Unitarized model of hadronic diffractive dissociation

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It is shown that in a supercritical Pomeron model the contribution of the triple-Pomeron diagrams violates the unitarity bound for the cross section even when taking into account the multiple Pomeron exchanges between the initial hadrons. The asymptotic behavior of the single diffractive dissociation cross section is calculated in the approximation where every Pomeron in the $3P$ diagram is eikonalized as well as an elastic interaction of initial hadrons is taken into account. In this approximation $\sigma^{\mathrm{SD}}/\sigma_{\mathrm{tot}}\to 0$, at $s\to\infty$.

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The interest in diffractive dissociation is caused by both comparatively new experimental data obtained at the Fermilab Tevatron [1] and by observations at the DESY ep collider HERA of deep inelastic processes with typical rapidity gaps [2]. The generally adopted interpretation of these similar enough phenomena is based on the following interaction mechanism. The incoming fast proton (or virtual photon in deep inelastic scattering) ''emits'' a Pomeron which interacts with the proton target producing a shower of hadrons. These hadrons are distributed in the rapidity scale at a large distance (rapidity gap) from the initial proton. The central point of the model, the interaction of the Pomeron with the proton target, is universal. It means that this subprocess does not depend on where a dissociation is considered: in a pure hadronic process or in a deep inelastic scattering.

In what follows we consider diffractive dissociation in a pure hadronic process $hh\to hX$. If the effective mass of the produced shower is large enough then with certain simplifying assumptions the cross section of the process may be presented (due to the generalized optical theorem) by a diagram with a triple-Pomeron vertex (Fig. 1).

The properties of the cross section of course depend on the specific model of Pomeron. The best known one at present, the so-called supercritical Pomeron, has a trajectory with the intercept $\alpha_P(0)=1+\Delta$ with $\Delta>0$. In particular the model of Donnachie and Landshoff with $\Delta=0.08$ based on a Pomeron-photon analogy describes quite well the hadronic data [3]. However the contribution of the supercritical Pomeron to the total cross section rises with energy as a power $\sigma\propto s^{\Delta}$, violating the Froissart-Martin bound $\sigma_{\mathrm{tot}}<\mathrm{const}\times\ln^2(s/s_0)$. The strict consistent procedure of unitarization is absent now, but there are some simple phenomenological ways to eliminate the rough contradictions with unitarity. For example, the eikonal [4], U-matrix [5] methods and their generalizations are often used for unitarization of the amplitude. A different approach to the problem was suggested in [6].

It is quite obvious that the three-Pomeron diagram also needs unitarity corrections, which should remove a too fast growing contribution of the supercritical Pomeron to the diffractive dissociation cross section (up to the $\ln s$ factors it is proportional to $s^{2\Delta}$). The $3P$ diagram seemed to be unitarized in the most simple way, taking into account multiple Pomeron exchanges between the incoming hadrons (initial-state interaction). This approach was suggested in Ref. [7]. In a more general context the problem of unitarization in diffractive dissociation was discussed a long time ago (see, for example [8]). It was claimed in [6] that, taking into account the initial-state interaction, the integrated cross section of diffractive dissociation rises logarithmically with energy, $\sigma^{\mathrm{SD}}\propto\ln s$, in accordance with the experimental data.

We will show that this conclusion of [7] is wrong, and taking into account the initial-state interaction by the eikonal (or by another) way does not allow one to eliminate the exceedingly fast growth of the cross section. We have performed asymptotic evaluations of a wider class of corrections, which indeed allows one to eliminate the explicit contradiction with unitarity bounds.

To make our arguments more clear we list a few well-known general statements and formulas. Because we are interested only in an asymptotic cross-section behavior, the contribution of f Reggeon is omitted in all expressions.

As in [6] we will work in the impact parameter representation. The normalization of the amplitude is

$$\frac{d\sigma}{dt}=\pi|f(s,t)|^2,\quad \sigma_{\mathrm{tot}}=4\pi\ \mathrm{Im}f(s,0). \tag{1}$$

An amplitude in the b representation is defined by the transformation

$$a(s,b)=\frac{1}{2\pi}\int d\vec{q}\ e^{-i\vec{q}\cdot\vec{b}}f(s,t),\quad t=-q^2. \tag{2}$$

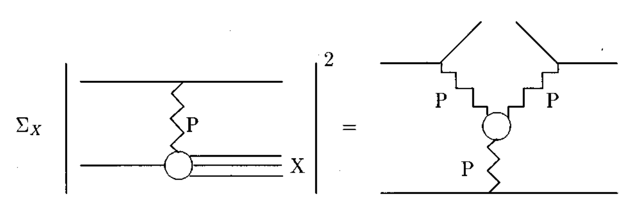

FIG. 1. Process of diffractive dissociation and $3P$ diagram.

