

## Dyonic BPS saturated black holes of heterotic string on a six-torus

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Within effective heterotic superstring theory compactified on a six-torus we derive minimum energy (super-symmetric), static, spherically symmetric solutions, which are manifestly invariant under the target space  $O(6,22)$  and the strong-weak coupling  $SL(2)$  duality symmetries with 28 electric and 28 magnetic charges subject to one constraint. The class of solutions with a constant axion corresponds to dyonic configurations subject to two charge constraints, with purely electric (or purely magnetic) and dyonic configurations preserving  $\frac{1}{2}$  and  $\frac{1}{4}$  of  $N=4$  supersymmetry, respectively. General dyonic configurations in this class have a space-time of extreme Reissner-Nordström black holes while configurations with more constrained charges have a null or a naked singularity.

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There has been accumulating evidence for the strong-weak coupling duality (referred to as the  $S$  duality) in string theory (see, for example, Refs. [1,2]), which relates supersymmetric vacua of a strongly coupled theory to supersymmetric vacua of a dual, weakly coupled, theory. A deeper understanding of these duality symmetries would provide us with a handle on the nonperturbative nature of superstring theory.

In four dimensions, string vacua with  $N=4$  low energy supersymmetry are conjectured to be self-dual, i.e., the string vacua of the heterotic string compactified on a six-torus transform into each other under the  $SL(2,Z)$  transformations.<sup>1</sup> The  $SL(2,Z)$  symmetry acts on charges, the axion, and the four-dimensional dilaton field, whose value determines the string loop expansion parameter and parametrizes the strength of the string coupling. In addition, the string world-sheet action can be cast in a manifestly  $O(6,22)$  symmetric form [5,6], referred to as the target space  $T$  duality symmetry.

Evidence for the  $S$  duality conjecture of  $N=4$  supersymmetric string vacua has been provided by demonstrating the  $S$  duality invariance of quantities which are not believed to be modified by string quantum corrections, e.g., the low energy effective field theory [7,6], allowed spectrum of electric and magnetic charges, Yukawa couplings between massless scalars and massive charged states as well as the Bogomol'nyi-Prasad-Sommerfeld (BPS) saturated mass spectrum of the corresponding nontrivial configurations in the effective theory [8,9].

The BPS saturated states within the effective theory compactified on a six-torus have been addressed for states with special charge configurations (see, for example, Refs. [8,10–13]), which in turn prevented one from establishing the full symmetry structure of such configurations as well as the full nature of their singularity structure. In this paper we shall

present the explicit form of general BPS saturated (super-symmetric), spherically symmetric, static configurations in the effective heterotic string theory compactified on a six-torus at generic points of moduli space, which can be obtained from the generating solution with, among scalar fields, only diagonal internal metric and the dilaton turned on. The BPS saturated spectrum is both  $O(6,22)$  and  $SL(2,R)$  invariant.<sup>2</sup> In addition, the explicit form of these configurations allows for a synthetic analysis of their singularity structures and their thermal properties within the class of solutions.

The effective field theory of massless bosonic fields for heterotic string on a Narain torus [14] can be obtained by compactifying the ten-dimensional  $N=1$  supergravity theory coupled to  $N=1$  super-Maxwell theory on a six-torus [6,15]. The ten-dimensional bosonic fields are given by  $\hat{G}_{MN}$ ,  $\hat{B}_{MN}$ ,  $\hat{A}_M^I$ , and  $\Phi$  ( $0 \leq M, N \leq 9$ ,  $1 \leq I \leq 16$ ), which correspond to ten-dimensional metric, two-form field, gauge fields of  $U(1)^{16}$ , and the dilaton field, respectively. The field strengths of  $\hat{A}_M^I$  and  $\hat{B}_{MN}$  are defined as  $\hat{F}_{MN}^I = \partial_M \hat{A}_N^I - \partial_N \hat{A}_M^I$  and  $\hat{H}_{MNP} = \partial_M \hat{B}_{NP} - \frac{1}{2} \hat{A}_M^I \hat{F}_{NP}^I + \text{c.p.}$ , respectively.

