# $SU(3)_L \times U(1)_X$ model of electroweak interactions without exotic quarks

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We consider an $SU(3)_L \times U(1)_X$ model of the electroweak interactions as a possible extension of the standard model to higher energies. The model contains new fermions with ordinary lepton and quark electric charges. There are new gauge bosons with masses in the TeV region. There exists a version of the model with identical low energy predictions as those of the standard model. [S0556-2821(96)01313-6]

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## I. INTRODUCTION

The standard model (SM) of the electromagnetic and weak interactions, the Glashow-Weinberg-Salam theory [1], is compatible with all known experimental data available today. Even though the SM is mathematically self-consistent it is not a fundamental theory. In a fundamental theory the undetermined physical parameters would not be put in by hand but rather would be determined by the theory itself. The fundamental theory would manifest itself through new physical phenomena at higher-energy scales beyond the SM. Ever since the advent of the SM various extensions of it have been considered. These extensions of the SM contain new physics such as extra neutral gauge bosons, nonstandard Higgs representations, extra fermions, left-right symmetry, compositeness, supersymmetry, etc. Since the SM is very well established, the deviations from it due to new physics should be small enough to lie within the present experimental errors and might be observable, if there exists any, in future more precise experiments.

Recently the old extensions of the SM based on the gauge group $SU(3)_L \times U(1)_X$ have been reconsidered [2–11]. The most distinguished feature of the gauge group $SU(3)_L \times U(1)_X$ is that the number of fermion families may be related to the number of quark colors to obtain an anomaly-free model. Most of the earlier and the recent such models contain exotic quarks, namely quarks with electric charge other than $-1/3$ and $2/3$ in units of $e$, as a result of which the new heavier gauge bosons are doubly charged. The variety of such models seems to have been all considered in the literature. On the other hand, little work has been done on models containing nonexotic quarks and the possibilities for model building have not been exhausted yet. It is the purpose of this paper to present and investigate the salient features of a model based on the gauge group $SU(3)_L \times U(1)_X$ with ordinary quarks. This paper is organized as follows.

In Sec. II we present the model. The gauge bosons of the model and their masses are discussed in Sec. III. The neutral current processes of the neutrinos and their interactions with the electrons are discussed in Sec. IV. Section V is devoted to the flavor-changing neutral-current interactions of the quarks. Finally, our conclusions are presented in Sec. VI.

## II. THE MODEL

In view of the anomaly cancellation, half the fermions of the three families are put in triplets and the other half are put in antitriplets of $SU(3)_L$. The electric charge operator $Q$ is embedded as

$$Q = \frac{1}{2}\lambda_L^3 - \frac{1}{2\sqrt{3}}\lambda_L^8 + \sqrt{\frac{3}{8}}X\lambda^9, \tag{1}$$

where the $\lambda_L^a (a=1,2,\ldots,8)$ are the $SU(3)_L$ generators and $\lambda^9 = \sqrt{2/3}\,\mathrm{diag}\,(1,1,1)$ is the $U(1)_X$ generator, all of which are normalized as $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$. The second and the third families of quarks are assumed to belong to the triplets (3, 2/3):

$$Q_{2L} = \begin{bmatrix} c^\alpha \\ s^\alpha \\ C^\alpha \end{bmatrix}_L, \quad Q_{3L} = \begin{bmatrix} t^\alpha \\ b^\alpha \\ T^\alpha \end{bmatrix}_L. \tag{2}$$

Their right-handed partners are $SU(3)_L$ singlets:

$$c_R^\alpha \sim (1,4/3), \quad s_R^\alpha \sim (1,-2/3), \quad C_R^\alpha \sim (1,4/3),$$

$$t_R^\alpha \sim (1,4/3), \quad b_R^\alpha \sim (1,-2/3), \quad T_R^\alpha \sim (1,4/3), \tag{3}$$

where $\alpha$ is the $SU(3)_c$ color index. The first family of quarks and all three families of leptons are assumed to belong to the antitriplets $(\bar{3},0)$ and $(\bar{3},-4/3)$, respectively:

$$Q_{1L} = \begin{bmatrix} d^\alpha \\ u^\alpha \\ D^\alpha \end{bmatrix}_L, \quad L_e = \begin{bmatrix} e^- \\ \nu_e \\ E^- \end{bmatrix}_L, \quad L_\mu = \begin{bmatrix} \mu^- \\ \nu_\mu \\ M^- \end{bmatrix}_L,$$

$$L_\tau = \begin{bmatrix} \tau^- \\ \nu_\tau \\ \Gamma^- \end{bmatrix}_L. \tag{4}$$

Their $SU(3)_L$-singlet right-handed partners are $e_R^- \sim (1,-2)$, $E_R^- \sim (1,-2)$, $\mu_R^- \sim (1,-2)$, $M_R^- \sim (1,-2)$, $\tau_R^- \sim (1,-2)$, and $\Gamma_R^- \sim (1,-2)$ for leptons and $d_R^\alpha \sim (1,-2/3)$, $u_R^\alpha \sim (1,4/3)$, and $D_R^\alpha \sim (1,-2/3)$ for quarks. The $C$, $T$, and $D$ are new heavy quarks with electric charges 2/3, 2/3, and $-1/3$. They are nonexotic as far as their electric charges are concerned. They are $SU(2)_L$ singlets and hence have different properties than the ordinary quarks under the weak interactions. The $E^-$, $M^-$, and $\Gamma^-$ are new heavy leptons. Note that there is an equal number of $3_L$ and $\bar{3}_L$ so that the pure $SU(3)_L$ anomaly vanishes. The remaining anomalies also cancel because

 

$\Sigma_f X_f = 0$ and $\Sigma_f (X_f)^3 = 0$ where the sum is over the fermions. Interesting enough, anomaly cancellation takes place between families, but not family by family as in the SM.

