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Master formula approach to chiral symmetry breaking and $\pi\pi$ scattering

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A master formula approach to chiral symmetry breaking is used to derive a general on-shell formula for $\pi\pi$ scattering. Comparison with experiment shows that the P-wave $\pi\pi$ scattering amplitude is strongly constrained by the electroproduction data in the ρ region.

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At low energies, chiral symmetry offers a powerful method for dealing with hadronic processes involving pions. In the past, various approaches have been formulated to implement its structures. Apart from a few exceptions however, most of them have relied either on off-shell extrapolation (the soft pion limit [1]), or an expansion starting from the chiral limit (chiral perturbation theory [2–4]).

In this Rapid Communication we show that such references to unphysical limits are not necessary. We derive a set of exact identities for pionic processes on shell, which naturally yields threshold theorems as well as corrections to them. In fact we find that all low energy theorems of current algebra are just variants of a single master formula.

Our strategy is to start from the gauge-covariant version of the PCAC (partial conservation of axial vector current) equation pioneered by Veltman and Bell [5], and convert it into an equation of motion for the pion field by a change of variables. Integration then yields a condition on the extended S matrix, which is the desired master formula. It has the structure of a reduction formula, and iteration allows an exact rewriting of scattering amplitudes in terms of correlation functions and form factors, some of which are measurable.

Our general results allow tests of new hypotheses far beyond the threshold region. An example in point is $\pi\pi$ scattering where we find that the P-wave amplitude in the ρ region is strongly constrained by electroproduction data and other sources.

Consider an action whose kinetic part is invariant under chiral $SU_L(2)\times SU_R(2)$ with a scalar-isoscalar mass term in the (2,2) representation. Examples are two-flavor QCD or sigma models. The symmetry properties of the theory may be expressed by gauging the kinetic part with c-number external fields v_μ^a and a_μ^a, and extending the mass term to include couplings with scalar and pseudoscalar fields s and p^a. For two-flavor QCD, the relevant part of the action reads

$$\mathbf{I}=+\int d^4x\ \bar{q}\gamma^\mu\left(i\partial_\mu+G_\mu+v_\mu^a\frac{\tau^a}{2}+a_\mu^a\frac{\tau^a}{2}\gamma_5\right)q - \frac{\hat{m}}{m_\pi^2}\int d^4x\ \bar{q}(m_\pi^2+s-i\gamma_5\tau^a p^a)q, \tag{1}$$

where m_π is the pion mass. We will assume that $\phi=(v_\mu^a,a_\mu^a,s,p^a)$ are smooth functions that fall off rapidly at infinity.

Currents and densities $\mathscr{O}=(\mathbf{V},\mathbf{A},f_\pi\sigma,f_\pi\pi)$ may be introduced as $\mathscr{O}(x)=\delta\mathbf{I}/\delta\phi(x)$ which obey the Veltman-Bell equations [5]

$$\nabla^\mu\mathbf{V}_\mu+\underline{a}^\mu\mathbf{A}_\mu+f_\pi\underline{p}\pi=0, \tag{2}$$

$$\nabla^\mu\mathbf{A}_\mu+\underline{a}^\mu\mathbf{V}_\mu-f_\pi(m_\pi^2+s)\pi+f_\pi p\sigma=0, \tag{3}$$

where $\nabla_\mu=\partial_\mu\mathbf{1}+\underline{v}_\mu$ is the vector covariant derivative, $\underline{a}_\mu^{ac}=\epsilon^{abc}a_\mu^b$, $\underline{p}^{ac}=\epsilon^{abc}p^b$, and f_π is the pion decay constant. In the above we have used the fact that the Bardeen anomaly [6] and the Wess-Zumino term [7] vanish for $SU_L(2)\times SU_R(2)$. Introducing the extended S matrix $\mathscr{S}$, holding the incoming fields fixed, and using the Schwinger action principle [8] implies

$$\langle\beta\ \text{in}|\delta\mathscr{S}|\alpha\ \text{in}\rangle=i\langle\beta\ \text{in}|\mathscr{S}\delta\mathbf{I}|\alpha\ \text{in}\rangle. \tag{4}$$

This result together with asymptotic completeness, yield the Peierls-Dyson formula [9]

$$\mathscr{O}(x)=-i\mathscr{S}^\dagger\frac{\delta\mathscr{S}}{\delta\phi(x)}. \tag{5}$$