The Kaluza-Klein compactification [16] of the original ten-dimensional action on a six-torus is obtained with the

following ansatz for the zehnbein:  $\hat{E}_M^A = \begin{pmatrix} e^{\frac{\phi}{2}} e^\alpha_\mu & A_\mu^{(1)m} e^a_m \\ 0 & e^a_m \end{pmatrix}$ ,

where  $A_\mu^{(1)m}$  ( $m=1, \dots, 6$ ) are Kaluza-Klein  $U(1)$  gauge fields and  $\phi \equiv \Phi - \ln \det e_m^a$  is the four-dimensional dilaton field.

The four-dimensional action [6,15] for massless bosonic fields contains the following fields: the graviton  $g_{\mu\nu}$ , the dilaton  $\phi$ , 28  $U(1)$  gauge fields  $\mathcal{A}_\mu^i \equiv (A_\mu^{(1)m}, A_{\mu m}^{(2)}, A_\mu^{(3)I})$  defined as  $A_{\mu m}^{(2)} \equiv \hat{B}_{\mu m} + \hat{B}_{mn} A_\mu^{(1)n} + \frac{1}{2} a_m^I A_\mu^{(3)I}$ ,  $A_\mu^{(3)I} \equiv \hat{A}_\mu^I - a_m^I A_\mu^{(1)m}$  with the field strengths  $\mathcal{F}_{\mu\nu}^i = \partial_\mu \mathcal{A}_\nu^i - \partial_\nu \mathcal{A}_\mu^i$ , the two-form field<sup>3</sup>  $B_{\mu\nu}$  with the field strength given by

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<sup>1</sup>The heterotic string compactified on a six-torus is conjectured to be dual to the type IIA string compactified on a  $T^2 \times K_3$  surface; this duality has its origin in the string-string duality conjecture [1–4] of the heterotic and the type IIA string theory in six dimensions.

<sup>2</sup>At the quantum level, these symmetries are integer valued.

<sup>3</sup>The four-dimensional two-form field is equivalent to a pseudo-scalar (the axion)  $\Psi$  through the duality transformation  $H^{\mu\nu\rho} = -(\sqrt{-g})^{-1} e^{2\phi} g^{\mu\nu\rho\sigma} \partial_\sigma \Psi$ .

$H_{\mu\nu\rho} = e_\mu^\alpha e_\nu^\beta e_\rho^\gamma \hat{E}_\alpha^M \hat{E}_\beta^N \hat{E}_\gamma^P \hat{H}_{MNP} = \partial_\mu B_{\nu\rho} - \frac{1}{2} \mathcal{A}_\mu^i L_{ij} \mathcal{F}_{\nu\rho}^j + \text{c.p.}$ , and a symmetric  $O(6,22)$  matrix  $M$  of scalar fields, which can be expressed in terms of the  $O(6,22)$  matrix

$$V = \begin{pmatrix} V^I \\ V^{II} \\ V^{III} \end{pmatrix} = \begin{pmatrix} E^{-1} & -E^{-1}C & -E^{-1}a^T \\ 0 & E & 0 \\ 0 & a & I_{16} \end{pmatrix} \quad (1)$$

as  $M = V^T V$ , where  $E \equiv [e_m^a]$ ,  $C \equiv [\frac{1}{2} \hat{A}_m^I \hat{A}_n^I + \hat{B}_{mn}]$ , and  $a \equiv [\hat{A}_m^I]$ .  $V$  plays a role of a vielbein in the  $O(6,22)$  target space.

