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Entropy perturbations due to cosmic strings

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We examine variations in the equation of state of the cosmic string portion of the cosmological fluid which lead to perturbations of the background matter density. These fluctuations in the equation of state are due to variations in the local density of cosmic string loops and gravitational radiation. Constructing a crude model of the distribution of entropy perturbations, we obtain the resulting fluctuation spectrum using a gauge-invariant formalism. We compute the resulting cosmic microwave background anisotropy, and estimate the effect of these perturbations on the cosmic string structure formation scenario. [S0556-2821(96)50110-4]

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I. INTRODUCTION

Cosmic strings are topological defects which may have formed in a grand unified theory era phase transition. Cosmic strings may play an important role in cosmology [1], inducing temperature anisotropies in the cosmic microwave background (CMB), and seeding perturbations for the formation of large scale structure (LSS).

In past work, Veeraraghavan and Stebbins [2] developed a formalism for the analysis of linearized perturbations to the cosmological fluid induced by cosmic strings. These techniques have been applied to the simulation of CMB temperature anisotropy [3], and to the analysis of string perturbations in cold dark matter for the formation of LSS [4]. In both applications, the effects of the strings on the linear density field were computed by convolving the string stress-energy tensor with the appropriate Green's function. As well, density perturbations, anticorrelated with the cosmic strings, were included to compensate for perturbations induced by strings at formation.

We propose to compute the effects of an additional source of cosmic string-generated perturbations, not explicitly included in previous analyses. These are entropy perturbations, caused by the inhomogeneous shifting of energy between cosmic string loops and gravitational radiation emitted by loops. The spatial variation in the equation of state of the total cosmological fluid is due to statistical fluctuations in the density of long cosmic strings, from which the distribution of small loops and gravitational radiation follow. Below a coherence scale ξ, the equation of state of the loops and radiation is uniform. The loops and radiation act together as a dilute gas, shifting the expansion rate within a volume ξ^3. Above ξ, the equation of state varies as rms statistical fluctuations, oscillating with wavelengths longer than the coherence scale.

The reason why these effects have not been explicitly treated in past work are the following. First, no simulation has included the gravitational radiation emitted by loops and strings in the cosmic string stress-energy tensor, for the purposes of CMB or LSS calculations. The effect of gravitational wave, tensor perturbations on the scalar, gravitational potential is a second-order effect. However, variations in the equation of state, as considered in this paper, result in a first-order effect in linearized gravity. Second, the perturbations resulting from the variations in the equation of state, although not genuine fluctuations in the total density, have not been modeled by the compensations employed to date [2]. The compensations included in CMB and LSS analyses have simply accounted for the initial distribution of strings and loops, not the subsequent shift in the form of matter, from loops and long strings to gravitational radiation.

Our goal in this paper is to estimate the importance of entropy perturbations for the formation of large-scale structures and CMB anisotropy, prior to carrying out a more detailed investigation. (Some preliminary work has been carried out in [5].) In the following, we present a simple analysis of the effects of these perturbations from cosmic strings. In Sec. II we present a model of the variations in the cosmic string equation of state. In Sec. III we give explicit solutions for the gauge-invariant potential, characterizing the density fluctuations induced by the entropy perturbations. We also compute the power spectrum, and comment on the effects on large-scale structure formation. In Sec. IV we estimate the effect of these perturbations on the CMB. We conclude in Sec. V.

II. MODEL OF ENTROPY PERTURBATIONS

Entropy perturbations to the cosmological fluid arise when there is an inhomogeneous shift in energy from one form of matter to another. Consider a perturbing component of the fluid $\delta\rho$ with pressure δp. The resulting perturbation to the entropy density is

$$\delta S = T^{-1}(\delta p - c_s^2 \delta\rho), \tag{2.1}$$

where c_s is the speed of sound in the background medium at temperature T. Hence, the presence of an additional form of matter for which $\delta p \neq c_s^2 \delta\rho$, that is the perturbing matter does not obey the equation of state of the background fluid, leads to a perturbation of the entropy density. In the cosmic string scenario, the long cosmic strings, loops, and emitted gravitational radiation,