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Eikonal summation of the high energy elastic Pomeron rescatterings can be realized with the amplitude $a(s,b)$ in the form

$$a(s,b)=i(1-e^{-\Omega(s,b)}),$$

$$\Omega(s,b)=-\frac{i}{2\pi}\int d\vec{q}e^{-i\vec{q}\cdot\vec{b}}f_0(s,t),$$

where $f_0(s,t)$ is an input elastic amplitude. Starting from a simplified model of the supercritical Pomeron with the trajectory $\alpha_P(t)=1+\Delta+\alpha'_Pt$,

$$f_0(s,t)=ig(t)\left(\frac{s}{s_0}\right)^{\alpha_P(t)-1}, \quad g(t)=ge^{-B_0t/4},$$

one can obtain

$$\Omega(s,b)=\nu(s/s_0)e^{-b^2/R^2(s/s_0)}, \tag{3}$$

where

$$\nu(s/s_0)=\frac{\sigma_0}{2\pi R^2(s/s_0)}\left(\frac{s}{s_0}\right)^{\Delta}=\frac{4g^2}{R^2(s/s_0)}\left(\frac{s}{s_0}\right)^{\Delta}, \tag{4}$$

$$R^2(s/s_0)=2B_0+4\alpha'\ln(s/s_0), \quad \sigma_0=8\pi g^2. \tag{5}$$

In this model $if_0(s,t)$ and $\Omega(s,b)$ are the real functions, but analyticity and crossing symmetry are restored by the substitution $s\to s\ \exp(-i\pi/2)$. It is easy to obtain from the above expressions that $\sigma_{\rm tot}\simeq 2\pi\Delta R^2(s/s_0)\ln(s/s_0)$, as $s/s_0\to\infty$. Thus, in a supercritical Pomeron model the eikonal corrections to one-Pomeron exchange remove the explicit violation of unitarity condition. The resulting cross sections satisfy the Froissart-Martin bound.

The expression for an integration over the t cross section of diffractive dissociation was written in [7] in the form

$$M^2\frac{d\sigma^{\rm SD}}{dM^2}=\sigma_0^2G_{PPP}\left(\frac{s}{M^2}\right)^{2\Delta}\left(\frac{M^2}{s_0}\right)^{\Delta}\frac{1}{[\pi\tilde{R}^2(s/M^2)]^2\pi\tilde{R}^2(M^2/s_0)}\int d\vec{b}\ d\vec{b}'\exp[-2\nu(s/s_0)e^{-b^2/R^2(s/s_0)}]$$
$$\times\exp\left(-2\frac{(\vec{b}-\vec{b}')^2}{\tilde{R}^2(s/M^2)}-\frac{b'^2}{\tilde{R}^2(M^2/s_0)}\right), \tag{6}$$

where ν,R^2 are defined by Eqs. (4) and (5),

$$\tilde{R}^2(z)=B_0+r^2+4\alpha'\ln(z) \tag{7}$$

and r is the radius of the triple-Pomeron vertex. The eikonal corrections due to Pomeron rescatterings in the initial state (Fig. 2) were accounted by the insertion of factor $\exp[-2\Omega(s,b)]=\exp[-2\nu(s/s_0)e^{-b^2/R^2(s/s_0)}]$ in the integrand of Eq. (6).

Unfortunately a mistake appeared in [7] when asymptotic evaluation of the integral was made. After integration over b and b' the differential cross section of diffractive dissociation becomes

$$\frac{M^2d\sigma^{\rm SD}}{dM^2}=\frac{\sigma_0^2}{2\pi\tilde{R}_1^2(s/M^2)}\times G_{PPP}\frac{a\gamma[a,2\nu(s)]}{[2\nu(s)]^a}\left(\frac{s}{M^2}\right)^{2\Delta}\left(\frac{M^2}{s_0}\right)^{\Delta}, \tag{8}$$

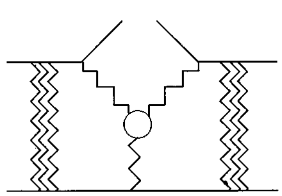

FIG. 2. $3P$ diagram which corresponds to interaction of hadrons in the initial state.

where

$$a=\frac{2R^2(s/s_0)}{\tilde{R}^2(s/M^2)+2\tilde{R}^2(M^2/s_0)} \tag{9}$$

and $\gamma[a,2\nu]$ is the incomplete Euler gamma function.

