition of $\alpha_{\overline{\text{MS}}}$[4].

	$Z_{\text{TAD}a}$	Z_{TAD1}	Z_{TAD2}	$Z_{\text{TAD}\pi}$	Z_{TADU_0}	Z_{TGF11}	$Z_{\text{boost}\pi}$
Z_ψ	$1-\frac{3\kappa}{4\kappa_c}$	$1-\frac{3\kappa}{4\kappa_c}$	$1-\frac{3\kappa}{4\kappa_c}$	$1-3\kappa 4\kappa c$	$1-\frac{3\kappa}{4\kappa_c}$	$1-\frac{3\kappa}{4\kappa_c}$	2κ
Tadpole	$1/8\kappa_c$	$1/8\kappa_c$	$1/8\kappa_c$	$1/8\kappa_c$	U_0	$1/8\kappa_c$	NO
q^*	2 GeV	$1/a$	$2/a$	π/a	$1/a$	$1/a$	π/a
$\alpha_s(q^*)$	0.204	0.193	0.152	0.134	0.190	0.181	0.133

$$\sqrt{Z_\psi^1 Z_\psi^2}Z_V^E=8\kappa_c\sqrt{1-3\kappa_1/4\kappa_c}\sqrt{1-3\kappa_2/4\kappa_c}$$
$$\times[1-1.038\alpha_{\overline{\text{MS}}}(q^*)],$$
$$\sqrt{Z_\psi^1 Z_\psi^2}Z_V^C=8\kappa_c\sqrt{1-3\kappa_1/4\kappa_c}\sqrt{1-3\kappa_2/4\kappa_c}.$$

We find that the results with the local current lie in between those from the extended and conserved currents, and have the best statistical signal. We therefore quote results from the local current as our best estimate and use the difference between them as an estimate of the systematic error.

V. LATTICE SCALE *a*

To convert lattice results to physical units, we use the lattice scale extracted by setting M_ρ to its physical value. This gives $1/a=2.330(41)$ GeV [3]. The variation of $1/a$ between the jackknife samples is folded into our error analysis; however, different ways of setting the scale are not. For example, using M_N to set the scale gives $1/a=2.062(56)$ GeV [3], while nonrelativistic QCD (NRQCD) simulations of the charmonium and Υ spectrum give $1/a=2.4(1)$ GeV [10]. As we show later, the scale determined from f_π is 2265(57) MeV. Thus estimates based on mesonic quantities such as M_ρ, heavy-heavy spectrum, and f_π all give consistent results. We take $1/a(M_\rho)=2.330(41)$ GeV and use the spread $\sim$70 MeV $\sim$3% as our best guess of the size of scaling violations relevant to the analysis of the decay constants. To reduce this error requires using an improved gauge and fermion action, which is beyond the scope of this work.

VI. SETTING THE QUARK MASSES

In order to extrapolate the lattice data to physical values of the quark mass, we have to fix $\overline{m}$, m_s, and m_c. The chiral limit is determined by linearly extrapolating the data for M_π^2 to zero using the six cases $\{U_iU_j\}$. Our best estimate is

$$\kappa_c=0.157\,131(9), \tag{6.1}$$

which is used in the calculation of Z_ψ.

To fix the value of κ_l corresponding to $\overline{m}$, we extrapolate the ratio M_π^2/M_ρ^2 to its physical value 0.031 82. The result is

$$\kappa_l=0.157\,046(9). \tag{6.2}$$

Thus our data are able to resolve between the chiral limit and $\overline{m}$. In [3] we had shown that a nonperturbative estimate of quark mass m_{np} calculated using the Ward identity, and $(1/2\kappa-1/2\kappa_c)$ are linearly related for light quarks; so either definition of the quark mass can be used for the extrapolation. We have chosen to use m_{np} in this paper.

The determination of the strange quark mass has significant systematic errors as shown below. We determine κ_s in three ways as described in [3]. We extrapolate M_K^2/M_π^2, M_{K^*}/M_ρ, and M_ϕ/M_ρ to $\overline{m}$ and then interpolate in the strange quark to match their physical value. In Table VI we give κ_s, the nonperturbative estimate $m_{\text{np}}a=m_sa$, and $m_s=Z_m(1/2\kappa-1/2\kappa_c)$ evaluated at 2 GeV in the $\overline{\text{MS}}$ scheme using the TAD1 matching between lattice and continuum. The data show a $\sim$20% difference between various estimates of m_s which cannot be explained away as due to statistical errors. Using M_K^2/M_π^2 to fix m_s implies that $m_s\equiv 25\overline{m}$ as we use the lowest order chiral expansion to fit the M_π^2 data. On the other hand, M_ϕ/M_ρ gives $m_s/\overline{m}\approx 30$. This estimate is not constrained by the chiral expansion and is in surprisingly good agreement with the next-to-leading chiral result [11]. In this paper we shall quote results for both $m_s(M_K)$ and $m_s(M_\phi)$, and take the values with $m_s(M_\phi)$ as our best estimates. The difference in results between using $m_s(M_K)$ and $m_s(M_\phi)$ will be taken as an estimate of the systematic error due to the uncertainty in setting m_s.

