Interactions of cosmic axions with Rydberg atoms in resonant cavities via the Primakoff process

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A quantum theoretical treatment of the interactions of cosmic axions with Rydberg atoms in resonant cavities via the Primakoff process is developed, by taking into account the finite temperature and the dissipation due to the finite damping time of the cavities. The time evolution of the number of the excited Rydberg atoms, evaluated numerically based on the theoretical formulations, enables us to obtain the detection efficiency of the axion-converted photons with a good signal-to-noise ratio. The optimum experimental setup to search for dark matter axions with Rydberg atoms is presented.

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Search for the so-called "invisible" axions [1,2] as nonbaryonic dark matter particles is one of the most challenging issues related to recent particle physics and cosmology [3,4]. The relevant mass window of such axions still open is between 1 μeV and 1 meV [3,5]. It is, however, extremely difficult to detect the cosmic axions due to their extraordinary weak interactions with ordinary matters, although pioneering tries were reported already [6,7].

A promising way to detect the cosmic axions is to convert firstly the axions into microwave photons in a resonant cavity under a strong magnetic field via the Primakoff process, as originally proposed by Sikivie [8]. A quite efficient scheme to search for the cosmic axions is then expected to be obtained by utilizing Rydberg atoms to detect the axion-converted microwave photons in the cavity [9]. A schematic diagram of the experimental system with Rydberg atoms is shown in Fig. 1. The axions are converted into photons in a cavity permeated by a strong magnetic field (called the conversion cavity), and then the photons transferred to another cavity (called the detection cavity) via a coupling hall are absorbed by the Rydberg atoms. The detection cavity is set to be free from the magnetic field to avoid the complexity of the energy levels due to the Zeeman splitting of the relevant levels. The Rydberg atoms, the transition frequency of which is tuned approximately to the cavity resonant frequency, are prepared by exciting alkaline atoms in the ground state with laser excitation just in front of the detection cavity. Thus the Rydberg atoms are coupled with the photons only in the detection cavity. The frequency width of lasers now available is so small (much less than 1 MHz) that it is possible to prepare all the Rydberg atoms in the lower state initially, thus being free from the inherent noise in the detection system in this sense. Only the Rydberg atoms thus excited to the upper state by absorbing the photons are detected quite efficiently with the selective field ionization method [10] just out of the cavity. By cooling the resonant cavities down to about 10 mK with a dilution refrigerator in high vacuum, all the relevant background photons are reduced to less than the expected number of the axion-converted photons. Thus we can expect that the Rydberg single-photon detector system provides a quite efficient detection scheme of the cosmic axions with the background noises suppressed sufficiently in themselves.

In order to obtain quantitative and rigorous results on the above-mentioned expectations we develop a quantum theoretical treatment of the interactions of cosmic axions with Rydberg atoms in resonant cavities via the Primakoff process, that is, via the axion-converted photons under a strong magnetic field. To the present author's knowledge, no such detailed, and especially quantum theoretical treatment of axion-atom interactions in resonant cavities has been published so far. We believe also that these theoretical formulations and calculations are interesting from the viewpoint of applications of the cavity quantum electrodynamics to fundamental research. Specifically, we take into account the finite temperature and the dissipation due to the finite damping time of the cavities. From these theoretical calculations we can determine the time evolution of the Rydberg atoms excited by absorbing both the background blackbody and axion-converted photons. This enables us to estimate the signal-to-noise (S/N) ratio in the detection of axion-converted photons. By taking into account the transit time of the Rydberg atoms through the cavity and the numerically evaluated S/N ratio, the minimum time required to observe the signals from the cosmic axions is estimated as a function of the collective coupling strength of Rydberg atoms with the radiation mode in the cavity. Thus we find the optimum experimental setup to search for the cosmic axions with the present scheme.

In this Rapid Communication we present the results of these theoretical analyses, and more detailed and practical design of the whole experimental system to search for the cosmic axions with Rydberg atoms based on these analyses will be presented separately in forthcoming papers.

