

## E(7) symmetric area of the black hole horizon

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Extreme black holes with 1/8 of unbroken  $N=8$  supersymmetry are characterized by the nonvanishing area of the horizon. The central charge matrix has four generic eigenvalues. The area is proportional to the square root of the invariant quartic form of  $E_{7(7)}$ . It vanishes in all cases when 1/4 or 1/2 of supersymmetry is unbroken. The supergravity nonrenormalization theorem for the area of the horizon in the  $N=8$  case protects the unique  $U$ -duality invariant. [S0556-2821(96)50210-9]

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$N=8$  supergravity has a hidden symmetry of equations of motion under the group  $E_{7(7)}$  as was discovered by Cremmer and Julia [1]. It was shown by Duff and Lu [2] that this hidden symmetry has its origin in supermembrane duality, which in turn implies the string duality by simultaneous dimensional reduction. More recently Hull and Townsend [3] provided some arguments that the discrete subgroup of  $E(7)$  may be an exact symmetry of the string theory. The corresponding discrete subgroup is called  $E_7(\mathbf{Z})$  and the symmetry is called  $U$  duality. It has been also emphasized by Witten [4] that the nonperturbative string dynamics has its deep origin in 11-dimensional supergravity, which is known to be the source of the hidden symmetry in four dimensions.

The purpose of this Rapid Communication is to show that the area of the horizon of the extreme black holes with unbroken supersymmetry can be studied from the perspective of  $N=8$  supergravity. It was understood some time ago that the supersymmetric bounds on the Arnowitt-Deser-Misner (ADM) mass  $M$  as well as the charge quantization are  $U$ -duality invariant [3]. However if  $U$  duality is indeed the symmetry of the theory, we may be able to establish the connection of this symmetry with the area of the extreme black hole horizons. The basic reason to look for such a connection comes from the fact that the canonical geometry of the black hole does not change under  $E_{7(7)}$  transformations. They affect only the scalars and the vectors of the theory. Thus one may guess that a simple formula for the area of the extreme black hole horizon may exist which has the following properties.

(i) In a generic case when all four values of the moduli of the eigenvalues of the central charge matrix  $|z_i|$  are different one has 1/8 of  $N=8$  supersymmetry unbroken,

$$M = |z_1|, \quad M > |z_2|, \quad M > |z_3|, \quad M > |z_4|, \quad A \neq 0, \quad (1)$$

and the area of the horizon  $A$  has to be  $E_{7(7)}$  symmetric, or, with an account taken of black hole charge quantization,  $E_7(\mathbf{Z})$  symmetric.

(ii) When all four moduli of the eigenvalues of the central charge matrix coincide, 1/2 of  $N=8$  supersymmetry is unbroken, and the area of the horizon  $A$  should vanish:

$$M = |z_1| = |z_2| = |z_3| = |z_4|, \quad A = 0. \quad (2)$$

(iii) For the particular case of (i) studied earlier with 1/4 of  $N=4$  supersymmetry unbroken,

$$M = |z_1|, \quad M > |z_2|, \quad |z_3| = |z_4| = 0, \quad (3)$$

the  $E_{7(7)}$  symmetric formula should reproduce the result [5,6]

$$A = 4\pi(|z_1|^2 - |z_2|^2). \quad (4)$$

It should of course vanish when two moduli of the eigenvalues of the central charge matrix coincide,

$$M = |z_1| = |z_2|, \quad A = 0. \quad (5)$$

This example shows that when  $|z_1| = |z_2|$  and the unbroken supersymmetry is thanks the area shrinks to zero. The corresponding two-dimensional diamondlike picture was presented in Fig. 1 in [5]. Inside the diamond the black holes are not extreme and do not have an unbroken supersymmetry. At each edge of the diamond there is 1/4 of supersymmetry unbroken (a different part of  $N=4$  for each side). At the vertices of a diamond the unbroken supersymmetry is always doubled, since each vertex has the supersymmetry of each adjoining edge of the diamond that enters into a given vertex. The original picture was drawn for the real values of central charges. Subsequently we found that the same picture appears to be valid upon  $SL(2, \mathbf{Z})$  rotation: the area as a function of the moduli of two central charges given in Eq. (4) is  $SL(2, \mathbf{Z})$  symmetric (see [6]).

