Possible effects of quark and gluon condensates in heavy quarkonium spectra

Y. B. Ding,[1,2,3] X. H. Guo,[3,5] X. Q. Li,[1,3,4] and P. N. Shen[1,3,5,6]
[1]*China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China*
[2]*Graduate School, Academia Sinica, P.O. Box 3908, Beijing 100039, China*
[3]*Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, China*
[4]*Department of Physics, Nankai University, Tianjin 300071, China*
[5]*Institute of High Energy Physics, Academia Sinica, P.O. Box 918(4), Beijing 100039, China*
[6]*Department of Physics and Mathematical Physics, the University of Adelaide, GPO Box 498, Adelaide, South Australia 5001*
(Received 13 March 1995; revised manuscript received 19 December 1995)

The Cornell potential with the best fitted parameters α_s and κ are modified by adding terms derived from nonperturbative QCD, which are characterized by a series of nonvanishing vacuum condensates of quarks and gluons. In terms of this potential, we study the system of heavy quarkonia. The results show that the correction caused by the additional terms reduces the deviation between the data and the values calculated with the pure Cornell potential and improves the splittings of energy levels. The achievements indicate that the nonperturbative effects induced by vacuum condensates play an important role for the correction to $1/q^2$, which in general was phenomenologically put in by hand. This result would be helpful for understanding nonperturbative QCD along a parallel direction to the QCD sum rules. [S0556-2821(96)04411-6]

PACS number(s): 12.39.Pn, 12.38.Mh, 14.40.Gx

I. INTRODUCTION

The success of the potential model in explaining hadronic spectra and hadronic properties has been remarkable. Especially, for the J/ψ and Υ families, various potential forms [1], in which both the Coulomb and confinement potentials are employed, can give results which are reasonably consistent with the data at the charm and bottom energy scales within a certain error. In general, it is believed that confinement comes from the nonperturbative effect of QCD, but unfortunately, so far, that is an unsolved problem.

Along another line, Shifman *et al.* [2] proposed to study hadronic properties in terms of the QCD sum rules where a few nonvanishing vacuum condensates of quarks and gluons $m_q\langle\psi_q\overline{\psi_q}\rangle$, $(\alpha_s/\pi)\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle$, etc., describe the nonperturbative effect. They started from a short distance, where the quark-gluon dynamics is essentially perturbative, and extrapolated the dynamics to a larger distance by introducing nonperturbative effects step by step [3]. The applications of the theory extrapolated in studying hadronic properties such as the spectrum, decay width, and hadronic matrix elements, etc., indicate that one can trust the validity of this approach.

Inspired by these successes, we have been trying to introduce nonperturbative QCD effects, characterized by nonvanishing quark and gluon vacuum condensates, into the traditional potential model [4], so that a deeper understanding of the hadronic structure and the underlying mechanisms which determine how quarks are bound into hadrons can be obtained. In our derivation of correction terms, the vacuum condensates were employed to modify the free gluon propagator. As a consequence, the quark-quark potential and the spectrum of heavy quarkonium are affected. Studying these possible effects is the main purpose of this paper. Meanwhile, we have noticed that beside the modification of the gluon propagator, characterized by the vacuum condensates, the closed-loop correction can also contribute a comparable effect, because these two kinds of corrections are in the same order of α_s. This has been shown by Gupta *et al.* [5,6], Fulcher [7] and Pantaleone *et al.* [8]. In this investigation, as a preliminary study, we temporarily treat the condensate effect alone and will collect the closed-loop correction in our later paper.

As to the condensate correction, there are two different results, ours [4] and Larsson's [9]. The effect of the difference may show up in the spin splittings of heavy quarkonia. Thus our additional effort in this paper will be dedicated to searching for the possible effects of these two approaches. We hope this investigation may help us to understand the condensate correction deeply.

As mentioned above, the nonvanishing vacuum condensates can only be used to describe an extrapolation from a short distance to a medium range [3]; therefore, the longer distance effect at the energy scale $\leqslant\Lambda_{\rm QCD}$ cannot be included in the scenario. In other words, this scenario is only valid within the intermediate range if only a finite number of vacuum condensates is kept, and for the bound states, the potential term responsible for the confinement should come from the larger distance $\geqslant 1/\Lambda_{\rm QCD}$. Therefore, we are attempting to introduce a reasonable picture where the main contribution to the confinement is caused by the interaction at $\leqslant\Lambda_{\rm QCD}$, where the physical picture is not clear yet. Since this part is not derivable at the present stage, we keep a phenomenological confinement form, generally the linear κr term which is the main part of the confinement potential and universal to all of the heavy flavors. Then we perturbatively introduce the corrections induced by the nontrivial vacuum condensates into our framework. It would make observable contributions to some hadronic properties of J/ψ and Υ families; for example, the spin splitting between 1 $^3S_1$ and $1\,^1S_0$ could be one of the sensitive quantities for the correction. Moreover, we will claim that there is no double counting between our derived correction and the contribution from the larger distance $1/\Lambda_{\rm QCD}$.