The axion-photon interaction under a strong static magnetic field (flux density $\boldsymbol{B}_0$) is described by the Lagrangian

$$\mathscr{L}_a = \epsilon_0 g_{a\gamma\gamma} \phi \boldsymbol{E} \cdot \boldsymbol{B}_0, \qquad (1)$$

where $g_{a\gamma\gamma}$ and ϕ are the coupling constant and the axion field operator, respectively. In this expression, the dielectric constant ϵ_0 is explicitly factored out so as to make $g_{a\gamma\gamma}$

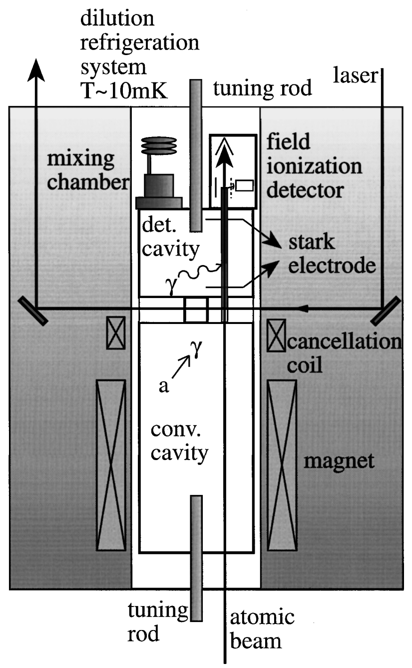

FIG. 1. Schematic diagram of the present experimental system to search for cosmic axions with Rydberg atoms in cooled resonant cavities. The axions are converted into photons in the conversion cavity permeated by a strong magnetic field, while the photons are absorbed by the Rydberg atoms in the detection cavity which is free from the magnetic field. Only the excited Rydberg atoms are ionized and detected with the selective field ionization method. The cavities are cooled down to $\sim$10 mK with a dilution refrigerator.

independent of the choice of the unit of electromagnetism. The electric field operator in the cavity $\mathscr{V}$ is given by

$$\boldsymbol{E}(\mathbf{x},t)=\sqrt{\frac{\hbar\omega_c}{2\epsilon_0}}[\boldsymbol{\alpha}(\mathbf{x})c(t)+\boldsymbol{\alpha}^*(\mathbf{x})c^\dagger(t)] \qquad (2)$$

for the radiation mode with a resonant frequency ω_c, where $c^\dagger$ and c are the creation and annihilation operators, and $\boldsymbol{\alpha}(\mathbf{x})$ is the mode vector field normalized by $\int_{\mathscr{V}}|\boldsymbol{\alpha}(\mathbf{x})|^2\,d^3x=1$. The whole cavity $\mathscr{V}$ may be viewed as a combination of two subcavities, the conversion cavity $\mathscr{V}_1$ with volume V_1 and the detection cavity $\mathscr{V}_2$ with volume V_2 coupled together: $\mathscr{V}=\mathscr{V}_1\oplus\mathscr{V}_2$. The axion-photon conversion takes place in $\mathscr{V}_1$, while the Rydberg atoms are excited by the photons in $\mathscr{V}_2$ where $\boldsymbol{B}_0(\mathbf{x})=\mathbf{0}$. Then, the mode vector may be divided as

$$\boldsymbol{\alpha}(\mathbf{x})=\boldsymbol{\alpha}_1(\mathbf{x})+\boldsymbol{\alpha}_2(\mathbf{x}), \qquad (3)$$

where $\boldsymbol{\alpha}_1(\mathbf{x})=\mathbf{0}$ for $\mathbf{x}\in\mathscr{V}_2$ and $\boldsymbol{\alpha}_2(\mathbf{x})=\mathbf{0}$ for $\mathbf{x}\in\mathscr{V}_1$, respectively. The normalization for $\boldsymbol{\alpha}(\mathbf{x})$ is written as

$$\int_{\mathscr{V}_1}|\boldsymbol{\alpha}_1(\mathbf{x})|^2\,d^3x+\int_{\mathscr{V}_2}|\boldsymbol{\alpha}_2(\mathbf{x})|^2\,d^3x=1. \qquad (4)$$