To determine the value of κ corresponding to m_c, we match M_D, M_{D^*}, and M_{D_s} as these are obtained from the same two-point correlation functions as used to determine the decay constants. Unfortunately, as shown in [3], the estimate of charmonium and D meson masses measured from the rate of exponential falloff of the two-point function (pole mass or M_1) and those from the kinetic mass defined as $M_2\equiv(\partial^2E/\partial p^2|_{p=0})^{-1}$ are significantly different. We find that the data for the heavy-heavy and heavy-light $q\overline{q}$ combinations are consistent with the nearest-neighbor symmetric-difference relativistic dispersion relation $\sinh^2(E/2)-\sin^2(p/2)=\sinh^2(M/2)$, in which case M_2, as de-

TABLE VI. Estimates of m_s using different combinations of hadron masses. We give the κ values, the quark mass determined by the Ward identity, and $m_s=Z_m(1/2\kappa-1/2\kappa_c)$ evaluated at 2 GeV in the $\overline{\text{MS}}$ scheme, and using the TAD1 tadpole subtraction procedure.

	κ_s	$m_{s,\text{np}}a$	m_s (2 GeV) (MeV)
M_K^2/M_ρ^2	0.155 03(7)	0.0372(14)	129(2)
M_{K^*}/M_ρ	0.154 79(19)	0.0421(36)	145(9)
M_ϕ/M_ρ	0.154 64(17)	0.0445(32)	154(8)

TABLE VII. Comparison of lattice estimates of D meson masses with the experimental data. We show results for M_1 and M_2 and for the two different ways of setting m_s described in the text.

	M_1	M_2	Expt.
M_D	1805(31)	1990(34)	1869
M_{D^*}	1876(32)	2085(35)	2008
$M_{D_s}[m_s(M_K)]$	1896(30)	2112(32)	1969
$M_{D_s}[m_s(M_\phi)]$	1914(26)	2137(27)	1969
$M_{D_s^*}[m_s(M_K)]$	1961(31)	2201(34)	2110?
$M_{D_s^*}[m_s(M_\phi)]$	1978(27)	2224(29)	2110?

fined above, is given by sinh M. The results for M_1 and M_2 for the D states are given in Table VII for $\kappa=0.135$ (we have simulated only one heavy quark mass). The data show that the experimental results lie between M_1 and M_2 for each of the three states, and the difference between M_1 and M_2 is large and statistically significant. The size of this systematic error and the uncertainty in setting the scale $1/a$ make it difficult to fix κ_{charm}. We simply assume that $\kappa=0.135$ corresponds to m_c and quote final results using M_2. As an estimate of systematic errors associated with not tuning m_c, we take the difference in results between using M_1 and M_2 since we do not have access to the rate of variation of decay constants in the vicinity of m_c.

VII. QUENCHED APPROXIMATION

In the last couple of years it has been pointed out by Sharpe and co-worker [12] and by Bernard and Golterman [13] that there exist extra chiral logarithms due to the η', which is also a Goldstone boson in the quenched approximation. These make the chiral limit of quenched quantities sick. To analyze the effects of quenching, Bernard and Golterman [13] have constructed the ratio

$$R \equiv \frac{f_{12}^2}{f_{11'} f_{22'}} \tag{7.1}$$

applicable in a four-flavor theory where $m_1 = m_{1'}$ and $m_2 = m_{2'}$. The advantage of this ratio in comparing full and quenched theories is that it is free of ambiguities due to the cutoff Λ in loop integrals and $O(p^4)$ terms in the chiral Lagrangian. The chiral expression for R in the quenched theory is

$$R^Q = 1 + \delta\left[\frac{m_{12}^2}{(m_{11'}^2 - m_{22'}^2)} \ln\frac{m_{11'}^2}{m_{22'}^2} - 1\right] + O((m_1 - m_2)^2), \tag{7.2}$$

where $\delta \equiv m_0^2/24\pi^2 f_\pi^2$ parametrizes the effects of the η'. The analogous expression in full QCD is

$$R^F = 1 + \frac{1}{32\pi^2 f^2}\left[m_{11'}^2 \ln\frac{m_{11'}^2}{m_{12}^2} + m_{22'}^2 \ln\frac{m_{22'}^2}{m_{12}^2}\right] + O((m_1 - m_2)^2). \tag{7.3}$$

The leading analytic corrections in both cases are $O((m_1 - m_2)^2)$ [14] and were not included in the analysis presented in Ref. [15]. The data, shown in Figs. 1 and 2, indicate the need for including them in the fits. [X_{quenched} is the coefficient of δ in Eq. (7.2), and X_{full} is the complete chiral logarithm term in Eq. (7.3).] The fit to the quenched expression, Fig. 1, gives $\delta=0.14(4)$. The fit to full QCD expression has smaller χ^2 if we leave the intercept as a free parameter. In that case the fit gives 1.69(45) and not unity as required by Eq. (7.3). Thus the effect of chiral logarithms is small, barely discernible from the statistical errors, and partly due to normal higher order terms in the chiral expansion. We shall therefore neglect the effects of quenched chiral logarithms in this study and only discuss deviations of f_P from a behavior linear in m_q at the appropriate places.

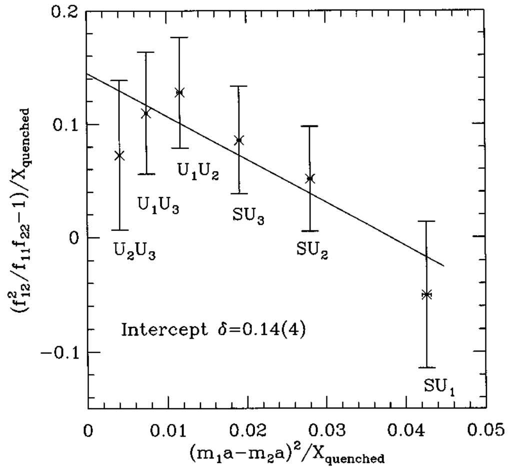

FIG. 1. Bernard-Golterman ratio R versus $(m_1 a - m_2 a)^2$. X_{quenched} is the coefficient of δ defined in Eq. (7.2). The intercept gives $\delta=0.14(4)$.

