M theory on $(K3\times S^1)/Z_2$

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We analyze M theory compactified on $(K3\times S^1)/Z_2$ where the Z_2 changes the sign of the three-form gauge field, acts on S^1 as a parity transformation, and on $K3$ as an involution with eight fixed points preserving SU(2) holonomy. At a generic point in the moduli space the resulting theory has as its low-energy limit $N=1$ supergravity theory in six dimensions with eight vector, nine tensor, and 20 hypermultiplets. The gauge symmetry can be enhanced (e.g., to E_8) at special points in the moduli space. At other special points in the moduli space tensionless strings appear in the theory. [S0556-2821(96)50412-1]

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During the past year it has been realized that the moduli space of string theories has a special point where the low-energy dynamics is governed by the $N=1$ supergravity theory in eleven dimensions [1–3]. This theory has subsequently been called the M theory, and it has been shown that some of the hidden symmetries of string theory become apparent when we view it as M-theory compactification on tori [4,5]. More general compactifications of this theory on orbifolds of S^1 [6] and of more general tori [7,8] have also been studied and have been shown to be dual to known string compactifications.

One of the main problems in studying M theory on orbifolds is that *a priori* there is no well-defined rule for computing the spectrum of twisted states, and one always has to rely on the requirement of anomaly cancellation to determine the massless spectrum from the twisted sector. For theories with a high number of supersymmetries the spectrum of massless states is fixed more or less uniquely by the requirement of supersymmetry and anomaly cancellation. This feature has been exploited to determine the spectrum of massless states in M theory from the twisted sector.

In this paper we shall focus on M theory on a Z_2 orbifold of $K3\times S^1$. The Z_2 changes the sign of the three form field C_{MNP}, acts as a parity transformation on S^1, and acts as an involution on $K3$—the same involution used in Ref. [9] to construct the dual of Chaudhuri-Hockney-Lykken (CHL) strings [10]. Even though the spectrum of massless states in this theory is not completely fixed by the requirement of anomaly cancellation, we shall be able to derive the spectrum by comparison with known results about M-theory compactification on T^5/Z_2. The result will be an $N=1$ supergravity theory in six dimensions with nine tensor multiplets, eight vector multiplets, and twenty hypermultiplets, a theory for which the anomalies cancel automatically [11–13].

Let us denote by τ the part of the Z_2 action that changes the sign of C_{MNP} and acts as a parity transformation on S^1, and by σ the involution on $K3$ that forms part of the Z_2 action. Thus the Z_2 symmetry is given by the product of τ and σ. The involution σ preserves SU(2) holonomy and has eight fixed points. Its action on the lattice of signature (3,19), representing the second cohomology elements of $K3$, exchanges the two E_8 factors in the lattice, leaving the (3,3) part invariant [14]. Thus it has eight negative and fourteen positive eigenvalues [9]. In particular it leaves the harmonic (0,2) and (2,0) forms, as well as twelve of the harmonic (1,1) forms invariant, and changes the sign of the other eight harmonic (1,1) forms. An example of such a $K3$ surface is given by the hypersurface [9,15] $\Sigma_{i=1}^4 (z_i)^4=0$ in CP^3, where z_i denote the homogeneous coordinates on CP^3. The involution σ is given by

$$z_1\to -z_1, \quad z_2\to -z_2, \quad z_3\to z_3, \quad z_4\to z_4. \qquad (1)$$

The eight fixed points are at

$$z_1=z_2=0, \quad (z_3/z_4)=\exp(2\pi ik/4), \quad k\in Z,$$
$$z_3=z_4=0, \quad (z_1/z_2)=\exp(2\pi ik/4), \quad k\in Z. \qquad (2)$$

Using the Lifschitz fixed-point theorem one can verify that this involution leaves fourteen of the harmonic two forms fixed while changing the sign of the other eight [9].