$$\delta\rho = \rho_\infty + \rho_{\rm loops} + \rho_{\rm gr}, \tag{2.2}$$

serve as the perturbing component of the cosmological fluid, driving the entropy perturbations in (2.1). More formally, we write

$$T\delta S = -\tfrac{1}{3}\Theta_{\mu\nu}g^{\mu\nu} + \tfrac{1}{3}(1-3c_s^2)\Theta_{\mu\nu}u^\mu u^\nu, \tag{2.3}$$

where $\Theta_{\mu\nu}(\eta,\vec{x})$ is the cosmic string stress-energy tensor, including the gravitational radiation, and u_μ is a unit timelike four-vector orthogonal to the time-constant spatial hypersurfaces. Given the source (2.3), we may carry out a procedure similar to that in [2–4]: convolve the source with the appropriate Green's function to obtain the gravitational potential. While it may be straightforward to adapt a cosmic string simulation to this task, in the present work we will construct an analytic model describing the cosmic string entropy perturbations.

Treating the cosmic string network as a dilute gas, we may compute the pressure contributed by the strings. In the dust-dominated era, only the gravitational radiation contributes to the entropy perturbations, δS in Eq. (2.1). Loops evolve as collisionless dust, so they do not contribute. Hence, it is the shifting of energy from cosmic strings into gravitational radiation that drives the entropy perturbations.

Based on a crude model of network evolution in which the strings rapidly approach a scaling solution, the energy injected into gravitational radiation, in the dust-dominated era, in a time interval given by the coherence time scale ξ is a constant fraction, $\mathcal{F}$, of the background energy density. Because the gravitational radiation is free streamed at the speed of light, we estimate that the coherence scale is given by the Hubble length, $\xi = \mathcal{H}^{-1}$. Hence, decomposing the perturbing energy density field in terms of an amplitude and spatial distribution, we write

$$T\delta S(\eta,\vec{x}) = \frac{1}{3}\mathcal{F}_{\rm gr}\rho_0 f(\eta,\vec{x}). \tag{2.4}$$

Here, η is the conformal time, ρ_0 is the background fluid energy density, $\mathcal{F}$ is the rms amplitude of the statistical fluctuations in the energy density as a fraction of the background, and $f(\eta,\vec{x})$ is a normalized distribution describing the spatial variation of the entropy perturbations. In the dust-dominated era, $\mathcal{F}_{\rm gr} \sim 4\pi\tilde{A}G\mu(1-2\langle v^2\rangle) \sim G\mu$. In this expression $\tilde{A}$, defined by

$$\tilde{A} \equiv \sqrt{\langle(\rho_\infty - \langle\rho_\infty\rangle)^2_\xi\rangle}\, t^2\mu^{-1}, \tag{2.5}$$

is the amplitude of the statistical fluctuations in the long string density ρ_∞, averaged on a length scale ξ, where t is physical time. The squared, average velocity along the string is $\langle v^2\rangle = 0.37$ in the dust-dominated era [6]. We caution that the decomposition (2.4) is a simplification. A more realistic analysis would use the stress-energy tensor of the cosmic strings, as determined by a numerical simulation, to compute δS.

The function $f(\eta,\vec{x})$ describes how the perturbations to the equation of state vary with position and time. We decompose this function in terms of Fourier harmonics:

$$f(\eta,\vec{x}) = (2\pi)^{-3}\int d^3k e^{i\vec{k}\cdot\vec{x}}\tilde{f}(\eta,k),$$
$$\langle\tilde{f}(\eta,k)\tilde{f}(\eta',k')\rangle = (2\pi)^3\delta(\vec{k}-\vec{k}')|\tilde{f}(\eta,k)\tilde{f}(\eta',k)|, \tag{2.6}$$