Now we would like to have an analogous picture in terms of the moduli of the four eigenvalues of the central charge matrix for  $N=8$  supersymmetry with various vertices of coinciding two or four central charges corresponding to the shrinking area of the black hole horizon. It is rather difficult to visualize this multidimensional figure with vertices describing the pattern of restoring double and/or quartic supersymmetry relative to edges.

Fortunately, the hidden symmetry of  $N=8$  supergravity helps to find the solution. There are actually not so many possibilities to verify: there exists exactly one quartic  $E_{7(7)}$

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invariant that can be built from one<sup>1</sup> central charge matrix  $Z_{AB}$ . To support our conjecture about this ‘‘generalized diamond’’ function we have to show that the area of the black hole horizon is proportional to the square root of this invariant.

The quartic invariant [1] can be represented in the simple form<sup>2</sup>

$$\diamond = \text{Tr}(ZZ\bar{Z})^2 - \frac{1}{4}(\text{Tr}ZZ\bar{Z})^2 + 4(\text{Pf}Z + \text{Pf}\bar{Z}). \quad (7)$$

Here

$$Z_{AB} = (q^{ab} + ip_{ab})(\Gamma^{ab})_A^B, \quad (8)$$

and  $(\Gamma^{ab})_A^B$  are the SO(8) matrices. The 28 electric  $q^{ab}$  and 28 magnetic  $p_{ab}$  charges are given in terms of the components of  $2 \times 28$  vector  $\hat{Z} = \hat{V}Z$ . Here  $\hat{V}$  is the constant value of the  $E_7(\mathbf{Z})$ -valued field  $V$ , and  $\mathcal{Z}$  is the  $2 \times 28$  vector of quantized electric and magnetic charges. The Pfaffian Pf of the antisymmetric complex matrix  $Z_{AB}$  is defined as  $\text{Pf}Z \equiv \epsilon^{ABCDEFGH} Z_{AB} Z_{CD} Z_{EF} Z_{GH}$ .

The group  $E_7$  acts on  $Z_{AB}$  as

$$\delta Z_{AB} = \Lambda_A^C Z_{CB} + \Lambda_B^C Z_{AC}, \quad (9)$$

$$\delta Z_{AB} = \Sigma_{ABCD} \bar{Z}^{CD}, \quad (10)$$

where  $\Lambda_A^C$  are 63 anti-Hermitian generators of SU(8), and  $\Sigma_{ABCD}$  are totally antisymmetric and self-dual generators of  $E_7$  orthogonal to SU(8),

$$\Sigma_{ABCD} = \frac{1}{24} \epsilon_{ABCDEFGH} \bar{\Sigma}^{EFGH}. \quad (11)$$

Only the discrete subgroup of  $E_{7(7)}$  is compatible with the quantization condition on dyon black hole charges. The quartic invariant of  $E_{7(7)}$  is also an  $U$ -duality invariant. Thus we satisfy condition (i) by construction.

We may now check our condition (ii). For example we may consider an  $a = \sqrt{3}$  extreme black hole, embedded into  $N=8$  supergravity. For an electrically charged solution with real positive central charges we have

$$z_1 = z_2 = z_3 = z_4 = M. \quad (12)$$

The quartic invariant can be calculated using

$$\text{Tr}(ZZ\bar{Z})^2 = 8M^4, \quad \text{Tr}ZZ\bar{Z} = 8M^2, \quad \text{Pf}Z = \text{Pf}\bar{Z} = M^4.$$

This gives

$$\diamond = 8M^4 - 16M^4 + 4M^4 + 4M^4 = 0. \quad (13)$$

For the pure magnetic case

<sup>1</sup>In the case of two central charges there exists a symplectic invariant for  $E_{7(7)}$  that was already used by Hull and Townsend [3] to verify that the quantization of charges for two dyon black holes is  $U$ -duality symmetric.

