Strangelets with finite entropy

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Strangelets with nonzero entropy are studied within the MIT bag model. Explicit account is taken of the constraints that strangelets must be color neutral and have a fixed total momentum. In general, masses increase with increasing entropy per baryon, and the constraints work so as to increase masses further. This has an important destabilizing effect on strangelets produced in ultrarelativistic heavy ion collisions. [S0556-2821(96)50209-2]

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Several ultrarelativistic heavy-ion collision experiments at Brookhaven and CERN are searching for (meta)stable lumps of roughly equal numbers of up, down, and strange quarks—so-called strangelets [1]. If created, strangelets are characterized by a very low charge-to-mass ratio, and they could provide one of the best indications of quark-gluon plasma formation.

An extensive body of literature [2] has studied the properties of strangelets at zero temperature, but the impact of nonzero entropy (temperature), which is certainly a condition to be expected in the hot environment of ultrarelativistic heavy-ion collisions, has not been investigated in detail. Clearly, the addition of thermal energy will lead to an increase in strangelet masses, and this is indeed what is demonstrated below. Furthermore, additional increases in the energy come about when one restricts the strangelets to be color singlets, and to have a fixed total momentum. All of this leads to a destabilization relative to zero entropy (temperature) calculations, which is of significant importance for the experimental production and detection of these objects.

In the present investigation we study strangelets in the finite entropy regime. We use the multiple reflection expansion approach [3] within the MIT bag model [4], and in order to study the important consequences of color singletness and definite momentum in a transparent manner, we set all quark masses equal to zero. Since the s-quark mass is expected to be in the range of 100–300 MeV we thereby get the ''most optimistic'' values possible (from a production point of view) for strangelet masses etc. Using zero quark masses and the multiple reflection expansion allows us to write many of our expressions in an analytical form, which is more transparent than the numerical integrals and sums otherwise obtained. On the other hand it prevents us from showing individual shell effects in the energy as a function of baryon number; only the mean effects of the shells are included. Preliminary results from a finite temperature shell-model calculation by Mustafa and Ansari [5] (without the color singlet and momentum restrictions) indicate that shells are washed out at temperatures exceeding 10–20 MeV, so above this temperature the two approaches should yield identical results.

For pedagogical reasons we first look at strangelet properties without imposing restrictions of color singletness and definite momentum. Here the general expression for the grand potential of particle species i is

$$\Omega_i = \mp g_i T \int_0^\infty dk \frac{dN}{dk} \ln(1 \pm \exp\{-[\epsilon(k)-\mu]/T\}), \quad (1)$$

where the upper sign is for fermions, the lower for bosons. μ and T are the chemical potential and temperature, k is the particle momentum, ϵ the corresponding energy, and g_i the statistical weight. The smoothed density of states, dN/dk, is given by the multiple reflection expansion with MIT bag model boundary conditions. For massless quarks $dN/dk = 6\{k^2V/2\pi^2 - C/24\pi^2\}$, where V and C denote volume and extrinsic curvature. For spherical strangelets characterized by $V=4\pi R^3/3$ and $C=8\pi R$ an integration gives, per flavor of massless quarks (including antiquarks),

$$\Omega_q = -\left(\frac{7\pi^2}{60}T^4 + \frac{\mu^2 T^2}{2} + \frac{\mu^4}{4\pi^2}\right)V + \left(\frac{T^2}{24} + \frac{\mu^2}{8\pi^2}\right)C, \quad (2)$$

with a corresponding net quark number, i.e., the number of quarks less the number of antiquarks,

$$N_q = -\left(\frac{\partial \Omega_q}{\partial \mu}\right)_{T,V} = \left(\mu T^2 + \frac{\mu^3}{\pi^2}\right)V - \frac{\mu}{4\pi^2}C. \quad (3)$$

For gluons with $dN/dk = 16\{k^2V/2\pi^2 - C/6\pi^2\}$,

$$\Omega_g = -\frac{8\pi^2}{45}T^4 V + \frac{4}{9}T^2 C. \quad (4)$$

Here and in the following we often explicitly write thermodynamical expressions in terms of μ, T, V, and C. One should notice, that since we concentrate on spherical systems, $C \equiv 8\pi(3/4\pi)^{1/3}V^{1/3}$, so V is the only independent ''shape'' variable. However, the use of C makes it more clear where finite-size corrections enter. Also, μ and T are sometimes functions of other variables, such as particle number N and entropy S.