The four-dimensional (4D) effective action is invariant under the  $O(6,22)$  transformations [6,15]

$$M \rightarrow \Omega M \Omega^T, \quad \mathcal{A}_\mu^i \rightarrow \Omega_{ij} \mathcal{A}_\mu^j, \quad g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad \phi \rightarrow \phi. \quad (2)$$

Here,  $\Omega \in O(6,22)$ , i.e.,  $\Omega^T L \Omega = L$ , where  $L$  is an  $O(6,22)$ -invariant matrix. In addition, the corresponding equations of motion and Bianchi identities have the invariance under the  $SL(2, \mathbb{R})$  transformations [17,15]:

$$S \rightarrow S' = \frac{aS+b}{cS+d}, \quad M \rightarrow M, \quad g_{\mu\nu} \rightarrow g_{\mu\nu},$$

$$\mathcal{F}_{\mu\nu}^i \rightarrow \mathcal{F}'_{\mu\nu}{}^i = (c\Psi+d)\mathcal{F}_{\mu\nu}^i + ce^{-\phi}(ML)_{ij}\tilde{\mathcal{F}}_{\mu\nu}^j, \quad (3)$$

where  $S \equiv \Psi + ie^{-\phi}$ ,  $\tilde{\mathcal{F}}^{i\mu\nu} = \frac{1}{2}(\sqrt{-g})^{-1}\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\rho\sigma}^i$ , and  $a, b, c, d \in \mathbb{R}$  satisfy  $ad - bc = 1$ .

The ten-dimensional supersymmetry transformations for a gravitino  $\psi_M$ , dilatino  $\lambda$ , and 16 gauginos  $\chi^I$  are expressed in terms of the four-dimensional fields as

$$\begin{aligned} \delta\hat{\psi}_\mu &= \partial_\mu \varepsilon + \frac{1}{4}\omega_{\mu\beta\gamma}\gamma^{\beta\gamma}\varepsilon + \frac{1}{4}e_\mu^\alpha \eta_{\alpha[\beta} e_{\gamma]}^\nu \partial_\nu \phi \gamma^{\beta\gamma}\varepsilon \\ &\quad - \frac{1}{8}e^{-\phi}H_{\mu\nu\rho}\gamma^{\nu\rho}\varepsilon + \frac{1}{8}(e_b^n \partial_\mu e_{nc} - e_c^n \partial_\mu e_{nb})I \otimes \Gamma^{bc}\varepsilon \\ &\quad - \frac{1}{4}e^{-\phi/2}[(VL)_{ci}^I + (VL)_{ci}^{II}]\mathcal{F}_{\mu\nu}^i \gamma^{\nu 5} \otimes \Gamma^c \varepsilon, \\ \delta\psi_d &= -\frac{1}{4}e^{-\phi/2}[e_d^m \partial_\mu e_{mb} + e_b^m \partial_\mu e_{md} - e_d^m e_b^n (\partial_\mu B_{mn} \\ &\quad + \frac{1}{2}a_m^I \partial_\mu a_n^I - \frac{1}{2}a_n^I \partial_\mu a_m^I)]\gamma^{\mu 5} \otimes \Gamma^b \varepsilon - \frac{1}{8}e^{-\phi}[(VL)_{di}^I \\ &\quad - (VL)_{di}^{II}]\mathcal{F}_{\mu\nu}^i \gamma^{\mu\nu} \varepsilon, \\ \delta\lambda &= e^{-\phi/2}\partial_\mu \Phi \gamma^\mu \varepsilon - \frac{1}{6}e^{-3/2\phi}H_{\mu\nu\rho}\gamma^{\mu\nu\rho}\varepsilon - \frac{1}{2}e^{-\phi/2}(\partial_\mu B_{mn} \\ &\quad + \frac{1}{2}a_m^I \partial_\mu a_n^I - \frac{1}{2}a_n^I \partial_\mu a_m^I)\gamma^\mu \otimes \Gamma^{mn}\varepsilon \\ &\quad - \frac{1}{2}e^{-\phi}(VL)_{di}^I \mathcal{F}_{\mu\nu}^i \gamma^{\mu\nu 5} \otimes \Gamma^d \varepsilon, \\ \delta\chi^I &= 2e^{-\frac{\phi}{2}}\partial_\mu a_m^I \gamma^{\mu 5} \otimes \Gamma^m \varepsilon + e^{-\phi}(VL)_{li}^{III} \mathcal{F}_{\mu\nu}^i \gamma^{\mu\nu} \varepsilon, \end{aligned} \quad (4)$$

where  $\delta\hat{\psi}_\mu \equiv \delta\psi_\mu - A_\mu^{(1)m} \delta\psi_m$  and  $\delta\psi_d \equiv e_d^m \delta\psi_m$ . Here,  $\gamma^\alpha$  and  $\Gamma^a$  satisfy the  $O(1,3)$  and  $O(6)$  Clifford algebras, respectively. The gamma matrices with curved indices are defined as  $\gamma^\mu \equiv e_\alpha^\mu \gamma^\alpha$  and  $\Gamma^m \equiv e_a^m \Gamma^a$ .