It should be emphasized that the amplitude of the wave function at the origin, which is essential to the spin splitting, depends on the potential model adopted. For instance, the Richardson potential [10] wave function at the origin is almost the half of Cornell's. In this investigation, we start with the Cornell potential, which has the simplest form among existant potential models, as the zeroth order approximation and then add in the condensate corrections in a perturbative way. Therefore, our wave function at the origin is close to that of the Cornell potential, and the difference between ours and Cornell's starts to show up only in the approximation higher than zeroth order. In our later work, we will study these differences and effects caused by adopting different model wave function as the zeroth order wave function.

In the next section, the potential corrections derived with vacuum condensates are briefly reviewed and different formulas are analyzed. In Sec. III, the numerical results are presented and compared with the data. Finally, our results are discussed and conclusions are drawn.

II. OUR MODEL

As Shifman, Vainshtain, and Zakharov (SVZ) [2] suggested, there are nonzero vacuum expectation values of the quark ($\langle \overline{\psi}_q \psi_q \rangle$) and gluon $[(\alpha_s/\pi)\langle GG\rangle]$ fields, and so the propagator of the gluon should include the effects of these condensates. In fact, in the propagator, all the terms associated with vacuum condensates are proportional to α_s^n $(n \geqslant 1)$. On the other hand, the contributions of higher order perturbative QCD corrections to the potential were discussed in Ref. [5]. In comparison with such corrections, the condensate terms do not suffer from the loop suppression. SVZ noticed that fact and then suggested that the condensates could be considered as a larger contribution. Therefore, at the order of α_s^2, we only include the condensates, but not the perturbative correction.

There have been various ways to modify the potential. Except the higher order perturbative QCD correction, all of them attempt to include some nonperturbative effects to the potential, because it is sure that such effects must be taken into account. Richardson proposed a form [10]

$$\widetilde{U}(q^2) = -\frac{4}{3}\frac{12\pi}{(33-2N_f)}\frac{1}{q^2}\frac{1}{\ln(1+q^2/\Lambda^2)},$$

where q^2 is the momentum of the gluon exchanged between quarks and N_f represents the flavor number, while the linear confinement still remains unchanged as κr. It gives an effective correction which indeed involves the nonperturbative effects. Besides, Fulcher [7] gave

$$V(r) = Ar - \frac{8\pi}{(33-2N_f)}\frac{1}{r}f(\Lambda r),$$

where $f(\Lambda r)$ has a very complicated integration form, which can be found in Ref. [7]. Recently, Gupta *et al.* considered not only the higher order perturbative radiative corrections, but also a more complicated nonperturbative term [6]. In all these works, the corrections related to nonperturbative effects are phenomenologically put in according to the observation or hint from lattice gauge results. Some very good results which coincide with the data within an error of a few MeV were reported [6]. We will discuss this problem in some detail in the last section.

On the other hand, in this work we are trying to understand such corrections in terms of some well-established theories which can handle nonperturbative QCD in a more natural way.

Within the QCD scenario, where the nonvanishing vacuum condensates of quarks and gluons characterize the nonperturbative effects, the modified gluon propagator in momentum space can be written as [4]

$$G_{\mu\nu} = \frac{-i}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F(q^2), \tag{1}$$

where

$$F(q^2) = 1 + \frac{1}{3}g_s^2 \sum_{\beta=u,d,s} \frac{m_\beta \langle \psi_\beta \overline{\psi}_\beta \rangle}{q^2(q^2 - m_\beta^2)} + \frac{9}{32}g_s^2 \langle G^2 \rangle \frac{1}{q^4}. \tag{2}$$

In this expression we only keep the lowest dimensional condensates $\langle \psi_q \overline{\psi}_q \rangle$ and $\langle GG \rangle$. We derived this expression in the standard way [4], in which the normal product operators such as $\psi\overline{\psi}(0)$ and $G^2(0)$ have nonvanishing matrix elements in the physical vacuum; i.e., $\langle \psi\overline{\psi} \rangle$ and $\langle G^2 \rangle$ are left as parameters to describe nonperturbative effects, and their values have already been determined in the literature [2].