The actual cavity is designed so that neglecting the small joint region the subcavities $\mathscr{V}_1$ and $\mathscr{V}_2$ admit the mode vectors $\boldsymbol{\alpha}_1^0(\mathbf{x})$ and $\boldsymbol{\alpha}_2^0(\mathbf{x})$ (up to the normalization and complex phase), respectively, whose frequencies are almost equal to some common value $\overline{\omega_c}$. In this situation, as confirmed by a numerical calculation, two nearby eigenmodes with the frequencies $\omega_c,\omega_c'\simeq\overline{\omega_c}$ are obtained for the whole cavity $\mathscr{V}$. Then, the mode vector $\boldsymbol{\alpha}(\mathbf{x})$ is constructed approximately of $\boldsymbol{\alpha}_1(\mathbf{x})\simeq\boldsymbol{\alpha}_1^0(\mathbf{x})$ and $\boldsymbol{\alpha}_2(\mathbf{x})\simeq\boldsymbol{\alpha}_2^0(\mathbf{x})$ with significant magnitudes in both $\mathscr{V}_1$ and $\mathscr{V}_2$. The conversion of the cosmic axions takes place predominantly for the radiation mode which is resonant with the axions satisfying the condition $|\omega_c-m_a|\lesssim\gamma$, as will be seen later, where γ is the cavity damping constant. The cavity can be designed so as to give the separation of $|\omega_c-\omega_c'|\gg\gamma$ for the nearby modes with strong enough coupling between $\mathscr{V}_1$ and $\mathscr{V}_2$. Therefore, in calculating the signal from the axion-photon conversion, the resonant mode can be extracted solely for the electric field in a good approximation, as given in Eq. (2), whose frequency is supposed to be close enough to the axion mass.

The cosmic axions, on the other hand, form a coherent state $|\eta_a\rangle$. Since the velocity dispersion of the cosmic axions is expected to be very small, $\beta_a\sim10^{-3}$ [3,8], the axions may be treated effectively with a single mode operator a corresponding to the coherent state. It is relevant to normalize the coherent mode with

$$\bar{n}_a=\langle\eta_a|a^\dagger a|\eta_a\rangle\simeq\lambda_a^3(\rho_a/m_a), \qquad (5)$$

where ρ_a is the axion energy density and the de Broglie wavelength of the axions is estimated to be $\lambda_a=h/(\beta_a m_a)\sim100$ m for the axion mass $m_a\sim10^{-5}$ eV.

The axion-photon conversion in the cavity $\mathscr{V}_1$ is well described in terms of the coherent axion mode a and the radiation mode c with the effective Hamiltonian

$$H_{ac}=-\hbar\kappa(a^\dagger c+ac^\dagger), \qquad (6)$$

which is obtained from the interaction Lagrangian (1). By considering the relations $|\langle\eta_a|a|\eta_a\rangle|=\bar{n}_a^{1/2}$ from Eq. (5) and $|\langle\eta_a|\phi_\pm|\eta_a\rangle|\simeq(\hbar\rho_a/m_a^2)^{1/2}$ in

$$\epsilon_0 g_{a\gamma\gamma}\langle\eta_a|(\textstyle\int_{\mathscr{V}_1}d^3x\;\phi_+\boldsymbol{E}_-\cdot\boldsymbol{B}_0)|\eta_a\rangle\simeq\hbar\kappa\langle\eta_a|ac^\dagger|\eta_a\rangle$$

("$\pm$" represent the positive and negative frequency parts, respectively) [11], the coupling constant κ is estimated for $\omega_c\simeq m_a$ in the unit of $c=\hbar=1$ [12] as

$$\kappa\simeq6\times10^{-26}\;\text{eV}\left(\frac{g_{a\gamma\gamma}}{1.4\times10^{-15}\;\text{GeV}^{-1}}\right)\left(\frac{\zeta_1 G B_0}{4\;\text{T}}\right)\times\left(\frac{\beta_a m_a}{10^{-3}\times10^{-5}\;\text{eV}}\right)^{3/2}\left(\frac{V_1}{10^4\;\text{cm}^3}\right)^{1/2}, \qquad (7)$$

where B_0 is the maximal density of the external magnetic flux. The form factor is defined by

$$G\equiv V_1^{-1/2}\zeta_1^{-1}\left|\int_{\mathscr{V}_1}d^3x\;\boldsymbol{\alpha}_1(\mathbf{x})\cdot[\boldsymbol{B}_0(\mathbf{x})/B_0]\right|, \qquad (8)$$

with

$$\zeta_1\equiv\left[\int_{\mathscr{V}_1}d^3x|\boldsymbol{\alpha}_1(\mathbf{x})|^2\right]^{1/2}. \qquad (9)$$