We introduce the complex Higgs fields $\Phi \sim (3,-2/3)$, $\phi_1 \sim (3,-2/3)$, and $\phi_2 \sim (3,\ 4/3)$. In the unitary gauge the Higgs fields can be written as

$$\Phi = \begin{bmatrix} 0 \\ 0 \\ \dfrac{V+H^0}{\sqrt{2}} \end{bmatrix} \sim (3,-2/3), \quad \phi_1 = \begin{bmatrix} \dfrac{\nu_1 + h_0^*}{\sqrt{2}} \\ h_1^- \\ \dfrac{\nu_1' + h_0^{*\prime}}{\sqrt{2}} \end{bmatrix} \sim (3,-2/3),$$

$$\phi_2 = \begin{bmatrix} 0 \\ \dfrac{\nu_2 + h^0}{\sqrt{2}} \\ h_2^+ \end{bmatrix} \sim (3,4/3). \tag{5}$$

Altogether there are ten massive Higgs particles; six of which are neutral and four of which are charged. Note that the Higgs fields $\Phi$ and $\phi_2$ not only provide mass to fermions but they also break the gauge symmetry. $\Phi$ breaks $SU(3)_L \times U(1)_X$ down to $SU(2)_L \times U(1)_Y$ at an energy scale $V$, and then $\phi_2$ breaks $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$ at an energy scale $\nu_2$. The Yukawa couplings of the (symmetry eigenstates of) fermions with the scalar multiplets before the symmetry breaking are

$$-L(\phi_1) = \overline{Q_{1L}} \sum_p G_{dp} p_R \phi_1^* + \overline{Q_{2L}} \sum_q G_{cq} q_R \phi_1 + \overline{Q_{3L}} \sum_q G_{tq} q_R \phi_1 + \text{H.c.}, \tag{6a}$$

$$-L(\phi_2) = \overline{Q_{1L}} \sum_q G_{uq} q_R \phi_2^* + \overline{Q_{2L}} \sum_p G_{sp} p_R \phi_2 + \overline{Q_{3L}} \sum_p G_{bp} p_R \phi_2 + \text{H.c.}, \tag{6b}$$

$$-L(\Phi) = \overline{Q_{1L}} \sum_p G_{Dp} p_R \Phi^* + \overline{Q_{2L}} \sum_q G_{Cq} q_R \Phi + \overline{Q_{3L}} \sum_q G_{Tq} q_R \Phi + \text{H.c.}, \tag{6c}$$

where $p = d,\ s,\ b,\ D$ and $q = u,\ c,\ t,\ C,\ T$. The $G$'s are the Yukawa coupling constants. The color index $\alpha$ and the primes from the symmetry eigenstates have been suppressed. Because of their identical transformation property the Higgs fields $\Phi$ and $\phi_1$ may provide mass to the same fermions through their Yukawa couplings. But it is known from the generalization of the Glashow-Iliopoulos-Maiani (GIM) mechanism [12] that to conserve quark flavors and hence to avoid flavor-changing neutral currents (FCNC's), quarks of given charge must receive their mass through the couplings of precisely one neutral Higgs boson [13]. The vacuum expectation values $\nu_1'$ of $\phi_1$ and $V$ of $\Phi$ [see Eq. (5)] cause (vertical) mixings [14] between the first (light) and the third (heavy) members of the $SU(3)_L$ triplets and antitriplets. It can be seen from Eq. (6a) that if $\nu_1' \neq 0$ there are off-diagonal interactions of $\phi_1$ such as $d + \phi_1 \to D$. The part of $-L(\phi_1)$ that will give rise to this off-diagonal interaction, if $\nu_1' \neq 0$, is

$$G_{dd} \overline{D_L'} d_R' \frac{\nu_1'}{\sqrt{2}},$$

where now we have retained the primes to denote the symmetry eigenstates. In terms of the mass eigenstates this is written as

$$G_{dd} \overline{D_L} (d_R U_{1d} + s_R U_{1s} + b_R U_{1b}) \frac{\nu_1'}{\sqrt{2}},$$

where $D \approx D'$ due to the fact that the $D$ quark is expected to be rather heavy, and $U$ is the rotation matrix which rotates the mass eigenstates to the symmetry eigenstates through

$$d_i' = U_{i\alpha} \alpha, \quad i = 1,2,3, \quad \text{and} \quad \alpha = d,s,b. \tag{7}$$

In Eq. (7), $d_1 = d$, $d_2 = s$, $d_3 = b$. Notice that $U$ is not the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$ because quarks are not treated universally. Thus the exchange of $\phi_1$ between $\underline{d} \leftrightarrow D$ currents can produce an effective Fermi interaction $D + d \to \phi_1 \to D + \bar{d}$. Even though there are no clues for such FCNC transitions in current experiments we may safely assume that such transitions are extremely suppressed just like the $d \leftrightarrow s$ transitions. The coupling $G_{\phi_1}$ of the effective Fermi interaction due to the $\phi_1$ exchange can be estimated to be [13]

$$G_{\phi_1} \approx \frac{m_D^2 |U_{1d}|^2}{\lambda \langle \phi_1 \rangle^4}.$$