It follows from the Veltman-Bell equations (2) and (3) that

$$\left(\nabla_\mu^{ac}\frac{\delta}{\delta v_\mu^c(x)}+\underline{a}_\mu^{ac}(x)\frac{\delta}{\delta a_\mu^c(x)}+\underline{p}^{ac}(x)\frac{\delta}{\delta p^c(x)}\right)\mathscr{S} =\left(\mathbf{X}_V^a(x)+\underline{p}^{ac}(x)\frac{\delta}{\delta p^c(x)}\right)\mathscr{S}=0, \tag{6}$$

$$\left(\nabla_\mu^{ac}\frac{\delta}{\delta a_\mu^c(x)}+\underline{a}_\mu^{ac}(x)\frac{\delta}{\delta v_\mu^c(x)}-[m_\pi^2+s(x)]\frac{\delta}{\delta p^a(x)}+p^a(x)\frac{\delta}{\delta s(x)}\right)\mathscr{S}=\left(\mathbf{X}_A^a(x)-[m_\pi^2+s(x)]\frac{\delta}{\delta p^a(x)}+p^a(x)\frac{\delta}{\delta s(x)}\right)\mathscr{S}=0, \tag{7}$$

where $\mathbf{X}_V$ and $\mathbf{X}_A$ are the generators of local $SU_L(2)\times SU_R(2)$. We further require

$$\langle 0|\mathbf{A}^a_\mu(x)|\pi^b(p)\rangle = i f_\pi \delta^{ab} p_\mu e^{-ip\cdot x}. \tag{8}$$

In the absence of stable axial vector or other pseudoscalar mesons, this is equivalent to the asymptotic conditions ($x^0 \to \mp\infty$)

$$\mathbf{A}^a_\mu(x) \to -f_\pi \partial_\mu \pi^a_{\text{in, out}}(x),$$
$$\partial^\mu \mathbf{A}^a_\mu(x) \to +f_\pi m_\pi^2 \pi^a_{\text{in, out}}(x), \tag{9}$$

where π_{in} and π_{out} are free incoming and outgoing pion fields. Comparison of (9) with (3) shows that π is a normalized interpolating field.

To incorporate (9) into (6) and (7) we introduce a modified action

$$\hat{\mathbf{I}} = \mathbf{I} - f_\pi^2 \int d^4x [s(x) + \tfrac{1}{2} a^\mu(x)\cdot a_\mu(x)], \tag{10}$$

the corresponding extended S matrix

$$\hat{\mathcal{S}} = \mathcal{S} \exp\left(-i f_\pi^2 \int d^4x [s(x) + \tfrac{1}{2} a^\mu(x)\cdot a_\mu(x)]\right), \tag{11}$$

and a change of variable $p = J/f_\pi - \nabla^\mu a_\mu$. Taking $\hat{\phi} = (v^a_\mu, a^a_\mu, s, J^a)$ as independent variables, modified currents, and densities $\hat{\mathcal{O}} = (\mathbf{j}_V, \mathbf{j}_A, f_\pi\hat{\sigma}, \hat{\pi})$ may be defined as

$$\hat{\mathcal{O}}(x) = \frac{\delta \hat{\mathbf{I}}}{\delta \hat{\phi}} = -i \hat{\mathcal{S}}^\dagger \frac{\delta \hat{\mathcal{S}}}{\delta \hat{\phi}}. \tag{12}$$

The chain rule yields

$$\mathbf{V}^a_\mu(x) = \mathbf{j}^a_{V\mu}(x) + f_\pi \underline{a}^{ac}_\mu(x)\hat{\pi}^c(x),$$
$$\mathbf{A}^a_\mu(x) = \mathbf{j}^a_{A\mu}(x) + f_\pi^2 a^a_\mu(x) - f_\pi(\nabla_\mu \hat{\pi})^a(x),$$
$$\sigma(x) = \hat{\sigma}(x) + f_\pi,$$
$$\pi^a(x) = \hat{\pi}^a(x). \tag{13}$$

Substitution into (2) gives

$$\nabla^\mu \mathbf{j}_{V\mu} + \underline{a}^\mu \mathbf{j}_{A\mu} + \underline{J}\pi = 0 \tag{14}$$

and therefore

$$\left(\mathbf{X}_V + \underline{J}\frac{\delta}{\delta J}\right)\hat{\mathcal{S}} = 0. \tag{15}$$