The second consequence of using the quenched approximation is that the coefficients in the chiral expansion are different in the quenched and full theories. This difference can be evaluated by comparing quenched and full QCD data, which is beyond the scope of this work. Thus we cannot provide any realistic estimates of errors due to using the quenched approximation.

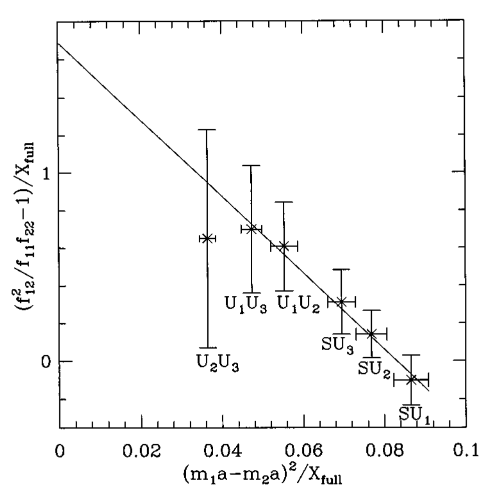

FIG. 2. Bernard-Golterman ratio R versus $(m_1 a - m_2 a)^2$. X_{full} is the chiral correction defined in Eq. (7.3). The linear fit gives an intercept of 1.69(45) instead of unity as indicated by Eq. (7.3).

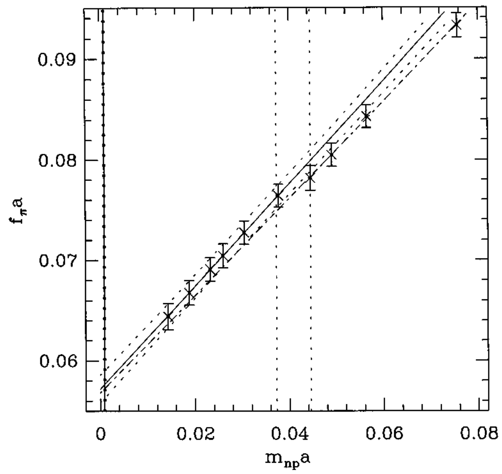

FIG. 3. Plot of data for $f_\pi a$ versus $m_{\rm np}a$. The linear fit, shown as a solid line, is to the six $\{U_iU_j\}$ points. The error estimate on the fit is shown by the dotted lines. The dot-dashed line is a linear fit to the four SS and $\{SU_i\}$ points. The vertical line at $m_{\rm np}a \approx 0$ represents $\overline{m}$, and the band at $m_{\rm np}a \approx 0.04$ denotes the range of m_s.

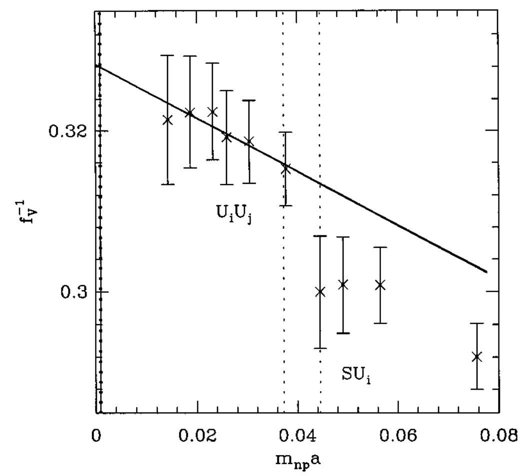

FIG. 5. Plot of data for f_V^{-1} versus $m_{\rm np}a$. The linear fit is almost identical for the six $\{U_iU_j\}$ or three $\{U_iU_i\}$ points. The SS and $\{SU_i\}$ points are also shown for comparison.

VIII. EXTRAPOLATION IN QUARK MASSES

In Fig. 3 we show the pseudoscalar data for $\{U_iU_j\}$ and $\{SS,SU_i\}$ combinations along with two different linear fits, one to the six $\{U_iU_j\}$ data points $[f_Pa=0.0572(14)+0.51(2)ma]$ and the other to the four SS and $\{SU_i\}$ points $[f_Pa=0.0568(14)+0.48(1)ma]$. Here m is the average mass of the quark and antiquark. The data show that even though the slopes for the two fits are different, the values after extrapolation are virtually indistinguishable. The size of the break between the $\{SS,SU_i\}$ and $\{U_iU_j\}$ cases at m_s is right at the 1σ level, and no such break is visible between the U_1U_i and the U_2U_2 cases. We thus extrapolate to f_π using $\{U_iU_j\}$ points and assume that the overall jackknife error adequately includes the uncertainty due to extrapolation.