Keeping in mind that the CKM matrix elements $V_{ud} = 0.9747–0.9759$, $V_{us} = 0.218–0.24$, and $V_{ub} = 0.002–0.007$ [15], we expect that $U_{1s}$ and $U_{1b}$ may be rather small but $U_{1d}$ is of the order of 1. Therefore taking, for example, $\lambda \approx 10^{-2}$, $\langle \phi_1 \rangle \approx 200$ GeV, $U_{1d} \approx 1$, we get $G_{\phi_1}/G_F \approx 1$ and $10^2$ for $m_D = 10$ and 100 GeV, respectively, where $G_F$ is the Fermi coupling. We see that an off-diagonal Higgs coupling $d + \phi_1 \to D$ would be expected to produce much too large a $\Delta D = 2$ effective Fermi interaction. This interaction must be suppressed. Since $\langle \phi_1 \rangle$ cannot be larger than 200 GeV (because $\nu_1$ and $\nu_1'$ are proportional to the $W^\pm$ mass), it seems that once it is allowed there is no way to suppress the $\Delta D = 2$ interaction even if we take $m_D = 1$ GeV. Thus this interaction must be eliminated [16]. This then forces us to impose the vacuum expectation value $\nu_1'$ of $\phi_1$ to vanish. This can be achieved by a judicious introduction of discrete symmetries. The symmetries that we impose on the dimension-four couplings in the Lagrangian are

$$(\phi_1, \phi_2) \to (\phi_1, \phi_2), \quad \Phi \to -\Phi,$$

$$Q_L \to Q_L,$$

$$f_R \to f_R, \quad F_R \to -F_R, \tag{8}$$

where $f$ and $F$ stand for ordinary and new fermions, respectively.

### III. GAUGE BOSONS

In this section we will consider the gauge bosons of this model. To start with there are an octet $W^a_\mu$ ($a=1,2,\ldots,8$) of massless gauge bosons associated with SU(3)$_L$ and a massless singlet $W^X_\mu$ associated with U(1)$_X$. The covariant derivatives are

$$D_\mu\varphi=\left(\partial_\mu+ig\,\frac{\lambda^a_L}{2}\,W^a_\mu+ig_x\frac{X(\varphi)}{2}\,W^x_\mu\right)\varphi, \tag{9}$$

where $X(\varphi)$ denotes the $X$ charge for the Higgs triplets $\varphi=\Phi,\phi_1,\phi_2$. $g$ and $g_x$ are the SU(3)$_L$ and U(1)$_X$ gauge couplings, respectively. At this stage we note that the electric charge operator of Eq. (1) can be written as

$$Q=\frac{1}{2}\,\lambda^3_L+\frac{1}{2}\,Y, \tag{10}$$

where

$$Y=-\frac{1}{\sqrt{3}}\,\lambda^8_L+X \tag{11}$$

is the hypercharge operator of the gauge group SU(2)$_L\times$U(1)$_Y$ of the SM. Note from Eqs. (10) and (11) that the coupling constants satisfy the matching conditions

$$g(M_W)=g_2(M_W),$$

$$\frac{1}{g^2_Y(M_W)}=\frac{1}{3g^2(M_W)}+\frac{1}{g^2_X(M_W)}, \tag{12}$$

where $g_2$ and $g_Y$ are the coupling constants of SU(2)$_L$ and U(1)$_Y$. The massless gauge boson $B$ associated with U(1)$_Y$ is

$$B_\mu=\frac{-g_XW^8_\mu+\sqrt{3}gW^X_\mu}{\sqrt{3g^2+g^2_X}}. \tag{13}$$

The orthonormal combination

$$Z'_\mu=\frac{\sqrt{3}gW^8_\mu+g_XW^X_\mu}{\sqrt{3g^2+g^2_X}}, \tag{14}$$

is useful in diagonalizing the neutral mass-squared matrix. The expression for the photon field can be read off from Eq. (1):

$$\frac{A_\mu}{e}=\frac{W^3_\mu}{g}-\frac{W^8_\mu}{\sqrt{3}g}+\frac{W^X_\mu}{g_X}, \tag{15}$$

which can be cast into the form

$$A_\mu=\sin\,\theta_WW^3_\mu+\cos\,\theta_WB_\mu, \tag{16}$$

where the mixing angle $\theta_W$ is given by

$$\sin\,\theta_W=\frac{\sqrt{3}g_X}{\sqrt{3g^2+4g^2_X}}. \tag{17}$$

The combination which is orthogonal to $A_\mu$ is

$$Z_\mu=\cos\,\theta_WW^3_\mu-\sin\,\theta_WB_\mu. \tag{18}$$

Apart from the massless photon in Eq. (15) the other mass eigenstates are

$$V^0_\mu=(W^4_\mu-iW^5_\mu)/\sqrt{2},\quad M^2_{V^0}=\frac{g^2}{4}\,(V^2+\nu^2_1),$$

$$\overline{V^0_\mu}=(W^4_\mu+iW^5_\mu)/\sqrt{2},\quad M^2_{\overline{V_0}}=M^2_{V^0},$$

$$V^\pm_\mu=(W^6_\mu\pm iW^7_\mu)/\sqrt{2},\quad M^2_{V^\pm}=\frac{g^2}{4}\,(V^2+\nu^2_2),$$

$$W^\pm_\mu=(W^1_\mu\mp iW^2_\mu)/\sqrt{2},\quad M^2_{W^\pm}=\frac{g^2}{4}\,(\nu^2_1+\nu^2_2). \tag{19}$$