<sup>2</sup>The detailed form in which it is presented in [1] is

$$\begin{aligned} \diamond &= Z_{AB} \bar{Z}^{BC} Z_{CD} \bar{Z}^{DA} - \frac{1}{4} Z_{AB} \bar{Z}^{AB} Z_{CD} \bar{Z}^{CD} \\ &+ \frac{1}{96} (\epsilon_{ABCDEFGH} \bar{Z}^{AB} \bar{Z}^{CD} \bar{Z}^{EF} \bar{Z}^{GH} + \epsilon^{ABCDEFGH} Z_{AB} Z_{CD} Z_{EF} Z_{GH}). \end{aligned} \quad (6)$$

$$z_1 = z_2 = z_3 = z_4 = iM, \quad (14)$$

and since the invariant is quartic in central charges, we get the same result: a vanishing area for the solutions with 1/2 of unbroken  $N=8$  supersymmetry.

To verify the condition (iii) we will use the fact that for this case we may consider  $Z_{12} = z_1$ ,  $Z_{34} = z_2$ , and have other elements of  $Z_{AB}$  vanishing. This leads to

$$\text{Tr}(ZZ\bar{Z})^2 = 2(|z_1|^4 + |z_2|^4), \quad \text{Tr}ZZ\bar{Z} = 2(|z_1|^2 + |z_2|^2),$$

$$\text{Pf}Z = \text{Pf}\bar{Z} = 0.$$

In this case

$$\diamond = 2(|z_1|^4 + |z_2|^4) - \frac{1}{4} [2(|z_1|^2 + |z_2|^2)]^2 = (|z_1|^2 - |z_2|^2)^2. \quad (15)$$

Thus we have learned that the area  $A$  of the SL(2; $\mathbf{Z}$ )-symmetric axion-dilaton horizon [5,6] in terms of the quartic invariant of  $E_7(\mathbf{Z})$  is given by

$$A = 4\pi \sqrt{|\diamond|}. \quad (16)$$

This shows again that the  $U$ -duality symmetric formula for the area indeed covers previously known solutions. The area is proportional to the square root of the quartic invariant. This is in complete agreement with the fact that  $E_7(\mathbf{Z})$  has SL(2; $\mathbf{Z}$ ) as a subgroup:

$$E_7(\mathbf{Z}) \supset \text{SL}(2; \mathbf{Z}) \times \text{SO}(6, 6; \mathbf{Z}). \quad (17)$$

It is interesting to check the area formula on more general solutions with all four central charges nonvanishing. For example, we can consider the truncation of  $N=8$  supergravity to the form describing  $N=4$  supergravity interacting with vector multiplets. We consider the action in the form [7]

$$\begin{aligned} S &= \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R - \frac{1}{2} [(\partial\eta)^2 + (\partial\sigma)^2 + (\partial\rho)^2] \right. \\ &\quad \left. - \frac{e^{-\eta}}{4} [e^{-\sigma-\rho}(F_1)^2 + e^{-\sigma+\rho}(F_2)^2 \right. \\ &\quad \left. + e^{\sigma+\rho}(F_3)^2 + e^{\sigma-\rho}(F_4)^2] \right). \end{aligned} \quad (18)$$