where $\tilde{f}(\eta,k) = |\tilde{f}(\eta,k)|e^{i\theta_{\vec{k}}}$. The fluctuations in the density of long strings lead to statistically homogeneous, isotropic variations in the density of loops and gravitational radiation. Because the long cosmic strings make a statistically random walk on scales larger than the string correlation length, the variations in the equation of state, and hence the phases $\theta_{\vec{k}}$, are randomly distributed. The product of the Fourier coefficients, $|\tilde{f}(\eta,k)\tilde{f}(\eta',k)|$, plays a role similar to that of the structure function used by Albrecht and Stebbins [4] to describe the matter perturbations directly induced by cosmic strings and loops. We normalize this distribution by smoothing the variance over a time scale ξ:

$$\sigma_f^2 \equiv \xi^{-1}\int_0^\xi d\xi'\langle f(\eta,\vec{x})f(\eta',\vec{x})\rangle = 1. \tag{2.7}$$

The reason for smoothing in time is that the cosmological fluid can only react to changes in the local equation of state due to the cosmic string loops and gravitational radiation on time scales greater than the coherence time scale. Hence we need only restrict the variance of the distribution f on the time scales which will lead to entropy perturbations.

We expect that the distribution does not vary on length scales smaller than the time ξ, so that $\tilde{f}(\eta,k>\xi^{-1})=0$. As a general distribution we write $\tilde{f}$ as a power law in wave number k, on scales above ξ:

$$|\tilde{f}(\eta,k)|=2\pi\sqrt{(\alpha+5/2)(\alpha+3/2)}\,\xi^{3/2}(k\xi)^{\alpha}\Theta(1-k\xi), \tag{2.8}$$

where $\xi/\eta=$ const. The numerical coefficients and dependence on ξ follow as a result of the normalization, in Eq. (2.7). The step function is defined as $\Theta(x)=1$ for $x\geqslant 0$ and $\Theta(x)=0$ for $x=0$. The parameter α determines how scales above the coherence scale ξ contribute to the distribution. As $\alpha\to\infty$,

$$\lim_{\alpha\to\infty}|\tilde{f}(\eta,k)|=2\pi\xi^{3/2}\delta(1-k\xi), \tag{2.9}$$

so that the distribution is characterized by a single, comoving scale. For finite, decreasing values of α, the range of comoving scales contributing to $\tilde{f}$ increases. In the limit $\alpha\to 0$, the distribution has support on all wavelengths above the comoving scale ξ. Because the network of long strings is distributed as a random walk on large scales, we expect the entropy perturbations to similarly have support on scales above ξ. Hence, we expect $\alpha\to 0$ to be more physically reasonable than $\alpha\to\infty$. However, in the following calculations we will take $\alpha\to\infty$ in order to simplify the calculations, noting that for other values of α, the results are not strongly affected. Having specified the properties of the entropy perturbations, we may proceed to determine the resulting fluctuation spectrum.

III. EVOLUTION OF PERTURBATIONS

To describe the perturbations of the background cosmological fluid, we use a gauge-invariant formalism [7,8]. We must supply an expression for the density perturbation relative to the hypersurface which represents the local rest frame of the cosmological fluid; this prescription gives as close as possible a ''Newtonian'' time slicing. For an entropy perturbation, that is a perturbation to the equation of state, we find the evolution of the gauge-invariant potential Φ is given by

$$\Phi''+3(1+c_s^2)\mathcal{H}\Phi'+[2\mathcal{H}'+(1+3c_s^2)\mathcal{H}^2]\Phi-c_s^2\nabla^2\Phi$$
$$=4\pi Ga^2(\delta p-c_s^2\delta\rho). \tag{3.1}$$

Here we work with conformal time η, where $'\equiv\partial/\partial\eta$, and $\mathcal{H}=a'/a$. The coordinates $(\eta,\vec{x})$ are dimensionless, as the expansion scale factor a carries units of length.

