Heavy quark potential and effective actions on blocked configurations

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Blocked SU(3) gauge configurations are analyzed to obtain a heavy quark potential and effective actions. Swendsen's optimized scale factor $b=2$ blocking scheme is used to generate blocked configurations. The heavy quark potential calculated on twice blocked configurations produced from $32^3\times 64$ lattices at $\beta=6.30$ shows good rotational invariance in contrast with the Wilson action. The determination of effective actions which are responsible for blocked configurations is carried out, in a space with up to nine coupling constants, by use of the canonical demon method. We find that the effective actions are not local. Apart from complicated actions, simple actions with two coupling constants are compared. [S0556-2821(96)02713-0]

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In lattice Monte Carlo simulations, improvements on the action are crucial in order to obtain accurate and reliable continuum results within reasonable CPU time. The history of improved actions is rather old. More than 10 years ago, Symanzik proposed the perturbative improved action [1] which eliminates errors of $O(a^2)$. On the other hand, early Monte Carlo tests on the Symanzik action showed no clear advantage to using it [2]. However, recent studies revitalized improved actions and offered the fixed point action [3] and the tadpole-improved Symanzik action [4]. These actions show good rotational invariance. Therefore they could be effective when used in Monte Carlo simulations. Their actual effectiveness for physical quantities has been investigated by Monte Carlo simulations [3–5].

It is still important to search for other improved actions which give us great improvements in Monte Carlo simulations since the effectiveness of existing improved actions is not fully confirmed. In this report, we analyze blocked SU(3) gauge configurations and try to obtain improved actions. The blocked configurations are generated by using the Monte Carlo renormalization group (MCRG) method [6] with Swendsen's optimized scale factor $b=2$ blocking scheme. The MCRG method was intensively used in determining the nonperturbative β function of SU(3) lattice gauge theory, in terms of the coupling shift $\Delta\beta$ [7–10]. The method uses the fact that each blocked trajectory (BT), starting from the Wilson axis, reaches the renormalized trajectory (RT) after enough blocking steps. On the RT, which stems from the fixed point on the critical surface, the gauge system has no lattice artifact [11]. Performing enough blocking steps or using the optimized blocking scheme which takes a blocked gauge system to the RT quickly, it may be possible to generate blocked configurations sitting on the RT. Such blocked configurations—let us call them *perfect configurations*—can give us information about a perfect action possessing good scaling behavior and rotational invariance. Even if the blocking is not enough to let the BT reach the RT, the blocked gauge system near the RT should have better behavior than the original (Wilson action) unblocked one.

In order to check the improvement by the MCRG method, we calculate the heavy quark potential on blocked configurations. The blocking scheme we take in the MCRG method is Swendsen's optimized scale factor $b=2$ blocking scheme [6,10]. The blocking scheme was optimized by multiplying a blocked link by a Gaussian SU(3) matrix, $\exp(i\Sigma_l\lambda_l c_l)$, where λ_l is a generator of SU(3) and c_l is a Gaussian random number generated with a probability $P(c_l)\sim\exp(-c_l^2/q^2)$. The parameter q can be adjusted to ensure fast convergence to the RT [10,11]. We found that the optimal value of q around $\beta=6.0$ is equal to zero [12]; i.e., in this case the Gaussian SU(3) matrix $\exp(i\Sigma_l\lambda_l c_l)$ is always unity. In order to fix the blocking scheme we take $q=0$ in our analysis. All the blocked configurations analyzed in this report were generated by the QCD-TARO Collaboration [10,13].

Figure 1 shows the heavy quark potential on $8^3\times 16$ blocked configurations. The blocked configurations were generated by two blocking steps from $32^3\times 64$ lattice at $\beta=6.30$. The solid curve is a fit to on-axis potential indicated by the circles. There is no visible difference between on-axis and off-axis potentials. From the fit to on-axis potential we obtain $\sigma a^2=0.287(26)$ which corresponds to the value of the string tension at $\beta\sim 5.56$ for the Wilson action. This is in very good agreement with the $\Delta\beta$ results [10] which predict $\Delta\beta\sim 0.75$ for a change of scale by 4, starting from $\beta=6.30$.