In the limit under consideration, $s\gg s_0,M^2/s_0,s/M^2\gg 1$, the ratio a tends to 2 and $\gamma[a,2\nu]$ tends to $\Gamma(2)$. Substituting these limits to expression (8) authors of [7] have obtained

$$\frac{M^2d\sigma^{\rm SD}}{dM^2}=\pi R^2(s/s_0)G_{PPP}\left(\frac{M^2}{s_0}\right)^{-\Delta}.$$

However, this result is wrong. Indeed, one can see using definitions (4), (5), (7), and (9) that, at $s,M^2,s/M^2\to\infty$

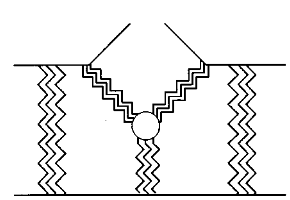

FIG. 3. Diagram with eikonalized Pomerons.

$$a=\frac{2R^2(s/s_0)}{\tilde{R}_1^2(s/M^2)+2\tilde{R}_1^2(M^2/s_0)}$$
$$=2\left[1-\frac{\ln(M^2/s_0)}{\ln(s/s_0)}+O\left(\frac{1}{\ln(s/s_0)}\right)\right].$$

Therefore the factor of expression (9) which violates the unitarity is transformed as

$$\left(\frac{s}{M^2}\right)^{2\Delta}\left(\frac{M^2}{s_0}\right)^{\Delta}[\nu(s)]^{-a}\simeq\exp\left\{2\Delta\ \ln\left(\frac{s}{M^2}\right)+\Delta\ \ln\left(\frac{M^2}{s_0}\right)-2\Delta\ \ln(s/s_0)\right\}\sim\left(\frac{M^2}{s_0}\right)^{\Delta},$$

conserving a too fast growth of the cross section at large M^2.

In what follows we investigate the diagrams which are important to restore the unitarity. In our opinion it is necessary not only to take into account an interaction of hadrons in the initial state but also to ‘‘eikonalize’’ each Pomeron in the diagram of Fig. 2. In another words we estimate an asymptotical contribution to σ^{SD} of diagrams of Fig. 3. Evidently it is impossible to calculate such diagrams in a general form without any simplified assumptions. At the same time there are two important and interesting points. First, is the violation of unitarity bound indeed removed after an eikonalization? Second, how fast does the eikonalized diffractive cross section σ^{SD} rise at $s\to\infty$?

Our model we define in the form, which corresponds to Fig. 3,

$$M^2\frac{d\sigma^{\mathrm{SD}}}{dM^2}=\sigma_0^{-1}\tilde{G}I,$$

where

$$I=\int d\vec{b}\ d\vec{b}'\exp[-2\Omega(s,b)]\times\left\{1-\exp\left[-\tilde{\Omega}\left(s_0\frac{s}{M^2},b'\right)\right]\right\}^2\times\{1-\exp[-\tilde{\Omega}(M^2,|\vec{b}-\vec{b}'|)]\}\qquad(10)$$

and $\tilde{G}$ includes all of the relevant couplings and constants. The value $\tilde{\Omega}(z,b)$ differs from the input one-Pomeron $\Omega(z,b)$ [Eq. (4)]. The difference is that one of the vertices g in it is changed for the part of triple-Pomeron vertex.

We note that it is not enough to eikonalize only Pomerons in $3P$ vertex in order to restore unitarity. If we do that, the factor $\exp[-2\Omega(s,b)]$ is absent in the integrand of (10). The integral is calculated easily:

$$M^2\frac{d\sigma^{\mathrm{SD}}}{dM^2}\propto R_1^2R_2^2\ \ln(s/M^2)\ln(M^2/s_0),\qquad \sigma^{\mathrm{SD}}\propto\ln^5(s/s_0).$$

The result evidently contradicts the Froissart-Martin bound.