This factor ζ_1 [<1 as seen from Eq. (4)] represents the effective reduction of the axion-photon conversion which is due to the fact that the magnetic field is applied only in $\mathscr{V}_1$. If a transverse magnetic (TM_{010}) cavity with $G=\sqrt{0.7}$ is taken typically for $\mathscr{V}_1$, as cited in Eq. (7), $\zeta_1 G B_0 = 4$ T is obtained for a magnet of $B_0 = 7$ T with the cavity volume factor $\zeta_1 = 0.7$.

The Rydberg atoms are excited by absorbing photons in the cavity $\mathscr{V}_2$, which is utilized for counting the axion-converted photons. In practice, by injecting a uniform atomic beam, certain number N of atoms are constantly present in the cavity interacting with the radiation mode. Then, such an ensemble of identical atoms is expected to behave as a collective system [13]. The effective coupling between the radiation mode c and the collective atomic mode b is given by

$$H_{bc} = -\hbar \Omega_N (b^\dagger c + b c^\dagger), \tag{10}$$

where $\Omega_N = \Omega \overline{N}^{1/2} \sim 10^{-9}$ eV$(\Omega/10^4\ \mathrm{s}^{-1})(N/10^5)^{1/2}$ with $\overline{N} \equiv \Sigma_i |\boldsymbol{\alpha}_2(\mathbf{x}_i)/\overline{\alpha}|^2 \sim N$ summed over the atomic position $\mathbf{x}_i$ $[\boldsymbol{\alpha}(\mathbf{x}_i) = \boldsymbol{\alpha}_2(\mathbf{x}_i)$ in $\mathscr{V}_2$, $|\boldsymbol{\alpha}_2(\mathbf{x}_i)| \leqslant \overline{\alpha}]$, and the single atom-field coupling is calculated by $\Omega = (d/\hbar)(\hbar\omega_c/2\epsilon_0)^{1/2}\overline{\alpha}$ at a position of $|\boldsymbol{\alpha}_2(\mathbf{x})| = \overline{\alpha}$ with the electric dipole matrix element d for the transition of frequency ω_b [13,14].

The time evolution of the axion-photon-Rydberg system is governed by the following equations of motion in the Heisenberg picture, when all of the Rydberg atoms are prepared initially in the lower state:

$$\frac{dz_i}{dt} = K_{ij} z_j + F_i, \tag{11}$$

where $z_i = (b, c, a)$, $F_i = (0, F_c, F_a)$, and

$$K = \begin{pmatrix} -i\omega_b & i\Omega_N & 0 \\ i\Omega_N & -i\omega_c - \frac{1}{2}\gamma & i\kappa \\ 0 & i\kappa & -i\omega_a - \frac{1}{2}\gamma_a \end{pmatrix} \tag{12}$$

with $\gamma = \omega_c/Q$ (Q is the quality factor of the cavity) and $\gamma_a \simeq \beta_a^2 m_a$, the energy dispersion of the cosmic axions. The external forces F_c and F_a are introduced for the Liouvillian relaxations of the photons and axions, respectively [13,14].