After we mod out by this Z_2 symmetry, the resulting theory has $N=1$ supersymmetry in six dimensions. The condition of anomaly cancellation by itself is not powerful enough to determine the spectrum of massless states in the theory completely, so we need to rely on some other principle for deriving the spectrum of massless states in the twisted sector. The principle that we shall adopt will be the following. Note that under the Z_2 transformation the manifold $S^1\times K3$ has sixteen fixed points—two from S^1 combining with the eight from $K3$. Near each of these fixed points the space looks like R^5/Z_2 where the Z_2 changes the sign of all the five coordinates. This is exactly the same structure that appears at the orbifold points of T^5/Z_2. Thus we would expect that the physics near these fixed points in $(K3\times S^1)/Z_2$ will be identical to the physics near the fixed points of T^5/Z_2. The latter case has already been analyzed in Refs. [7,8]. In particular it was shown in Ref. [8] that each of the fixed points acts as a source of $-1/2$ unit of magnetic three-form charge. The total magnetic charge is cancelled by

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putting $n_F/2$ five-branes moving on the internal manifold where n_F is the total number of fixed points. Each of these five-branes in turn gives rise to a massless tensor multiplet of the chiral $N=2$ supersymmetry algebra in six dimensions, which corresponds to a tensor multiplet and a hypermultiplet of the $N=1$ supersymmetry algebra. Since here $n_F=16$, we conclude that the massless spectrum from the twisted sector corresponds to eight tensor multiplets and eight hypermultiplets.[1] Following [8] we can also give a physical interpretation of the scalars coming from the hypermultiplet and tensor multiplets. The scalars in the tensor multiplet represent the location of the five-branes along S^1, whereas those in the hypermultiplet represent the location of the five-branes along $K3$.[2]

Having thus derived the spectrum of massless states in the twisted sector, we now turn to the easier task of deriving the spectrum of massless states in the untwisted sector. Let us denote by μ,ν the six noncompact coordinates ($0\leqslant\mu$, $\nu\leqslant 5$), by m,n the coordinates on $K3$ ($6\leqslant m$, $n\leqslant 9$) and by 10 the coordinate on S^1. Then we get one more tensor multiplet associated with the antiselfdual component of $C_{(10)\mu\nu}$ (the self-dual component being part of the gravity multiplet), and eight vector multiplets associated with $C_{mn\mu}$ with (mn) denoting one of the eight two cycles on $K3$ that are odd under σ. (Note that C_{MNP} changes sign under the Z_2 involution). Finally we get 49 scalars [34 from the moduli of σ-invariant $K3$, one from the radius of S^1, and 14 from $C_{(10)mn}$ with (mn) denoting a two cycle of $K3$ even under σ.] One of these scalars is part of the tensor multiplet coming from the untwisted sector, the others form 12 hypermultiplets of the $N=1$ supersymmetry algebra. The particular combination of scalars that becomes part of the tensor multiplet coming from the untwisted sector is given by $\lambda=\sqrt{V/R}$ where V is the volume of $K3$ and R is the radius of S^1.

Combining the spectrum from the twisted and the untwisted sectors we see that the massless sector corresponds to an $N=1$ supergravity theory in six dimensions with nine tensor multiplets, eight vector multiplets, and 20 hypermultiplets. It can easily be seen that this theory is anomaly free [12,13], which is a reflection of the fact that M theory has no gravitational anomaly on a smooth manifold, and we have ensured that there is no anomaly from the fixed points by putting $-1/2$ unit of magnetic charge at each of the fixed points [8]. Thus, this provides another example of a consistent M theory compactification. In fact the analysis based on M theory provides us with a nice geometrical picture of the hyper- and tensor-multiplet moduli spaces. The scalars in 12 of the 20 hypermultiplets label the moduli space of σ-invariant $K3$ together with background values of $C_{(10)mn}$ on cycles of $K3$ that are even under σ, those in the other eight hypermultiplets label the location of the eight five-branes on $K3$, and the scalars in the eight tensor multiplets coming from the twisted sector label the locations of the eight five-branes on S^1.

Note that at a generic point in the moduli space there are no massless charged states. There are however massive charged states that arise from membranes wrapped around the two cycles of $K3$. It is certainly possible that they could become massless at special points in the moduli space, but it would be difficult to predict such events directly, since the spectrum of states in this theory is not controlled by Bogomol'nyi bound. However, before the Z_2 projection, the theory had $N=2$ supersymmetry and hence BPS states, so we can first study the spectrum in this theory and then project into Z_2 invariant states. As we shall see, this provides a fruitful approach to determining points of enhanced gauge symmetries in this theory.