Static configurations, which saturate the Bogomol'nyi bound for their masses, i.e., the minimum energy configurations in their class, correspond to bosonic backgrounds

which preserve the supersymmetry transformations (4) [8], thus also referred to as supersymmetric configurations. The Killing spinor equations, which are obtained by setting the supersymmetry transformations (4) to zero, provide constraints on the Killing spinors  $\varepsilon$  and a set of coupled first-order differential equations for the supersymmetric bosonic backgrounds.

Our aim is to obtain general supersymmetric, spherically symmetric, static configurations with a general allowed charge content associated with the 28  $U(1)$  gauge fields. The four-dimensional space-time metric is chosen to be of the form

$$g_{\mu\nu} dx^\mu dx^\nu = \lambda dt^2 - \lambda^{-1} dr^2 - R(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

and the scalar fields  $M$ ,  $\phi$ , and  $\Psi$  depend on the radial coordinate  $r$ , only. The Maxwell's equations and Bianchi identities determine the  $U(1)$  field strengths to be

$$\mathcal{F}_{tr}^i = \frac{e^\phi}{R}[M_{ij}\tilde{Q}_j + \Psi(ML)_{ij}P_j], \quad \mathcal{F}_{\theta\phi}^i = P_i \sin\theta, \quad (6)$$

where  $P_i$ 's correspond to the physical magnetic charges and the physical electric charges [18] are given by  $Q_i = e^{\phi_\infty}[M_{ij\infty}\tilde{Q}_j + \Psi_\infty(ML)_{ij\infty}P_j]$ .

One can show that with the above static, spherically symmetric ansatz the Killing spinors are invariant under the  $O(6,22)$  transformations and transform as  $\varepsilon \rightarrow [\cos(\Delta/2) + i\gamma^5 \sin(\Delta/2)]\varepsilon$  under the  $SL(2, \mathbb{R})$ . Here,  $\tan\Delta = -ce^{-\phi}/(c\Psi+d)$ . The first-order differential equations are thus invariant under both transformations and therefore one can generate a new class of supersymmetric solutions by imposing  $O(6,22)$  and  $SL(2, \mathbb{R})$  transformations on the known supersymmetric solution. One can bring the arbitrary asymptotic values of the scalar fields to the forms  $M_\infty = I$  and  $S_\infty = i$  by imposing the following  $O(6,22)$  and  $SL(2, \mathbb{R})$  transformations:

$$M_\infty \rightarrow \hat{M}_\infty = \hat{\Omega} M_\infty \hat{\Omega}^T = I, \quad S_\infty \rightarrow \check{S}_\infty = (aS_\infty + b)/d = i, \quad (7)$$

where  $\hat{\Omega} \in O(6,22)$ ,  $ad = 1$ , and in the quantized theory the charge lattice vectors live in the new transformed lattice. Then, the subsets of  $O(6,22)$  and  $SL(2, \mathbb{R})$  transformations that preserve the above new asymptotic values of  $M$  and  $S$  are  $O(6) \times O(22)$  and  $SO(2)$ , respectively. To obtain solutions with arbitrary asymptotic values of  $M$  and  $S$ , one has to undo the above transformations.

We are going to find the general solution for configurations where, from the scalar fields, only the diagonal internal metric and the dilaton field are nonzero. We shall refer to such configurations as generating ones, since all the other configurations in this class can be obtained by performing a subset of  $O(6) \times O(22) \subset O(6,22)$  and  $SO(2) \subset SL(2, \mathbb{R})$  transformations on the generating ones. Note, that configurations obtained in that manner have the same four-dimensional space-time structure and thus the same singularity and thermal properties as the generating solution.