One can use various versions of the known double dyon solutions with 1/4 of unbroken supersymmetry of  $N=4$  theory [9,7]. These solutions are described by two different central charges. The detailed description of the corresponding two central charges from the heterotic point of view as well as from the point of view of the type II theory compactified on  $K^3$  is given in Ref. [8]. The simplest solution with 1/8 of unbroken  $N=8$  supersymmetry characterized by four different central charges is

$$ds^2 = -e^{2U} dt^2 + e^{-2U} dx^2, \quad e^{4U} = \psi_1 \psi_2 \chi_1 \chi_2,$$

$$e^{-2\eta} = \frac{\psi_1 \psi_3}{\chi_2 \chi_4}, \quad e^{-2\sigma} = \frac{\psi_1 \chi_4}{\chi_2 \psi_3}, \quad e^{-2\rho} = \frac{\psi_1 \chi_2}{\psi_3 \chi_4},$$

$$F_1 = \pm d\psi_1 \wedge dt, \quad \tilde{F}_2 = \pm d\chi_1 \wedge dt,$$

$$F_3 = \pm d\psi_3 \wedge dt, \quad \tilde{F}_4 = \pm d\chi_4 \wedge dt, \quad (19)$$

where

$$\begin{aligned}\psi_1 &= \left(1 + \frac{|q|_1}{r_1}\right)^{-1}, & \chi_2 &= \left(1 + \frac{|p|_2}{r_2}\right)^{-1}, \\ \psi_3 &= \left(1 + \frac{|q|_3}{r_3}\right)^{-1}, & \chi_4 &= \left(1 + \frac{|p|_4}{r_4}\right)^{-1},\end{aligned}\quad (20)$$

and magnetic potentials correspond to  $\tilde{F}_{2/4} = e^{-\eta^\pm(-\sigma+\rho)} F_{2/4}^*$ , where the asterisk denotes the Hodge dual. Charges in each gauge group could be placed either in various places ( $r_1 \neq r_2 \neq r_3 \neq r_4$ ) or in just one place ( $r_1 = r_2 = r_3 = r_4$ ). The signs of all charges could take any values, without correlation between various gauge groups. The resulting configuration is characterized by four different central charges (with  $4G=1$ ):

$$\begin{aligned}z_1 &= (q_1 + q_3) + (p_2 + p_4), \\ z_2 &= (q_1 + q_3) - (p_2 + p_4), \\ z_3 &= (q_1 - q_3) + (p_2 - p_4), \\ z_4 &= (q_1 - q_3) - (p_2 - p_4).\end{aligned}\quad (21)$$

The mass is equal to the largest of the moduli of the eigenvalues of the central charge matrix  $M = \max|z_i|$ ,  $i=1,2,3,4$ . The area is proportional to the square root of the absolute value of the product of electric and magnetic charges,

$$A = 4\pi |q_1 p_2 q_3 p_4|^{1/2}, \quad (22)$$

and in terms of  $N=8$  central charges we have found the area to be equal to

$$A = 4\pi \left( \sum_i z_i^4 - 2 \sum_{i>j} z_i^2 z_j^2 + 8z_1 z_2 z_3 z_4 \right)^{1/2}. \quad (23)$$

Again we see that for  $z_3 = z_4 = 0$  our formula is reduced nicely to  $A = 4\pi(|z_1|^2 - |z_2|^2)$ . The crucial check comes here: Will the diamond formula (7) reproduce this expression? Yes, it does. One can verify that Eq. (7) in the case  $Z_{12} = z_1$ ,  $Z_{34} = z_2$ ,  $Z_{56} = z_3$ ,  $Z_{78} = z_4$  reproduces Eq. (23).

We would like to make two comments on the black hole solutions in  $N=8$ . The first one is related to the cosmic censorship conjecture. It says that naked singularities cannot appear as a result of gravitational collapse. This does not help much for charged stringy black holes since there are no elementary particles that would carry the corresponding charges, and therefore these black holes cannot appear as a result of gravitational collapse anyway. Supersymmetry, which leads to the Bogomol'ni bound, sometimes implies the absence of naked singularities [5]. However, the link between supersymmetry and cosmic censorship is not universal, it does not exist, e.g., for  $a = \sqrt{3}$  black holes. It is interesting that for  $N=8$  supersymmetry broken down spontaneously all the way to 1/8 of supersymmetry plays the role of a cosmic censor. Indeed, as long as one keeps away from all the vertices where unbroken supersymmetry is doubled or quadrupled, the singularities in canonical geometry are protected by the horizon, exactly as was observed in Ref. [5] in  $N=4$  theory. One may try to develop the idea of Rahmfeld [7] that the nonsingular black holes of the Reissner-Nordström-type can be build out of the elementary constituents (for example from four  $a = \sqrt{3}$  solutions) that