On the other hand, substitution into (3) gives

$$\nabla^\mu \mathbf{j}_{A\mu} + \underline{a}^\mu \mathbf{j}_{V\mu} = -f_\pi^2 \nabla^\mu a_\mu + f_\pi \nabla^\mu \nabla_\mu \pi - f_\pi \underline{a}^\mu \underline{a}_\mu \pi + f_\pi(m_\pi^2 + s)\pi - (J - f_\pi \nabla^\mu a_\mu)(\hat{\sigma} + f_\pi). \tag{16}$$

This equation may be integrated by introducing the retarded and advanced Green's functions

$$(-\Box - m_\pi^2 - \mathbf{K}) G_{R,A} = \mathbf{1}, \tag{17}$$

$$\mathbf{K} = 2\underline{v}^\mu \partial_\mu + (\partial^\mu \underline{v}_\mu) + \underline{v}^\mu \underline{v}_\mu - \underline{a}^\mu \underline{a}_\mu + s \tag{18}$$

where we have adopted a condensed matrix notation. We have the Yang-Feldman-Kallen-type equations [10]

$$\pi = (1 + G_R\mathbf{K})\pi_{\text{in}} - G_R J + G_R(\nabla^\mu a_\mu - J/f_\pi)\hat{\sigma} - \frac{1}{f_\pi} G_R(\nabla^\mu \mathbf{j}_{A\mu} + \underline{a}^\mu \mathbf{j}_{V\mu})$$
$$= (1 + G_A\mathbf{K})\pi_{\text{out}} - G_A J + G_A(\nabla^\mu a_\mu - J/f_\pi)\hat{\sigma} - \frac{1}{f_\pi} G_A(\nabla^\mu \mathbf{j}_{A\mu} + \underline{a}^\mu \mathbf{j}_{V\mu}). \tag{19}$$

Noting that $\pi_{\text{out}} = \hat{\mathcal{S}}^\dagger \pi_{\text{in}} \hat{\mathcal{S}}$, and using (12) we arrive at

$$\frac{\delta}{\delta J}\hat{\mathcal{S}} = -iG_R J\hat{\mathcal{S}} + i\hat{\mathcal{S}}(1 + G_R\mathbf{K})\pi_{\text{in}} + \frac{1}{f_\pi} G_R(\nabla^\mu a_\mu - J/f_\pi)\frac{\delta \hat{\mathcal{S}}}{\delta s} - \frac{1}{f_\pi} G_R \mathbf{X}_A \hat{\mathcal{S}}$$
$$= -iG_A J\hat{\mathcal{S}} + i(1 + G_A\mathbf{K})\pi_{\text{in}}\hat{\mathcal{S}} + \frac{1}{f_\pi} G_A(\nabla^\mu a_\mu - J/f_\pi)\frac{\delta \hat{\mathcal{S}}}{\delta s} - \frac{1}{f_\pi} G_A \mathbf{X}_A \hat{\mathcal{S}}. \tag{20}$$

Evidently, any result which is a consequence of (9) and symmetry (6) and (7) must be contained in (15) and (20). Since (15) simply represents local isospin invariance, the nontrivial results of current algebra must be basically contained in (20).

To show that this is the case and that (20) is the desired master formula, we note that

$$G_{R,A} = \Delta_{R,A} + \Delta_{R,A}\mathbf{K}G_{R,A}$$
$$= \Delta_{R,A} + G_{R,A}\mathbf{K}\Delta_{R,A}, \tag{21}$$

where $\Delta_{R,A}$ are the Green's functions for free fields. Multiplying (20) by $(1 + G_A\mathbf{K})^{-1} = 1 - \Delta_A\mathbf{K}$ and Fourier decomposing yield

$$[a^a_{\text{in}}(k), \hat{\mathcal{S}}] = \int d^4y\, d^4z\, e^{ik\cdot y}(1 + \mathbf{K}G_R)^{ac}(y,z) \times\left(-i\hat{\mathcal{S}}(\mathbf{K}\pi_{\text{in}})^c(z) + i\hat{\mathcal{S}}J^c(z) - \frac{1}{f_\pi}(\nabla^\mu a_\mu - J/f_\pi)^c(z)\frac{\delta \hat{\mathcal{S}}}{\delta s(z)} + \frac{1}{f_\pi}\mathbf{X}^c_A(z)\hat{\mathcal{S}}\right), \tag{22}$$