Let us now calculate the asymptotic behavior of integral I at the limit $s\to\infty$. To do that let us rewrite it in the form

$$I=4\pi\int_0^\infty db\int_0^\infty db'\int_0^\pi d\phi\ bb'[1-e^{-\nu_1e^{-b'^2/R_1^2}}][1-e^{-\nu_2e^{-b^2/R_2^2}}]^2\exp\{-2\nu_0\exp[-(b^2+b'^2+2bb'\cos\phi)/R_0^2]\}.$$

To avoid being cumbersome in the following formulas we use the notation

$$\nu_0\equiv\nu(s/s_0),\qquad \nu_1=\frac{4g\upsilon}{R_1^2}\left(\frac{M^2}{s_0}\right)^{\Delta},\qquad \nu_2=\frac{4g\upsilon}{R_2^2}\left(\frac{s}{M^2}\right)^{\Delta},$$

$$R_0^2\equiv R^2(s/s_0),\qquad R_1^2\equiv\tilde{R}_1^2(M^2/s_0),\qquad R_2^2\equiv\tilde{R}_2^2(s/M^2),$$

where $R^2(s/s_0)$ is defined by Eq. (5) and

$$\tilde{R}_1^2(M^2/s_0)=B_0+r_0^2+4\alpha'\ln(M^2/s_0),$$

$$\tilde{R}_2^2(s/M^2)=B_0+r_{1,2}^2+4\alpha'\ln(s/M^2).$$

Making the substitution of the integration variables

$$\nu_1\exp(-b^2/R_1^2)=z_1,$$

$$\nu_2\exp(-b^2/R_2^2)=z_2,$$

we get

$$I=2\pi R_1^2R_2^2\int_0^{\nu_1}\frac{dz_1}{z_1}\int_0^{\nu_2}\frac{dz_2}{z_2}(1-e^{-z_1})^2(1-e^{-z_2})\int_0^\pi d\phi\ \exp\left(-2\nu_0\exp\left\{-\left[\rho_1\ln\left(\frac{\nu_1}{z_1}\right)+2\sqrt{\rho_1\ln\left(\frac{\nu_1}{z_1}\right)\rho_2\ln\left(\frac{\nu_2}{z_2}\right)}\cos\phi+\rho_2\ln\left(\frac{\nu_2}{z_2}\right)\right]\right\}\right),$$

where

$$\rho_1=\tilde{R}_1^2/R_0^2, \quad \rho_2=\tilde{R}_2^2/R_0^2.$$

It is convenient to divide both integrals over z_1 and z_2 into two parts, one is from 0 up to 1 and the second is from 1 up to ν_i:

$$I=I_{0,0}^{1,1}+I_{1,0}^{\nu_1,1}+I_{0,1}^{I,\nu_2}+I_{1,1}^{\nu_1,\nu_2}.$$

Consider, for example, first, where the integrations over z's go from 0 up to 1 (remaining ones are similarly estimated). The small z_1 and z_2 contribute to it. Therefore replacing $1-\exp(-z_i)$ by z_i and using the new variables

$$x_i=-\frac{\ln z_i}{\ln \nu_i}, \quad i=1,2,$$

we write $I_{0,0}^{1,1}$ in the form

$$I_{0,0}^{1,1}=2\pi R_1^2R_2^2 \ln\nu_1\ln\nu_2\int_0^\pi d\phi\int_0^\infty dx_1\exp(-2x_1\ln\nu_1)\int_0^\infty dx_2\exp(-x_2\ln\nu_2)\exp\left\{-2\nu_0\exp\left(-(\beta_1\sqrt{1+x_1}+\beta_2\sqrt{1+x_2})^2 +4\beta_1\beta_2\sqrt{(1+x_1)(1+x_2)}\sin^2\frac{\phi}{2}\right)\right\},$$

where

$$\beta_1=\sqrt{\rho_1\ln\nu_1}, \quad \beta_2=\sqrt{\rho_2\ln\nu_2}. \tag{11}$$

Before further calculation let us pay attention to the behavior of ρ, β, ν, at $s\to\infty$. It is easy to see that for an arbitrary but fixed ρ_i,

$$\ln\nu_i=\rho_i\ln\nu_0\left[1-\frac{\ln(R_0^2/\sigma_0)}{\ln\nu_0}+O\left(\frac{1}{\ln(s/s_0)}\right)\right], \tag{12a}$$

$$\beta_i=\rho_i\sqrt{\ln\nu_0}\left[1-\frac{1}{2}\frac{\ln(R_0^2/\sigma_0)}{\ln\nu_0}+O\left(\frac{1}{\ln(s/s_0)}\right)\right], \tag{12b}$$

$$\rho_1+\rho_2=1+\frac{r_0^2+r_{1,2}^2}{R_0^2}, \quad (\beta_1+\beta_2)^2=\ln\nu_0-\ln\left(\frac{R_1^2R_2^2}{\sigma_0R_0^2}\right)+O(1). \tag{12c}$$