In Fig. 4 we show the extrapolation for heavy-light mesons for three cases of ''heavy'' (C,S,U_1) quarks. The linear fits in the light quark mass,

$$f_Pa=0.103(3)+0.33(5)m_{\rm np}a \quad (CU_i),$$

$$f_Pa=0.074(1)+0.26(1)m_{\rm np}a \quad (SU_i),$$

$$f_Pa=0.067(1)+0.25(1)m_{\rm np}a \quad (U_1U_i), \qquad (8.1)$$

fit the data extremely well in each of the three cases. Deviations from linearity are apparent if the ''light'' quark mass is taken to be S as shown by the fourth point at $m_{\rm np}a=0.076$. These can be taken into account by including corrections, i.e., chiral logarithms and/or a quadratic term. A fit including a quadratic term fits all four points exceedingly well; however, the extrapolated value changes by $<0.2\sigma$ in all three cases. Also, the change in curvature between U_1U_i and CU_i is within the error estimates. Considering that the form of the correction term is not unique and that the linear and quadratic fits give essentially the same result, we consider it sufficient to use a linear fit to the three U_i points to extrapolate the heavy-light decay constants to $\overline{m}$.

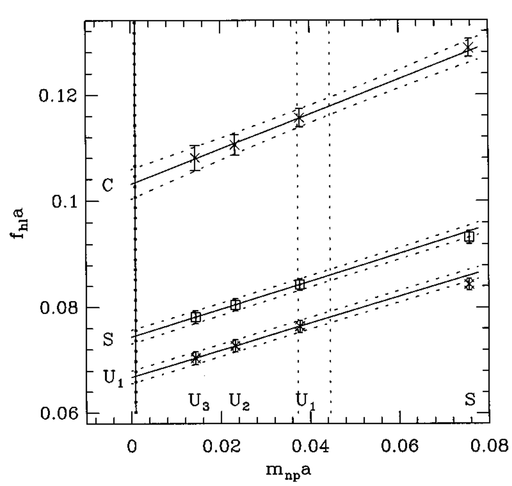

FIG. 4. Extrapolation of heavy-light pseudoscalar decay constants for three cases of ''heavy'' C, S, U_1 quarks. The linear fits are to the three ''light'' U_i quarks, and the fourth point (light quark is S) is included to show the breakdown of the linear approximation.

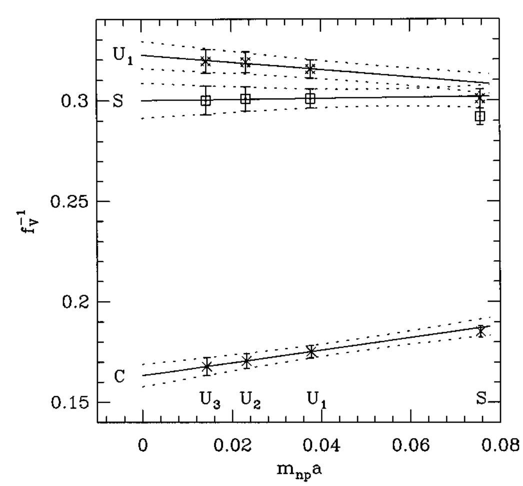

FIG. 6. Extrapolation of heavy-light vector decay constants for three cases of ''heavy'' C, S, U_1 quarks. The linear fits are to the three ''light'' U_i quarks, and the fourth point (light quark is S) is included to show the breakdown of the linear approximation.

TABLE VIII. Summary of results for pseudoscalar decay constants in lattice units. The variations with m_s (set by M_K or M_ϕ) and the heavy-light meson mass (M_1 or M_2) are shown explicitly. The jackknife error estimates include statistical and a part of systematic errors due to extrapolation in quark masses.

		$Z_{\mathrm{TAD}a}$	Z_{TAD1}	Z_{TAD2}	$Z_{\mathrm{TAD}\pi}$	$Z_{\mathrm{TAD}U_0}$	Z_{TGF11}	$Z_{\mathrm{boost}\pi}$
(M_1)	f_π	0.057(01)	0.058(01)	0.058(01)	0.059(01)	0.054(01)	0.058(01)	0.060(01)
(M_1,M_K)	f_K	0.067(01)	0.067(01)	0.068(01)	0.068(01)	0.063(01)	0.067(01)	0.068(01)
(M_1,M_ϕ)	f_K	0.068(01)	0.068(01)	0.069(01)	0.070(01)	0.064(01)	0.069(01)	0.069(01)
(M_1,M_K)	f_K/f_π	1.161(11)	1.161(11)	1.161(11)	1.161(11)	1.161(11)	1.161(11)	1.127(10)
(M_1,M_ϕ)	f_K/f_π	1.186(16)	1.186(16)	1.186(16)	1.186(16)	1.186(16)	1.186(16)	1.145(14)
(M_1)	f_D	0.103(03)	0.103(03)	0.105(03)	0.105(03)	0.097(03)	0.104(03)	0.083(02)
(M_2)	f_D	0.098(02)	0.098(03)	0.100(03)	0.100(03)	0.092(02)	0.099(03)	0.079(02)
(M_1)	f_D/f_π	1.793(49)	1.793(49)	1.793(49)	1.793(49)	1.793(49)	1.793(49)	1.388(38)
(M_2)	f_D/f_π	1.705(45)	1.705(45)	1.705(45)	1.705(45)	1.705(45)	1.705(45)	1.320(35)
(M_1,M_K)	f_{D_s}	0.115(02)	0.115(02)	0.117(02)	0.118(02)	0.108(02)	0.116(02)	0.091(01)
(M_2,M_K)	f_{D_s}	0.109(02)	0.109(02)	0.111(02)	0.111(02)	0.102(02)	0.110(02)	0.086(01)
(M_1,M_ϕ)	f_{D_s}	0.118(02)	0.118(02)	0.120(02)	0.120(02)	0.110(02)	0.118(02)	0.092(01)
(M_2,M_ϕ)	f_{D_s}	0.111(02)	0.112(02)	0.113(02)	0.114(02)	0.104(02)	0.112(02)	0.087(01)
(M_1,M_K)	f_{D_s}/f_D	1.117(19)	1.117(19)	1.117(19)	1.117(19)	1.117(19)	1.117(19)	1.088(18)
(M_2,M_K)	f_{D_s}/f_D	1.112(18)	1.112(18)	1.112(18)	1.112(18)	1.112(18)	1.112(18)	1.083(17)
(M_1,M_ϕ)	f_{D_s}/f_D	1.141(22)	1.141(22)	1.141(22)	1.141(22)	1.141(22)	1.141(22)	1.106(20)
(M_2,M_ϕ)	f_{D_s}/f_D	1.135(21)	1.135(21)	1.135(21)	1.135(21)	1.135(21)	1.135(21)	1.100(19)