The equations of motion (11) are solved by determining the eigenmodes λ_m with factorization

$$\det(s\mathbf{1} - K) = (s - \lambda_0)(s - \lambda_+)(s - \lambda_-). \tag{13}$$

The initial particle numbers $\langle z_i^\dagger(0) z_j(0) \rangle = \delta_{ij} \overline{n}_j$ are taken as follows: $\overline{n}_a$ is given in Eq. (5), $\overline{n}_b = 0$ for the number of excited atoms (all the Rydberg atoms are prepared in the lower state), and $\overline{n}_c = (e^{\hbar\omega_c/k_B T_c} - 1)^{-1}$ for the thermal photons at the cavity temperature T_c. Then, by taking into account the correlation properties of F_c and F_a [13,14], the particle numbers at $t > 0$ are determined in terms of $\overline{n}_c$ and $\overline{n}_a$:

$$n_i(t) = \langle z_i^\dagger(t) z_i(t) \rangle = r_{ic}(t)\overline{n}_c + r_{ia}(t)\overline{n}_a, \tag{14}$$

where

$$r_{ij}(t) = \sum_{m,n} g_{ij}^{m*} g_{ij}^{n} \left[\left(1 - \frac{\xi_j}{\nu_{mn}}\right) e^{-\nu_{mn} t/\tau_\gamma} + \frac{\xi_j}{\nu_{mn}} \right] \tag{15}$$

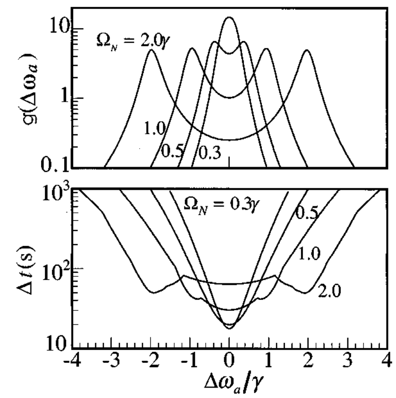

FIG. 2. (a) The form factor $g(\Delta\omega_a)$ and (b) the measurement time $\Delta t(s)$ required to search for the cosmic axion signal with 3σ confidence level, respectively, as a function of the detuning frequency $\Delta\omega_a/\gamma$ for various values of Ω_N. The optimal transit time is chosen (see Fig. 3) for each $\Delta\omega_a$. The relevant parameters used are $\omega_c = \omega_b = 10^{-5}$ eV, $Q = 5\times 10^4$, $\mathscr{V}_1 = 10^4$ cm^3, $B_0 = 7$ T, and $T_c = 10$ mK.

with $g_{ij}^m = \lim_{s\to\lambda_m}(s - \lambda_m)(s\mathbf{1} - K)_{ij}^{-1}$, $\xi_a = \gamma_a/\gamma$, $\xi_c = 1$, $\nu_{mn} = -(\lambda_m^* + \lambda_n)/\gamma$, and $\tau_\gamma = \gamma^{-1}$.

In the following analyses we assume the condition $\Delta\omega_b \equiv \omega_b - \omega_c = 0$ for simplicity, where $r_{bc}(\infty) = 1$ is obtained [13]. All the results obtained below are, however, not sensitive to the difference $\Delta\omega_b$ as long as $|\Delta\omega_b| \lesssim \gamma/2$ is satisfied. The factors relevant to the atomic signal $r_{ba}(t)\overline{n}_a$ from the axion-photon conversion are calculated as $g_{ba}^m = -\kappa\Omega_N(\lambda_m - \lambda_k)^{-1}(\lambda_m - \lambda_l)^{-1}$ $(m \neq k, l;\ k \neq l)$. Then, by factorizing $r_{ba}(\infty)$ with $(\kappa/\gamma)^2$, the atomic excitation is given asymptotically for $t \gg \tau_\gamma$ by

$$n_b(\infty) = \overline{n}_c + (\kappa/\gamma)^2 g(\Delta\omega_a)\overline{n}_a, \tag{16}$$

where $\Delta\omega_a \equiv \omega_a - \omega_c$. Figure 2(a) shows typical behavior of the form factor $g(\Delta\omega_a)$ numerically calculated for various values of the coupling constant Ω_N. The form factor $g(\Delta\omega_a)$ has peaks corresponding to the resonant conditions $\lambda_0 \simeq \lambda_\pm$ for g_{ba}^m. Especially, when $\Omega_N > \gamma/4$, it has two peaks (Rabi splitting) around $\Delta\omega_a \simeq \pm\Omega_N$ with width $\simeq \gamma$.