The Killing spinor equations for the configuration with only nonzero scalar fields given by the diagonal internal metric ( $e_a^m = \delta_a^m e_a$ ) and the dilaton take the form

$$\begin{aligned}
\sqrt{\lambda}R[\partial_r \ln \lambda + \partial_r \phi] \varepsilon_{u,\ell} &= \pm \sum_{a=1}^6 (\mathbf{Q}_a^{(1)} + \mathbf{Q}_a^{(2)}) \Gamma^a \varepsilon_{\ell,u}, \\
\sqrt{\lambda}R[\partial_r \ln \lambda - \partial_r \phi] \varepsilon_{u,\ell} &= i \sum_{a=1}^6 (\mathbf{P}_a^{(1)} + \mathbf{P}_a^{(2)}) \Gamma^a \varepsilon_{\ell,u}, \\
2\sqrt{\lambda}R \partial_r \ln e_a \Gamma^a \varepsilon_{u,\ell} &= [\mp (\mathbf{Q}_a^{(1)} - \mathbf{Q}_a^{(2)}) \\
&\quad + i(\mathbf{P}_a^{(1)} - \mathbf{P}_a^{(2)})] \varepsilon_{\ell,u}, \\
a=1, \dots, 6, \quad \partial_r \sqrt{\lambda}R &= 0, \tag{8}
\end{aligned}$$

where  $\mathbf{Q}_a^{(1)} \equiv e^{\phi/2} e_m^a \tilde{Q}_m$ ,  $\mathbf{Q}_a^{(2)} \equiv e^{\phi/2} e_m^a \tilde{Q}_{6+m}$ ,  $\mathbf{P}_a^{(1)} \equiv e^{-\phi/2} e_m^a P_m$ , and  $\mathbf{P}_a^{(2)} \equiv e^{-\phi/2} e_m^a P_{6+m}$ . And from  $\delta\chi^I = 0$ , one has  $P_I^{(3)} = 0 = Q_I^{(3)}$ . It can be shown [19] that out of  $2 \times 28$  dyonic charges, only two magnetic and two electric charges can be nonzero with electric and magnetic charges arising from different U(1) factors, with one set of electric and magnetic charges arising from the Kaluza-Klein sector and the other set arising from the two-form gauge fields with the same corresponding indices. Without loss of generality we choose the nonzero charges to be  $P_1^{(1)}, P_1^{(2)}, Q_2^{(1)}, Q_2^{(2)}$ .

The upper  $\varepsilon_u$  and lower  $\varepsilon_\ell$  two-component spinors are subject to the constraints:  $\Gamma^1 \varepsilon_{u,\ell} = i \eta_P \varepsilon_{\ell,u}$  if  $\mathbf{P}_1^{(1)} \neq 0$  and/or  $\mathbf{P}_1^{(2)} \neq 0$ , and  $\Gamma^2 \varepsilon_{u,\ell} = \mp \eta_Q \varepsilon_{\ell,u}$  if  $\mathbf{Q}_2^{(1)} \neq 0$  and/or  $\mathbf{Q}_2^{(2)} \neq 0$ . Here,  $\eta_P$  and  $\eta_Q$  are  $\pm 1$ . Note, that nonzero magnetic and electric charges each break  $\frac{1}{2}$  of the remaining supersymmetries. Thus, purely electric (or magnetic) configurations preserve  $\frac{1}{2}$ , while dyonic solutions preserve  $\frac{1}{4}$  of  $N=4$  supersymmetry in four dimensions. The first and the second sets of configurations fall into vector- and hypersupermultiplets, respectively.