In another way, by comparing the $(2n+1)$-point Green's function (n is the number of external legs of ψ or A^μ) and the n-point Green's function with the insertion of the operators $\psi\overline{\psi}(0)$ or $G^2(0)$ (these Green's functions are with respect to the physical vacuum), Larsson achieved [9]

$$D_{\mu\nu} = \left[1 - \sum_\beta \frac{g_s^2 m_\beta \langle \psi_\beta \overline{\psi}_\beta \rangle}{q^2(q^2 + m_\beta^2)} + \frac{5 g_s^2 \langle G^2 \rangle}{288 q^4}\right]^{-1} \times \frac{(-i)}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right), \tag{3}$$

where the propagator was derived in Euclidean space. Expanding the first piece with respect to α_s, one notices that not only are the coefficients at the lowest order of α_s different from those in Eq. (2), but also the sign of the coefficient associated with the gluon condensate is different from that in Eq. (2). These differences arise from different approaches used to deal with nonperturbative effects and perhaps due to the the improper usage of the fixed point gauge [2] which violates translational invariance [11].

In this work, we also determine phenomenologically whether Eq. (2) or (3) is more consistent with the data.

We write down a proper scattering amplitude between two quarks as

$$M=(-ig_s)^2\overline{u_q}(p_1)\gamma_\mu\frac{\lambda^a}{2}u_q(p_1')D^{\mu\nu}(q^2)\overline{u_{q'}}(p_2)\gamma_\nu \times\frac{\lambda^a}{2}u_{q'}(p_2'), \quad (4)$$

with

$$p_1-p_1'=p_2'-p_2=q,$$

and carry out the Breit-Fermi expansion with the spinors $u_q(p_i)$ being the solutions of free quarks. In deriving an effective potential, the spontaneous approximation $q_0=0$ has been taken. It should be mentioned that this approximation is a traditional treatment in the literature although it is not quite valid in the potential derivation. Moreover, in Ref. [12], the term $q_\mu q_\nu$ was kept as $q_i q_j$ $(q_0=0)$. If the conserved vector current (CVC) theorem is respected, i.e., $q^\mu\overline{u_q}(p_1)\gamma_\mu u_q(p_1')=0$, the term $q_\mu q_\nu/q^2$ vanishes, and then the derived potential will have a small difference with that without CVC theorem, even though it is not very extravagantly apart. Then one can apply the three-dimensional Fourier transformation to convert the propagator in momentum space to coordinate space and derive an effective potential between quarks.

The potential derived in the way used in Ref. [4], in which leading order nonperturbative QCD effects are considered, together with the phenomenological linear confinement can be written in the form

$$V(r)=-\frac{4\alpha_s}{3r}+\kappa r+V_1^{\rm corr}(r)+V_2^{\rm corr}(r)+V_3^{\rm corr}(r),$$

where $V_1^{\rm corr}$ is the correction from the nontrivial physical vacuum condensates, while $V_2^{\rm corr}$ and $V_3^{\rm corr}$ are the Breit-Fermi corrections to the Coulomb and linear confinement terms, respectively. Because of the lengthy expressions for these potentials, we do not present them here. The explicit forms of these potentials can be found in the Appendix.

We note the following.

(1) In the previous literature, for instance Ref. [14], a comparison with the experimental data shows that the potential $\kappa r-4\alpha_s/3r$, with universal values of κ and α_s, is applicable to both J/ψ and Υ in solving the Schrödinger equation. Hence, this confining potential should be independent of the quark mass m_c or m_b. In other words, it corresponds to the potential in the limit $m_Q\to\infty$ $(m_Q\gg\Lambda_{\rm QCD})$. It is universal and exists in all $Q\overline{Q}$ systems. On the other hand, all correction terms associated with vacuum condensates are of the order of $1/m_Q^2$, which can be directly read from the Feynman diagrams. Therefore, they appear as mass-dependent corrections to $\kappa r-4\alpha_s/3r$. Consequently, there is no double counting involved in $\kappa r-4\alpha_s/3r$. Moreover, the Coulomb term $-4\alpha_s/3r$ comes from one-gluon exchange and represents a short distance effect where perturbative QCD works perfectly well. Therefore the correction induced by the condensates which manifest the nonperturbative effects does not overlap with the pure Coulomb part either. The κr term may be understood in the following picture. When m_Q is very large, one can define the total momentum k of the quark Q as $P_Q=m_Q v+\delta$ where Q is almost on the mass shell, v is the four-velocity of Q, and δ is the so-called residual momentum of the order of $\Lambda_{\rm QCD}$ [13]. From Eq. (4), one finds that in the large m_Q limit the emitted gluons are soft. This leads to the conclusion that the confinement term κr plays a role at the energy scale $\Lambda_{\rm QCD}$. In this picture, κr is independent of m_Q.