The transit time of the atoms passing through the cavity places the effective time for the atom-field interactions. It is typically $t_{\mathrm{tr}} = 10^{-5}$ s$(L/10^{-1}\ \mathrm{m})(10^4\ \mathrm{ms}^{-1}/v)$, where L is the cavity length and v the mean velocity of atoms. This transit time also determines the number of atoms per unit time, N/t_{tr}, which are measured at the exit of the cavity. Thus, the counting rates of the excited atoms which are produced by the axion-photon conversion and thermal noise are given, respectively, by

$$R_s = \frac{r_{ba}(t_{\mathrm{tr}})\overline{n}_a}{t_{\mathrm{tr}}}, \quad R_n = \frac{r_{bc}(t_{\mathrm{tr}})\overline{n}_c}{t_{\mathrm{tr}}}. \tag{17}$$

The measurement time required to search for the signal at the confidence level of $m\sigma$ is then estimated by

$$\Delta t = \frac{m^2(1 + R_n/R_s)}{R_s}. \tag{18}$$

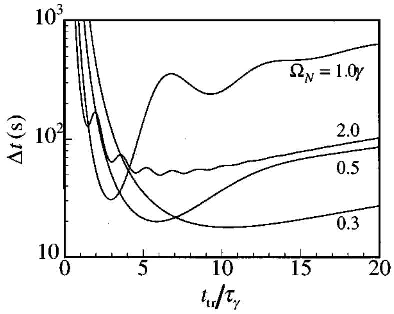

FIG. 3. Transit time dependence of the measurement time $\Delta t(s)$. The relevant parameters are the same as in Fig. 2.

Shown in Fig. 3 is the transit time dependence of Δt for $m=3$ which is numerically calculated for various values of Ω_N. In this calculation we have assumed the cavity parameters $T_c=10$ mK ($\overline{n}_c \simeq 9\times10^{-6}$) and $\gamma=2\times10^{-10}$ eV ($\tau_\gamma \simeq 3\times10^{-6}$ s) with $\omega_c=10^{-5}$ eV and $Q=5\times10^4$. The axion-photon coupling κ is taken from Eq. (7), and $\overline{n}_a \simeq 5.7\times10^{25}$ is calculated in Eq. (5) by assuming ρ_a equal to our galactic halo density $\rho_{\text{halo}} \simeq 0.3$ GeV cm^{-3} with $m_a=10^{-5}$ eV and $\beta_a=10^{-3}$. As seen in Fig. 3, Δt takes the minimum value ~ 10 s if t_{tr} is taken to be several times of τ_γ for $\Omega_N \gtrsim 0.3\gamma$. If $t_{tr} \gtrsim 10\tau_\gamma$, Δt is increasing proportional to t_{tr} with $r_{ba}(t_{tr}) \simeq r_{ba}(\infty)$. On the other hand, if $t_{tr} \lesssim \tau_\gamma$, there is not enough time for the atom-field interactions to provide a significant atomic signal.

The atomic transit time t_{tr} thus should be adjusted to be several times τ_γ for the case of $\Omega_N \gtrsim 0.3\gamma$, in order to realize the optimal measurement time Δt. In this situation, the atom-field interactions last long enough in the cavity, so that the asymptotic values are almost achieved for the atomic excitations; $r_{ba}(t_{tr}) \simeq (\kappa/\gamma)^2 g(\Delta\omega_a)$ and $r_{bc}(t_{tr}) \simeq 1$. The S/N ratio is then found to be significant from a rough estimate $R_s/R_n \simeq g(\Delta\omega_a)(\kappa/\gamma)^2 \overline{n}_a/\overline{n}_c(T_c) \simeq 2$–6 with the maximal value of $g(\Delta\omega_a)=g_{\max} \simeq 4$–10 as shown in Fig. 2(a). This estimate has actually been confirmed in the numerical calculation of Δt shown in Fig. 3, where the S/N ratio is calculated precisely from Eq. (17) at each value of t_{tr} with the specific value of $\Delta\omega_a$ to give $g(\Delta\omega_a)=g_{\max}$.

