# Nucleon tensor charges in the SU(2) chiral quark-soliton model

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We investigate the singlet $g_T^{(0)}$ and isovector $g_T^{(3)}$ tensor charges of the nucleon, which are deeply related to the first moment of the leading twist transversity quark distribution $h_1(x)$, in the SU(2) chiral quark-soliton model. With rotational $O(1/N_c)$ corrections taken into account, we obtain $g_T^{(0)}=0.69$ and $g_T^{(3)}=1.45$ at a low normalization point of several hundreds MeV. Within the same approximation and parameters the model yields $g_A^{(0)}=0.36$, $g_A^{(3)}=1.21$ for axial charges and a correct octet-decuplet mass splitting. We show how the chiral quark-soliton model interpolates between the nonrelativistic quark model and the Skyrme model. [S0556-2821(96)00309-6]

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The complete information about the quark structure of the nucleon in leading-order hard processes is contained in three twist-two parton distributions. Two of them [$f_1(x)$ and $g_1(x)$] have been studied extensively theoretically and measured in deep-inelastic scattering experiments [1]. The third transversity quark distribution $h_1(x)$ is inaccessible for measurements in inclusive deep-inelastic experiments. However, the $h_1(x)$ plays an essential role in polarized Drell-Yan processes [2] and other exclusive hard reactions [3–5]. The measurement of the $h_1(x)$ has been proposed recently by the BNL Relativistic Heavy Ion Collider (RHIC) Spin Collaboration [6] and HERMES Collaboration at the DESY $ep$ collider HERA [7].

The evolution equation for $h_1(x)$ has been derived in Refs. [8,9]. Also it was shown by Jaffe and Ji [5] that the first moment of $h_1(x)$ is related to the nucleon tensor charge,

$$\int_0^1 dx[h_1(x)-\overline{h_1}(x)]=g_T^f, \tag{1}$$

where $f$ is a flavor index ($f=u,d,s,\ldots$) and the tensor charge $g_T^f$ is defined as the forward nucleon matrix element [5,10]:

$$\langle N|\overline{\psi_f}\sigma_{\mu\nu}\psi_f|N\rangle=g_T^f\overline{U}\sigma_{\mu\nu}U, \tag{2}$$

where $U(p)$ is a standard Dirac spinor and $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$. It is convenient to introduce singlet and isovector tensor charges:

$$g_T^{(3)}=g_T^u-g_T^d,\quad g_T^{(0)}=g_T^u+g_T^d. \tag{3}$$

The tensor charges depend on the renormalization scale, and the corresponding anomalous dimension at one loop has been calculated in Refs. [2,9]: $\gamma=2\alpha_s/3\pi$.

Our aim is to calculate the tensor charges (2) in the chiral quark-soliton model ($\chi$QSM, often called the semibosonized Nambu–Jona-Lasinio model) at a low normalization point of several hundreds MeV.

The $\chi$QSM has been successful in reproducing the static properties of the baryons such as the octet-decuplet mass splitting [11–14], axial charges [15–18], and magnetic moments [19,20] and their form factors [19,21] (for details, see the recent review [22]). The baryon in this model is regarded as a bound state of $N_c$ quarks bound by a nontrivial chiral field configuration. Such a semiclassical picture of baryons can be justified in the $N_c\to\infty$ limit in line with more general arguments by Witten [23]. A remarkable virtue of $\chi$QSM is that the model interpolates between the nonrelativistic quark model (NRQM) and the Skyrme model [24]. In particular, due to such an interplay, it enables us to examine the dynamical difference between the axial and tensor charges of the nucleon.

In the following, we employ the effective QCD partition function from the instanton picture of QCD in the limit of low momenta. It is given by a functional integral over pseudoscalar and quark fields [25],

$$\mathcal{Z}=\int\mathcal{D}\Psi\mathcal{D}\Psi^\dagger\mathcal{D}\pi^A\exp\left(i\int d^4x\Psi^\dagger iD\Psi\right), \tag{4}$$

where $iD$ and $U^{\gamma_5}$ denote the Dirac differential operator and the pseudoscalar chiral field, respectively:

$$iD=\beta(-i\partial\!\!\!/+MU^{\gamma_5}+\overline{m}\mathbf{1}),\quad U^{\gamma_5}=e^{i\pi^A\tau^A\gamma_5}. \tag{5}$$

