

Balitskii-Fadin-Kuraev-Lipatov QCD Pomeron in high energy hadron collisions: Inclusive dijet production

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We calculate the inclusive dijet production cross section in high energy hadron collisions within the BFKL resummation formalism for the QCD Pomeron. Unlike the previous calculations with the Pomeron developing only between tagging jets, we include also the Pomerons which are adjacent to the hadrons. With these adjacent Pomerons we define a new object, the BFKL structure function of the hadron, which enables one to calculate the inclusive dijet production for any rapidity intervals.

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At present, much attention is being paid to the perturbative QCD Pomeron obtained by Balitskii, Fadin, Kuraev, and Lipatov (BFKL) [1]. One of the reasons is that it relates hard processes ($-t = Q^2 \gg \Lambda_{\text{QCD}}^2$) and semihard ones ($s \gg -t = Q^2 \gg \Lambda_{\text{QCD}}^2$): It sums up leading energy logarithms of perturbative QCD into a singularity in the complex angular momentum plane. Several proposals to find direct manifestations of the BFKL Pomeron are available in the literature, see, e.g., [2–9], but it is still difficult to get the necessary experimental data.

Among those proposals the first one was made by Mueller and Navelet [2]. They pointed out that the inclusive dijet production in high energy hadron collisions may serve as a probe for the BFKL Pomeron; namely, a specific exponential growth of the cross section K factor with the rapidity interval of tagged jets was predicted.

This idea was further developed in [8,9], where it was found out that the relevant object may be the azimuthal angle correlation of jets.

Note that these studies were restricted to a consideration of some special configuration of the inclusive dijet production; namely, in [2,8,9] only the production cross section of the most forward and most backward jets is considered [Fig. 1(a)]. However, unlike the usual hard QCD processes with strong $k_{\perp}$ ordering, in BFKL Pomeron kinematics [10] there are strong rapidity ordering and weak $k_{\perp}$ diffusion. This means, in particular, that the most forward or backward jets do not need to have the hardest $k_{\perp}$. Therefore, there is no warranty that one can tag most forward or backward jets without a dedicated full-acceptance detector [11].

Unfortunately, the available detectors [Collider Detector at Fermilab (CDF) and D0] at the Fermilab Tevatron as well as forthcoming detectors at the CERN Large Hadron Collider (LHC) have a limited acceptance in $k_{\perp}$ ($k_{\perp \text{min}} \sim \text{tens GeV}$)

and (pseudo)rapidity for tagging jets. So, one cannot compare the results of [2,8,9] with the preliminary D0 data on the large rapidity interval dijets [12] without an analysis of jet radiation beyond the acceptance of the detectors.

In this Rapid Communication we study, within the BFKL approach, the inclusive dijet cross section in high energy hadron collisions without any restrictions on untagging jets. Our results provide an opportunity to confront BFKL Pomeron predictions on the inclusive dijet production with data which could be extracted from the existing CDF and D0 jet event samples after a modification of the jet analysis algorithms. This may be decisive in checking applicability of the factorization hypothesis for high energy hadron collisions

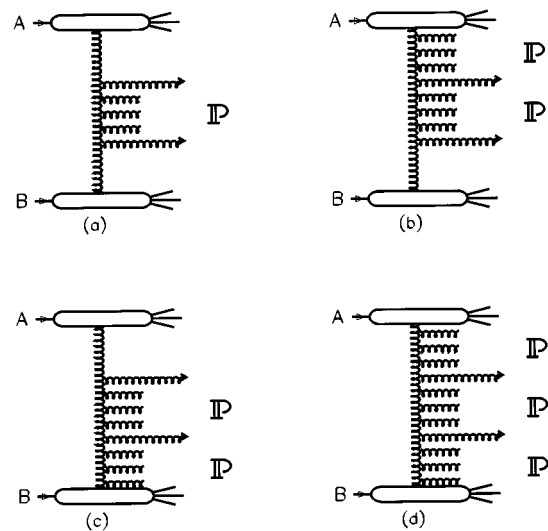


FIG. 1. Subprocesses for the dijet production in a collision of hadrons A and B; vertical curly lines correspond to the Reggeized gluons; horizontal ones to the real gluons radiated into the rapidity intervals; all subprocesses contain the inner Pomeron. (a) Subprocess without the adjacent Pomeron; (b) subprocess with a Pomeron adjacent to the hadron A; (c) same for the hadron B; (d) subprocess with two adjacent Pomerons.

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(arguments against it, see [13]) and the leading logarithm approximation.