The remaining two neutral mass eigenstates are obtained by diagonalizing the neutral mass-squared matrix in the $Z-Z'$ basis:

$$M^2=\begin{pmatrix} M^2_Z & M^2_{ZZ'} \\ M^2_{ZZ'}, & M^2_{Z'} \end{pmatrix}, \tag{20}$$

where with $t=g^2_X/g^2=\sin^2\theta_W/[1-(4/3)\sin^2\theta_W]$,

$$M^2_Z=\frac{g^2}{4\cos^2\,\theta_W}\,(\nu^2_1+\nu^2_2),$$

$$M^2_{Z'}=\frac{g^2}{36(3+t)}\,[4V^2(3+t)^2+\nu^2_1(3-2t)^2+\nu^2_2(3+4t)^2],$$

$$M^2_{ZZ'}=\frac{g^2}{12\,\cos\,\theta_W\sqrt{3+t}}\,[\nu^2_1(3-2t)-\nu^2_2(3+4t)]. \tag{21}$$

The eigenvalues of (20) are

$$\frac{1}{2}\,\{M^2_Z+M^2_{Z'}\pm[(M^2_{Z'}-M^2_Z)^2+4(M^2_{ZZ'})^2]^{1/2}\}, \tag{22}$$

where the $(+)$ and $(-)$ signs are identified, respectively, with $M^2_{Z_2}$ and $M^2_{Z_1}$. The mass eigenstates are

$$Z_{1\mu}=\cos\,\theta Z_\mu-\sin\,\theta Z'_\mu,$$

$$Z_{2\mu}=\sin\,\theta Z_\mu+\cos\,\theta Z'_\mu, \tag{23}$$

where $Z_1$ replaces the neutral gauge boson of the SM and $Z_2$ is a heavy partner. In Eq. (23) $\theta$ is given by

$$\tan^2\,\theta=\frac{M^2_Z-M^2_{Z_1}}{M^2_{Z_2}-M^2_Z}. \tag{24}$$

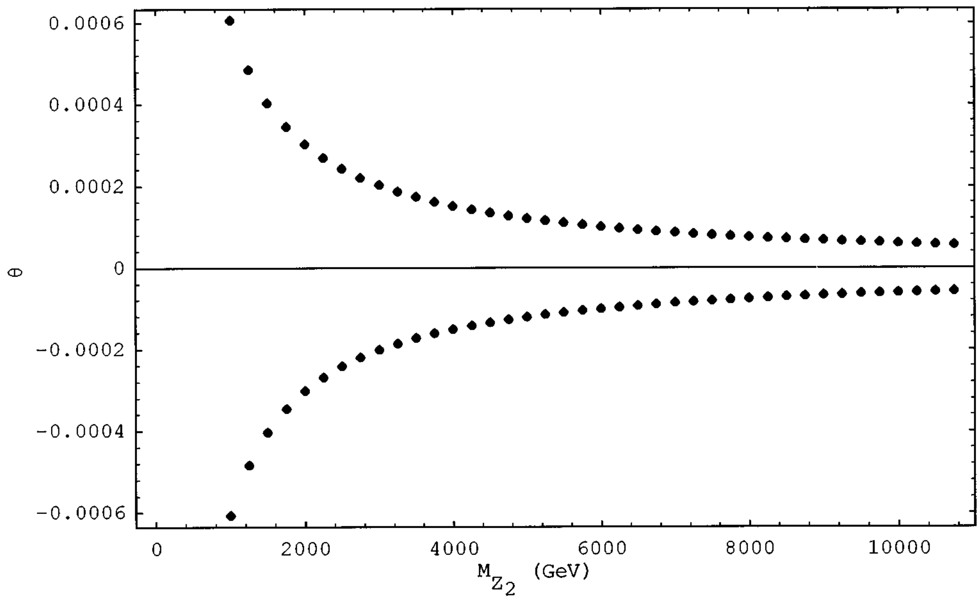


FIG. 1. Plot of the mixing angle $\theta$ against $M_{Z_2}$.