The explicit form for the static, spherically symmetric generating solution is given by<sup>4</sup>

$$\begin{aligned}
\lambda &= r^2 / [(r - \eta_P P_1^{(1)})(r - \eta_P P_1^{(2)})(r - \eta_Q Q_2^{(1)}) \\
&\quad \times (r - \eta_Q Q_2^{(2)})]^{1/2}, \\
R &= [(r - \eta_P P_1^{(1)})(r - \eta_P P_1^{(2)})(r - \eta_Q Q_2^{(1)})(r - \eta_Q Q_2^{(2)})]^{1/2}, \\
e^\phi &= [(r - \eta_P P_1^{(1)})(r - \eta_P P_1^{(2)}) / (r - \eta_Q Q_2^{(1)}) \\
&\quad \times (r - \eta_Q Q_2^{(2)})]^{1/2}, \\
g_{11} &= \left( \frac{r - \eta_P P_1^{(2)}}{r - \eta_P P_1^{(1)}} \right), \quad g_{22} = \left( \frac{r - \eta_Q Q_2^{(1)}}{r - \eta_Q Q_2^{(2)}} \right), \\
g_{mm} &= 1 \quad (m \neq 1, 2). \tag{9}
\end{aligned}$$

Here, the radial coordinate is chosen so that the horizon is at  $r=0$ . The requirement that the ADM mass of the above configuration saturates the Bogomol'nyi bound restricts the choice of parameters  $\eta_{P,Q}$  such that  $\eta_P \text{sgn}(P_1^{(1)} + P_1^{(2)})$

$= -1$  and  $\eta_Q \text{sgn}(Q_2^{(1)} + Q_2^{(2)}) = -1$ , thus yielding non-negative BPS saturated ADM mass of the form  $M_{\text{BPS}} = |P_1^{(1)} + P_1^{(2)}| + |Q_2^{(1)} + Q_2^{(2)}|$ . In order to have regular BH solution with singularity behind or on the horizon, one has to choose the relative signs of two magnetic and two electric charges to be the same.<sup>5</sup> Thus, the solution has always nonzero BPS saturated Arnowitt-Deser-Misner (ADM) mass.

A general class of solutions with zero axion can be obtained from the generating ones by performing a subset of  $O(6) \times O(22) \subset O(6,22)$  transformations which generate new types of solutions from the generating one. These transformations correspond to  $SO(6)/SO(4)$  transformations with  $(6 \times 5 - 4 \times 3)/2 = 9$  parameters and  $SO(22)/SO(20)$  transformations with  $(22 \times 21 - 20 \times 19)/2 = 41$  parameters, which along with the original 4 charges provide a configuration with  $56 - 2 = 54$  charges; namely, those are configurations with 28 electric  $\vec{Q}$  and 28 magnetic  $\vec{P}$  charges subject to the following two constraints (in the basis where the asymptotic value of  $M$  takes an arbitrary value):

$$\vec{P}^T \mathcal{M}_{R,L} \vec{Q} = 0 \quad [\mathcal{M}_{R,L} \equiv (LML)_\infty \mp L]. \tag{10}$$

The  $SO(2) \subset SL(2, \mathbb{R})$  transformation provides one with one more parameter  $\tan \Delta = -ce^{-\phi}/(c\Psi + d)$ , which replaces the two constraints (10) with one  $SL(2, \mathbb{R})$  and  $O(6,22)$  invariant constraint on charges :

$$\vec{P}^T \mathcal{M}_L \vec{Q} [\vec{Q}^T \mathcal{M}_R \vec{Q} - \vec{P}^T \mathcal{M}_R \vec{P}] + (R \leftrightarrow L) = 0. \tag{11}$$

Therefore, the general configurations in this class have  $2 \times 28 - 1 = 55$  charge degrees of freedom.<sup>6</sup>

The ADM mass for a general configuration in this class can be obtained from the one for the generating solutions and can be cast in the following  $O(6,22)$  and  $SL(2, \mathbb{R})$  invariant form:<sup>7</sup>

<sup>5</sup>Note, that the case of opposite relative signs for the two electric charges (and two magnetic charges) and the equal magnitude of the two electric (and two magnetic charges) would yield zero ADM mass as pointed out in a related context by Hull and Townsend [20]. Such purely electrically (or purely magnetically) charged configurations were found and studied in Refs. [21], while dyonic ones and their implications for enhanced symmetries [20] at special points of moduli space were addressed in Ref. [22]. Such configurations are *not* regular; they have a naked singularity.