$$[\hat{\mathcal{S}}, a^{a\dagger}_{\text{in}}(k)] = \int d^4y\, d^4z\, e^{-ik\cdot y}(1 + \mathbf{K}G_R)^{ac}(y,z) \times\left(-i\hat{\mathcal{S}}(\mathbf{K}\pi_{\text{in}})^c(z) + i\hat{\mathcal{S}}J^c(z)\hat{\mathcal{S}} - \frac{1}{f_\pi}(\nabla^\mu a_\mu - J/f_\pi)^c(z)\frac{\delta \hat{\mathcal{S}}}{\delta s(z)} + \frac{1}{f_\pi}\mathbf{X}^c_A(z)\hat{\mathcal{S}}\right), \tag{23}$$

where $a^a_{\rm in}(k)$ and $a^{a\dagger}_{\rm in}(k)$ are the annihilation and creation operators of incoming pions with momentum k and isospin a. Iterations give the two and higher pion reduction formulas, e.g., up to order $\hat{\phi}$:

$$[a^b_{\rm in}(k_2),[\hat{\mathscr{S}},a^{a\dagger}_{\rm in}(k_1)]]=\int d^4y\ e^{-ik_1\cdot y}\frac{1}{f_\pi}\mathbf{X}^a_A(y)\times[a^b_{\rm in}(k_2),\hat{\mathscr{S}}]. \tag{24}$$

The Bogoliubov causality condition [11] implies that

$$T^*[\hat{\mathscr{O}}(x_1)\cdots\hat{\mathscr{O}}(x_n)]=(-i)^n\hat{\mathscr{S}}^\dagger\frac{\delta^n}{\delta\hat{\phi}(x_1)\cdots\delta\hat{\phi}(x_n)}\hat{\mathscr{S}}. \tag{25}$$

With this in mind, using (22)–(24), sandwiching between nucleon states, and switching off the external fields, give the familiar πN scattering formula [12]

$$\begin{aligned}\langle N(p_2)|[a^b_{\rm in}(k_2),[\mathbf{S},a^{a\dagger}_{\rm in}(k_1)]]|N(p_1)\rangle\\ =-\frac{i}{f_\pi}m^2_\pi\delta^{ab}\int d^4y\ e^{-i(k_1-k_2)\cdot y}\langle N(p_2)|\hat{\sigma}(y)|N(p_1)\rangle\\ -\frac{1}{f^2_\pi}k^\alpha_1k^\beta_2\int d^4y_1d^4y_2e^{-ik_1\cdot y_1+ik_2\cdot y_1}\\ \times\langle N(p_2)|T^*[\mathbf{j}^a_{A\alpha}(y_1)\mathbf{j}^b_{A\beta}(y_2)]|N(p_1)\rangle\\ +\frac{1}{f^2_\pi}k^\alpha_1\int d^4y\ e^{-i(k_1-k_2)\cdot y}\epsilon^{abe}\langle N(p_2)|\mathbf{V}^e_\alpha(y)|N(p_1)\rangle,\end{aligned} \tag{26}$$

where $\mathbf{S}=\hat{\mathscr{S}}|_{\phi=0}$ is the on-shell S matrix. The disconnected part in (26) can be checked to cancel. At threshold, (26) yields the Tomozawa-Weinberg relation [13].

The extension to $\pi\pi$ scattering is straightforward in principle, although lengthy in practice. We find that the transition amplitude $i\mathscr{T}(p_2d,k_2b\leftarrow k_1a,p_1c)$ is a sum of four contributions

$$i\mathscr{T}_{\rm tree}=\frac{i}{f^2_\pi}(s-m^2_\pi)\delta^{ac}\delta^{bd}+2\ \text{perm}, \tag{27}$$

$$i\mathscr{T}_{\rm vector}=\frac{i}{f^2_\pi}\epsilon^{abe}\epsilon^{cde}(s-u)\left(\mathbf{F}_V(t)-1-\frac{t}{4f^2_\pi}\mathbf{\Pi}_V(t)\right)+2\ \text{perm}, \tag{28}$$