In Eqs. (21) and (24) $M_Z$ corresponds to the mass of the neutral gauge boson of the SM. Its value is predicted very accurately in the SM as $M_Z$=91.177 GeV [17]. On the other hand, due to the mixing between $Z$ and $Z'$, Eq. (23), the mass of $Z_1$, $M_{Z_1}$, is slightly less than $M_Z$. Considering the experimental error in $M_Z$ we take $M_{Z_1}$=91.175 GeV [17]. Next we plot the mixing angle $\theta$ against $M_{Z_2}$ in Fig. 1, where it is seen that as $M_{Z_2}$ gets larger $\theta$ becomes smaller. As expected, $\theta$ vanishes for $M_{Z_2}=\infty$. However, $\theta$ may vanish ''accidentally'' in a different way without $M_{Z_2}$ being infinity. A glance at Eq. (21) reveals that the $Z-Z'$ mixing vanishes if

$$\nu_1^2=\left(\frac{3+4t}{3-2t}\right)\nu_2^2, \tag{25}$$

in which case we would have $Z=Z_1$, $Z'=Z_2$, and

$$M_Z^2=M_{Z_1}^2=\frac{g^2}{2\cos^2\theta_W}\left(\frac{3+t}{3-2t}\right)\nu_2^2,$$

$$M_{Z'}^2=M_{Z_2}^2=\frac{g^2}{18}\left[2V^2(3+t)+\nu_2^2(3+4t)\right],$$

$$M_{ZZ'}^2=0. \tag{26}$$

Even though Eq. (25) remains as a plausible and attractive possibility unless feature experiments detect sizable deviations in the predictions of the SM, without any symmetry principle protecting it, Eq. (25) can only be enforced through fine-tuning. However, we will base our discussion in the following not on the ''accidental'' case of Eq. (25) but on the general case of Eq. (23).

## IV. NEUTRAL-CURRENT PROCESSES

The neutral-current Lagrangian apart from the electromagnetic Lagrangian is

$$L^{\mathrm{NC}}=-\frac{g}{c\theta_W}\{f_3J_{3L}^{\mu}-f_Qs^2\theta_WJ_{\mathrm{EM}}^{\mu}+f_8J_{8L}^{\mu}\}Z_{1\mu}$$
$$-\frac{g}{c\theta_W}\{f_3'J_{3L}^{\mu}-f_Q's^2\theta_WJ_{\mathrm{EM}}^{\mu}+f_8'J_{8L}^{\mu}\}Z_{2\mu}, \tag{27}$$

where

$$f_3=c\theta-\frac{1}{\sqrt{3}}\frac{s\theta s^2\theta_W}{\sqrt{1-(4/3)s^2\theta_W}}, \tag{28}$$

$$f_Q=c\theta-\frac{1}{\sqrt{3}}\frac{s\theta}{\sqrt{1-(4/3)s^2\theta_W}},$$

$$f_8=\frac{s\theta c^2\theta_W}{\sqrt{1-(4/3)s^2\theta_W}},$$

$$f_3'=-s\theta-\frac{1}{\sqrt{3}}\frac{c\theta s^2\theta_W}{\sqrt{1-(4/3)s^2\theta_W}},$$

$$f_Q'=-s\theta-\frac{1}{\sqrt{3}}\frac{c\theta}{\sqrt{1-(4/3)s^2\theta_W}},$$

$$f_8'=\frac{c\theta c^2\theta_W}{\sqrt{1-(4/3)s^2\theta_W}}.$$

In Eqs. (27) and (28) $c\omega$ and $s\omega$ stand for cos $\omega$ and sin $\omega$, $\omega=\theta$, and $\theta_W$. In Eq. (27) the particle currents are given by

$$J_{aL}^{\mu}=\sum_i\overline{\Psi}_{iL}\gamma^{\mu}\frac{\lambda_L^a(i)}{2}\Psi_{iL}\quad(a=1,2,\ldots,8),$$

$$J_{\mathrm{EM}}^{\mu}=\sum_i\overline{\Psi}_i\gamma^{\mu}Q(i)\Psi_i, \tag{29}$$

where $\lambda_L^a(i)$ and $Q(i)$ are, respectively, the eigenvalue of $\lambda_L^a$ and electric charge of fermion $i$. In the literature the neutral-current couplings are expressed as

$$L^{\rm NC}=-\frac{g}{c\,\theta_W}\left(\sum_i\,[\epsilon_L(i)\overline{\Psi}_{iL}\gamma^\mu\Psi_{iL}+\epsilon_R(i)\overline{\Psi}_{iR}\gamma^\mu\Psi_{iR}]Z_{1\mu}+\sum_i\,[\epsilon'_L(i)\overline{\Psi}_{iL}\gamma^\mu\Psi_{iL}+\epsilon'_R(i)\overline{\Psi}_{iR}\gamma^\mu\Psi_{iR}]Z_{2\mu}\right)$$
$$=-\frac{g}{c\,\theta_W}\left(\sum_i\,[\overline{\Psi}_i\gamma^\mu(g_V^i+g_A^i\gamma^5)\Psi_i]Z_{1\mu}+\sum_i\,[\overline{\Psi}_i\gamma^\mu(g_V'^i+g_A'^i\gamma^5)\Psi_i]Z_{2\mu}\right), \tag{30}$$

where

$$g'_{V,A}\equiv\epsilon_L(i)\pm\epsilon_R(i),$$
$$\epsilon_L(i)=f_3\,\frac{\lambda_L^3(i)}{2}-f_QQ(i)s^2\theta_W+f_8\,\frac{\lambda_L^8(i)}{2},$$
$$\epsilon_R(i)=-f_QQ(i)s^2\theta_W, \tag{31}$$

and the expressions for $\epsilon'_L(i)$ and $\epsilon'_R(i)$ are obtained from Eq. (31) by replacing the $f_j$ by $f'_j$. Noting that $\overline{\lambda_L^3}=-\lambda_L^3=-\mathrm{diag}(1,-1,0)$, and $\overline{\lambda_L^8}=-\lambda_L^8=-(1/\sqrt{3})\mathrm{diag}(1,1,-2)$ we obtain for the $Z_{1\mu}$ couplings