We will also compute the power spectrum of the gauge-invariant potential generated by the entropy perturbations in the dust-dominated era. We define the correlation function of the potential to be the second moment of Φ:

$$\xi(\eta,r)\equiv\langle\Phi(\eta,\vec{x})\Phi(\eta,\vec{x}')\rangle=4\pi\int k^2dk\frac{\sin(kr)}{kr}\mathcal{P}_\Phi(\eta,k). \tag{3.2}$$

Here $r=|\vec{x}-\vec{x}'|$ is a conformal distance. Recall that the density contrast δ due to the gauge-invariant potential is given by

$$\delta\equiv\frac{2}{3}\mathcal{H}^{-2}(\nabla^2\Phi-3\mathcal{H}\Phi'-3\mathcal{H}^2\Phi). \tag{3.3}$$

For modes well within the horizon, $k\gg\mathcal{H}$, the density contrast is dominated by the gradient term. Thus, we have the relationship

$$\mathcal{P}_\delta=\frac{4k^4}{9\mathcal{H}^4}\mathcal{P}_\Phi \quad \text{for } k\gg\mathcal{H} \tag{3.4}$$

relating the power spectrum in the potential to the power in the density contrast.

In the dust-dominated era, the scale factor behaves as $a\propto\eta^2$ and the speed of sound c_s for the cold, collisionless dust vanishes so that the evolution equation for the gauge-invariant potential simplifies greatly. We may solve the differential equation as

$$\Phi(\eta,\vec{x})=4\pi G\int_{\eta_{\rm eq}}^{\eta}d\eta'\,\eta'^{-6}\int_{\eta_{\rm eq}}^{\eta'}d\eta''\,\eta''^6a^2(\eta'')$$
$$\times[\delta p(\eta'',\vec{x})-c_s^2\delta\rho(\eta'',\vec{x})]+(\eta-\eta_{\rm eq})$$
$$\times\Phi'(\eta_{\rm eq},\vec{x})+\Phi(\eta_{\rm eq},\vec{x}). \tag{3.5}$$

Hence, given the boundary terms Φ and Φ' at $\eta_{\rm eq}$, at the onset of the dust-dominated era, it is straightforward to obtain the potential Φ at any later time.

The source of the entropy perturbations is the gravitational radiation emitted by oscillating cosmic string loops. Using (2.4), we may obtain the dust-era solution for Φ. We will make the simplification that the boundary terms are unimportant. This is reasonable provided we are interested in length scales due to perturbations that enter the horizon after radiation-matter equality. Hence, we find

$$\mathcal{P}_\Phi(\eta,k)=\frac{4\mathcal{F}_{\rm gr}^2}{(2\pi)^3}\left|\int_{\eta_{\rm eq}}^{\eta}d\eta'(\eta')^{-6}\int_{\eta_{\rm eq}}^{\eta'}d\eta''(\eta'')^4\tilde{f}(\eta'',k)\right|^2. \tag{3.6}$$

In the case $\alpha\to\infty$, the power is

$$\lim_{\alpha\to\infty}\mathcal{P}_\Phi(\eta,k)=\frac{2\mathcal{F}_{\rm gr}^2}{25\pi}k^{-3}[1-x^{-5}]^2\Theta(x-1)\Theta(1-x_{\rm eq}). \tag{3.7}$$

Here, $x=k\xi$. For other values of α the power spectrum also behaves as $\mathcal{P}_\Phi\propto k^{-3}$ for modes well within the horizon, generating a scale-invariant isocurvature spectrum. Fluctuations generated during the dust-dominated era will have wavelengths from $\sim 10h^{-2}$ Mpc, the horizon scale at radiation-matter equality, up to the present horizon scale. These fluctuations will contribute to the large angle CMB anisotropy and the formation of the largest structures.

We may compare the power in entropy-generated density fluctuations with the power due to the perturbations induced directly by the cosmic strings. Referring to (3.4), we find

$$4\pi k^3\mathcal{P}_\delta(k)=\frac{288}{25}\mathcal{F}_{\rm gr}^2(kh)^4, \tag{3.8}$$

where k has units h^2/Mpc and h is the Hubble parameter normalized to $H_0 = 100$ km/s/Mpc. Compare this to the power spectrum due to cosmic strings in cold dark matter (CDM) (Eq. (8) of [4]):