Removing the restriction on tagging jets to be most forward or backward one should take into account additional contributions to the cross section with jets more rapid than the tagging ones. There are three such contributions: two with a couple of Pomerons [Figs. 1(b), 1(c)] and one with three [Fig. 1(d)]. In this Rapid Communication we will call the Pomerons developing between colliding hadrons and their descendant jets the adjacent Pomerons and the Pomeron developing between the tagging jets the inner Pomeron. These additional contributions contain extra power of α_s per extra Pomeron but hardly could they be regarded as corrections since they are also proportional to a kinematically dependent factor which one can loosely treat as the number of partons in the hadron moving faster than the descendant tagging jet.

Contribution to the cross section of Fig. 1(a) considered by Mueller and Navelet [2] is

$$\begin{aligned} & \frac{x_1 x_2 d\sigma_{\{P\}}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}} \\ &= \frac{\alpha_s C_A}{k_{1\perp}^2} \frac{\alpha_s C_A}{k_{2\perp}^2} x_1 F_A(x_1, \mu_1^2) x_2 F_B(x_2, \mu_2^2) \\ & \times f^{\text{BFKL}}(k_{1\perp}, k_{2\perp}, y), \end{aligned} \quad (1)$$

where the subscript on $\sigma_{\{P\}}$ labels the contribution to the cross section as a single Pomeron; $C_A=3$ is a color group factor; x_i are the longitudinal momentum fractions of the tagging jets; $k_{i\perp}$ are the transverse momenta; $F_{A,B}$ are the effective structure functions of colliding hadrons; $y=\ln(x_1 x_2 s/k_{1\perp} k_{2\perp})$ is the relative rapidity of tagging jets and, finally, f^{BFKL} is the solution for the BFKL equation.

If one retains in $f^{\text{BFKL}}(k_{1\perp}, k_{2\perp}, y)$ of Eq. (1) only the leading α_s -independent contribution to its α_s expansion, one gets the result of [14]

$$\begin{aligned} & \frac{x_1 x_2 d\sigma_{\{P\}}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}} = \frac{\alpha_s C_A}{k_{1\perp}^2} \frac{\alpha_s C_A}{k_{2\perp}^2} \\ & \times x_1 F_A(x_1, \mu_1^2) x_2 F_B(x_2, \mu_2^2) \\ & \times \frac{1}{2} \delta^{(2)}(k_{1\perp} - k_{2\perp}). \end{aligned} \quad (2)$$

An interesting feature of this result is that it does not contain specific contributions from gluons and quarks; they turn out to be the same up to a simple group factor in the high energy limit. This provides the possibility to hide all the nonperturbative physics in the pair of effective structure functions $x F_{A,B} = x(G_{A,B} + (C_F/C_A)Q_{A,B} + (C_F/C_A)Q_{A,B})$. What distinguishes the cross section of Eq. (1) from the analogous one of Eq. (2) is a systematic resummation of $(\alpha_s y)$ corrections to the hard subprocess cross section, which is necessary when the relative rapidity of jets, y , is not small.

The solution for the BFKL equation has the integral representation [1]

$$\begin{aligned} & f^{\text{BFKL}}(k_{1\perp}, k_{2\perp}, y) \\ &= \sum_{n=-\infty}^{\infty} \int d\nu \chi_{n,\nu}(k_{1\perp}) e^{y\omega(n,\nu)} \chi_{n,\nu}^*(k_{2\perp}), \end{aligned} \quad (3)$$

where the star means complex conjugation,

$$\chi_{n,\nu}(k_{\perp}) = \frac{(k_{\perp}^2)^{-1/2 + i\nu} e^{i\nu\phi}}{2\pi} \quad (4)$$

are Lipatov's eigenfunctions, and

$$\omega(n, \nu) = \frac{2\alpha_s C_A}{\pi} \left[\psi(1) - \text{Re} \psi \left(\frac{|n|+1}{2} + i\nu \right) \right]$$

are Lipatov's eigenvalues. Here ψ is the logarithmic derivative of the Euler Γ function.