In carrying out this analysis, we shall make use of known dualities between the M theory compactification on $K3\times S^1$ and heterotic compactification on T^4 [3]. For our purpose it will be most convenient to make this duality map in two stages: first regard M theory on S^1 as a type IIA theory, and then use the duality between type IIA theory on $K3$ and the heterotic string theory on T^4. In this language, the transformation τ corresponds to the $(-1)^{F_L}$ symmetry of the type IIA theory, which in turn corresponds to a total inversion of the (4,20) lattice of charges in the heterotic string theory [21]. On the other hand the transformation σ acts by exchanging the two E_8 factors in the charge lattice of the heterotic string theory [9]. Thus if we denote a vector in the charge lattice of the heterotic string theory by $(\vec{k},\vec{k}_1,\vec{k}_2)$, where $\vec{k}$ is an eight-dimensional vector representing momentum and winding in the compact direction, and $\vec{k}_1$ and $\vec{k}_2$ are vectors in the two E_8 root lattices respectively, the net effect of the Z_2 action will be the map $(\vec{k},\vec{k}_1,\vec{k}_2)\to(-\vec{k},-\vec{k}_2,-\vec{k}_1)$. Now consider being at a special point in the moduli space of heterotic string theory on T^4 where the $E_8\times E_8$ gauge symmetry is unbroken. In this case we can get massless gauge bosons invariant under Z_2 by taking the combination

$$|(\vec{0},\vec{\alpha},\vec{0})\rangle_V+|(\vec{0},\vec{0},-\vec{\alpha})\rangle_V, \tag{3}$$

where $\vec{\alpha}$ is a root vector of E_8, and the subscript V denotes that we are considering space-time vectors. We also get Z_2 invariant massless charged scalars by considering the combination

$$|(\vec{0},\vec{\alpha},\vec{0})\rangle_S-|(\vec{0},\vec{0},-\vec{\alpha})\rangle_S. \tag{4}$$

The extra minus sign compensates for the fact that the vertex operator for the scalar has a component proportional to one of the right moving internal currents that are odd under Z_2. Similarly we can get massless charged spinors states.

Although we have constructed these states in heterotic string theory on T^4, by the chain of dualities, these states

[1]Note that conventional heterotic string compactification on $K3$ only gives theories with one tensor multiplet arising from the antisymmetric tensor field. But supergravity theories with more than one tensor multiplet have been considered in the past [16–18]. The possibility of getting extra tensor multiplets in six-dimensional theories by putting five-branes moving on internal manifolds has already been discussed in Ref. [19]. Here we have an explicit realization of this idea. Other related six-dimensional string compactification has been discussed in Ref. [20].

[2]Since the Z_2 identification acts simultaneously on S^1 and on $K3$, we see that the tensor- and the hyper-multiplet moduli spaces do not factorize globally.

must also exist in M theory on $K3\times S^1$ at appropriate points in the moduli space, and are Z_2 invariant there as well.[3] Thus when we make the Z_2 projection, these states are expected to survive, and will give us massless states in the resulting theory. (As we shall argue later, quantum corrections do not make these states massive.) In particular the massless charged vector states, together with appropriate fermionic states, and the eight U(1) vector multiplets, give us the E_8 gauge multiplet. On the other hand, the massless charged scalars, together with appropriate spinors, and eight neutral hypermultiplets associated with Z_2 invariant $E_8\times E_8$ Wilson lines (in the language of heterotic string theory), will form a hypermultiplet in the adjoint representation of E_8. Thus the resulting theory will have nine tensor multiplets, an E_8 vector multiplet, a hypermultiplet in the adjoint representation of E_8, and 12 neutral hypermultiplets as its massless spectrum.

Are there new massless charged states coming from the twisted sector? To answer this question, we need to look at the geometry of $K3$ near the enhanced symmetry points. As has been argued in Refs. [3,22,23], in M theory on $K3\times S^1$, enhanced gauge symmetries occur at points in the $K3$ moduli space where some two cycles collapse to zero size. In particular, the enhanced $E_8\times E_8$ gauge symmetry will occur at a point where two sets of nonintersecting two cycles, one associated with the first E_8, and the other associated with the second E_8, collapse to zero size. The action of the involution σ does not leave any of these two sets of cycles invariant, but exchanges them. As a result the fixed-point singularities introduced due to the orbifolding by Z_2 are distinct from the singularities in $K3$ arising from collapsed two cycles. Thus we would expect that the massless spectrum from the twisted sector, associated with the fixed points of σ, will not be affected by the collapse of the two cycles. This argument continues to hold even when we move the five-branes away from the fixed points, at least as long as they do not hit the collapsed two cycles. The net result of this is that the twisted sector does not generically give rise to new charged massless states in the spectrum. This is just as well, since the spectrum of states that have already been found gives an anomaly-free theory.

