

Dicyclic horizontal symmetry and supersymmetric grand unification

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It is shown how to use as horizontal symmetry the dicyclic group $Q_6 \subset SU(2)$ in a supersymmetric unification $SU(5) \otimes SU(5) \otimes SU(2)$ where one $SU(5)$ acts on the first and second families, in a horizontal doublet, and the other acts on the third. This can lead to acceptable quark masses and mixings, with an economic choice of matter supermultiplets, and charged lepton masses can be accommodated.

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The smallness of most of the quark masses and mixing parameters and the strong hierarchy among them is one of the most interesting puzzles in particle physics. Spontaneously broken horizontal symmetry is the most popular candidate theory for understanding the flavor structure, including in supersymmetric models. In the context of the minimal supersymmetric standard model (MSSM), a horizontal symmetry may also give a viable alternative to build in a super-Glashow-Iliopoulos-Maiani (GIM) mechanism to suppress flavor-changing neutral currents (FCNC) induced by supersymmetric particles [1–3]. Attempts has also been made to use horizontal symmetry to address the μ problem [2,4], the strong CP problem [5], FCNC due to light leptoquarks [6], and baryon number violation in supersymmetry (SUSY) [7]. There is hence a growing interest in the topic.

However, as global symmetries are in general not respected by gravitational effects [8], the horizontal symmetry should be gauged. Canceling the gauge anomalies then imposes a strong constraint on model building [9–12]. For a simple non-Abelian symmetry we are left with essentially only $SU(2)$ and its discrete dicyclic subgroups Q_{2N} [12–15].

Now we consider an extra desirable ingredient, compatibility with supersymmetric vertical (grand) unification, such as $SU(5)$. The only grand unified theory (GUT) compatible gauged horizontal symmetry model proposed so far is incompatible with SUSY [12]. Here we provide the first SUSY-GUT compatible such model.

Inspired by the antiunification approach to quark masses [16], models with separate GUT groups for each of the three families has been introduced [17]. Here we consider instead only two $SU(5)$'s for horizontal singlet and doublet families. The structure then gives, to the first approximation, rank-one quark mass matrices. We show that, with judiciously chosen heavy scalar vacuum expectation values (VEV's), the full hierarchical and phenomenologically viable quark mass matrix textures can be generated, using nonrenormalizable gravitational interactions [18].

Our model has gauged $SU(5) \otimes SU(5) \otimes SU(2)$, with this symmetry broken to a diagonal $SU(5)$ (SUSY-)GUT group around and above the GUT scale. The full pattern of symmetry breaking is illustrated in Fig. 1.

The assignment of the three families of quarks and leptons to $[SU(5) \otimes SU(5) \otimes Q_6]$ is thus

$$\text{3rd family } (\bar{5} + 10, 1, 1),$$

$$\text{1st and 2nd families } (1, \bar{5} + 10, 2_1).$$

Upon breaking to diagonal $SU(5)$ this becomes a normal three-family SUSY-GUT.

The Higgs fields which will break electroweak symmetry are in $(5 + \bar{5}, 1, 1)$ and so couple only to the third family in a renormalizable fashion. Scalar VEV's in $(\bar{5}, 5)$ or $(5, \bar{5})$ will break to the diagonal subgroup. There will also be $SU(5) \otimes SU(5)$ singlets, nontrivial under Q_6 . Beyond these scalars, it will be necessary only to introduce an extra $(15, 10, 2_1)$ and $(\bar{15}, 10, 2_1)$ multiplets to complete the model [19].

Taking as an expansion parameter $\lambda \sim \sin \theta_C \sim 0.22$ we will use two scales below M_{Planck} which are taken as $M_1 \sim \lambda \tilde{M}_{\text{Planck}}$ which characterizes the VEV of a $(1, 1, 2_1)$ and $M_2 \sim \lambda^3 M_{\text{Planck}}$ which sets the $SU(5) \otimes SU(5)$ breaking VEV's. In fact, M_2 lies just above the usual $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV, as the effective Planck mass $\tilde{M}_{\text{Planck}}$ is given by $M_{\text{Planck}} / \sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV. Thus, the hierarchy of the observed quark masses at accessible energy merely reflect the existence of the superheavy scalars laying near $\tilde{M}_{\text{Planck}}$ with VEV's at the somewhat lower horizontal-symmetry-breaking scales M_1 and M_2 ; it remains to be seen whether this sequence of horizontal-symmetry breaking can be derived from a reasonable Higgs boson potential.