are singular when free. One may satisfy the condition of the absence of naked singularities if one assumes that four of these singular constituents,  $F$  electropole,  $H$  electropole,  $F$  monopole, and  $H$  monopole may be confined inside the non-singular black hole. Each of the elementary black hole solutions carries the central charges as follows. The first one has  $z_1 = z_2 = z_3 = z_4$ , the second one has  $z_1 = z_2 = -z_3 = -z_4$ , the third one has  $z_1 = -z_2 = z_3 = -z_4$  and finally the last one has  $z_1 = -z_2 = -z_3 = z_4$ . Each of the elementary constituents breaks 1/2 of the  $N=8$  supersymmetry, however, each one breaks a different part of it. Four of them can be placed in four different points in space. When all charges are placed at one point in space we have a configuration described by the geometry of the Reissner-Nordström type with the singularity protected by the horizon, whose area is given by the unique formula  $A = 4\pi\sqrt{|\diamond|}$ . The unbroken supersymmetry of all four elementary constituents forms only 1/8 of the  $N=8$  supersymmetry, which is the maximum common part of the unbroken supersymmetry of all four constituents. As long as all four elementary black holes are at one point, we have a configuration with the singularity covered by the horizon. If we take one of the  $a = \sqrt{3}$  solutions outside this area, we would have a naked singularity. Thus, if one really wants to avoid the violation of the cosmic censorship (which may not be necessary in application to stringy solitons) one may conjecture that  $N=8$  supersymmetry has to be broken spontaneously down to 1/8. The elementary black holes have to be confined inside the horizon for this purpose. This picture is consistent with the idea of black holes as elementary particles, suggested by Holzhey and Wilczek [10] in the context of  $\sqrt{3}$  extreme black holes.

The second comment is about the difference between black hole solutions in  $N=8$  and  $N=4$  theories. If all four supersymmetric positivity bounds of  $N=8$  supersymmetry are respected for the black hole solutions, there are no non-trivial massless solutions, since the mass has to be larger than all four eigenvalues of the central charge matrix. However, in  $N=4$  supersymmetry we have only two positivity bounds to respect, and some combination of charges (left-handed in the heterotic theory or some specific combination in type II string on  $K^3$ ) do not enter the central charge matrix anymore. Therefore they do not have to vanish simultaneously with the vanishing ADM mass, and the massless solitons become available [11].

It is interesting to note that the quartic invariant in  $E_{7(7)}$  was constructed by Cartan [12,1] in a form that is different from the one that was found later by Cremmer and Julia and that we used here. It is believed that these two forms are proportional to each other, however, to the best of our knowledge, no proof of this is available. Cartan's quartic form is

$$\begin{aligned}J &= x^{ij} y_{jk} x^{kl} y_{li} - \frac{1}{4} x^{ij} y_{ij} x^{kl} y_{kl} \\ &+ \frac{1}{96} (\epsilon^{ijklmnop} y_{ij} y_{kl} y_{mn} y_{op} + \epsilon_{ijklmnop} x^{ij} x^{kl} x^{mn} x^{op}).\end{aligned}\quad (24)$$

This alternative quartic form suggests a very nice interpretation of the fact that the area is a product of four charges in Eq. (22). For this purpose we have to perform the dual transformation from  $SU(8)$  version of the  $N=8$  supergravity