Since in this case there is no supersymmetry to protect the massless charged states against gaining mass (by absorbing the charged hypermultiplets), we must examine carefully the fate of these states under quantum corrections. Although quantum corrections in M theory are not properly understood yet, we can make the following observation. Let us consider the case where both $K3$ and S^1 have large volume, but we are at the ($E_8\times E_8$) singular point of $K3$. In this case, before the Z_2 projection the massless states come from membranes wrapped around the vanishing two cycles of $K3$. Thus these states are localized on $K3$, but not on S^1, and we obtain a set of states in the six-dimensional theory by quantizing propagation of these massless states on the internal S^1. Before the Z_2 projection, there is enough supersymmetry to guarantee that the zero modes of these fields along S^1 give rise to zero mass states in the resulting six-dimensional theory (even after taking into account interactions). Now let us consider the effect of Z_2 modding. As has been argued before, Z_2 has the effect of exchanging two sets of vanishing cycles on $K3$. Thus if we denote by M the neighborhood of the singular point on $K3$, then after the Z_2 modding, the internal space still has the structure $M\times S^1$ near the singular point on $K3$. As a result, the spectrum of the resulting six-dimensional theory is still obtained by quantizing propagation of the massless charged fields on the internal S^1. Since this is the same problem as before, we would expect that the zero modes of these fields on S^1 would still produce massless states in the resulting six-dimensional theory. Put another way, the quantum problem that we need to solve to determine if the spectrum contains the massless charged states do not know about supersymmetry breaking, and hence will continue to give massless charged states in the theory even after the Z_2 modding.[4] Of course, in this case we shall get only half of the states compared to what we had previously, since instead of having two copies of the singular region $M\times S^1$, we now have only one copy.

The point of enhanced gauge symmetry also gives us a way of determining the coupling between the tensor and vec-

[3]In carrying out the argument further, we are implicitly assuming that in the theory obtained after the Z_2 modding, there is no phase transition between the weak-coupling region of M theory, where the model was constructed, and the weak-coupling region of the heterotic string theory, where one can see the appearance of massless charged states. To get further insight, we note that the heterotic coupling constant λ in six dimensions is related to the volume V of $K3$ and the radius R of S^1 measured in the M-theory metric as $\lambda=\sqrt{V/R}$. Thus we can keep λ small even when R and V are both large, by keeping the ratio V/R small. This of course does not mean that the heterotic string theory is weakly coupled in this region, since one or more of the radii of T^4 becomes large in this limit, so that the correct description in the heterotic theory is as a higher-dimensional theory where it is strongly coupled. However, $\sqrt{V/R}$ becomes the scalar component of the tensor multiplet from the untwisted sector after the Z_2 projection, and the moduli associated with T^4 in the heterotic description become part of the hypermultiplet moduli space after the projection. So the above analysis does show that we can go from the region of weakly coupled M theory to the region of weakly coupled heterotic theory by remaining at a fixed point in the tensor-multiplet moduli space (i.e., keeping V/R fixed), and moving in the hypermultiplet moduli space. Thus we do not expect phase transitions of the kind discussed in Ref. [19] to appear as we move from the region where the M theory is weakly coupled to the region where the heterotic string theory is weakly coupled.