$$\epsilon_L(u)=\frac{1}{2}\,f_3-\frac{2}{3}\,f_Qs^2\theta_W-\frac{1}{2\sqrt{3}}\,f_8, \tag{32}$$
$$\epsilon_L(c)=\epsilon_L(t)=\frac{1}{2}\,f_3-\frac{2}{3}\,f_Qs^2\theta_W+\frac{1}{2\sqrt{3}}\,f_8,$$
$$\epsilon_L(d)=-\frac{1}{2}\,f_3+\frac{1}{3}\,f_Qs^2\theta_W-\frac{1}{2\sqrt{3}}\,f_8,$$
$$\epsilon_L(s)=\epsilon_L(b)=-\frac{1}{2}\,f_3+\frac{1}{3}\,f_Qs^2\theta_W+\frac{1}{2\sqrt{3}}\,f_8,$$
$$\epsilon_L(C)=\epsilon_L(T)=-\frac{2}{3}\,f_Qs^2\theta_W-\frac{1}{\sqrt{3}}\,f_8,$$
$$\epsilon_L(D)=\frac{1}{3}\,f_Qs^2\theta_W+\frac{1}{\sqrt{3}}\,f_8,$$
$$\epsilon_L(l^-)=-\frac{1}{2}\,f_3+f_Qs^2\theta_W-\frac{1}{2\sqrt{3}}\,f_8,$$
$$\epsilon_R(u)=\epsilon_R(c)=\epsilon_R(t)=-\frac{2}{3}\,f_Qs^2\theta_W,$$
$$\epsilon_R(d)=\epsilon_R(s)=\epsilon_R(b)=\frac{1}{3}\,f_Qs^2\theta_W,$$
$$\epsilon_R(C)=\epsilon_R(T)=-\frac{2}{3}\,f_Qs^2\theta_W,$$
$$\epsilon_R(D)=\frac{1}{3}\,f_Qs^2\theta_W,$$
$$\epsilon_R(l^-)=f_Qs^2\theta_W,$$

where in Eqs. (32) $l^-$ stands for $e^-$, $\mu^-$, and $\tau^-$. Again the couplings $\epsilon'_L(i)$ and $\epsilon'_R(i)$ for $Z_2$ are obtained from Eqs. (32) by replacing the $f_j$ by $f'_j$ which are given in Eq. (28).

An estimate of the mixing angle $\theta$ can be obtained from the low-energy $\nu_\mu e$ scattering experiments. The interaction between $\nu_\mu$ and electrons is described by

$$L^{\nu_\mu e}=-\frac{G_F}{\sqrt{2}}\,\overline{\nu_\mu}\gamma^\mu(1+\gamma^5)\nu_\mu J^e_\mu, \tag{33}$$

where in this model

$$J^e_\mu=\overline{e}\gamma_\mu\left[g_V^e+\frac{M^2_{Z_1}}{M^2_{Z_2}}\,g_V'^e+\left(g_A^e+\frac{M^2_{Z_1}}{M^2_{Z_2}}\,g_A'^e\right)\gamma^5\right]e. \tag{34}$$

Here

$$g_V^e=\epsilon_L(e)+\epsilon_R(e)=\left(-\frac{1}{2}+2s^2\theta_W\right)c\,\theta+\frac{5}{2\sqrt{3}}\,\frac{s\,\theta}{\sqrt{1-(4/3)s^2\theta_W}}, \tag{35}$$

and

$$g_A^e=\epsilon_L(e)-\epsilon_R(e)=-\frac{1}{2}\,c\,\theta+\frac{1}{2\sqrt{3}}\,\frac{s\,\theta}{\sqrt{1-(4/3)s^2\theta_W}}, \tag{36}$$

are due to the $Z_1$ exchange. The contribution of the $Z_2$ exchange is suppressed by $M^2_{Z_1}/M^2_{Z_2}$. Hence we can neglect the $g_V'^e$ and $g_A'^e$ terms in $J^e_\mu$. The experimental value for $g_V^e$ is $-0.045\pm0.022$ [17]. Therefore,

$$\frac{5}{2\sqrt{3}}\,\frac{|s\,\theta|}{\sqrt{1-(4/3)s^2\theta_W}}\leqslant 0.022, \tag{37}$$

from which we get

$$|\theta|\leqslant 0.013. \tag{38}$$

On the other hand, the experimental value of $g_A^e=-0.513\pm0.025$ yields $|\theta|\leqslant 0.072$ which is less stringent than (38). In the next section we will use (38) to obtain a lower bound on the $Z_2$ mass.