$$4\pi k^3 \mathcal{P}_\delta(k) = 4\pi(6.8)^2(kh)^4(G\mu)^2. \tag{3.9}$$

Here we have used the I model of Albrecht and Stebbins [4] motivated by the string simulation results of Bennett and Bouchet, and Allen and Shellard [6], on length scales well above the $8h^{-1}$ Mpc normalization scale. Constructing a ratio of the LSS power due to the perturbations induced directly in the cosmological fluid by cosmic strings, to those generated by entropy perturbations, we obtain

$$\mathcal{P}_{\rm direct}/\mathcal{P}_{\rm entropy} \sim 50(G\mu)^2 \mathcal{F}_{\rm gr}^{-2} = 3.2\tilde{A}^{-2}. \tag{3.10}$$

We see that for $\mathcal{F}_{\rm gr} \sim 7G\mu$ or, using the estimate for $\mathcal{F}_{\rm gr}$ preceding Eq. (2.5), for $\tilde{A} \sim 1.8$ the power for large-scale structures are comparable. As $\alpha \to 0$, such that the entropy perturbations have support on a wide range of length scales, the power due to entropy perturbations increases slightly, and the powers are comparable for $\tilde{A} \sim 1.4$. The reason for the increase is that entropy perturbations generated at a range of times, rather than at a single time as with the $\alpha \to \infty$ model, now contribute to the power in a single mode, on these scales. Numerical simulations indicate that the mean energy density in long strings during the dust-dominated era is $\rho_\infty = (4 \pm 1)\mu/t^2$, [6] obtained by averaging over multiple realizations of the string simulation. If the quoted uncertainty in ρ_∞ measures the amplitude of the statistical fluctuations, averaged on a scale ξ, then $\tilde{A} \sim 1$. Modeling the long string network as a random walk on large scales, we expect $\tilde{A} \sim 0.6$ [5]. Hence, we expect $\tilde{A} \sim 0.6 - 1$ to be a reasonable range for the amplitude of entropy perturbations. We caution that these results are intended as a preliminary estimate; a more reliable calculation will use the string simulations directly to compute the effects of entropy perturbations.

IV. CMB ANISOTROPY

The CMB anisotropies induced by the entropy-generated perturbations may be computed from the gauge-invariant potential. The dominant contribution on large angular scales, as is common for isocurvature perturbations, is due to the time variation of the potential Φ, integrated along the line of sight to the surface of last scattering:

$$\frac{\Delta T}{T}(\hat{n}) = 2\int_{\eta_{ls}}^{\eta_0} d\eta\, \Phi'(\eta, |\eta_0 - \eta|\hat{n}). \tag{4.1}$$

Here, $\hat{n}$ is a unit vector on the celestial sphere. The correlation function is expressed in the usual form:

$$C(\theta) \equiv \left\langle \frac{\Delta T}{T}(\hat{n}) \frac{\Delta T}{T}(\hat{n}') \right\rangle = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos\theta). \tag{4.2}$$

The amplitude of the multipole moments is given by C_l, and $\hat{n}\cdot\hat{n}' = \cos\theta$. Evaluating the large-angle CMB anisotropy due to dust-era entropy perturbations only, in the $\alpha \to \infty$ case, we find $l^2 C_l \sim 6\mathcal{F}_{\rm gr}^2$ for $10 \lesssim l \lesssim 40$. Recall that $\mathcal{F}_{\rm gr} \sim 4\tilde{A}G\mu$, so that for $\tilde{A} \sim 2.0$ these multipole moments are comparable to the $l^2 C_l \sim 400(G\mu)^2$ perturbations measured in the numerical simulations [3]. This is roughly consistent with the result for $\tilde{A}(\sim 1.8)$ found in the previous section, in Eq. (3.10), in order that the LSS power due to the entropy perturbations is comparable to the power due to the perturbations induced directly by cosmic strings.