[4]In principle the world volume instantons [24], wrapped around S^1, and a two cycle on $K3$ that encloses both sets of collapsed two cycles, can give an exponentially small contribution to the mass of these charged particles. This is possible since such configurations do ''know'' about the Z_2 projection. In this case we would have to interpret this as the charged vector multiplet (and the charged hypermultiplet) acquiring small mass through nonperturbative effects in M theory. Put another way, the hypermultiplet moduli space will contain a point where the gauge symmetry is almost restored (in the sense that the charged vector- and hyper-multiplets have small mass) but is never completely restored. E. Witten has suggested that it might be possible to argue that such a phenomenon does not occur in this theory, since it will be hard to write down a low-energy effective field theory satisfying this requirement.

tor multiplets. For this let us consider a limit where we increase the size of $K3$ by remaining at the point of enhanced $E_8\times E_8$ symmetry, and at the same time, adjust the radius R of S^1 so as to keep the scalar in the tensor multiplet coming from the untwisted sector fixed. In this case the coupling between the states in the twisted sector tensor multiplet, which live on the five-branes, and the charged vector multiplets, which live on the collapsed two cycles of $K3$, can be made to vanish by taking the location of the five branes on $K3$ to be far away from the location of the collapsed two cycles on $K3$. But this coupling cannot depend either on the location of the five-brane on $K3$, or the size of $K3$ (appropriately scaled by the radius of S^1), since these belong to the hypermultiplet moduli space. Thus the coupling between the tensor multiplets coming from the twisted sector, and the E_8 gauge multiplets [including the $U(1)^8$ factor] must vanish identically. Only the tensor multiplet belonging to the untwisted sector couples to the vector multiplets. Using the fact that the anomaly polynomial vanishes, and the results of Ref. [18], this coupling can be shown to be of the form $\sqrt{-g}e^{-\Phi}\mathrm{tr}(F^2)$, where g denotes the canonical metric, and $\Phi(=\ln\sqrt{V/R})$ is the scalar component of the tensor multiplet in the untwisted sector.

One might try to get further enhancement of gauge groups by going to special points in the moduli space of the (4,4) lattice where the theory before the Z_2 projection develops enhanced gauge symmetries. If $\vec{\beta}$ denotes any nonzero element of such a special (4,4) lattice representing a massless charged particle in the theory before the Z_2 projection, then we can get new Z_2 invariant massless vectors by considering the combination

$$|(\vec{\beta},\vec{0},\vec{0})\rangle_V+|(-\vec{\beta},\vec{0},\vec{0})\rangle_V. \tag{5}$$

Similarly, Z_2 invariant massless scalars are obtained by taking the combination

$$|(\vec{\beta},\vec{0},\vec{0})\rangle_S-|(-\vec{\beta},\vec{0},\vec{0})\rangle_S. \tag{6}$$

The spectrum obtained this way is free from gravitational anomalies since we get an equal number of vector- and hyper- multiplets. However, typically the spectrum suffers from gauge anomalies. Consider for example, going to a special point in the (4,4) lattice where the theory before the Z_2 projection develops an enhanced $SU(n)$ gauge symmetry $(n\leqslant 5)$. The action of Z_2 on the group elements can be represented as complex conjugation of the $SU(n)$ matrices. Thus the gauge group after the Z_2 projection will be $SO(n)$, generated by real $SU(n)$ matrices. The vectors given in (5), together with appropriate spinors, constitute the vector multiplet in the adjoint of $SO(n)$. On the other hand, the scalars given in (6), together with the moduli fields that need to be tuned to reach the $SU(n)$ point, and appropriate spinors, constitute a hypermultiplet in the symmetric rank two tensor representation of $SO(n)$.

Is there a possibility of getting massless states from the twisted sector? In the M theory on $K3\times S^1$, enhanced gauge symmetries of the type discussed above occur at points in the $K3$ moduli space where a set of two cycles associated with the (3,3) part of the lattice of second cohomology elements collapses to zero size.[5] These cycles are mapped on to themselves under the action of σ. As a result, the singularities introduced by the orbifolding procedure are certainly going to interfere with the singularities introduced on $K3$ by collapsing two cycles, and we would, in general, expect new charged massless states from the twisted sector.