The difference in slope between fits to $\{U_iU_j\}$ and $\{SU_i\}$ points does effect the value of f_K. We therefore calculate it in two ways; the central value is taken by extrapolating the $\{SU_i\}$ and $\{U_1U_i\}$ data in the light quark to $\overline{m}$ and then interpolating in the ''heavy'' to m_s. In the second way we use the slope determined from $\{U_iU_j\}$ points and extrapolate to $\overline{m}+m_s$. The two give consistent results, and we use the difference as an estimate of the systematic error.

The analogous plots for f_V^{-1} are shown in Figs. 5 and 6. To extract f_ρ^{-1} we make linear fits to the six $\{U_iU_j\}$ and the three $\{U_iU_i\}$ points. As shown in Fig. 5, these two fits are almost identical $[f_Va=0.328(10)+0.33(23)m_{\mathrm{np}}a]$ and neither of them fits the data very well. The $\{SU_i\}$ points show a very significant break from the $\{U_iU_j\}$ points, and so to extract f_K*,f_D* we use the fits shown in Fig. 6. As in the case of f_P, a linear fit to the three cases (CU_i,SU_i,U_1U_i) works well. The fit parameters are

$$f_Va=0.163(6)+0.31(10)m_{\mathrm{np}}a \quad (CU_i),$$

$$f_Va=0.300(9)+0.026(14)m_{\mathrm{n}}a \quad (SU_i),$$

$$f_Va=0.322(7)-0.18(10)m_{\mathrm{np}}a \quad (U_1U_i). \qquad (8.2)$$

Note that the slope changes sign between the SU_i and U_1U_i cases. Since the points at $m_{\mathrm{np}}a=0.076$ (S) show deviations from the linear fits, we do not include this point in our analysis.

IX. RESULTS AT β=6.0

The results for the pseudoscalar decay constants, in lattice units, are given in Table VIII for each of the seven renormalization schemes. The table also shows the variation with respect to the two choices of m_s and whether one uses M_1 or M_2 for the heavy-light meson mass. For our best estimates we use Z_{TAD1} and convert this data to MeV using $1/a(M_\rho)$. The results are summarized in Table IX where we again display variation with respect to m_s and the heavy-light meson mass.

Our final results are shown in Table X along with the estimates of the various systematic errors discussed above. Thus, at β=6.0, the value of f_π come out about 3% larger. Using f_π data to set the lattice scale gives $1/a(f_\pi)=2265(57)$ MeV, whereas $1/a(M_\rho)=2330(41)$ MeV [3]. Even ignoring the various systematic errors, the two estimates differ by roughly 1σ.

The ratio $f_K/f_\pi=1.186(16)$ is about 2σ smaller than the experimental value 1.223 if one ignores all systematic errors.

TABLE IX. Results for decay constants in the Z_{TAD1} scheme as a function of m_s and heavy-light meson masses. The data have been converted to MeV using M_ρ to set the scale. Only the jackknife error estimates are given.

	M_1 and $m_s(m_K)$	M_1 and $m_s(m_\phi)$	M_2 and $m_s(m_K)$	M_2 and $m_s(m_\phi)$
f_π	134.4(41)			
f_K	156.1(37)	159.4(33)		
f_D	241.0(75)		229.2(70)	
f_{D_s}	269.1(54)	275.0(46)	254.8(51)	260.1(44)

TABLE X. Our final results using TAD1 scheme along with estimates of statistical and various systematic errors as described in the text. All dimensionful numbers are given in MeV with the scale set by M_ρ. For the systematic errors due to m_s, m_c and q^*, we also give the sign of the effect. We cannot estimate the uncertainty due to using the quenched approximation. Also, we do not have useful estimates for entries marked with a question mark (?).

	Best estimate	Statistical and extrapolation	Tuning m_s	Tuning m_c	q^*	Tuning a (3%)	Z_A
f_π	134	4			+2	4	10
f_K	159	3	−3		+3	5	10
f_D	229	7		+12	+4	7	14
f_{D_s}	260	4	−5	+15	+4	8	20
f_K/f_π	1.19	0.02	−0.025				0
f_D/f_π	1.71	0.05		+0.09			?
f_{D_s}/f_D	1.135	0.021	−0.023	+0.006			0

(The systematic error in fixing, m_s would tend to lower our estimate, i.e., further increasing the difference.) An under estimate of this ratio in the quenched approximation is consistent with predictions of quenched chiral perturbation theory (CPT) [12,13].