Making use of the above-introduced objects we rewrite Eq. (1) as

$$\begin{aligned} & \frac{x_1 x_2 d\sigma_{\{P\}}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}} \\ &= \frac{\alpha_s C_A}{k_{1\perp}^2} \frac{\alpha_s C_A}{k_{2\perp}^2} \sum_n \int d\nu x_1 F_A(x_1, \mu_1^2) \\ & \times [\chi_{n,\nu}(k_{1\perp}) e^{y\omega(n,\nu)} \chi_{n,\nu}^*(k_{2\perp})] x_2 F_B(x_2, \mu_2^2). \end{aligned} \quad (5)$$

As one can guess, subprocesses of Figs. 1(b)–1(d) with the adjacent Pomerons contribute to the effective structure functions, i.e., one can account for them by just adding some “radiation corrections” to the structure functions of Eq. (5):

$$\begin{aligned} & x_1 F_A(x_1, \mu_1^2) \Rightarrow x_1 \Phi_A(x_1, \mu_1^2, n, \nu, k_{1\perp}) \\ & \equiv x_1 F_A(x_1, \mu_1^2) \\ & + x_1 D_A(x_1, \mu_1^2, n, \nu, k_{1\perp}), \end{aligned} \quad (6)$$

$$\begin{aligned} & x_2 F_B(x_2, \mu_2^2) \Rightarrow x_2 \Phi_B^*(x_2, \mu_2^2, n, \nu, k_{2\perp}) \\ & \equiv x_2 F_B(x_2, \mu_2^2) \\ & + x_2 D_B^*(x_2, \mu_2^2, n, \nu, k_{2\perp}). \end{aligned} \quad (7)$$

The complex conjugation on Φ_B could be understood if one looks at the right-hand side of Eq. (5) as a matrix element of a t -channel evolution operator with the relative rapidity, y , as an evolution parameter and F_B as a final state; (n, ν) are then “good quantum numbers” conserved under the evolution; this makes room for (n, ν) dependence of the corrected structure functions. We note also that the corrected structure functions may depend on the transverse momenta of the tagging jets.

To get an explicit expression for the radiation correction to the effective hadron structure functions, let us consider, for example, the contribution of Fig. 1(b). In terms of the radiation correction, D_A , it is

$$\frac{x_1 x_2 d\sigma_{\{PP\}A}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}}$$

$$= \frac{\alpha_S C_A}{k_{1\perp}^2} \frac{\alpha_S C_A}{k_{2\perp}^2} \sum_n \int d\nu x_1 D_A(x_1, \mu_1^2, n, \nu, k_{1\perp}) \times [\chi_{n,\nu}(k_{1\perp}) e^{y\omega(n,\nu)} \chi_{n,\nu}^*(k_{2\perp})] x_2 F_B(x_2, \mu_2^2). \quad (8)$$

On the other hand, BFKL summation of the leading energy logarithms yields

$$\frac{x_1 x_2 d\sigma_{\{PP\}A}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}}$$

$$= \frac{\alpha_S C_A}{k_{1\perp}^2} \frac{\alpha_S C_A}{k_{2\perp}^2} \frac{2\alpha_S C_A}{\pi^2} \int_{x_1}^1 d\xi x_1 F_A(\xi, \mu_1^2) \int_{\mu_1}^{\xi\sqrt{s}} \frac{d^2 q_{1\perp}}{q_{1\perp}^2} \times \int d^2 q_{2\perp} f^{\text{BFKL}}(q_{1\perp}, q_{2\perp}, y_1(\xi, q_{1\perp}))$$

$$\times f^{\text{BFKL}}(q_{2\perp} + k_{1\perp}, k_{2\perp}, y) x_2 F_B(x_2, \mu_2^2), \quad (9)$$

where $\xi, q_{1\perp}$ parametrize the momentum of the most rapid untagging jet moving in the same direction as hadron A [see Fig. 1(a)]

$$y_1(\xi, q_{1\perp}) = \ln \frac{\xi k_{1\perp}}{x_1 q_{1\perp}} \quad (10)$$

is the relative rapidity measuring the rapidity lapse spanned by the adjacent Pomeron. We cut the infrared-divergent transverse momentum integration by the normalization point, μ_1 , of F_A . Other integration cuts in Eq. (9) are evident from the kinematics.¹

Splitting the Pomerons into the product of Lipatov's eigenfunctions and making the transverse momentum integrations in Eq. (9) one gets that, first, Eqs. (8) and (9) are compatible and, second,²

$$D_A(x_1, \mu_1^2, n, \nu, k_{1\perp}) = i \frac{\alpha_S C_A}{\pi^2} \frac{\Gamma(|n|/2 + 1/2 + i\nu)}{\Gamma(|n|/2 + 1/2 - i\nu)} \int_{-\infty}^{\infty} d\lambda \frac{\Gamma(|n|/2 + 1 - i(\nu - \lambda))}{\Gamma(|n|/2 + 1 + i(\nu - \lambda))} \frac{\Gamma(1/2 - i\lambda)}{\Gamma(1/2 + i\lambda)} \frac{1}{(i|n|/2 + \nu - \lambda + i\epsilon)} \times \int_x^1 \frac{d\xi}{x} \left(\frac{\xi}{x} \right)^{\omega(0,\lambda)} F_A(\xi, \mu_1^2) \frac{(k_{1\perp}/\mu_1)^{1+\omega(0,\lambda)-2i\lambda}}{1+\omega(0,\lambda)-2i\lambda} \left[1 - \left(\frac{\mu_1}{\xi\sqrt{s}} \right)^{1+\omega(0,\lambda)-2i\lambda} \right]. \quad (11)$$