To keep track of the bookkeeping for the components of the Q_6 couplings we find it most convenient to assign to the two components of a 2_n doublet the values $\pm n$, reflecting the eigenvalues of $2T_3 = \pm n$ in the natural embedding $SU(2)$

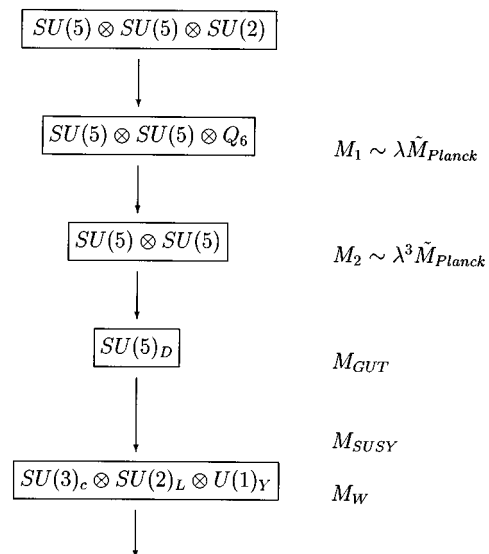


FIG. 1. Illustration of the symmetry-breaking pattern of the model.

$\supset Q_{2N}$. Recall that for even-dimensional $SU(2)$ irreducible representations

$$d \rightarrow 2_1 + 2_3 + \dots + 2_{d-1},$$

while, for odd d ,

$$d \rightarrow \text{singlet} + 2_2 + 2_4 + \dots + 2_{d-1};$$

the singlet is 1 for $d=1,5,9,\dots$ and $1'$ for $d=3,7,11,\dots$. Of course we will need only $d=1, 2, 3$, and 4 for Q_6 .

With this bookkeeping we find that the mass matrix textures emerge from VEV's as follows: the only scalar with VEV at scale M_1 is $(1,1,[+1])$; the VEV's at scale M_2 are

$$\begin{aligned} &-(\bar{5},5,[-3]), \\ &-(5,\bar{5},[-2]) \text{ and } (5,\bar{5},[-1]), \\ &-(15,\bar{10},[-1]), \end{aligned}$$

where the Q_6 entry implies the $2T_3$ eigenvalue. These large VEV's are assumed to be in vectorlike supermultiplets [19]. We assume the most general superpotential containing all renormalizable and nonrenormalizable couplings compatible with the symmetry of the model. Tracking down all the entries of the mass matrices to the lowest order in λ we have the result

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^9 \\ \lambda^6 & \lambda^4 & \lambda^7 \\ \lambda^9 & \lambda^7 & 1 \end{pmatrix}, \tag{1}$$

$$M_d \sim \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix}. \tag{2}$$

The authors of [20] have analyzed all possible symmetric quark mass matrices with the maximal (six) and next-to-maximal (five) number of texture zeros, and concluded that only five models, denoted by the roman numerals I to V in their work, are phenomenologically viable. Note that the symmetric structure is just an input assumption. In our case, the GUT structure enforced a symmetric mass matrix for the up sector, but leaves that for the down sector arbitrary. While $U(1)$ flavor symmetry constructions for quark mass matrices with nonsymmetric hierarchical textures have been attempted [2,10,21], the full list of such phenomenologically viable quark mass matrices is not yet available. However, we can simply exploit the fact that low-energy physics is unaffected by an arbitrary rotation of the right-handed quark fields. Discarding the large order entries and imposing a rotation on the right-handed second and third down-quark fields with an angle $\sim \lambda^3$, we obtain, in the symmetric basis,

$$M_u \sim \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

$$M_d \sim \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ 0 & \lambda^3 & 1 \end{pmatrix}, \tag{4}$$

corresponding to case I of [20], hence showing that the (asymmetric) quark mass texture is phenomenologically viable.

Now we turn to the charged lepton mass matrix. The simplest way to accommodate it to obtain the Georgi-Jarlskog pattern [22] by replacing the scalar VEV $(5,\bar{5},[-2])$, which is responsible for the $(M_d)_{22}$ entry, with a $(5,4\bar{5},[-2])$. If one wants to avoid having a $4\bar{5}$, there is the alternative suggested by Ellis and Gaillard [18]. While the diagonal $SU(5)$ singlet from the $(5,\bar{5})$ contributions to the quark and lepton masses are the same, the other $SU(3) \otimes SU(2) \otimes U(1)$ singlet in the adjoint 24 of the diagonal $SU(5)$ gives quark and lepton masses in the ratio $-3/2$. Ellis and Gaillard showed that if both the singlet and the 24 contribute, with partial cancellation in the lepton sector, the $\bar{5}$ VEV could fit both the quark and lepton sectors. The 24 VEV is of course GUT-breaking, which is needed anyway. In our case, its contribution has to be smaller by about a factor of M_{GUT}/M_2 . Without going into detail, a simple comparison with the Ellis-Gaillard analysis shows that this can be successful.