$\tau^A$ are Pauli matrices and $M$ is the dynamical quark mass which arises as a result of the spontaneous chiral symmetry breaking and is momentum dependent. The momentum dependence of $M$ introduces the natural ultraviolet cutoff (inverse average instanton size $1/\rho\sim600$ MeV) [25] for the theory given by Eq. (4) and simultaneously brings a renormalization scale to the model. The $\overline{m}$ stands for the current quark mass defined by $\overline{m}=(m_u+m_d)/2$ with isospin symmetry assumed. The operator $iD$ is expressed in Euclidean space in terms of the Euclidean time derivative $\partial_\tau$ and the Dirac one-particle Hamiltonian $H(U)$:

$$iD=\partial_\tau+H(U), \tag{6}$$

with

$$H(U)=\frac{\vec{\alpha}\cdot\nabla}{i}+\beta MU+\beta\overline{m}\mathbf{1}. \tag{7}$$

One can relate the hadronic matrix element equation (2) to a correlation function:

$$\langle 0|J_B(\vec{x},T)\overline{\psi}\sigma_{\mu\nu}\tau^a\psi J_B^\dagger(\vec{y},0)|0\rangle \tag{8}$$

at large Euclidean time $T$. The baryon current $J_B$ can be constructed from quark fields:

$$J_B=\frac{1}{N_c!}\varepsilon^{i_1\cdots i_{N_c}}\Gamma_{II_3}^{\alpha_1\cdots\alpha_{N_c}}\psi_{\alpha_1 i_1}\cdots\psi_{\alpha_{N_c}i_{N_c}}, \tag{9}$$

where $\alpha_1\cdots\alpha_{N_c}$ are spin–isospin indices, $i_1\cdots i_{N_c}$ are color indices, and the matrices $\Gamma_{II_3}^{\alpha_1\cdots\alpha_{N_c}}$ are chosen in such a way that the quantum numbers of the corresponding current are equal to $II_3$. The correlation function (8) can be calculated in the effective chiral quark model defined by Eq. (4) using $1/N_c$ expansion. The related technique can be found in [11,22,26]. Here we give a result for the tensor charges to the next to leading order of the $1/N_c$ expansion:

$$g_T^{(0)}=\frac{\alpha}{I},\quad g_T^{(3)}=\beta+\frac{\delta}{I}, \tag{10}$$

where $\alpha,\ \beta,\ \delta$ and $I\sim N_c$ are given by

$$\alpha=\frac{iN_c}{2}\int d^3x\int\frac{d\omega}{2\pi}\mathrm{tr}\left\langle x\left|\frac{1}{\omega+iH}\tau_i\frac{1}{\omega+iH}\gamma_5\gamma^i\right|x\right\rangle, \tag{11a}$$

$$\beta=-\frac{N_c}{6}\int d^3x\int\frac{d\omega}{2\pi}\mathrm{tr}\left\langle x\left|\frac{1}{\omega+iH}\tau_i\gamma_5\gamma^i\right|x\right\rangle, \tag{11b}$$

$$\delta=\frac{iN_c}{6}\int d^3x\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}\mathscr{P}\frac{1}{\omega-\omega'}\varepsilon^{ijk}\times\mathrm{tr}\left\langle x\left|\frac{1}{\omega+iH}\tau_i\frac{1}{\omega'+iH}\tau_j\gamma_5\gamma_k\right|x\right\rangle, \tag{11c}$$

$$I=\frac{N_c}{2}\int d^3x\int\frac{d\omega}{2\pi}\mathrm{tr}\left\langle x\left|\frac{1}{\omega+iH}\tau_i\frac{1}{\omega+iH}\tau^i\right|x\right\rangle. \tag{11d}$$

Having examined Eqs. (11a)–(11d) in large $N_c$ limit, we find that $g_T^{(0)}\sim N_c^0$ and $g_T^{(3)}\sim N_c$, which are the same as in case of NRQM. In NRQM the tensor charges are equal to the corresponding axial charges. Though $N_c$ dependence of the tensor charges given above are equal to that of the corresponding axial ones, we shall show that the tensor and axial charges have a different behavior in the limit of large soliton size (large constituent quark mass).

Before studying the tensor charges, let us discuss how the surprisingly small value of the singlet axial charge (so called ‘‘*spin crisis*’’) is related to its asymptotic behavior in the limit of large soliton size in the present model. The suppression in the ratio of the axial charges $g_A^{(0)}/g_A^{(3)}\sim 1/N_c$ in large $N_c$ limit does not provide a solution to the ‘‘*spin crisis*,’’ since NRQM shows the same $N_c$ behavior of the singlet-isovector ratio but it simultaneously gives $g_A^{(0)}=1$. Hence, in order to understand the ‘‘*spin crisis*,’’ it is necessary to seek an additional suppression in the singlet-isovector ratio of axial charges. In the Skyrme model the ratio of the axial charges $g_A^{(0)}/g_A^{(3)}\sim 1/N_c^2$ is suppressed by the additional powers of the $1/N_c$ in comparison with NRQM [28] which was suggested as a solution to the ‘‘*spin crisis*.’’ However, this additional $1/N_c$ suppression is lifted in extensions of the Skyrme model by inclusion of vector mesons [29]. In $\chi$QSM [17,27] the ratio of axial charges is given by $g_A^{(0)}/g_A^{(3)}\sim 1/N_c$ in contrast to the Skyrme model. The difference is due to the nonlocality of the effective action for pions [Eq. (4)] in $\chi$QSM. In other words, higher gradient terms neglected in the Skyrme model give a nonvanishing contribution to the singlet $g_A^{(0)}$ in large $N_c$ limit.