<sup>6</sup>The constraint (11) on charges signals that the obtained class of configurations may not be the most general supersymmetric one. This constraint is removed for the supersymmetric (nonextreme) states by applying an additional subset of  $SO(8,24)$  transformations [13,23] on the corresponding supersymmetric (nonextreme) generating solutions.  $O(8,24)$  is the symmetry of the effective three-dimensional action for the corresponding stationary solutions. An analogous procedure was used [24] to generate all the black holes in Abelian Kaluza-Klein theory.

<sup>7</sup>We thank A. Sen for pointing out to us the procedure to derive such a mass.

<sup>4</sup>Such a solution is obtained along the similar lines as the generating solution for the supersymmetric, spherically symmetric solutions in Abelian Kaluza-Klein theory [19].

$$M_{\text{BPS}}^2 = e^{-\phi_\infty} \{ \vec{P}^T \mathcal{M}_R \vec{P} + \vec{Q}^T \mathcal{M}_R \vec{Q} + 2[(\vec{P}^T \mathcal{M}_R \vec{P})(\vec{Q}^T \mathcal{M}_R \vec{Q}) - (\vec{P}^T \mathcal{M}_R \vec{Q})^2]^{1/2} \}. \quad (12)$$

Note, that when the magnetic  $\vec{P}$  and electric  $\vec{Q}$  charges are parallel in the SO(6,22) sense, this ADM mass is the bound for configurations that preserve  $\frac{1}{2}$  of  $N=4$  supersymmetry [8,11–13,25,26], i.e., the corresponding generating solution is either purely electric or purely magnetic. In the case when the magnetic and electric charges are not parallel, the mass is larger and the configurations preserve  $\frac{1}{4}$  of  $N=4$  supersymmetry.

We now turn to the discussion of the thermal and global space-time properties of such configurations, which can be classified according to the number of nonzero charges of the generating solutions.

(i) The case with *all the four charges nonzero*<sup>8</sup> corresponds to black holes (BH's) with a horizon at  $r=0$  and a timelike singularity hidden behind the horizon, i.e., the global space-time structure is that of the extreme Reissner-

<sup>8</sup>Solutions with two electric (and two magnetic) charges equal correspond to configurations with constant  $M$ . A class of such configurations was obtained by Kallosh *et al.* [10]. For the case where all the four charges are equal, all the scalars are constant and the four-dimensional metric reduces to that of Reissner-Nordström BH's, which is also pointed out in Ref. [27].

Nordström BH's. The corresponding Hawking temperature  $T_H = \partial_r \lambda|_{r=0} / (2\pi)$  is zero and the entropy ( $\equiv \frac{1}{4}$  of the area of the event horizon) is finite  $S = \pi \sqrt{|P_1^{(1)} P_1^{(2)} Q_2^{(1)} Q_2^{(2)}|}$ .

(ii) The case with three nonzero charges corresponds to solutions with a singularity located at the horizon ( $r=0$ ),  $T_H=0$ , and  $S=0$ .

(iii) The case with two nonzero charges,<sup>9</sup> say,  $P_1^{(1)} \neq 0 \neq P_1^{(2)}$ , corresponds to singular solutions with the horizon and the singularity coinciding at  $r=0$ ,  $T_H = 1/(4\pi \sqrt{|P_1^{(1)} P_1^{(2)}|})$ , and  $S=0$ .

The case with one nonzero charge<sup>10</sup> corresponds to BH's with a naked singularity,  $T_H = \infty$  and  $S=0$ .

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<sup>9</sup>Configurations with two nonzero electric charges were constructed by Sen [13], and shown by Peet [26] to be supersymmetric. The supersymmetric configurations with  $P_1^{(1)} \neq 0$  and  $Q_2^{(1)} \neq 0$ , found by the authors [19], correspond to configurations in the Kaluza-Klein sector of the theory.

<sup>10</sup> $P_1^{(1)} \neq 0$  case corresponds to the Kaluza-Klein monopole solution of Gross and Perry, and Sorkin [28], and were shown to be supersymmetric by Gibbons and Perry [25]. The case when  $P_1^{(2)} \neq 0$  corresponds to the  $H$ -monopole solution [12].

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