Some comments on the above formula are in order. The $i\epsilon$ defines the right way to deal with the singularity at $n = \nu - \lambda = 0$. The dependence on the energy $\sqrt{s}$ is weak, we check it for the Tevatron and LHC energies performing the numerical calculations (see below) with and without suppression of the last factor in Eq. (11).

To make a local summary we have got the following compact and significant form for the dijet production inclusive cross section:

$$\frac{x_1 x_2 d\sigma_{\text{dijet}}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}} = \frac{\alpha_S C_A}{k_{1\perp}^2} \frac{\alpha_S C_A}{k_{2\perp}^2} \sum_n \int d\nu x_1 \Phi_A(x_1, \mu_1^2, n, \nu, k_{1\perp}) \times [\chi_{n,\nu}(k_{1\perp}) e^{y\omega(n,\nu)} \chi_{n,\nu}^*(k_{2\perp})] \times x_2 \Phi_B^*(x_2, \mu_2^2, n, \nu, k_{2\perp}), \quad (12)$$

where the new structure functions $x\Phi_{A,B}$ that depend on Lipatov's quantum numbers (n, ν) , we call them BFKL structure functions, can be read off Eqs. (7) and (11).

We expect that Eq. (12) may serve as an example of factorization matching the Regge limit of QCD. It requires fur-

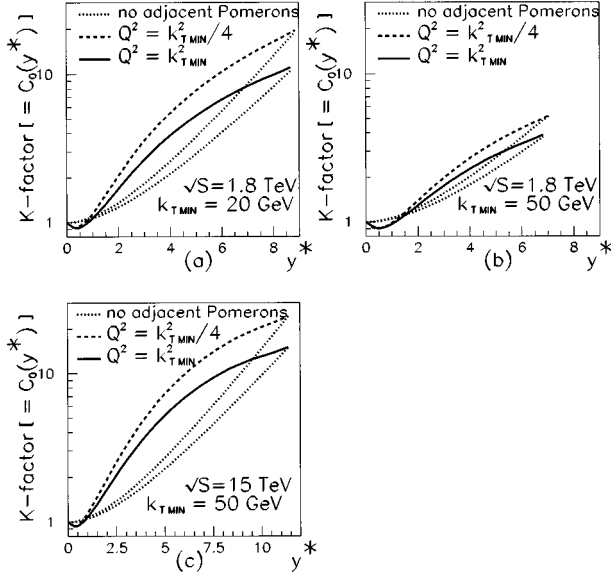
ther work to develop a factorization scheme in the spirit of [16] which would be able to accommodate nonleading corrections to Eq. (12).

Equation (12) gets simpler for x -symmetric dijet production with $x_1 = x_2$ after integration over transverse momenta squared larger than a cut value, $k_{\perp\text{min}}^2 = \mu_1^2 = \mu_2^2 = Q^2$. Thus, following [2,8,9] we consider

$$\int_{k_{\perp\text{min}}^2}^{e^{y^*} k_{\perp\text{min}}^2} dk_{1\perp}^2 dk_{2\perp}^2 \left(\frac{x_1 x_2 d\sigma_{\text{dijet}}}{dx_1 dx_2 d^2 k_{1\perp} d^2 k_{2\perp}} \right)_{x_1=x_2} = \frac{(\alpha_S C_A)^2}{s} (e^{y^*} - 1) F_A(e^{y^*/2} k_{\perp\text{min}} / \sqrt{s}, k_{\perp\text{min}}^2) \times F_B(e^{y^*/2} k_{\perp\text{min}} / \sqrt{s}, k_{\perp\text{min}}^2) \sum_n \frac{e^{in\phi}}{2\pi} C_n(y^*, k_{\perp\text{min}}). \quad (13)$$

¹We consider here only a kinematically simple case when the rapidities of the tagging jets have different signs in the center-of-mass frame.

²A simplified analog of this formula was found in a triple-jet inclusive production study [15].