to SO(8) version. This type of double analysis was used to study extreme black holes in Ref. [5] where we used in parallel the SU(4) and the SO(4) version of  $N=4$  supergravity. The electric and magnetic charges in the first theory become either two electric or two magnetic charges in the other one. In  $N=8$  case this will lead us to reinterpret all four charges as either magnetic or electric. For example we will get either a pure electric solution with  $x^{12}=q_1$ ,  $x^{34}=p_2$ ,  $x^{56}=q_3$ ,  $x^{56}=p_4$ , or a pure magnetic one with  $y_{12}=q_1$ ,  $y_{34}=p_2$ ,  $y_{56}=q_3$ ,  $y_{56}=p_4$ . In both cases the area is reproduced by Cartan's quartic invariant. In the first pure electric case we have the contribution only from the fourth term in  $J$ , while in the pure magnetic case only the third term contributes. In both cases we get the same result:

$$J = J_{\text{el}} = J_{\text{magn}} = 4 q_1 p_2 q_3 p_4. \quad (25)$$

This makes it plausible that the E(7) symmetric formula (16) for the area is in addition proportional to the square root of the Cartan's quartic invariant:

$$A \sim \sqrt{|J|}. \quad (26)$$

The first two terms do not contribute to (25), since we have used a very simple solution. However, some more complicated examples of solutions of the heterotic string theory were found recently by Cvetič and Tseytlin [9] where the area does not reduce to the product of four charges. It would be very interesting to promote this solution to  $N=8$  theory with 1/8 of unbroken supersymmetry and verify the area formula for them. Various black hole solutions with a different number of unbroken supersymmetry of  $N=8$  theory have been studied also in Ref. [13]. Khuri and Ortín [14] have recently classified various supersymmetric embeddings into  $N=8$  theory of the known  $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$  extreme black holes. This study suggests various possibilities for analyzing our area formula. In particular, the puzzle of the existence of the nonsupersymmetric dyon embedding can be understood from the E(7) point of view.<sup>3</sup> It is clear from (24) why the

<sup>3</sup>The supersymmetric embedding of the  $a=0$  black hole in  $N=4$  theory that was performed in Ref. [5] is a particular case of the solution described above with  $2Q_R = q_1 + q_3$ ,  $2P_R = p_2 + p_4$ ,  $2Q_L = q_1 - q_3 = 0$ ,  $2P_L = p_2 - p_4 = 0$  and the area  $\sim |Q_R P_R|$ . Thus it also required at least four vector fields, from the perspective of SO(8) theory, to be present in the solution.

E(7) symmetry requires at least four electric or four magnetic charges in SO(8) version to be nonvanishing to get a nonvanishing area simultaneous with 1/8 of the unbroken supersymmetry. This reflects the fact that the on-shell superfields of  $N=8$  supergravity are E(7) symmetric.

So far all checks on the extreme black holes in  $N=8$  theory with 1/8 of unbroken supersymmetry completely confirm the area formula (16), (26). Hopefully, more elaborate solutions will lead us to all possible realizations of the  $U$  duality.

The arguments in favor of the nonrenormalization theorem for the extreme black hole area of the horizon were presented in Ref. [5]. They were based on the fact that the unbroken supersymmetry of the bosonic solution of supergravity is equivalent to the existence of the fermionic isometries in the corresponding superspace. Therefore the calculation of the on-shell action cannot produce quantum corrections as long as the corrections come from the supersymmetric invariants, be they local or nonlocal. Because of the presence of the fermionic isometries all invariants given by the full superspace integrals are guaranteed to vanish due to Berezin's rules of integration over the anticommuting variables. Apart from possible supersymmetry anomalies (which are not expected in  $N=8$  supergravity) and possible integrals over the submanifold (which were studied before and do not seem to challenge the solutions with unbroken 1/8 supersymmetry), the nonrenormalization theorem for extreme black holes area seems to have a pretty solid basis. In view of this it is particularly satisfying that this theorem protects a unique quartic invariant of  $E_7(\mathbf{Z})$ .

The main conclusion of this work is the following. The largest hidden symmetry of supergravity with 133 parameters becomes manifest if one looks into the structure of the extreme black hole horizon. 20 years ago Ramond gave a talk [15] with the following title: "Is there an exceptional group in your future? E(7) and the travails of the symmetry breaking." This prediction most certainly worked for extreme supersymmetric black holes.

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