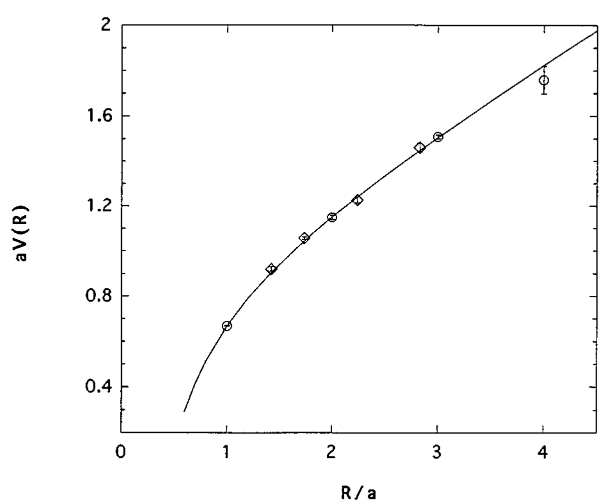

FIG. 1. Heavy quark potential on blocked configurations ($8^3\times 16$ lattice) at $\beta=6.3$. The circles (diamonds) correspond to on (off-) axis potentials. The solid curve is a fit to the on-axis potentials.

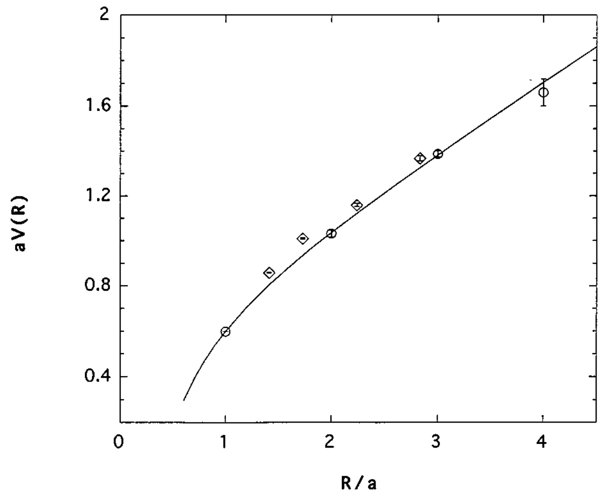

FIG. 2. Same as in Fig. 1, but for the Wilson action at $\beta=5.55$.

In order to evaluate the improvement we also calculate the heavy quark potential for the Wilson action at $\beta=5.55$. Figure 2 shows the heavy quark potential calculated on $8^3\times16$ lattices. The solid curve is a fit to on-axis potentials, which give us $\sigma a^2=0.300(27)$. A large deviation from the on-axis potentials to the off-axis ones can be seen, which is evidence of lack of rotational invariance. The definite improvement of the rotational invariance would indicate that already after two blocking steps the gauge system of our blocked configurations is close to the RT.

We now turn to the question of obtaining effective actions from the blocked configurations. Once the effective action is obtained it can be used, as an improved action, in Monte Carlo simulations to directly generate improved gauge configurations without blocking from larger lattices. However, determination of an effective action is quite a hard task since we do not know the exact form of the effective action and it might be a complicated one. In determining the effective action, we use the canonical demon method which is shown to be efficient [14,15]. Originally Creutz [17] proposed the microcanonical demon method and it was utilized for determination of an effective action for SU(2) gauge theory [16]. An improved version, the canonical demon method, was discussed by Hasenbush *et al.* [14] who determined effective actions for O(3) nonlinear σ model with accuracy. It was also applied for SU(3) gauge theory and effective actions were successfully obtained [15].

For our purpose we prepared 35(30) configurations blocked twice from $32^3\times64$ lattice at $\beta=6.0(6.3)$. Each of those configurations are used in the canonical demon method. Our implementation of the canonical demon method is briefly described as follows (see Refs. [14–17] for more details). First we choose one of those configurations and introduce demons associated with coupling constants corresponding to a given ansatz for the effective action. The joint system of demons and links is updated by the microcanonical simulation. After 100 microcanonical sweeps the demons move into the next configuration selected from the rest. Values of the demons energy are recorded during the simulation and average values will be converted to the values of the coupling constants. Here let us assume that the effective action of the configurations takes the form

TABLE I. Path of Wilson loops.