The major uncertainty in the results for the heavy-light cases f_D and f_{D_s} comes from the uncertainty in Z_A and in setting the charm mass. These corrections can be significant, and we need to reduce the various sources of systematic errors in order to extract reliable continuum estimates.

In Tables XI and XII we give the values for the vector decay constant f_V^{-1}, extrapolated to the masses of a number of vector states even though some of them do not decay electromagnetically to l^+l^-. These tables also give the variation with respect to setting m_s, the heavy-light meson mass (M_1 or M_2), q^*, Z_A, and the dependence on the lattice current. The criterion that the three types of currents should give consistent results justifies using the Lepage-Mackenzie procedure for V_i^C also, as pointed out by Bernard in [16]. Using the $\sqrt{2\kappa}$ normalization for V_i^C [i.e., the same normalization as V_4^C (p_μ=0), which is constrained by the value of the conserved charge] gives significantly smaller values for cases with C quarks.

X. INFINITE VOLUME CONTINUUM RESULTS

In the companion paper analyzing the meson and baryon spectrum [3], we show that there are no noticeable differences between results obtained on 24^3 (earlier calculations) and our 32^3 lattices. Thus we do not apply any finite size corrections to our data. To extract results valid in the continuum limit, we combine our data with those from the GF11 (β=5.7,5.93,6.17) [4], JLQCD (β=6.1,6.3) [5], and APE (β=6.0,6.2) [6] Collaborations. We have attempted to correct for as many systematic differences; however, some like differences in lattice volumes, range of quark masses analyzed, and fitting techniques remain.

Bernard, Labrenz, and Soni have previously carried out a systematic study of heavy-light decay constants with Wilson fermions [17]. They obtained f_D=208(9)±35±12 MeV and f_{D_s}=230(7)±30±18 MeV after extrapolation to a=0. Overall, within errors, their raw lattice data at common parameter values agree with the numbers presented here. However, since their best β=6.0 data is based on only eight $24^3\times40$ lattices and the various sources of systematic errors are handled differently, we do not discuss it any further.

We first compare the data for f_π and f_K from the different collaborations as shown in Figs. 7 and 8. The various calculations have similar statistics (within a factor of 2) and the two largest physical volumes used are by GF11 (24^3 at β=5.7) and LANL (32^3 at β=6.0) Collaborations. To facilitate comparison we make three changes: (a) We switch to the convention in which f_π=93 MeV, (b) use the Z_{TGF11} scheme, and (c) set m_s using M_K. A noticeable difference in

TABLE XI. Results for f_V^{-1} extrapolated to the masses of a number of vector states specified within square brackets as a function of the renormalization schemes, m_s (M_K or M_ϕ), and meson mass (M_1 or M_2).

		$Z_{\mathrm{TAD}a}$	Z_{TAD1}	Z_{TAD2}	$Z_{\mathrm{TAD}\pi}$	$Z_{\mathrm{TAD}8k}$	Z_{TGF11}	$Z_{\mathrm{boost}\pi}$
(M_1)	[M_ρ]	0.324(10)	0.328(10)	0.340(11)	0.346(11)	0.305(09)	0.331(10)	0.345(11)
(M_1,M_K)	[M_{K^*}]	0.319(06)	0.322(06)	0.335(07)	0.340(07)	0.300(06)	0.325(06)	0.331(06)
(M_1,M_ϕ)	[M_{K^*}]	0.315(06)	0.318(06)	0.330(06)	0.336(06)	0.296(06)	0.321(06)	0.325(06)
(M_1,M_K)	[M_ϕ]	0.312(04)	0.316(04)	0.328(05)	0.333(05)	0.293(04)	0.318(04)	0.316(04)
(M_1,M_ϕ)	[M_ϕ]	0.308(04)	0.311(04)	0.323(05)	0.328(05)	0.289(04)	0.314(04)	0.309(05)
(M_1)	[M_{D^*}]	0.162(05)	0.164(05)	0.170(06)	0.173(06)	0.152(05)	0.165(06)	0.134(04)
(M_2)	[M_{D^*}]	0.137(04)	0.139(04)	0.144(04)	0.147(04)	0.129(04)	0.140(04)	0.114(03)
(M_1,M_K)	[$M_{D_s^*}$]	0.173(03)	0.175(03)	0.182(03)	0.185(03)	0.163(03)	0.177(03)	0.140(03)
(M_2,M_K)	[$M_{D_s^*}$]	0.145(02)	0.147(02)	0.152(03)	0.155(03)	0.136(02)	0.148(02)	0.117(02)
(M_1,M_ϕ)	[$M_{D_s^*}$]	0.175(03)	0.177(03)	0.184(03)	0.187(03)	0.164(03)	0.179(03)	0.141(03)
(M_2,M_ϕ)	[$M_{D_s^*}$]	0.146(02)	0.148(02)	0.154(03)	0.156(03)	0.138(02)	0.149(02)	0.118(02)

TABLE XII. Results for f_V^{-1} as a function of the different discretizations of the vector current. We also show the dependence on m_s (M_K or M_ϕ) and meson mass (M_1 or M_2).