The total Ω can be found from summing the terms above plus the bag energy BV and other thermodynamical quantities such as the free energy F and the internal energy E, can be derived. For three massless quark flavors of equal chemical potential (this gives the lowest possible energy and electrical neutrality, so that no Coulomb energy needs to be taken into account) one finds

$$\Omega(T,V,\mu)=\left(-\frac{19\pi^2}{36}T^4-\frac{3}{2}\mu^2T^2-\frac{3}{4\pi^2}\mu^4+B\right)V+\left(\frac{41}{72}T^2+\frac{3}{8\pi^2}\mu^2\right)C, \tag{5}$$

$$F(T,V,N)=\left(-\frac{19\pi^2}{36}T^4+\frac{3}{2}\mu^2T^2+\frac{9}{4\pi^2}\mu^4+B\right)V+\left(\frac{41}{72}T^2-\frac{3}{8\pi^2}\mu^2\right)C, \tag{6}$$

$$E(S,V,N)=\left(\frac{19\pi^2}{12}T^4+\frac{9}{2}\mu^2T^2+\frac{9}{4\pi^2}\mu^4+B\right)V-\left(\frac{41}{72}T^2+\frac{3}{8\pi^2}\mu^2\right)C, \tag{7}$$

where the entropy $S\equiv-\partial\Omega/\partial T|_{V,\mu}$.

Strangelets are in mechanical equilibrium when $\partial F/\partial V|_{T,N}=\partial\Omega/\partial V|_{T,\mu}=\partial E/\partial V|_{S,N}=0$, corresponding to

$$BV=\left(\frac{19\pi^2}{36}T^4+\frac{3}{2}\mu^2T^2+\frac{3}{4\pi^2}\mu^4\right)V-\left(\frac{41}{216}T^2+\frac{1}{8\pi^2}\mu^2\right)C. \tag{8}$$

Thus in mechanical equilibrium one gets the following expressions for the grand potential, free energy, internal energy, and baryon number:

$$\Omega=\left(\frac{41}{108}T^2+\frac{1}{4\pi^2}\mu^2\right)C, \tag{9}$$

$$F=\left(3\mu^2T^2+\frac{3}{\pi^2}\mu^4\right)V+\left(\frac{41}{108}T^2-\frac{1}{2\pi^2}\mu^2\right)C, \tag{10}$$

$$E=4BV, \tag{11}$$

$$A=\left(\mu T^2+\frac{1}{\pi^2}\mu^3\right)V-\frac{\mu}{4\pi^2}C. \tag{12}$$

Equation (11) follows directly from Eqs. (7) and (8), and it is in fact a general result for ultrarelativistic particles in a bag, since the energy density of a relativistic gas is 3 times the particle pressure, which equals B, so $E=3BV+BV=4BV$. For massive quarks this result no longer holds.

Dotted curves in the figures illustrate the behavior of energy per baryon as a function of baryon number and temperature or entropy per baryon derived from the equations above.

So far we have not explicitly taken into account the fact that strangelets have to be color singlets, and must have a definite total momentum. To do this we use the color singlet and fixed momentum projected grand canonical partition function of Elze and Greiner [6], which we have independently checked. This partition function, calculated using the group theoretical projection method [7], is derived in a saddle-point approximation valid at high temperature and/or chemical potential. The partition function is

$$Z=\Pi_{\rm color}\Pi_{p=0}Z^{(0)}, \tag{13}$$

where $\Pi_{\rm color}$ is the correction factor due to the color singlet constraint, and $\Pi_{p=0}$ is the correction factor due to the fixed momentum constraint, here taken at zero total momentum [8]. This factorization is only valid in the saddle-point approximation. $Z^{(0)}$ is the unprojected partition function for a collection of noninteracting massless quarks, anti-quarks, and gluons in a spherical MIT bag. [The grand potential in Eq. (5) equals $-T\ln Z^{(0)}$.] Both the partition function and the projection factors are calculated with a density of states based on the multiple reflection expansion. The color projection factor is given by

$$(2\pi\sqrt{3}\Pi_{\rm color})^{-1/4}=VT^3\left\{2+\mathcal{N}_q\left[\frac{1}{3}+\left(\frac{\mu}{\pi T}\right)^2\right]\right\}+CT\frac{12-\mathcal{N}_q}{12\pi^2}, \tag{14}$$

and the factor due to the zero-momentum constraint is

$$\pi\Pi_{p=0}^{-2/3}=VT^3\pi^2\left\{\mathcal{N}_q\left[\frac{7}{30}+\left(\frac{\mu}{\pi T}\right)^2+\frac{1}{2}\left(\frac{\mu}{\pi T}\right)^4\right]+\frac{16}{45}\right\}-CT\left\{\frac{\mathcal{N}_q}{72}\left[1+3\left(\frac{\mu}{\pi T}\right)^2\right]+\frac{4}{27}\right\}. \tag{15}$$

Terms proportional to $\mathcal{N}_q$, which is the number of massless quark flavors, originate from quarks, while the remaining terms are due to gluons.