(2) Since all the correction terms associated with the vacuum condensates are proportional to $1/m_Q^2$, under this meaning, they are of the same order as the relativistic corrections in the Breit-Fermi expansion.

(3) Since the higher order terms are omitted, the derived potential is not appropriate in dealing with higher resonances.

(4) Our numerical results indicate that beyond the $2S$ state the calculated values would deviate from the data more and more, but for the $1S$, $1P$, and $2S$ states, these values indeed make sense (see below).

III. NUMERICAL RESULTS

In this work, to elucidate the significance of the correction, we present a few very typical quantities which are calculated in the framework of QCD.

The Cornell potential $-4\alpha_s/3r+\kappa r$ with corresponding parameters α_s and κ, [14] which gave the best fit to the J/ψ and Υ family data, is adopted as a basic condition, and the values of vacuum condensates are taken from Ref. [2]. Thus there are no free parameters at all in the derived expressions (2) and (3).

To our understanding, the term κr is universal to J/ψ and Υ and dominates the confinement part. This is consistent with the consideration in the literature which deals with nonperturbative corrections. Then one can consider the additional part from condensates as a $1/m_Q^2$ correction to the potential. It is noticed that, as discussed in most of the previous literature, one always assumed that κr was caused by a scalar exchange. As a consequence, it would not induce a spin splitting. On the other hand, the only term responsible for the spin splitting is the Coulomb term which contributes as a vector potential. It should be mentioned that this assumption was based on phenomenological requirements — i.e., for the Cornell potential, if κr was induced by scalar exchange, a better fit to the spin splitting data could be obtained. Now, the new correction induced by the condensates contributes to the spin splitting, and so the whole picture changes. In particular, this modification demands that κr come not only from scalar exchange, but also from vector exchange. Thus, we can write

$$\kappa r=\beta\kappa r+(1-\beta)\kappa r,\quad 0\leqslant\beta\leqslant 1, \tag{5}$$

where the factor β characterizes the fraction of the confinement potential which comes from vector exchange, while $(1-\beta)$ denotes that from scalar exchange. If κr is fully caused by the scalar exchange, then $\beta=0$ and it is the same assumption as in the previous literature. The explicit value of β can be fixed by data fitting. Then the spin splitting $\Delta=(M_{1^3S_1}-M_{1^1S_0})$ for $c\bar{c}$ and $b\bar{b}$ systems can be calculated. The correction term which contributes to the spin splitting can be read as

$$-\left(\frac{g^2}{4\pi}\right)\frac{\langle\lambda^a\lambda^a\rangle}{4}\left[\left(\frac{1}{6}\sum_\beta a_\beta\frac{1}{r}-\frac{1}{12}br-\frac{1}{6}\sum_\beta a_\beta\frac{e^{-m_\beta r}}{r}\right)+\frac{\pi}{3}c\,\delta(\vec{r})\right](\vec{\sigma}_1\cdot\vec{\sigma}_2), \tag{6}$$

where

$$c=\frac{2}{m_1m_2}+\sum_\beta\frac{A'_\beta}{m_\beta^3m_1m_2},\quad a_\beta=\frac{A'_\beta}{m_1m_2m_\beta},\quad b=\frac{B'}{m_1m_2},$$

$$A'_\beta=\begin{cases}\frac{g_s^2}{3}\langle\psi_\beta\bar{\psi}_\beta\rangle, & \text{Ref. [4]},\\ g_s^2\langle\psi_\beta\bar{\psi}_\beta\rangle, & \text{Ref. [9]},\end{cases}$$

$$B'=\begin{cases}\frac{9}{32}\,g_s^2\langle G^2\rangle, & \text{Ref. [4]},\\ -\frac{5}{288}g_s^2\langle G^2\rangle & \text{Ref. [9]},\end{cases}$$

and $m_1=m_2$ are the masses of the heavy quarks in quarkonium.

It is noted that the terms with a_β in Refs. [4] and [9] have the same sign, but the numerical value of this term in Ref. [9] is 3 times larger than that in Ref. [4]. Since the contribution from this term has the same sign (though $\langle\psi\bar{\psi}\rangle$) as that from the term with $\delta(\vec{r})$ ($\langle\psi\bar{\psi}\rangle>0$), it enhances the spin splitting between $1\,^1S_0(\langle\vec{\sigma}\cdot\vec{\sigma}\rangle=-3)$ and $1\,^3S_1(\langle\vec{\sigma}\cdot\vec{\sigma}\rangle=1)$. However, for the terms with b in Refs. [4] and [9], they have not only different numerical values, but also opposite signs. Therefore, the term with b given by Ref. [9] tends to increase the spin splitting, while the corresponding term in Ref. [4] reduces the spin splitting. Since