$$\begin{aligned}i\mathscr{T}_{\rm scalar}=-\frac{2im^2_\pi}{f_\pi}\delta^{ab}\delta^{cd}\left(\mathbf{F}_S(t)+\frac{1}{f_\pi}-\frac{1}{2f^2_\pi}\langle 0|\hat{\sigma}|0\rangle\right)\\ +\frac{m^4_\pi}{f^2_\pi}\delta^{ab}\delta^{cd}\int d^4y\ e^{-i(k_1-k_2)\cdot y}\\ \times\langle 0|T^*[\hat{\sigma}(y)\hat{\sigma}(0)]|0\rangle_{\rm conn}+2\ \text{perm},\end{aligned} \tag{29}$$

$$\begin{aligned}i\mathscr{T}_{\rm rest}=+\frac{1}{f^4_\pi}k^\alpha_1k^\beta_2p^\gamma_1p^\delta_2\int d^4y_1d^4y_2d^4y_3\\ \times e^{-ik_1\cdot y_1+ik_2\cdot y_2-ip_1\cdot y_3}\\ \times\langle 0|T^*[\mathbf{j}^a_{A\alpha}(y_1)\mathbf{j}^b_{A\beta}(y_2)\mathbf{j}^c_{A\gamma}(y_3)\mathbf{j}^d_{A\delta}(0)]|0\rangle_{\rm conn},\end{aligned} \tag{30}$$

where s,t,u are the Mandelstam variables,

$$\langle 0|a^d_{\rm in}(p_2)\mathbf{V}^e_\alpha(y)a^{c\dagger}_{\rm in}(p_1)|0\rangle_{\rm conn}=i\epsilon^{dec}(p_1+p_2)_\alpha\mathbf{F}_V(t)e^{-i(p_1-p_2)\cdot y} \tag{31}$$

is the pion electromagnetic form factor,

$$i\int d^4x\ e^{iq\cdot x}\langle 0|T^*[\mathbf{V}^a_\alpha(x)\mathbf{V}^b_\beta(0)]|0\rangle=\delta^{ab}(-g_{\alpha\beta}q^2+q_\alpha q_\beta)\mathbf{\Pi}_V(q^2) \tag{32}$$

is the isovector correlation function, and

$$\langle 0|a^d_{\rm in}(p_2)\sigma(y)a^{c\dagger}_{\rm in}(p_1)|0\rangle_{\rm conn}=\delta^{cd}\mathbf{F}_S(t)e^{-i(p_1-p_2)\cdot y} \tag{33}$$

is the scalar form factor.

So far we have made use of only chiral symmetry and general principles. The appearance of the vector quantities (31) and (32) is a direct manifestation of the $SU_L(2)\times SU_R(2)$ structure. Fortunately, these quantities (modulo subtraction constants) are experimentally accessible, and are well described by ρ dominance. The scalar contributions (29) have the wrong quantum numbers for ρ to be significant. The remainder (30) is unknown, but since $\mathbf{j}_A$ is one-pion reduced, we assume it has no strong resonant behavior. Hence, near the ρ peak, we expect the $\pi\pi$ scattering amplitude to be dominated by (27) and (28).

This point is confirmed by the data. Ignoring (29) and (30), and using a one-resonance fit to $\mathbf{F}_V$ and $\mathbf{\Pi}_V$ [1] [14], we obtain the isospin $I=1$ and angular momentum $l=1$ amplitude as shown in Fig. 1 for the real and imaginary parts. The results are in excellent agreement with the $\pi\pi$ data [15], justifying *a posteriori* our neglect of (29) and (30). A similar analysis can be carried out for the $I=0$, $l=0$ channel [16].

It is also interesting to go to low energies, where the unknown terms (29) and (30) can be estimated by expanding in $1/f_\pi$. The master equation (20) then truncates to

$$\begin{aligned}\frac{\delta\hat{\mathscr{S}}_0}{\delta J}&=-i\hat{\mathscr{S}}_0G_RJ+i\hat{\mathscr{S}}_0(1+G_R\mathbf{K})\pi_{\rm in}\\ &=-i\hat{\mathscr{S}}_0G_AJ+i(1+G_A\mathbf{K})\pi_{\rm in}\hat{\mathscr{S}}_0\end{aligned} \tag{34}$$

corresponding to the quadratic action

[1] $\mathbf{\Pi}_V$ follows from e^+e^- annihilation data via a once-subtracted dispersion relation. The subtraction constant is fixed by the one-loop analysis discussed below.