We now have the ingredients necessary to calculate the energy per baryon for a zero-momentum, color-neutral drop of quark matter at finite temperature (entropy). As discussed earlier we concentrate on three flavors of massless quarks with equal chemical potentials. We introduce the constrained grand potential

$$\Omega_{\rm con}(T,V,\mu)=-T\ln Z(T,V,\mu). \tag{16}$$

For each baryon number A, we then solve the equations of mechanical equilibrium,

$$\left(\frac{\partial\Omega_{\rm con}}{\partial V}\right)_{T,\mu}=0, \tag{17}$$

fixed baryon number,

$$-\left(\frac{\partial\Omega_{\rm con}}{\partial\mu}\right)_{T,V}=3A, \tag{18}$$

and fixed entropy per baryon,

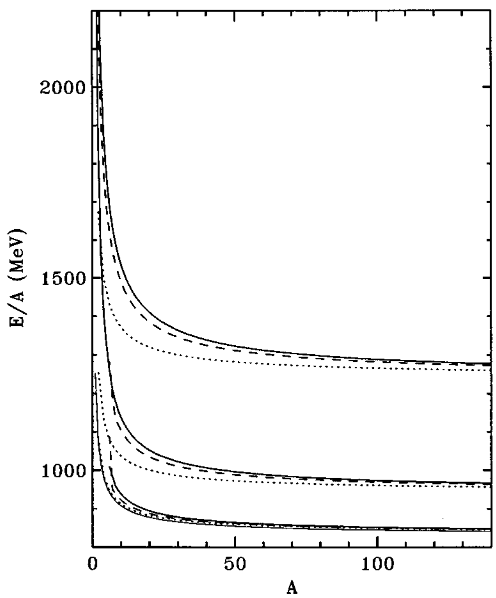

FIG. 1. Energy per baryon as a function of baryon number for strangelets with equal numbers of massless up, down, and strange quarks. $T=0$ results are shown by the downmost, thin curve. Otherwise, dotted curves are results without constraints, dashed curves with the color singlet restriction, and full curves with both color singlet and zero-momentum constraint. The entropy per baryon is 10 for the upper set of curves, 5 for the set in the middle, and 1 for the lowest set. The bag constant was chosen as $B^{1/4}=145$ MeV. For other choices of B the energy scales in proportion to $B^{1/4}$.

$$-\frac{1}{A}\left(\frac{\partial\Omega_{\mathrm{con}}}{\partial T}\right)_{V,\mu}=\frac{S}{A}, \tag{19}$$

with respect to T, μ, and V.

Using $E=4BV$ we then calculate the energy per baryon as a function of baryon number and show the results for fixed S/A in Fig. 1, where dashed curves include the color singlet constraint without the fixed momentum constraint, and full curves include both color singlet and fixed momentum constraints. As expected (when calculated for fixed natural variable S) both constraints lead to an increase in energy. For very low A there is no solution to Eqs. (17)–(19). The lack of a solution for small A ($A\leqslant 1$–7, depending on S/A; for $S=0$ solutions exist for all A) is a feature common to the saddle-point approximation and the numerical calculation (see Fig. 3). It can be traced to the breakdown of the approximation to the density of states for very low values of the product of particle momentum and bag radius kR. Below the value of kR corresponding to the ground state, the density of states becomes spuriously negative. For low A this part of the spectrum is weighted relatively more.

Results for fixed T [i.e., without imposing Eq. (19)] are shown in Fig. 2, where the results for the unprojected case, each of the two projections alone, and together are superimposed for different temperatures. For sufficiently high temperature and low baryon number, the effect of (mainly) the color singlet constraint is equivalent to a lowering of the temperature by as much as 10 MeV. In other words, the curve including color singlet corrections (or both corrections) crosses the curves for the unconstrained calculation at lower temperatures. It is also seen that the color singlet constraint is the most important of the two in terms of the effect on the energy per baryon.