The effects of the entropy-generated perturbations on small-scale CMB anisotropy are estimated using the analytic techniques developed by Hu and Sugiyama [9]:

$$C_l = \frac{2}{\pi}\int \frac{dk}{k} k^3 |\theta_l(\eta_0, k)|^2,$$

$$\begin{aligned}\theta_l(\eta_0,k) = {} & (\theta_0(\eta_{ls},k) - \tilde{\Phi}(\eta_{ls},k)) j_l(k|\eta_0 - \eta_{ls}|) \\ & + \theta_1(\eta_{ls},k)(2l+1)^{-1}(l j_{l-1}(k|\eta_0-\eta_{ls}|) \\ & -(l+1) j_{l+1}(k|\eta_0 - \eta_{ls}|)) \\ & -2\int_{\eta_{ls}}^{\eta_0} d\eta\, \tilde{\Phi}'(\eta,k) j_l(k|\eta_0 - \eta|).\end{aligned} \tag{4.3}$$

In the above equations (see Eqs. (12) and (14) of [9]), θ_0 and θ_1 are the monopole and dipole moments of the perturbations to the baryon-photon fluid. The final term, the integrated Sachs-Wolfe effect, dominates on large angular scales, as with isocurvature models. Modeling only the dust-era entropy perturbations, the resulting contribution to the CMB anisotropy due to monopole and dipole oscillations of the baryon-photon fluid is small. However, if we artificially lengthen the duration of the prerecombination, matter-dominated epoch as a way of extending the validity of our model to smaller scales and earlier times, we find that significant anisotropy may result on small angular scales. For the $\alpha \to \infty$ model with a coherence scale $\xi = \mathcal{H}^{-1}$, a series of peaks develops, beginning near $l \sim 200$, before diffusion damps out the perturbations. Smaller values of the coherence scale lead to a decrease in the peak amplitude and a shift in the peak location to higher l values. It is important to note that the location of the peak in the spectrum is sensitive to the value of the comoving coherence scale ξ for the entropy perturbations. Magueijo *et al.* [10] have carried out a similar calculation of the small-angle anisotropies using the Albrecht-Stebbins LSS model as an effective source for fluctuations in the baryon-photon fluid. There, they found that the location of the peak shifts to larger values of l as the coherence length scale decreases. This behavior may be an important tool for discriminating between inflationary and defect-based models. Furthermore, perturbations generated by scale-dependent variations in the equation of state, as considered in this paper, may be a generic property of defect-based cosmological scenarios. For global defects, however, the generation of entropy perturbations will be due to the distribution of Goldstone, rather than gravitational, radiation. Similar calculations have been carried out for textures by Crittenden and Turok [11].

V. CONCLUSION

In this paper we have analyzed the effects of entropy perturbations in the cosmic string scenario. These perturbations arise from spatial variations in the density of cosmic string loops and gravitational radiation, leading to a field of approximately Gaussian fluctuations in the equation of state of the cosmological fluid. This source of anisotropy has not been explicitly considered in previous analyses, because the contribution to the cosmic string stress-energy tensor by gravitational waves has not been included. By constructing a crude model of this source, we have been able to estimate the effects of the resulting perturbations on large-scale structure formation and the cosmic microwave background.

Let us comment now on this analytic model. We claim that the distribution function used to describe the variations in the density of gravitational radiation and loops, and hence the variations in the equation of state, is a reasonable first approximation for the purposes of estimating the relative importance of this effect. There are certain shortcomings of this construction, among them the uncertainties in the amplitude of the rms fluctuations $\widetilde{A}$, the size of the comoving coherence scale ξ, and the dependence of the distribution f on wave number k, which we have condensed into the parameter α. For large-scale perturbations, the amplitude and shape of the spectrum does not depend strongly on α or ξ, as a scale-invariant isocurvature power spectrum results. For what we expect to be a reasonable range for the amplitude, $\widetilde{A} \sim 0.6-1$, the entropy perturbations make a non-negligible contribution to the LSS and large-angle CMB spectrum. In summary, we have crudely modeled the essential features: a statistically homogeneous, isotropic distribution of fluctuations in the equation of state, varying on length scales above the comoving horizon radius. It remains to compare this model with the distribution observed in a realistic cosmic string simulation. We plan to carry out these advances in the near future.

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