Having discussed both the quark and charged-lepton mass matrix texture construction, a few comments are in order.

The breaking of the horizontal $SU(2)$ through the discrete dicyclic subgroup Q_6 is needed to avoid the otherwise large D -term contributions to the scalar quark masses in the $SU(2)$ breaking [3]. Our model may otherwise be considered only in the $SU(2)$ framework. However, the D -term contributions would lift any assumed degeneracy among the squarks and cause unacceptable FCNC in, for example, $K-\bar{K}$ mixing (see [23] and references therein). The strongest FCNC constraint can be expressed as an upper limit on the (12) entry of the left-handed down-squark \tilde{m}_{LL}^2 matrix, in the quark-mass eigenbasis

$$\delta \tilde{m}_{ds}^2 = \tilde{m}_1^2 K_{11} K_{12}^\dagger + \tilde{m}_2^2 K_{12} K_{22}^\dagger + \tilde{m}_3^2 K_{13} K_{32}^\dagger, \tag{5}$$

where \tilde{m}_i^2 are the three eigenvalues and K the unitary transformation matrix that diagonalizes \tilde{m}_{LL}^2 . In the limit that $K_{13} K_{32}^\dagger$ is negligible, this reduces to

$$\delta \tilde{m}_{ds}^2 \approx (\tilde{m}_2^2 - \tilde{m}_1^2) K_{12}, \tag{6}$$

hence a degeneracy condition between \tilde{m}_1^2 and \tilde{m}_2^2 , unless the mixing K_{12} is itself exceedingly small [24]. As recently advocated by some authors [25], the $2+1$ family structure, gives a natural first-order degeneracy between \tilde{m}_1^2 and \tilde{m}_2^2 , and is therefore flavorable from the perspective. The degeneracy is however lifted as the horizontal symmetry is broken. In our model, the lifting is of order λ^2 which is too large. Extra mechanisms, as proposed in Ref. [3], are needed to help suppress the FCNC.

In principle, the nonrenormalizable mass terms may be obtained, alternatively, from the Froggatt-Nielsen mechanism [26]. In that case, one needs $M_1/M_0 \sim \lambda$ and $M_2/M_0 \sim \lambda^3$ where M_0 is the mass scale of the vectorlike

fermions mediating the Yukawa vertices involving the chiral fermions. However, M_0 cannot really be brought down much below M_{Planck} because the proliferation of heavy supermultiplets may lead to a nonperturbative gauge coupling [2]. Contributions of the supermultiplets that contain the necessary large VEV's at M_1 and M_2 to the renormalization group (RG) β functions can be dangerous too if their masses are much below M_{Planck} . However, their vectorlike nature allows, in principle, an arbitrarily large mass. Here, we take a conservative strategy and assume these masses to be real large, thus rendering their contributions to the β functions vanishingly small.

The supermultiplets that contain the $SU(5) \otimes SU(5)$ breaking VEV's in the model are assumed to be vectorlike. Hence they are heavy and have no contribution to gauge anomalies. The supermultiplet $(1,1,2_1)$ can have heavy Majorana mass. It has a contribution which helps to cancel the otherwise nontrivial global- $SU(2)$ anomaly. Local gauge anomaly cancellation in our model is completely straightforward with no additional states.

The $(1,1,2_1)$ can be identified naturally as a right-handed neutrino supermultiplet. If an extra $(1,1,1)$ is added, the family structure of the right-handed neutrinos is then the same as the quarks and leptons. While this appears natural, the neutrino masses and mixings hence derived, assuming no extra

VEV's, do not look very good. However, the right-handed neutrino sector need not have the same family structure as the quarks and leptons, so long as the global- $SU(2)$ anomaly condition is satisfied; and there could be some extra multiplets with or without VEV's among them that modify the neutrino masses and mixings without upsetting the quark and charged-lepton mass textures.

Like the minimal SUSY- $SU(5)$, the model has dimension-4 baryon number violating operators that have to be removed by imposing R parity or otherwise. In particular the $(\bar{5},1,1)$ Higgs supermultiplet definitely has to be distinguished from the quark-lepton one in the same representations to avoid fast proton decay. Finally, the infamous doublet-triplet splitting problem has not been addressed.

In conclusion, we illustrate here the possible construction of a quark (and charged-lepton) mass matrix texture model from a gauged (non-Abelian) horizontal symmetry in the supersymmetric grand unification framework. To the best of our knowledge, the model is the first of its kind [27]. We hope at least some features of the model may contribute toward our understanding of flavor physics.

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