One notices that the effect of color singletness goes away for small T (S/A). This is how it should be, because for $T=0$ there is no problem in constructing a color neutral

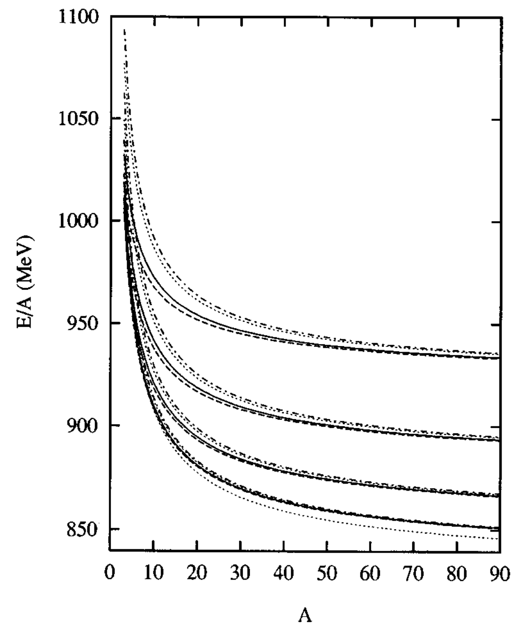

FIG. 2. The energy per baryon in the unprojected case (dotted lines), including the zero-momentum constraint (dashed-dotted lines), including the color-singlet constraint (dashed lines), and with both constraints (full lines). The calculations were again done for $B^{1/4}=145$ MeV and 3 massless quark flavors. From bottom to top: $T=0,10,20,30,40$ MeV. The lower end point of all curves is at $A=3$.

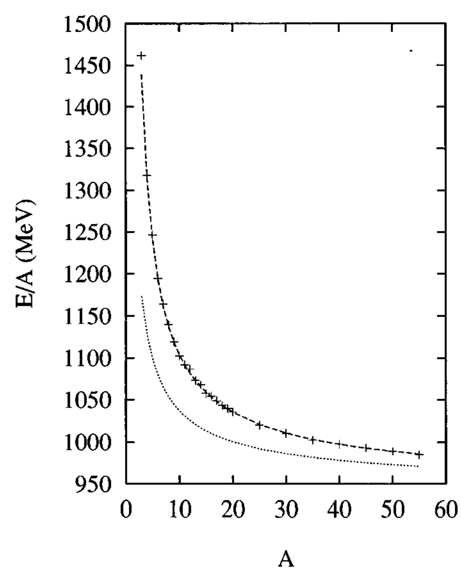

FIG. 3. The energy per baryon for $S/A=5$ and $B^{1/4}=145$ MeV including only the color singlet constraint. The saddle-point approximation (dashed line) is seen to be in very good agreement with the result obtained from a numerical evaluation of the color projected partition function (shown as +). No solutions were found for $A\leqslant 2$. The unprojected result is shown for reference (dotted line).

strangelet by placing quarks in the lowest energy levels (e.g., constructing a strangelet with $A=6$ from 2 blue, 2 green, and 2 red up quarks, and similarly for down and strange quarks, with all quarks in the $1S_{1/2}$ ground state). For $T>0$ quarks are statistically distributed over energy levels, and the constraints reduce the number of possible configurations, forcing the energy up. Also, the constraints are only important for $A<100$.

The validity of the saddle-point approximation is demonstrated in Fig. 3, where the approximation is compared with the result of a numerical integration, for a particular choice of parameters. In the high chemical potential regime relevant to the present investigation the approximation is very good.

As stated in the introduction we have assumed zero quark masses in the interest of clarity and ease. Nonzero quark masses, in particular that of the s quark, will of course further increase strangelet masses. For $T=0$ this increase manifests itself as a well-understood shift in E/A [9]. We expect a similar shift to occur for $T>0$. Since there are more up and down quarks (and gluons, at high T) than there are strange quarks in the ground state strangelets for $m_s>0$, there is no reason to believe that the quantitative effects of finite entropy, color singlet, and momentum constraints differ significantly from those found in the present paper.

We have shown that the mass of strangelets increases with the entropy per baryon, or temperature, of the system. At fixed entropy per baryon the mass is further increased when the objects are constrained to be color singlets, and (to a lesser extent) by the requirement of a definite total momentum (taken to be zero in the calculations [8]). The total magnitude of the effect is of order 80 MeV/baryon for temperatures of 40 MeV (which for high baryon numbers corresponds to roughly 4 units of entropy/baryon), and increases rapidly for higher entropy (temperature). This important change in energy (and other corresponding thermodynamical parameters) must be taken into account in models for production and detection of strangelets in ultrarelativistic heavy ion collisions. In particular, it is worth noting that the relative importance of the effects grows in the low-baryon number regime that is easiest to probe in laboratory experiments. This will make strangelet searches in these regimes more difficult, but metastability still seems possible for some parameter choices [10], and in any case more detailed production and evolution models for strangelets are needed to give a definite prediction of the experimental signature. The effects described in this investigation also play a role in relation to quark matter formation in other circumstances, such as protoneutron stars.

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[10] Naively, metastability is possible for E/A below the Λ mass, $m_\Lambda=1116$ MeV.