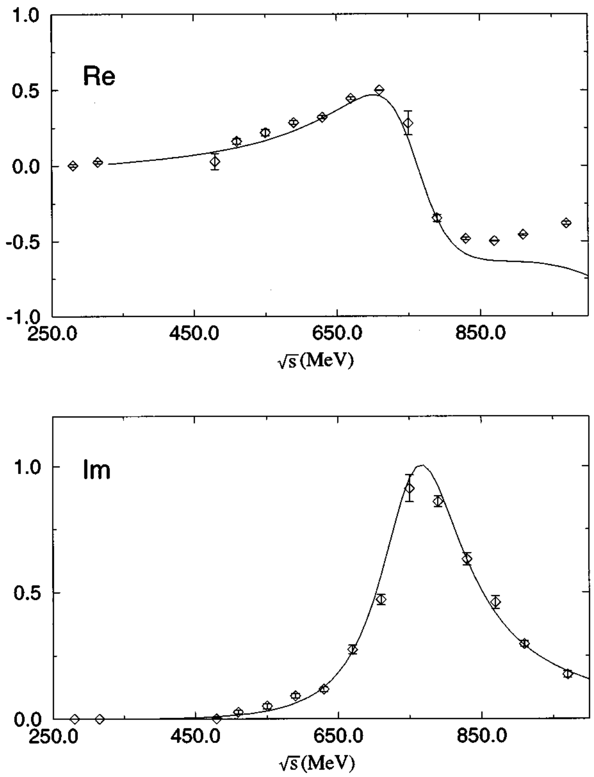

FIG. 1. Real (Re) and imaginary (Im) parts of the P-wave $\pi\pi$ amplitude versus $\sqrt{s}$. The solid line is the result following from (27) and (28) using a one-resonance fit to the available data for $\boldsymbol{F}_V$ and $\boldsymbol{\Pi}_V$ from Ref. [14]. The $\pi\pi$ data are from Ref. [15].

$$\mathbf{I}_Q=\frac{1}{2}\int d^4x[+(\nabla^\mu\pi)^a(\nabla_\mu\pi)^a-(\underline{a}^\mu\pi)^a(\underline{a}_\mu\pi)^a -(m_\pi^2+s)\pi^a\pi^a]+\int d^4x\ J^a\pi^a. \qquad (35)$$

In (34) and (35), s and a_μ^a enter only through the combination $\hat{s}=s\mathbf{1}-\underline{a}_\mu\underline{a}^\mu$. If we take this to be true for $\hat{\mathcal{S}}_0$, we obtain a two-parameter fit to pionic data at one-loop level, which reproduces the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [17]. Also, since $\hat{s}$ is isospin symmetric, (30) contributes only to the isospin 0 and 2 channels by (25), and the bulk of the P-wave amplitude comes from (28) as claimed.

We may also relax the condition on s and a_μ. The result then coincides with ordinary chiral perturbation theory [4]. For the P-wave scattering lengths (in units of m_π^{-2}) we have[2]

$$a_1^1(\text{tree}) = 0.0300\,,$$

$$a_1^1(\text{vector}) = 0.0049\,,$$

$$a_1^1(\text{scalar}) = 0.0039\,,$$

$$a_1^1(\text{rest}) = 0.0000\,, \qquad (36)$$

which shows that (30) is again small at threshold.

To summarize, we have derived a master formula for chiral symmetry breaking (20), and in particular an exact on-shell formula (27)–(30) for $\pi\pi$ scattering. Our formula (20) is strictly equivalent to the chiral symmetry equation (3) and the one-particle formula (8). Therefore any result of chiral symmetry and (8) at low momenta, must be contained in (20). Indeed, we have explicitly checked that Weinberg's [12] on-shell pion-nucleon scattering formula (26) follows from (20) by iteration, and that chiral perturbation theory follows also from (20) by expanding in inverse powers of the pion decay constant. Empirically, we find that (30) is small at ρ energies compared to resonant (vector) terms dictated by chiral symmetry. Since (30) is small near threshold we conclude that the bulk of the ρ contribution to $\pi\pi$ scattering is given by the vector term (28) in a model-independent way, a question which has attracted some attention in the literature [18].

A comprehensive discussion of the present formulation, further applications, and detailed comparison with previous work by other authors will be given elsewhere [19]. Extension to $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ is currently under investigation.

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[2]The uncertainty in the evaluation of the scalar contribution is large (0.0051). This uncertainty follows from the uncertainties associated with the various fitting parameters.

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