The measurement time Δt is also shown in Fig. 4 as a function of Ω_N. Here the optimal transit time t_{tr}, as shown in Fig. 3, is taken for the respective values of Ω_N. It is found

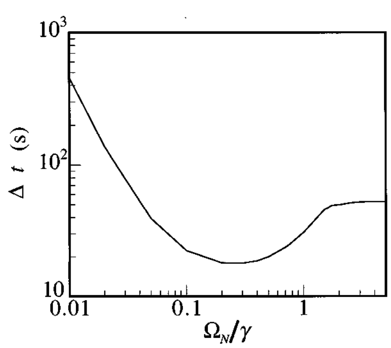

FIG. 4. The measurement time $\Delta t(s)$ as a function of the coupling constant Ω_N. The relevant parameters are the same as in Fig. 2.

in Fig. 4 that the choice of Ω_N/γ around 0.1–1.0 is the most favorable. Close inspection of the relevant transition strengths and the level schemes of alkaline Rydberg atoms [10] shows that the realization of the collective atom-field coupling Ω_N in this range can be obtained by selecting appropriate Rydberg states and controlling suitably the number of injected atoms. The optimal transit time t_{tr}, on the other hand, can be obtained by adjusting the atomic velocity.

Since the axion mass is unknown we have to search for the axions by varying the cavity resonant frequency and also the transition frequency of the Rydberg atoms. The tuning of the cavity resonant frequency can be obtained by moving an insulating rod inserted in the cavity. The tuning of the transition frequency of the Rydberg atoms are accomplished in two ways: rough tuning of the frequency is done by selecting appropriate levels of the atoms, i.e., by changing the principal quantum number n of the Rydberg states, while fine-tuning of the frequency is performed by applying an electric field in the detection cavity, thus inducing the Stark shift of the transition energy of the Rydberg states. The frequency step of ω_c (with keeping $\omega_b=\omega_c$) to be changed in the search for axions with unknown mass may be taken to be

$$\Delta\omega_c = \gamma/2, \tag{19}$$

as suggested from the form of $g(\Delta\omega_a)$ shown in Fig. 2(a). This can also be seen by plotting the $\Delta\omega_a$ dependence of the measurement time Δt with the optimal t_{tr}, as shown in Fig. 2(b) for various values of Ω_N. If the axion mass m_a ($\simeq \omega_a$) lies in the frequency range scanned with this step, then the atomic signal will become significant at a certain step to give the resonant value of $\Delta\omega_a$; $g(\Delta\omega_a) \geq g_{\max}/2$ as found in Fig. 2(a), and $\Delta t \simeq 50$ s or shorter as seen in Fig. 2(b).

In this situation, the total scanning time over a frequency range of $0.1\omega_c$ is estimated to be

$$t_{\text{tot}} = \frac{0.1\omega_c}{\Delta\omega_c}\Delta t \simeq 5 \text{ days} \tag{20}$$

in the experimental search for the cosmic axions with $g_{a\gamma\gamma} \sim 1.4\times10^{-15}$ GeV^{-1} [15] and $m_a \sim 10^{-5}$ eV. Taking into account all the axion-mass dependence of the related parameters, the above Δt and t_{tot} depend on the axion mass approximately as

$$\Delta t \simeq 50 \text{ s}\left(\frac{m_a}{10\ \mu\text{eV}}\right)^{2.5}, \quad t_{\text{tot}} \simeq 5 \text{ days}\left(\frac{m_a}{10\ \mu\text{eV}}\right)^{1.5}. \tag{21}$$

From these dependences, the present method could be applied for the axion of mass up to about 50 μeV with only one conversion cavity. Use of multiple cavities for axion conversion [7] will enable us to extend the present method further to more massive axions.

In conclusion, we have developed a quantum theoretical treatment of the axion-Rydberg atom interactions in resonant cavities via the Primakoff process. From the time evolution of the excited Rydberg atoms due to the absorption of the axion-converted and thermal background photons, the transit time dependence of the detection efficiency of the axions is estimated numerically for actual experimental circumstances. Then, the optimum conditions are found for the setup of the experimental arrangement in the present scheme. These analyses clearly indicate that the Rydberg-atom single-photon detection scheme in resonant cavities is a quite sensitive method to search for the cosmic axions, enabling us to reach the theoretical limit of the cosmic axion density especially of the Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) type [2,15] within a reasonable measuring time.

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