$\chi$QSM interpolates between NRQM and the Skyrme one, i.e., in the limit of small soliton size it reproduces the results of NRQM, whereas in the opposite limit of large soliton size it mimics the Skyrme model. Besides the $1/N_c$ suppression, the ratio $g_A^{(0)}/g_A^{(3)}$ in our model is quenched in the limit of large soliton size (large constituent quark mass) by the inverse powers of the soliton size (quark mass). Indeed the numerical calculations for the self-consistent soliton give $g_A^{(0)}\approx 0.36$ [17], which is a relatively small number and is compared well with the experimental value $0.31\pm 0.07$ [30].

Reviewing Eqs.(11a)–(11d) in the limit of large soliton size (large constituent quark mass), one can easily find that $\alpha\sim(MR_0)^2$, $I\sim(MR_0)^3$, and $\beta,\delta\sim MR_0$. Therefore, the ratio of the tensor charges $g_T^{(0)}/g_T^{(3)}\sim 1/(MR_0)^2$ is sizably reduced in the limit of large soliton size, while the analogous analysis of the axial charges [17,24] gives even much stronger suppression in the ratio $g_A^{(0)}/g_A^{(3)}\sim 1/(MR_0)^6$. This observation of the different behaviors between the axial and tensor charges leads to a conclusion that the tensor charges might deviate from the axial ones remarkably.

In the limit of $MR_0\to 0$, $\chi$QSM corresponds to NRQM and yields $g_T^{(0)}=g_A^{(0)}=1$, $g_T^{(3)}=g_A^{(3)}=(N_c+2)/3$ (derivation for the axial charges, see Ref. [24]). Note that it is of great importance to take into account the rotational $1/N_c$ corrections [$\delta$ contribution in Eq. (10)] to derive this result in $O(N_c^0)$ order. The soliton in $\chi$QSM has a radius $MR_0\sim 1$, so that one could expect a deviation from NRQM predictions as well as from the Skyrme model results. In Fig. 1 we show the dependence of the tensor and axial charges on the soliton size. The results were obtained by calculating the functional traces in Eqs. (11a)–(11d) according to the Kahana and Ripka method [31] with a simple variational Ansatz for the profile function. We take advantage of the inverse-tangent profile function $P(r)=2\arctan(R_0^2/r^2)$ which has the correct asymptotic behavior of the profile function at small and large distances.

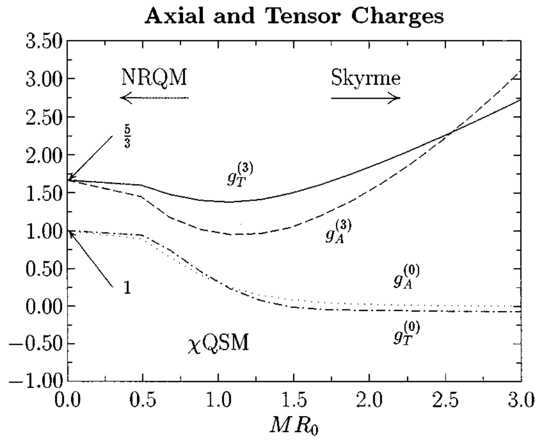


FIG. 1. The dependence of the axial and tensor charges on the soliton size. The solid curve represents the $g_T^{(3)}$, while the dashed curve draws the $g_A^{(3)}$. The dot-dashed curve depicts the $g_T^{(0)}$, whereas the dotted curve illustrates the $g_A^{(0)}$. The small arrows stand for the values of $g_T^{(3)}=g_A^{(3)}=5/3$ and $g_T^{(0)}=g_A^{(0)}=1$ in NRQM, respectively. The large arrows denote NRQM and Skyrme limit of the present model. The constituent quark mass for this figure is $M=370$ MeV to be consistent with Ref. [24].

From Fig. 1 we observe that the axial and tensor charges starting from the same values of $(N_c+2)/3\approx 1.67$ for the isovector case and 1 for the singlet one at small soliton size have qualitatively different behavior for larger $MR_0$: the dependence of the tensor charges on soliton size is weaker than the corresponding dependence of the axial charges. This qualitative difference is in accordance with the asymptotics of the charges in large soliton size:

$$g_A^{(3)}\sim(MR_0)^2,\quad g_T^{(3)}\sim MR_0,$$

$$g_A^{(0)}\sim\frac{1}{(MR_0)^4},\quad g_T^{(0)}\sim\frac{1}{MR_0}.$$

We see that indeed the asymptotic dependence of the tensor charges is weaker than the corresponding dependence of the axial charges. From this one can conclude that the tensor charges are closer to their values of $g_T^{(0)}=1$ and $g_T^{(3)}=5/3\approx 1.67$ in NRQM than the corresponding axial charges. Similar conclusions were obtained in the bag model [5].