Wilson loop	Path $(\mu\neq\nu\neq\sigma\neq\gamma)$
1×1	$\mu,\nu,-\mu,-\nu$
1×2	$\mu,\nu,\nu,-\mu,-\nu,-\nu$
2×2	$\mu,\mu,\nu,\nu,-\mu,-\mu,-\nu,-\nu$
Chair	$\mu,\nu,\sigma,-\nu,-\mu,-\sigma$
Sofa	$\mu,\nu,\sigma,\sigma,-\nu,-\mu,-\sigma,-\sigma$
Twist	$\mu,\nu,\sigma,-\mu,-\nu,-\sigma$
$4D$twist	$\mu,\nu,\sigma,\gamma,-\mu,-\nu,-\sigma,-\gamma$

$$S=\sum_i \beta_i S_i[U]. \tag{1}$$

In this case demons d_i corresponding to each of the coupling constants β_i should be introduced. The distribution of the demon energy E_{d_i} during the canonical demon simulation is expected to be

$$P(E_{d_i})\sim\exp(-\beta_i E_{d_i}). \tag{2}$$

Thus the average demon energy is given by

$$\langle E_{d_i}\rangle=\frac{1}{\beta_i}-E_m/\tanh(\beta_i E_m), \tag{3}$$

provided that the demon energy E_{d_i} is restricted to a region of $-E_m<E_{d_i}<E_m$. This relation will be used to calculate the values of the coupling constants numerically. In our simulation E_m was set to 10.

We consider the following ansatz action for an effective action with nine coupling constants:

$$S=\mathrm{Re}\Big(\beta_{1\times1}\sum \mathrm{Tr}U_{1\times1}+\beta_{1\times2}\sum \mathrm{Tr}U_{1\times2}+\beta_{2\times2}\sum \mathrm{Tr}U_{2\times2}+\beta_{\mathrm{chair}}\sum \mathrm{Tr}U_{\mathrm{chair}}+\beta_{\mathrm{sofa}}\sum \mathrm{Tr}U_{\mathrm{sofa}}+\beta_{\mathrm{twist}}\sum \mathrm{Tr}U_{\mathrm{twist}}$$
$$+\beta_{4D\mathrm{twist}}\sum \mathrm{Tr}U_{4D\mathrm{twist}}+\beta_6\sum\left[\tfrac{3}{2}(\mathrm{Tr}U_{1\times1})^2-\tfrac{1}{2}\mathrm{Tr}U_{1\times1}\right]+\beta_A\sum\left[\tfrac{9}{8}|\mathrm{Tr}U_{1\times1}|^2-\tfrac{1}{8}\right]\Big), \tag{4}$$

TABLE II. Results for twice-blocked configurations at $\beta=6.0$. The asterisks indicate truncation, i.e., those not considered in the canonical demon simulation.

Case	A	B	C	D	E	F
$\beta_{1\times1}$	5.0643(35)	6.1564(53)	6.036(14)	6.484(16)	6.282(12)	6.619(11)
$\beta_{1\times2}$	*	-0.6241(23)	-0.6547(30)	-0.6249(32)	-0.7257(37)	-0.7236(32)
$\beta_{\rm chair}$	*	*	-0.0711(19)	*	-0.1055(17)	-0.1079(14)
$\beta_{\rm twist}$	*	*	0.3004(23)	*	0.2103(38)	0.2087(27)
$\beta_{2\times2}$	*	*	*	*	0.0835(32)	0.0847(37)
$\beta_{\rm sofa}$	*	*	*	*	0.0327(16)	0.0319(10)
$\beta_{4D\rm twist}$	*	*	*	*	0.1058(08)	0.1065(11)
β_6	*	*	*	-0.339(12)	*	-0.325(13)
β_A	*	*	*	-0.125(27)	*	-0.143(15)

where $\mathrm{Tr}U_i$ are normalized to unity and $i=\{1\times1, 1\times2, \ldots, 4D\mathrm{twist}\}$ indicate Wilson loop types whose paths are summarized in Table I. For this action nine demons are used in the canonical demon method. The results obtained with the canonical demon method are listed in Table II for $\beta=6.0$ and in Table III for $\beta=6.3$. We also consider several forms of the action truncated from Eq. (4). The coupling constants indicated by a asterisk in the tables are truncated, i.e., not used in the canonical demon simulation. In general it is expected that starting from the smallest Wilson loop (1×1) and adding larger Wilson loops we could see smaller values of the coupling constants for the larger Wilson loops. If in the adding process the coupling constants corresponding to the larger Wilson loops have no contribution to the effective action, the values of the corresponding coupling constants obtained with the canonical demon method should be negligibly small. We find out, however, that even for Wilson loops with eight links (2×2 and sofa), the corresponding coupling constants are still rather large. This implies that the effective action of the blocked configurations we took here is not local.