## V. FLAVOR-CHANGING NEUTRAL-CURRENT PROCESSES

We note that by imposing the symmetries in Eq. (8) the vertical mixings of the ordinary fermions with the new (heavy) fermions are forbidden. There are FCNC's coupled to both $Z_1$ and $Z_2$. Note from (32) that $c$ and $t$, $s$ and $b$, and $C$ and $T$ have identical couplings. Therefore the GIM mechanism operates in the neutral currents coupled to $Z_1$ and $Z_2$ only in the sectors $(c,t)$, $(s,b)$, and $(C,T)$. There are FCNC's between $u\leftrightarrow(c,t)$ and $d\leftrightarrow(s,b)$ in the couplings to $Z_1$ and $Z_2$. These currents must be suppressed. The suppression of, for example, $d\leftrightarrow s$ current provides a lower limit on the $Z_2$ mass. The $K_L^0-K_S^0$ mass difference gets contributions from the exchanges of $Z_1$ and $Z_2$ between $d\leftrightarrow s$ currents which is described by

$$\begin{aligned}L_{d\leftrightarrow s}=&-\frac{g}{2c\theta_W}\,|U_{1s}^*U_{1d}|\{[\overline{s}\gamma^\mu(g_V^d+g_A^d\gamma^5)d\\&-\overline{s}\gamma^\mu(g_V^s+g_A^s\gamma^5)d]Z_{1\mu}+[\overline{s}\gamma^\mu(g_V'^d+g_A'^d\gamma^5)d\\&-\overline{s}\gamma^\mu(g_V'^s+g_A'^s\gamma^5)d]Z_{2\mu}\},\end{aligned}\tag{39}$$

where now $d$ and $s$ are mass eigenstates. $U$ is the matrix that rotates the mass eigenstates to the weak (symmetry) eigenstates [see Eq. (7)]. At low energies Eq. (39) gives rise to the effective interaction

$$\begin{aligned}L_{\text{eff}}=&\frac{g^2}{4c^2\theta_W}\,|U_{1s}^*U_{1d}|^2\left(\frac{1}{M_{Z_1}^2}\,[\overline{s}\gamma^\mu(c_V+c_A\gamma^5)d]^2\right.\\&\left.+\frac{1}{M_{Z_2}^2}\,[\overline{s}\gamma^\mu(c_V'+c_A'\gamma^5)d]^2\right),\end{aligned}\tag{40}$$

where

$$\begin{aligned}c_V&=g_V^d-g_V^s=-\frac{1}{\sqrt{3}}\,f_8,\\c_A&=g_A^d-g_A^s=c_V,\\c_V'&=g_V'^d-g_V'^s=-\frac{1}{\sqrt{3}}\,f_8',\\c_A'&=g_A'^d-g_A'^s=c_V'.\end{aligned}\tag{41}$$

On the other hand, in the SM the contribution of the $c$ quark is [18]

$$L_{\text{eff}}=-\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\frac{m_c^2}{M_W^2s^2\theta_W}|V_{us}^*V_{ud}|^2\left(\overline{s}\gamma^\mu\frac{1}{2}(1-\gamma^5)d\right)^2,\tag{42}$$

where in Ref. [18] only two-family mixing has been assumed in which case $|V_{us}^*V_{ud}|=s\theta_Cc\theta_C$, where $\theta_C$ is the Cabibbo angle. To suppress the FCNC's we must assume that the contribution of (40) to the $K_L^0-K_S^0$ mass difference are less than the contribution of the $c$ quark. We then obtain

$$M_{Z_1}^2>\left(\frac{|U_{1s}^*U_{1d}|^2}{|V_{us}^*V_{ud}|^2}\,t^2\theta_W\,\frac{M_W^4}{m_c^2}\,\frac{32\pi}{\alpha}\right)\frac{f_8^2}{3},\tag{43}$$

$$M_{z_2}^2>\left(\frac{|U_{1s}^*U_{1d}|^2}{|V_{us}^*V_{ud}|^2}\,t^2\theta_W\,\frac{M_W^4}{m_c^2}\,\frac{32\pi}{\alpha}\right)\frac{f_8'^2}{3},\tag{44}$$

where $G_F/\sqrt{2}=g^2/8M_w$ has been used. Taking $M_{Z_1}=91.175$ GeV, $M_W=80$ GeV, $m_c=1.5$ GeV, $s^2\theta_W=0.23$, we get from (43) the upper bound

$$\frac{|U_{1s}^*U_{1d}|}{|V_{us}^*V_{ud}|}<\frac{1.3\times10^{-3}}{s\theta}.\tag{45}$$

Equation (44) gives

$$M_{Z_2}>\frac{|U_{1s}^*U_{1d}|}{|V_{us}^*V_{ud}|}\,1.4\times10^5\ \text{GeV},\tag{46}$$

where $c\theta_c\approx1$ has been used. We tabulate in Table I the lower bound (46) as a function of the ratio $|U_{1s}^*U_{1d}|/|V_{us}^*V_{ud}|$ of the mixing angles. We see from Table I that $M_{Z_2}$ can be as light as 700 GeV or may be even lighter. However, it seems more likely that $M_{Z_2}$ lies in the TeV range. An upper limit on the $V^0$ and $V^\pm$ masses can be obtained in terms of the $Z_2$ mass by making use of the symmetry-breaking hierarchy $V\gg\nu_1,\nu_2$. From Eqs. (19) and (21) we obtain

$$M_{V^0,V^\pm}^2<\frac{9}{4(3+t)}\,M_{Z'}^2<\frac{9}{4(3+t)}\,M_{Z_2}^2,\tag{47}$$

which for $s^2\theta_W=0.23$ gives

$$M_{V^0,V^\pm}<0.8M_{Z_2}.\tag{48}$$

TABLE I. Lower bound on $M_{Z_2}$ as a function of the ratio of the mixing angles in the present model to those in the SM.