$$\left.\frac{(5/288)\langle G^2\rangle r^2}{\langle\psi_\beta\bar{\psi}_\beta\rangle/m_\beta}\right|_{r\sim 0.4\ \mathrm{fm}}\sim 0.016\left(\frac{\pi}{\alpha_s}\right)$$

is a small number, the existence of $\langle GG\rangle$ does not give rise to a more significant influence to the spin splitting than that of $\langle\psi_q\bar{\psi}_q\rangle$. Anyway, a measurement of the spin splitting between $1\,^1S_0$ and $1\,^3S_1$ tells us that the nonperturbative effects characterized by the nonvanishing vacuum condensates indeed play an important role and so manifest themselves in phenomenology.

The spin splitting of the $1\,^3P_J$ state is also influenced by the above-mentioned mentioned correction as well as the parameter β. The correction terms can be rewritten as

$$V_1^{\rm corr}(r)=V_1^{\rm cen}(r)+V_1^{\rm SO}(r)+V_1^{\rm ten}(r)+\cdots,$$
$$V_2^{\rm corr}(r)=V_2^{\rm cen}(r)+V_2^{\rm SO}(r)+V_2^{\rm ten}(r)+\cdots, \tag{7}$$
$$V_3^{\rm corr}(r)=V_3^{\rm SO}(r)+V_3^{\rm ten}(r),$$

with

$$V_3^{\rm SO}=\frac{1}{2m_Q^2}\left[3\frac{V'}{r}-\frac{S'}{r}\right],$$

$$V_3^{\rm ten}=\frac{1}{12m_Q^2}\left[\frac{V'}{r}-V''\right], \tag{8}$$

where the superscripts cen, SO, and ten denote the central, spin-orbit, and tensor parts of the corresponding $V_i^{\rm corr}(r)$, respectively, and V and S represent the vector and scalar parts of the confinement κr in Eq. (5), respectively. Then the spectrum of $1\,^3P_J$ states can be expressed as

$$M(1\,^3P_2)=\bar{M}+f-\frac{2}{5}g,$$
$$M(1^3p_1)=\bar{M}-f+2g, \tag{9}$$
$$M(1^3p_0)=\bar{M}-2f-4g,$$

where

$$f=\langle V_1^{\rm SO}(r)\rangle+\langle V_2^{\rm SO}(r)\rangle+\langle V_3^{\rm SO}(r)\rangle,$$
$$g=\langle V_1^{\rm ten}(r)\rangle+\langle V_2^{\rm ten}(r)\rangle+\langle V_3^{\rm ten}(r)\rangle, \tag{10}$$

and $\bar{M}$ is the weighted average for the $1\,^3P_J$ states.

It is noticed that in Eq. (10) the V and S terms are of opposite sign and the newly achieved correction term is opposite in sign to the Coulomb term. Then one may easily be convinced that to keep a good fit to data, the appearance of nonperturbative correction terms requires $\beta\neq 0$. In fact, Gupta *et al.* [6] also noticed that with a reasonable contribution from nonperturbative effects, the linear confinement κr must be a superposition of two parts as given in Eq. (5). As a result, they obtained an approximate β value of 0.25 with their model potential. Alternatively, with our modified potential, we have $\beta\approx 0.5$–0.6 for a better fit.

The numerical calculation is performed in the following way. The Cornell potential which is universal to c and b families, i.e., independent of m_Q, is considered as the domi-

TABLE I. (a) $c\bar{c}$ system. (b) $b\bar{b}$ system. In this table, $E_{20} \equiv M_{1\,^3p_2} - M_{1\,^3p_0}$, $E_{21} \equiv M_{1\,^3p_2} - M_{1\,^3p_1}$, $\Delta_{ss}^{(1)} \equiv M_{1\,^3s_1} - M_{1\,^1s_0}$ and $\Delta_{ss}^{(2)} \equiv M_{2\,^3s_1} - M_{2\,^1s_0}$. The experimental data are taken from 1994 Particle Data Group.