Making use of the above properties of ρ, β, ν, one can argue that the main contribution in the integrals over x_i,ϕ is determined by small x_1, x_2, ϕ. Keeping linear in x_1, x_2, ϕ^2 terms in the internal exponential and substituting

$$u_1=\exp[-\beta_1(\beta_1+\beta_2)x_1], \quad u_2=\exp[-\beta_2(\beta_1+\beta_2)x_2],$$

we obtain

$$I_{0,0}^{1,1}=2\pi\frac{R_1^2R_2^2 \ln\nu_1\ln\nu_2}{\beta_1\beta_2(\beta_1+\beta_2)^2}\int_0^\pi d\phi\int_0^1\frac{du_1}{u_1}\int_0^1\frac{du_2}{u_2}u_1^{a_1}u_2^{a_2}\exp\left\{-2\frac{R_1^2R_2^2}{\sigma_0R_0^2}\exp(\beta_1\beta_2\phi^2)u_1u_2\right\}$$

$$\simeq 2\pi R_1^2R_2^2\frac{\sqrt{\pi}\ln\nu_1\ln\nu_2}{(\beta_1\beta_2)^{3/2}(\beta_1+\beta_2)^2(a_1-a_2)}\left\{\frac{\Gamma(a_2)}{\sqrt{a_2}}\left(2\frac{R_1^2R_2^2}{\sigma_0R_0^2}\right)^{-a_2}-\frac{\Gamma(a_1)}{\sqrt{a_1}}\left(2\frac{R_1^2R_2^2}{\sigma_0R_0^2}\right)^{-a_1}\right\},$$

where

$$a_1=\frac{\beta_1/\rho_1}{\beta_1+\beta_2}, \quad a_2=\frac{2\beta_2/\rho_2}{\beta_1+\beta_2}. \tag{13}$$

Since $a_1=1+O[1/\ln(s/s_0)]$ and $a_2=2+O[1/\ln(s/s_0)]$, as it follows from (12) and (13) we finally obtain, for the integral $I_{0,0}^{1,1}$,

$$I_{0,0}^{1,1}=\pi\sqrt{\pi}\sigma_0R_0^2\frac{\ln\nu_1\ln\nu_2}{(\beta_1\beta_2)^{3/2}(\beta_1+\beta_2)^2}\left[1+O\left(\frac{1}{\ln(s/s_0)}\right)\right].$$

Similar calculations for other terms of the integral I give rise to

$$I_{0,1}^{1,\nu_2}\simeq I_{0,0}^{1,1}, \quad I_{1,0}^{\nu_1,1}\simeq\frac{1}{2\sqrt{2}}\frac{\sigma_0R_0^2}{R_1^2R_2^2}I_{0,0}^{1,1},$$

$$I_{1,1}^{\nu_1,\nu_2}<\text{const}\times\exp(-R_0^2/\sigma_0).$$

Thus, as $s\to\infty$,

$$M^2 \frac{d\sigma^{\mathrm{SD}}}{dM^2} \simeq \pi\sqrt{\pi}\tilde{G}(R_0^2)^2 \sqrt{\frac{\Delta\ \ln(s/s_0)}{R_1^2 R_2^2}} \simeq \mathrm{const}\frac{[\ln(s/s_0)]^{3/2}}{\sqrt{\ln(s/M^2)\ln(M^2/s_0)}}. \tag{14}$$

Integration of Eq. (14) over M^2 in the domain where $\rho_{1,2} \not\to 0$, gives

$$\sigma^{\mathrm{SD}} \propto \ln^{3/2}(s/s_0), \quad \sigma^{\mathrm{SD}}/\sigma_{\mathrm{tot}} \to 0, \quad \text{at } s\to\infty.$$

Thus we have proposed and investigated the simplified eikonal model for a process of hadronic single diffractive dissociation. We have shown that eikonalization of each Pomeron in the $3P$ diagram and an account of the elastic interaction of hadrons in the initial state allow us to restore the unitarity which is violated by an input supercritical Pomeron. Our result is only an asymptotical one. The numerical calculations and account of the nonasymptotical contributions are needed to compare the considered model with experiment.

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