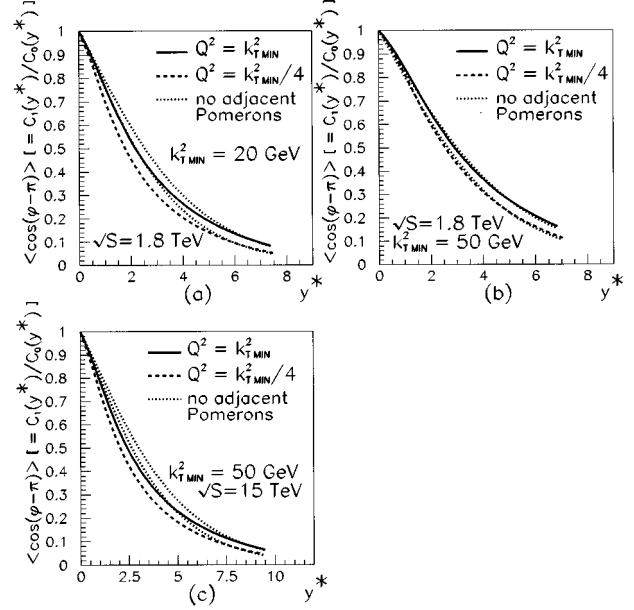
FIG. 2. The y^* dependence of the dijet K factor.

This depends on the azimuthal angle ϕ between the tagging jets and an effective relative rapidity,³ $y^* \equiv \ln(x_1 x_2 s / k_{\perp \min}^2)$. We compute the Fourier coefficients of the azimuthal angle dependence, $C_n(y^*, k_{\perp \min})$. The normalization of Eq. (13) makes $C_0(y^*, k_{\perp \min})$ equal to the K factor; the ratio of the cross section integrated over the azimuthal angle, to the Born one. The exponential growth of this quantity with rapidity was predicted in [2]. Another quantity of interest is the average cosine of the azimuthal angle between tagging jets, $\langle \cos(\phi - \pi) \rangle = C_1(y^*, k_{\perp \min}) / C_0(y^*, k_{\perp \min})$ [8,9] ($\phi = \pi$ for back-to-back jets). We plot our predictions for the K factor in Fig. 2 and $\langle \cos(\phi - \pi) \rangle$ in Fig. 3. The leading order CTEQ3L structure functions [17] with $\Lambda_{\text{QCD}}^{(5)} = 132$ MeV have been used.

We point out the following qualitative features of our numerical results: (i) the contribution of the adjacent Pomerons to the cross section is significant, up to 60% (20%) at $k_{\perp \min} = 20$ GeV (50 GeV) at the Tevatron energy; (ii) the contribution of the adjacent Pomerons slowly dies out as y^* approaches its kinematical bounds; (iii) growth of the energy as well as the decrease of the lower cutoff on the transverse momenta of the tagging jets causes the contribution of the adjacent Pomerons to be even more significant; (iv) the azimuthal angle decorrelation (deviation of the average cosine from unit) is less sensitive to the contribution of the adjacent Pomerons.

As is apparent from our plots, the resummation effects are significant not only for large rapidity intervals. Thus, taking into account an unknown rapidity dependence of the non-leading corrections, it remains an open question which re-

³Note that in the leading logarithm approximation we have $k_{\perp} \sim k_{\perp \min}$ and hence $y \sim y^*$.

FIG. 3. The y^* dependence of the average azimuthal angle cosine between the tagging jets.

gion of rapidity intervals is more promising for BFKL Pomeron manifestation searches.

We should note here that the extraction of data on high- $k_{\perp}$ jets from the event samples in order to compare them with the BFKL Pomeron predictions should be different from the algorithms directed to a comparison with perturbative QCD predictions for the hard processes. These algorithms, motivated by the strong $k_{\perp}$ ordering of the hard QCD regime, employ hardest- $k_{\perp}$ jet selection (see, e.g., [18]). It is doubtful that one can reconcile these algorithms with the weak $k_{\perp}$ diffusion and the strong rapidity ordering of the semihard QCD regime, described by the BFKL resummation. We also note that our predictions should not be compared with the preliminary data [12] extracted by the most forward or backward jet selection criterion. Obviously, one should include for tagging all the registered pairs of jets (not only the most forward-backward pair) to compare with our predictions. In particular, to make a comparison with Figs. 2 and 3 one should sum up all the registered x -symmetric dijets ($x_1 = x_2$) with transverse momenta harder than $k_{\perp \min}$.

Based on our study we draw a conclusion that the adjacent BFKL Pomerons can play a decisive role in high energy hadron collisions, as it may be seen in inclusive dijet production.

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