	Expt.	$V^{\text{Cornell}} = \frac{-4\alpha_s}{3r} + \kappa r$	$V^{\text{Cornell}} + V_2^{\text{corr}} + V_3^{\text{corr}} + V_1^{\text{corr}}$ (V_1^{corr} in Ref. [4]) $\beta = 0.6$	$V^{\text{Cornell}} + V_2^{\text{corr}} + V_3^{\text{corr}} + V_1^{\text{corr}}$ (V_1^{corr} in Ref. [9]) $\beta = 0.6$	$V^{\text{Cornell}} + V_2^{\text{corr}}$	$V^{\text{Cornell}} + V_2^{\text{corr}} + V_3^{\text{corr}}$ $\beta = 0.25$
			(a)			
$1\,^1s_0$	2978.8 ± 1.9	3074.0	2979.3	3026.6	3010.8	2999.5
$2\,^1s_0$	3594.0 ± 5.0	3662.1	3446.3	3676.0	3625.0	3618.4
$1\,^3s_1$	3096.88 ± 0.04	3074.0	3090.6	3157.7	3095.1	3098.8
$2\,^3s_1$	3686.00 ± 0.09	3662.1	3493.1	3754.0	3674.5	3676.7
$1\,^3p_0$	3415.1 ± 1.0		3321.2	3452.7	3440.8	3418.0
$1\,^3p_1$	3510.53 ± 0.12	$(1P_c)$3497.1	3395.1	3533.1	3489.3	3480.7
$1\,^3p_2$	3556.17 ± 0.13		3445.1	3605.8	3514.4	3527.5
E_{20}	141.07	0	123.9	153.1	73.6	109.4
E_{21}	45.64	0	50.0	72.7	25.1	46.7
$\Delta_{ss}^{(1)}$	118.08	0	111.3	131.1	84.3	99.3
$\Delta_{ss}^{(2)}$	92.0	0	46.8	78.0	49.5	58.3
			(b)			
$1\,^3s_1$	9460.37 ± 0.21	9427.0	9466.8	9503.8	9452.8	9453.7
$2\,^3s_1$	100023.30 ± 0.31	10007.0	9987.7	10073.0	10017.0	10017.0
1^3p_0	9859.8 ± 1.3		9843.6	9907.6	9865.8	9861.4
$1\,^3p_1$	9891.9 ± 0.7	$(1P_c)$9912.8	9886.1	9950.3	9902.5	9900.8
$1\,^3p_2$	9913.2 ± 0.6		9920.5	9985.9	9929.1	9931.7
E_{20}	53.4	0	76.9	78.3	63.3	70.3
E_{21}	21.3	0	34.4	35.6	26.6	30.9

nant part of the potential, and the corresponding parameters α_s and κ are determined before adding in the corrections. The values of α_s and κ are 0.381 and 0.182 GeV^2, respectively. And, then, the newly derived corrections due to the nonvanishing vacuum condensates as well as the Breit-Fermi corrections are treated as a perturbation adding onto the dominant part. The resultant values for the $c\bar{c}$ and $b\bar{b}$ systems are tabulated in Tables I(a) and I(b), respectively.

The decay width $\Gamma(J/\psi \to e^+e^-)$ is also a good test for various models. For $\Gamma(e^+e^-)$, the annihilation process is related to the zero-point wave function of J/ψ. Considering the QCD correction, we have

$$\Gamma(e^+e^-) = \Gamma_0(e^+e^-)(1 - 16\alpha_s/3\pi)$$

and

$$\Gamma_0(e^+e^-) = \frac{16\pi e_Q^2 \alpha^2}{M^2} |\phi(0)|^2,$$

where $\phi(0)$ is the zero-point value of the J/ψ wave function. In this scenario, the wave function $\phi(0)$ undergoes a modification due to the addition of the nonperturbative QCD terms. With expression (2) one has 4.85 keV, while with expression (3), 4.72 keV and the experimental data are 4.69 keV. Since the zero-point wave function is not sensitive to the new correction, the modified $\Gamma(e^+e^-)$ does not deviate far from that calculated directly with the Cornell potential.

IV. DISCUSSION AND CONCLUSION

The potential model is successful in explaining the hadronic spectra and other properties of heavy quarkonia. First, it was believed that quarks are confined inside hadrons; therefore, the potential must include a confinement term. The simplest form is the linear confinement kr. Besides, at short distance, where perturbative QCD works well, one-gluon exchange provides a Coulomb-type potential. By including these two extreme sides, the Cornell potential, in the form of $V(r) = -4\alpha_s/3r + \kappa r$, where α_s and κ are treated as free parameters, indeed gives reasonable results for both $c\bar{c}$ and $b\bar{b}$ families. However, there must be some nonperturbative effects which are not included in the linear term of the Cornell potential and they definitely make substantial contributions to the evaluation of spectra and other properties.

In the general approaches of Richardson, Fulcher, and others, the universal linear confinement κr was usually kept unchanged, but the simple propagator of the gluon, $(-i/q^2)(g_{\mu\nu}-q_\mu q_\nu/q^2)$, was modified by multiplying a q-dependent factor F which is model dependent. Usually, this factor was phenomenologically introduced based on some physical arguments or hints from the lattice calculations or obtained from higher order perturbative QCD corrections [6].