It is conceivable that the spectrum from the twisted sector arranges itself so as to give an anomaly free theory. However, as we shall now argue, in this case there is no guarantee that the extra charged massless states obtained from the untwisted sector survive quantum corrections, and hence a more likely possibility is that we do not get any gauge-symmetry enhancement at all in the quantum theory. To see this we follow the same approach as in the case of the $E_8\times E_8$ singular points. In this case the massless charged states are again localized on $K3$ near the vanishing cycles. Let us denote by N the region in $K3$ near the vanishing cycles. Thus before the Z_2 projection the internal space near the singular point had the structure $N\times S^1$, and the quantum problem to be solved involved the propagation of the massless charged fields on an internal S^1. Supersymmetry guaranteed that the zero modes of these massless fields along S^1 give rise to strictly massless states in six dimensions. The situation after the Z_2 modding is quite different however. Since in this case the region N is mapped onto itself under the Z_2 action, the region near the singular point after the Z_2 modding no longer has the structure $N\times S^1$. In particular, if P denotes the singular point onto which the two cycles have collapsed, then at P the internal space has the form $P\times(S^1/Z_2)$. Thus in this case the quantum problem to be solved for determining the spectrum of massless charged states in the six-dimensional theory reduces to studying the propagation of these massless fields not on an internal S^1, but on an internal S^1/Z_2. This of course is a different problem, and now there is not enough supersymmetry in the problem to guarantee that we get massless states in the six-dimensional theory even after taking into account quantum corrections. Thus we see that the massless charged states that we have constructed in the untwisted sector, do not in general remain massless after we take into account quantum corrections.

Interesting phenomena might occur at other points in the moduli space where the heterotic string theory develops more general enhanced gauge symmetries involving the full (4,20) lattice. But again in this case one or more of the collapsed cycles are invariant under the Z_2 action, and hence our ability to analyze the spectrum of massless states breaks down.

This theory has several T-duality symmetries. One class of such symmetries involve the permutation of the eight five-branes, which acts on the eight tensor multiplets arising from the twisted sector as a permutation group. We also have a set of duality symmetries that exchange the eight vector multip-

[5]This can also involve a special limit where the size of $K3$, measured in type IIA metric, is of order unity, and the $C_{(10)mn}$ fields are adjusted appropriately. This would correspond to enhanced symmetries appearing from the (1,1) part of the lattice that is not associated with the second cohomology elements of $K3$, but to the zeroth and fourth cohomology elements.

lets among each other − this is part of the global diffeomorphism group O(3,19;Z) of $K3$ that commutes with the Z_2 projection.

Upon compactification on a circle, both the vector and the tensor multiplets give rise to vector multiplets in the resulting five-dimensional theory. In the spirit of Ref. [25] it is tempting to conjecture that this theory has a U-duality group that exchanges the vectors coming from the six-dimensional vector multiplet with the ones coming from the six-dimensional tensor multiplet. It will be interesting to study what kind of moduli space [26] arises as a result of this compactification.

Finally one might ask if the M-theory compactification discussed here has a string theory dual. It is clear that the dual in this case cannot be a heterotic string compactification on $K3$ since that always gives only one tensor multiplet. It turns out that Dabholkar and Park have independently constructed six-dimensional string theories with precisely the same spectrum of massless states at a generic point in the moduli space [27]. Thus we expect these M-theory compactifications to be dual to the ones constructed in Ref. [27] in a nonperturbative sense.

There are several apparently singular regions in the moduli space of this theory that deserve further study. One of these, already discussed earlier, corresponds to the region where the $K3$ becomes singular due to collapsed two cycles, and the locations of some of the fixed points of $K3$ under the involution σ coincide with the locations of the collapsed two cycles. Another interesting region would be where the locations of two or more five-branes in the internal space coincide. There can be open membranes stretched between two five-branes in the M theory [28,29], which would appear as self-dual strings to a six-dimensional observer. These string are expected to become tensionless in the limit where the relative separation of two or more five-branes vanish [28], giving rise to a phenomenon similar to the one observed in type IIB string theory compactified on $K3$ [30]. Note that reaching these special points in the moduli space requires simultaneous adjustment of hyper-multiplet moduli (the locations of the five-branes on $K3$) and tensor multiplet moduli (the locations of the five-branes on S^1). Upon further compactification on a circle, these tensionless strings, wrapped around the extra circle, will give rise to new massless states in the theory, and might further enhance the gauge symmetry. This lends further support to the conjecture that the five-dimensional theory might have a duality symmetry that exchanges the vectors arising from the six-dimensional vector multiplet with those arising from the six-dimensional tensor multiplet.

Note added: After this paper was submitted, an interesting paper appeared that obtains a model with the same low-energy spectrum from an F-theory compactification [31]. This model is likely to be another dual of the model discussed here.

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