In order to obtain the complete effective action which reproduces the blocked gauge configurations, it would be necessary to enlarge the coupling constant space. However, we are not interested in such a complicated action since it may be useless in Monte Carlo simulations; i.e., one needs not only much computational time but also intricate programming skills. Recent studies showed that rather simple actions, such as Iwasaki's two couplings action, can greatly improve unwanted behavior in the deconfinement transition with the dynamical Wilson fermion [18]. Similar improvements on finite temperature physics are also seen for the Symanzik improved actions [20]. It might be interesting to see if the improvement is achieved only by the interplay of the dominant coupling constants.

In Table IV, we summarize the ratio between dominant two couplings (i.e., 1×1 and 1×2) for various improved actions. ''SY'' means the tree level Symanzik action [1], which has the form

$$S=\beta\sum \mathrm{ReTr}\left(\tfrac{5}{3}U_{1\times1}-\tfrac{1}{12}U_{1\times2}\right). \tag{5}$$

''TAD-1(2)'' is the tadpole-improved *tree-level* Symanzik action [4], which corrects the coefficient of the 1×2 loop coupling constant as

$$S=\beta\sum \mathrm{ReTr}\left(\frac{5}{3}U_{1\times1}-\frac{1}{12\langle U_{1\times1}\rangle^{1/2}}U_{1\times2}\right), \tag{6}$$

where $\langle U_{1\times1}\rangle$ is the average value of the plaquette. For TAD-1(2), we use the average values of the plaquette on the blocked configurations, which are 0.5407 (0.4103) at $\beta=6.3$ (6.0). ''IW'' is the renormalization-group-improved action by Iwasaki [14] and ''MCRG-1(2)'' are taken from our study. For all the cases the $\beta_{1\times2}$ coupling constant is negative and it contributes (5–11)% of the $\beta_{1\times1}$ coupling constant. Compared to the tree-level Symanzik-improved action, which gives $\beta_{1\times2}/\beta_{1\times1}=-0.05$, the others which may include some nonperturbative effects seem to prefer a larger percentage, especially, our results are more than 10%. Moreover, note that the magnitude of the ratio in-

TABLE III. Same as in Table I, but at $\beta=6.3$.

Case	A	B	C	D	E	F
$\beta_{1\times1}$	5.6804(37)	7.986(13)	8.652(20)	8.464(15)	9.443(29)	9.979(21)
$\beta_{1\times2}$	*	-0.9169(41)	-0.9241(41)	-0.9226(32)	-1.1698(31)	-1.1627(47)
$\beta_{\rm chair}$	*	*	-0.1648(38)	*	-0.2213(45)	-0.2329(37)
$\beta_{\rm twist}$	*	*	0.2907(62)	*	0.1108(54)	0.1195(66)
$\beta_{2\times2}$	*	*	*	*	0.1603(73)	0.1606(49)
$\beta_{\rm sofa}$	*	*	*	*	0.0516(20)	0.0524(11)
$\beta_{4D\rm twist}$	*	*	*	*	0.0987(21)	0.0972(21)
β_6	*	*	*	-0.357(70)	*	-0.368(35)
β_A	*	*	*	-0.150(72)	*	-0.140(68)

TABLE IV. Ratio of $\beta_{1\times 2}$ and $\beta_{1\times 1}$. MCRG-1(2) is taken from case B in Table II(III).

	SY	TAD-1	TAD-2	IW	MCRG-1	MCRG-2
$\beta_{1\times 2}/\beta_{1\times 1}$	-0.05	-0.068	-0.078	-0.091	-0.10	-0.11

creases from MCRG-1 to MCRG-2, even though β increases. This would show that nonperturbative effects still dominate in the region we are studying.

There exists another possibility to improve gauge system, which is to directly use the blocked configurations themselves for measurements of physical quantities. As we have seen in Fig. 1, rotational invariance is quite well recovered on the blocked configurations, which means that the gauge system of the blocked configurations has less artifact. The hadron spectroscopy using the same blocked configurations we analyzed here has been already performed [21] and it was shown that the hadron masses are in reasonable agreement with ones calculated on fine configurations when an appropriate improved fermion action is used.

In summary, we analyzed the blocked gauge configurations and then obtained the heavy quark potential and the effective actions. The heavy quark potential showed good rotational invariance. Although the several forms of the effective action were obtained with the canonical demon method, they only represent truncated forms of the effective action. However, we think such truncated effective actions still hold some improvements when used in Monte Carlo simulations.

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