Along the other line [2], the QCD sum rules are also successful in explaining hadronic effects. It implies that there should be crossing between two lines. Thus the nonvanishing vacuum condensates which characterize the nonperturbative effects must be somewhat involved in the modification factor F and may be the dominant piece or at least an important one.

In this work, by including the effects of quark and gluon condensates, we derive the modified gluon propagator. Namely, a q-dependent factor F which is similar to that shown in the recent literature is obtained in the framework of QCD. There are no free parameters in the derived expressions.

It is important to notice that as pointed out by Shifman, this framework is an extrapolatation from short distances where perturbative QCD is reliable. Therefore, one cannot expect that this factor F can include as much as a purly phenomenological ansatz. But it does shed light on the physical picture and enrich our understanding of the physical mechanism which binds quarks into hadrons.

For fitting experimental data, it is required that the linear confinement come not only from the scalar exchange but also from the vector exchange. This is consistent with the model of Gupta *et al.* However, the value of β depends on the model. In Ref. [6], it is about 0.25, but in our case, it must be a value of 0.5–0.6; otherwise, the result is not meaningful. This indicates that the vector exchange gives a large contribution to the linear potential.

Our numerical results show that by considering the effects of the quark and gluon condensates, a more reasonable result, especially a better fit to the spin splitting E_{20}, E_{10}, and $\Delta_{ss}^{(1)}$, can be obtained. However, $\Delta_{ss}^{(2)}$ does not change in the right direction. The reason is that we only take the lower dimensional condensates $\langle q\bar{q}\rangle$ and $\langle G^2\rangle$ into consideration. For higher excited states, the interaction range becomes larger, we then have to extrapolate the scenario to the larger distance by introducing higher dimensional condensates such as $\langle q\bar{q}G\rangle$, $\langle GGG\rangle$, etc.

There is a discrepancy between Refs. [4] and [9] on the sign of the coefficient of term $\langle GG\rangle$. As we pointed above, maybe it is caused by using different methods or the fixed-point gauge. Unfortunately, the contribution from this term is small compared to that from $\langle q\bar{q}\rangle$; therefore, the numerical values calculated by using the formulas of Refs. [4] and [9] are not very far apart. We are going to pursue this problem based on both first principles and phenomenology in our next work.

In our evaluation, the closed-loop corrections are omitted since we only try to analyze the nonperturbative effect in the present paper. However, the closed-loop corrections are comparable with those from the quark and gluon condensates. Therefore, a complete analysis where both the condensate and loop contributions would be considered will be the aim of our next work. It will be helpful to clarify that the result as well as the method in Ref. [4] is better or those in Ref. [9] are more appropriate.

It should be mentioned that in this work we take the Cornell potential as the zeroth order approximation. Because of the fact that the wave function at the origin depends on the model sensitively, it will be interesting to choose other models as the zeroth order approximation and compare the calculated results. This will also be done in our next work.

Finally, nonperturbative effects are considered by a using perturbative treatment. Although it is simple, sometimes it does not work well. Since our purpose is to illustrate the effects of nonperturbative QCD effects, it would be not a serious issue. It is also interesting to employ other ways such as the variational method adopted by Gupta *et al.* to carry out again the analysis. It deserves further study.

ACKNOWLEDGMENTS

One of the authers (P.S.) would like to thank Professor A.W. Thomas for valuable suggestions and kind hospitality during his stay in the Department of Physics and Mathematics, University of Adelaide. This work was partly supported by the National Natural Science Foundation of China (NSFC) and the Australian Research Council.

APPENDIX

By using the method mentioned in the text and Ref. [4], the quark-quark potential is rederived. The explicit expression of the revised quark-quark potential can be written as

$$U(\vec{p}_1,\vec{p}_2,\vec{r})=\frac{g^2}{4\pi}\frac{(\lambda_1^a\cdot\lambda_2^{4a})}{4}\Bigg\{A_1\delta(\vec{r})+A_2\frac{1}{r}+A_3r+A_4r^3+A_{5\beta}\frac{e^{-m_\beta r}}{r}+\left[A_6\frac{1}{r^3}+A_7\frac{1}{r}+A_8r+A_{8\beta}\frac{m_\beta^2r^2+3m_\beta r+3}{r^3}e^{-m_\beta r}\right]$$
$$\times S_{12}+\left[A_9\frac{1}{r^3}+A_{10}\frac{1}{r}+A_{11}r+A_{11\beta}\frac{m_\beta r+1}{r^3}e^{-m_\beta r}\right]\cdot[\vec{f}_1\cdot(\vec{r}\times\vec{p}_1)-\vec{f}_2\cdot(\vec{r}\times\vec{p}_2)]+p\text{-dependent term}\Bigg\},$$