		Local	Extended	Conserved
(M_1)	$[M_\rho]$	0.328(10)	0.335(26)	0.304(18)
(M_1,M_K)	$[M_{K*}]$	0.322(06)	0.338(13)	0.297(10)
(M_1,M_ϕ)	$[M_{K*}]$	0.318(06)	0.334(12)	0.293(09)
(M_1,M_K)	$[M_\phi]$	0.316(04)	0.332(06)	0.291(06)
(M_1,M_ϕ)	$[M_\phi]$	0.311(04)	0.327(06)	0.287(05)
(M_1)	$[M_{D*}]$	0.164(05)	0.171(06)	0.151(05)
(M_2)	$[M_{D*}]$	0.139(04)	0.146(05)	0.128(04)
(M_1,M_K)	$[M_{D_s^*}]$	0.175(03)	0.182(03)	0.163(03)
(M_2,M_K)	$[M_{D_s^*}]$	0.147(02)	0.152(03)	0.137(03)
(M_1,M_ϕ)	$[M_{D_s^*}]$	0.177(03)	0.183(03)	0.164(03)
(M_2,M_ϕ)	$[M_{D_s^*}]$	0.148(02)	0.153(03)	0.138(03)

the data shown in Figs. 7 and 8 is that the APE points at β=6.0 lie about 1σ higher than LANL's and the value of $m_\rho a$ is also larger. We believe that the difference is partly a result of extrapolation from heavier quarks (the APE Collaboration use a linear fit to extrapolate data at κ=0.153,0.1540.155 to the chiral limit). We find that both f_P and M_V [3] data show negative curvature, and a linear extrapolation using only SS and U_1U_1 points increases the LANL estimates, accounting for the full difference in M_ρ and a part of that in f_π. The more important feature of the data, however, is that neither plot shows a clear a dependence. Nevertheless, a linear fit to all data, assuming that lattice spacing errors are $O(a)$, gives

$$\frac{f_\pi}{M_\rho}=0.111(5) \quad (\text{expt. } 0.120),$$

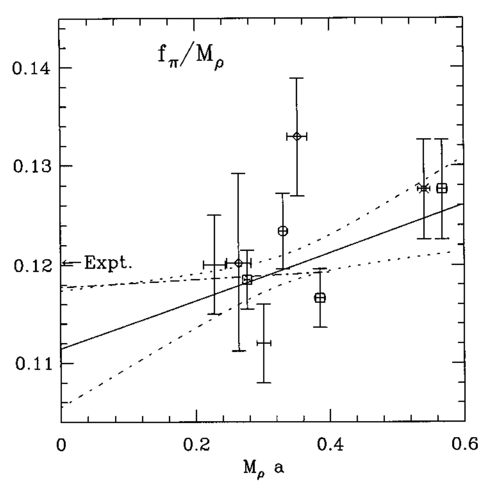

FIG. 7. Linear extrapolation to the continuum limit of the ratios f_π/M_ρ. Our data are shown with the symbol octagon, squares and fancy squares are the points from the GF11 Collaboration [4], diamonds are APE Collaboration points [6], and the plus symbol labels JLQCD [5] data. The two GF11 points at $M_\rho a \approx 0.56$ represent 16^3 (squares) and 24^3 (fancy squares) lattices at β=5.7. The solid line is a linear fit to all the data with error estimates shown by the dotted lines. The dot-dashed line is a fit to the data excluding β=5.7 points.

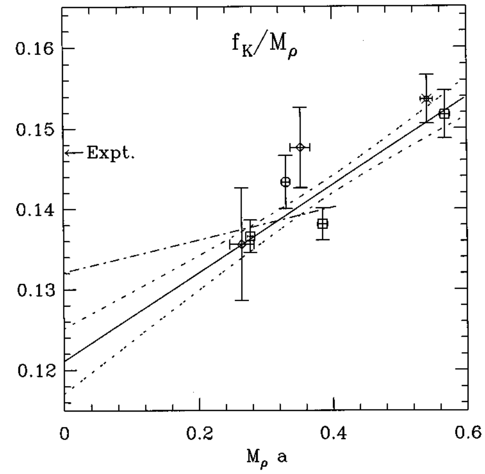

FIG. 8. Linear extrapolation to the continuum limit of the ratios f_K/M_ρ. Our data are shown with the symbol octagon, the squares and fancy squares are the points from the GF11 Collaboration [4], and the diamond labels APE [6] data. The solid line is a linear fit to all the data with error estimates shown by the dotted lines. The dot-dashed line is a fit to the data excluding β=5.7 points.

$$\frac{f_K}{M_\rho}=0.121(4) \quad (\text{expt.}0.147), \tag{10.1}$$

with $\chi^2/N_{\rm DF}$=1.6 and 1.7, respectively. The change from the GF11 results is marginal as the fit is still strongly influenced by the point at β=5.7, which may lie outside the domain of validity of the linear extrapolation. A linear extrapolation excluding the β=5.7 data gives

$$\frac{f_\pi}{M_\rho}=0.118(10),$$

$$\frac{f_K}{M_\rho}=0.132(8), \tag{10.2}$$

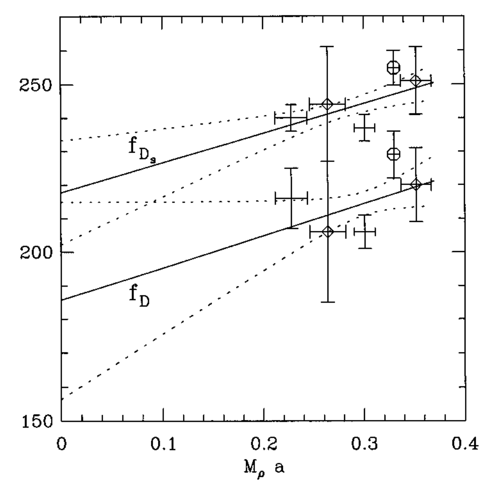

FIG. 9. Extrapolation to the continuum limit of f_D and f_{D_s} expressed in MeV. Error estimates on the linear fits are shown by the dotted lines. Our data are shown with the symbol octagon, the plus points are from the JLQCD Collaboration [5], and the diamonds label the APE Collaboration [6] data.