| $\frac{\lvert U_{1s}^*U_{1d}\rvert}{\lvert V_{us}^*V_{ud}\rvert}$ | $M_{Z_2}$ (GeV) |
|---|---|
| 1 | $1.4\times10^5$ |
| $10^{-1}$ | $1.4\times10^4$ |
| $10^{-2}$ | $1.4\times10^3$ |
| $0.5\times10^{-2}$ | $0.7\times10^3$ |

The FCNC's coupled to $Z_1$ and $Z_2$ are not the only ones. There are also those coupled to the Higgs bosons. As we have discussed before, the flavor-changing neutral-current Higgs (FCNH) interactions between ordinary and new heavy quarks are eliminated by imposing the discrete symmetry in (8). However, there exists FCNH interactions involving ordinary quarks and the light Higgs multiplets $\phi_1$ and $\phi_2$. This is because, as is seen in Eq. (6), one of the three families gets mass from a different Higgs boson than the other two. These FCNH interactions too have to be eliminated, or at least suppressed, which can be achieved by a judicious choice of Yukawa couplings or by fine tuning of the vacuum expectation values $\nu_1$ and $\nu_2$. We will not address this issue further because the problem is similar to the case in models with exotic quarks [5].

Before concluding this paper, we will obtain an upper bound on the mass $M_{Z_2}$ of the heavy partner $Z_2$ of $Z_1$. As is seen from Eq. (28), the $f_j$ and $f_j'$ which appear in the neutral-current Lagrangian $L^{\text{NC}}$ in Eq. (27) are inversely proportional to the square-root factor $\sqrt{1-(4/3)\sin^2\theta_W}$. The finiteness of the neutral-current processes requires that $\sin^2\theta_W$ at the symmetry limit $M_{Z'}$ be less than 3/4. Considering the one-loop running of $\sin^2\theta_W$, the upper bound on $M_{Z'}$, and hence on $M_{Z_2}$, can be calculated from the constraint $\sin^2\theta_W(M_{Z'})<3/4$. Including the Higgs multiplets in the evolution of $\sin^2\theta_W$, and taking $\sin^2\theta_W(M_Z)=0.2333$ and $\alpha^{-1}=127.9$ we find

$$M_{Z_2}\leqslant 3.6\times 10^3 \ \ \text{TeV}. \tag{49}$$

It is interesting and instructive to compare this upper bound with that in the SU(3)$_L\times$U(1)$_X$ models containing exotic quarks, namely quarks with electric charge $\pm 4/3$, $\pm 5/3$ in units of $e$. In these models the $f_j$ and $f'_j$ in Eq. (28) are inversely proportional to the factor $\sqrt{1-4\sin^2\theta_W}$. Therefore, $\sin^2\theta_W$ must be less than 1/4 at the energy scale $M_{Z'}$. Computing the evolution of $\sin^2\theta_W$, it has been shown in Ref. [4] that $M_{Z_2}\leqslant 1$ TeV, while more detailed calculations in Ref. [9] has given $M_{Z_2}\leqslant 3.1$ TeV. The reason for the large difference on the upper bounds of $M_{Z_2}$ in the two versions, namely exotic vs non-exotic quarks versions, of the SU(3)$_L\times$U(1)$_X$ theory is due to the different coefficients of $\sin^2\theta_W$ in the square-root factors above. The coefficients are different in the two versions because of the different embeddings of the electric charge operator. New physics in the present model, if it exists, may thus occur at an arbitrarily high-energy scale, much beyond the scope of future TeV accelerators. Hence it is impossible to rule out the present model as far as the low-energy experiments are concerned.

## VI. CONCLUSIONS

We have presented an SU(3)$_L\times$U(1)$_X$ model of the electroweak interactions as an extension of the SM to higher energies. Contrary to most of the existing SU(3)$_L\times$U(1)$_X$ models in the literature, the model presented here does not contain exotic quarks, as a result of which the theory does not have doubly charged massive gauge bosons. The gauge bosons are either neutral or singly charged. We obtained an upper limit on the $Z-Z'$ mixing from the scattering experiments. There are FCNC's coupling both to the $Z_1$ and $Z_2$ gauge bosons, as well as to Higgs bosons. Suppression of these currents provided us with a lower limit on the mass of $Z_2$ in terms of the ratio of the mixing angles in this model to those in the SM. As is the case in most SU(3)$_L\times$U(1)$_X$ models, $M_{Z_2}$ is more likely to lie in the TeV range. Regarding the massess of the new gauge bosons, the present model is not as restrictive as the ones with exotic quarks and doubly charged gauge bosons. The heavy partner $Z_2$ of $Z_1$ might have mass in the $10^3$ TeV range in the present version of the SU(3)$_L\times$U(1)$_X$ theory without exotic quarks. We have argued that the reason for such a high upper bound is the coefficient 4/3 of $\sin^2\theta_W$ in the factor $1-(4/3)\sin^2\theta_W$. The same coefficient turns out to be 4 in models with exotic quarks and allows the upper bound to be 3.1 TeV only. Future experiments may confront the present model with its competitors, and confirm either this one or the others, or rule out all the SU(3)$_L\times$U(1)$_X$ models. Thus the need for the accelerators in the TeV range stands more than ever.

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