with

$$A_1=-\pi\left[\frac{1}{2m_1^2}+\frac{1}{2m_2^2}-\frac{1}{m_1m_2}+\left(\frac{2}{3m_1m_2}+\frac{A'_\beta}{3m_\beta^3m_1m_2}\right)\times(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right],$$

$$A_2=1-\frac{A'_\beta}{m_\beta}\left[\frac{1}{8m_1^2}+\frac{1}{8m_2^2}-\frac{1}{4m_1m_2}+\frac{1}{m_\beta^2}+\frac{1}{6m_1m_2}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\right],$$

$$A_3=-\left[\frac{A'_\beta}{2m_\beta}-B'\left(\frac{1}{16m_1^2}+\frac{1}{16m_2^2}-\frac{1}{8m_1m_2}+\frac{(\vec{\sigma}_1\cdot\vec{\sigma}_2}{12m_1m_2}\right)\right],$$

$$A_4=\frac{1}{24}B',$$

$$A_{5\beta}=\frac{A'_\beta}{m_\beta}\left[\frac{1}{8m_1^2}+\frac{1}{8m_2^2}-\frac{1}{4m_1m_2}+\frac{1}{m_\beta^2}+\frac{(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{6m_1m_2}\right],$$

$$A_6=-\left[\frac{1}{4m_1m_2}-\frac{A'_\beta}{4m_1m_2m_\beta^3}\right],$$

$$A_7=-\frac{A'_\beta}{24m_1m_2m_\beta},$$

$$A_8=-\frac{1}{96m_1m_2}b',$$

$$A_{8\beta}=-\frac{A'_\beta}{12m_1m_2m_\beta^3},$$

$$A_9=-\left[1-\frac{A'_\beta}{m_\beta^3}\right],$$

$$A_{10}=\frac{A'_\beta}{2m_\beta},$$

$$A_{11}=\frac{1}{8}B',$$

$$A_{11\beta}=-\frac{A'_\beta}{m_\beta^3},$$

where the summation over β is implied and the expressions of A'_β and B' are given in the text.

[1] W. Lucha, F. Schoberl, and D. Gromes, Phys. Rep. **200**, 127 (1991); E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980); H. Suura *et al.*, *ibid.* **21**, 3204 (1980); J. Richardson, Phys. Lett. **82B**, 272 (1979); T. Liu, Z. Chen, and T. Huang, Z. Phys. C **46**, 133 (1990); G. Bhanot and S. Rudaz, Phys. Lett. **78B**, 119 (1979).
[2] M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B147**, 385 (1979); V. Shuryak, Phys. Rep. **115**, 151 (1984).
[3] M. Shifman, in *QCD—20 Years Later*, Proceedings of the workshop, Aachen, Germany, 1992, edited by P. M. Zerwas and H. A. Kastrup (World Scientific, Singapore, 1993).
[4] P. Shen, X. Li, and X. Guo, Phys. Rev. C **45**, 1894 (1992); J. Bian and T. Huang, Commun. Theor. Phys. **16**, 337 (1991); J. Liu *et al.*, Phys. Rev. D **49**, 3474 (1994).
[5] S. Gupta, S. Radford, and W. Repko, Phys. Rev. D **26**, 3305 (1982); S. Gupta and S. Radford, *ibid.* **24**, 2309 (1981); **39**, 974 (1989).
[6] S. Gupta *et al.*, Phys. Rev. D **49**, 1551 (1994).
[7] L. Fulcher, Phys. Rev. D **42**, 2337 (1990); **44**, 2079 (1991).
[8] J. Pantaleone, S. Tye, and Y. J. Ng, Phys. Rev. D **33**, 777 (1986).
[9] T. Larsson, Phys. Rev. D **32**, 956 (1985).
[10] J. Richardson, Phys. Lett. **26**, 272 (1979); Y. Ding, Z. Chen, and T. Huang, Phys. Lett. B **196**, 191 (1987); T. Liu, Z. Chen, and T. Huang, Z. Phys. C **46**, 133 (1990).
[11] X. Henz, presented at the international symposium on Quark-Gluon-Plasma, Wuhan, China, 1994 (unpublished).
[12] L. D. Landau and E. M. Lifshitz, *Quantum Electrodynamics* (Pergamon Press, New York, 1982).
[13] H. Georgi, Phys. Lett. B **264**, 447 (1991).
[14] D. B. Lichtenberg, R. Roncaglia, J. G. Wills, E. Predazzi, and M. Rosso, Z. Phys. C **46**, 75 (1990).