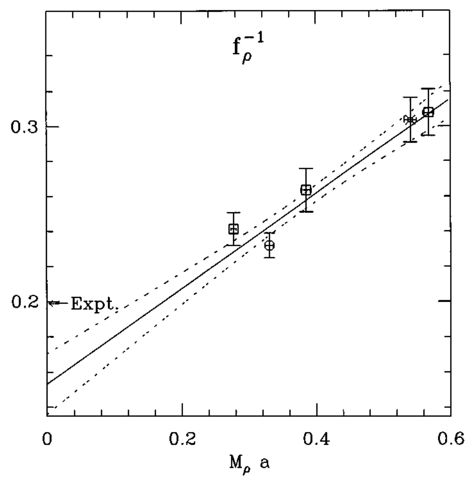

FIG. 10. Linear extrapolation to the continuum limit of f_ρ^{-1}. Our data are shown with the symbol octagon, and the rest of the points are from the GF11 Collaboration [4].

with $\chi^2/N_{\rm DF}$=2.1 and 1.9, respectively. Using $m_s(M_\phi)$ would increase f_K by $\sim$2%. Given the large difference in the extrapolated value depending on whether the data at β=5.7 is included or not makes it clear that more data are required to make a reliable extrapolation to the continuum limit.

The f_D and f_{D_s} data are combined with results from JLQCD [5] and APE [6] Collaborations as shown in Fig. 9. The results are in the TAD1 scheme, and for comparison we use $m_s(M_K)$. Also, from here on we switch back to the convention in which f_π=131 MeV. The APE Collaboration use M_1 for the meson mass. For consistency we have shifted their data to M_2 using our estimates given in Table X. A linear extrapolation to $a=0$ then gives

$$f_D = 186(29) \ \ \text{MeV},$$

$$f_{D_s} = 218(15) \ \ \text{MeV}, \tag{10.3}$$

with $\chi^2/N_{\rm DF}$=2.2 and 2.0, respectively. Using $m_s(M_\phi)$ would increase f_{D_s} to 224(16) MeV. The quality of the fits is, however, not very satisfactory. We feel that in order to improve the reliability of estimates in Eq. (10.3) one needs to reduce the various systematic errors that have not been included in the $a\rightarrow 0$ extrapolations presented above.

The status of experimental measurements of f_D and f_{D_s} is summarized in the recent review by Richman and Burchat (see Table VI in Ref. [18]). The statistical errors are large and there is no consensus yet. Most experiments give numbers in the range 225–350 MeV for f_{D_s}, while for f_D there is only an upper limit, $<$290 MeV. Thus it is important to extract reliable lattice values for the heavy-light decay constants.

Finally, a linear fit to f_ρ^{-1} data is shown in Fig. 10. The extrapolated value 0.153(17) with $\chi^2/N_{\rm DF}$=1.8 is smaller than the experimental value 0.199(5) and also smaller than that from a fit to just the GF11 data, which gives 0.18(2) [4].

XI. CONCLUSIONS

We have presented a detailed analysis of the decay constants involving light-light and heavy-light (up to charm) quarks. We find that the various sources of systematic errors (due to setting the quark masses, renormalization constant, and lattice scale) are now larger than the statistical errors. Work is under progress to address these issues. Our best estimates for the pseudoscalar decay constants and the various sources of error, without extrapolation to the continuum limit, are given in Table X.

We would like to stress that by including all of the present high statistics large lattice data, the extrapolation to the continuum limit is, in all cases, not very reliable. For the Wilson action the corrections are $O(a)$, and one expects that a linear extrapolation should suffice starting at some β. We find that in all cases the combined world data do not show an unambiguous linear behavior in a. Since different groups analyze the data in different ways, there is no clean way of including the systematic errors in individual points in the fits. We therefore cannot resolve whether the poor quality of the linear fits is due to the various systematic and statistical errors or due to the presence of higher order corrections. As a result, our overall conclusion is that precise data at a few more values of β are required in order to extract reliable results in the $a\rightarrow 0$ limit.

We have made linear fits to the data with and without including the point at the strongest coupling, β=5.7. A linear fit to combined world data gives f_π=120(6) MeV and f_K=135(5) MeV. Excluding the β=5.7 point changes these estimate to f_π=128(6) MeV and f_K=146(5) MeV. Our best estimates for heavy-light meson, f_D=186(29) MeV and f_{D_s}=218(15) MeV in the continuum limit, are from a linear fit to data at $\beta\geqslant$6.0. The above estimates are using $m_s(M_K)$. Using $m_s(M_\phi)$ (our preferred value) would increase f_K and f_{D_s} by $\approx$2%.

We study three lattice transcriptions of the vector current to calculate f_V^{-1}. Using the Lepage-Mackenzie scheme to calculate Z_V for each of the three currents yields results that are consistent to within 10%. We extrapolate f_ρ^{-1} to the continuum limit by combining with results from the GF11 Collaboration. The result is 0.153(17) compared to the experimental value of 0.199(5).

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