In the above lines, we considered the dependence of $g_A^{(0)}$ and $g_A^{(3)}$ ($g_T^{(0)}$ and $g_T^{(3)}$, respectively) on $MR_0$. This can be translated into a dependence on the Dirac radius $R_1$ and allows then a direct comparison with recent results of Brodsky and Schlumpf [32]. For this we extracted $R_1$ from our self-consistent calculation [21] with several constituent quark masses and plotted in the vicinity of the physical point ($R_1=0.74$fm) $g_A^{(0)}$ and $g_A^{(3)}$ vs $M_N R_1$ with $M_N$ being the proton mass. We find that the slopes of these curves agree well with those of [32], though in Ref. [32] the value of $g_A^{(0)}=0.6$ appears to be larger than our 0.36 and experimental value $0.31\pm 0.07$ [30]. It is interesting to note that those models having quite different origins show comparable features. A detailed investigation will be presented elsewhere.

In order to evaluate the tensor charges numerically, we employ the self-consistent profile function obtained by diagonalizing the Dirac Hamiltonian in a box (we choose the radial box size $D\simeq 10$ fm to achieve good accuracy) and solving the self-consistent equations by iteration. The technical details can be found in Refs. [33,34].

TABLE I. The tensor charges of the nucleon $g_T^{(0)}$ and $g_T^{(3)}$ as varying the constituent quark mass $M$.

| $M$ | 370 MeV | 420 MeV | 450 MeV |
|---|---|---|---|
| $g_T^{(0)}$ | 0.756 | 0.688 | 0.686 |
| $g_T^{(3)}$ | 1.446 | 1.449 | 1.466 |

We have calculated the tensor charges for different values of the constituent quark mass, which is the only free parameter of the model. The corresponding results are reported in Table I. As our preferred value of the constituent quark mass, we choose $M=420$ MeV at which the model reproduces with good accuracy many nucleon observables: octet-decuplet mass splitting [14], isospin splittings in baryon octet and decuplet [35], singlet axial charge [17,27], magnetic moments, isovector axial charge [16], and electromagnetic form factors [19].

Finally, we obtain,

$$g_T^{(3)}\approx 1.45,\quad g_T^{(0)}\approx 0.69, \tag{12}$$

or

$$g_T^{(u)}\approx 1.07,\quad g_T^{(d)}\approx -0.38. \tag{13}$$

We find that our results are close to those in the bag model [5] and consistent with QCD sum rule calculations of Refs. [10,36].

It is worth noting that the dependence of the tensor charges on the normalization point is rather weak:

$$g_T^{(f)}(\mu)=\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{4/29} g_T^{(f)}(\mu_0), \tag{14}$$

as $\mu\to\infty$ the tensor charges slowly vanish. One can use this equation to recalculate the tensor charges at higher normalization points using the values of tensor charges (12) at low normalization points. The value of normalization point $\mu_0$ pertinent to our model is not uniquely determined from first principles; one has to choose $\mu_0$ of the order of $\rho^{-1}\simeq 600$ MeV, but there may be a factor of order unity. To do this quantitatively we follow the approach of Ref. [37] and define $\mu_0=a/R$ ($R$ is the average distance between instantons $\sim 1/200$ MeV$^{-1}$) with a dimensionless parameter $a$ to be varied in the variational estimate of bulk properties of the instanton medium. According to [37] the parameter $a$ can be varied without significant change of parameters of the effective low-energy theory [Eq. (4)] from $a\simeq 3$ to $a\simeq 7$. In this region of $a$ the one-loop QCD coupling constant varies in region (see Table I of Ref. [37]):

$$\frac{\alpha_s(\mu_0)}{2\pi}=0.098\pm 0.035. \tag{15}$$

Using these numbers and evolution Eq. (14) one can estimate an uncertainty of the tensor charge at high normalization points due to the uncertainty in the determination of the low normalization point $\mu_0$ pertinent to our model:

$$\frac{\Delta g_T(Q^2)}{g_T(Q^2)} \simeq \frac{4}{29} \frac{\Delta \alpha_s(\mu_0)}{\alpha_s(\mu_0)} = 0.05. \tag{16}$$

From this analysis we see that owing to the weak dependence of the tensor charges on the normalization point our results Eq. (13) for the tensor charges at low-energy normalization points acquire an additional error of about 5% being evolved to high normalization points.

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