PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOLUME 53, NUMBER 1 **1 JANUARY 1996**

RAPID COMMUNICATIONS

Rapid Communications are intended for important new results which deserve accelerated publication, and are therefore given priority in the editorial office and in production. A Rapid Communication in **Physical Review D** *should be no longer than five printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but because of the accelerated schedule, publication is generally not delayed for receipt of corrections unless requested by the author.*

Rare kaon decays with "missing energy"

William J. Marciano and Zohreh Parsa

Brookhaven National Laboratory, P.O. Box 5000, Upton, New York 11973-5000

and National Institute for Nuclear Theory, Box 351550, University of Washington, Seattle, Washington 98195-1550

(Received 30 August 1995)

Rare kaon decays due to the loop-induced standard model operator $\bar{s}_L\gamma_\mu d_L\bar{\nu}_L\gamma^\mu\nu_L$ are examined. Isospin-violating mass effects and electroweak radiative corrections are shown to reduce $B(K^+\to\pi^+\nu\bar{\nu})$ and $B(K_L\to\pi^0\nu\bar{\nu})$ relative to $B(K^+\to\pi^0e^+\nu_e)$ by 10% and 5.6%, respectively. Predicted branching ratios for $(K_L\to\nu\bar{\nu}\gamma)$ and $(K_L\to\nu\bar{\nu})$ (if neutrinos have mass) are given. The sensitivity of "missing energy" rare K decays to new interactions or the emission of light weakly interacting neutral particles, other than neutrinos, is also briefly discussed.

PACS number(s): 13.20. Eb, 12.15.Lk

In the standard model, flavor-changing neutral current amplitudes are absent at the tree level due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [1]. They are, however, induced at the quantum loop level [2,3]. An interesting case is the $\bar{s}d\to\nu_i\bar{\nu}_i$, $i=e,\mu,\tau$, transition amplitudes. They give rise to the effective interaction Lagrangian

$$\mathscr{L}_{\rm eff}=-\frac{G_\mu\alpha(m_Z)}{\sqrt{2}\pi\sin^2\theta_W}\sum_{i=e,\mu,\tau}2C_i\bar{s}_L\gamma_\mu d_L\bar{\nu}_{i_L}\gamma^\mu\nu_{i_L}+\text{H.c.},\qquad(1)$$

where

$$G_\mu=1.166\,39(1)\times10^{-5}\ \text{GeV}^{-2},\qquad(2)$$

$$\alpha(m_Z)\simeq1/128,\qquad(3)$$

$$\sin^2\theta_W\simeq0.23,\qquad(4)$$

$$\Psi_L\equiv\frac{1-\gamma_5}{2}\Psi,\qquad(5)$$

and the C_i are complex coefficients which depend on the charm and top quark masses, Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements, and perturbative QCD corrections [4–6].

A thorough study of the C_i has been carried out by Buchalla and Buras [4,7]. From their work, one finds that the C_i, $i=e,\mu$, are well approximated by the following expression which depends on the Wolfenstein parametrization [8] λ, A, ρ, η of the CKM matrix

$$C_e=C_\mu=X_t\left(\frac{m_t^2}{m_W^2}\right)\lambda^5A^2\left\{1-\rho-i\eta-\lambda^2\left[\eta^2-\frac{\rho}{2}+i\eta\left(\frac{1}{2}-\rho\right)\right]\right\}+(2.4\pm0.5)\times10^{-4},\qquad(6)$$

$$X_t(y)=0.985\frac{y}{8}\left(\frac{2+y}{y-1}+\frac{3y-6}{(1-y)^2}\ln y\right).\qquad(7)$$

The $2.4\pm0.5\times10^{-4}$ term in (6) stems from charm quark loops and the error is due to charm mass and QCD uncertainties [4]. In the case of $i=\tau$, the τ-lepton mass enters the loop calculations such that

$$\text{Re}C_\tau\simeq\text{Re}C_e-7.6\times10^{-5},\quad\text{Im}C_\tau\simeq\text{Im}C_e.\qquad(8)$$

Employing $m_t\equiv m_t(m_t)_{\overline{\rm MS}}=170$ GeV, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme, $\lambda=0.22$, $A=0.83$ (the product $\lambda^5A^2=|V_{us}||V_{cb}|^2$ currently has an uncertainty of about $\pm 15\%$) along with the less certain central values $\rho = 0$, $\eta = 0.36$, one finds

$$|\mathrm{Re}C_e| = |\mathrm{Re}C_\mu| = 1.1|\mathrm{Re}C_\tau| = 7.81\times 10^{-4}, \tag{9}$$

$$|\mathrm{Im}C_i| = 2.01\times 10^{-4}, \quad i = e,\mu,\tau. \tag{10}$$

Those quantities currently carry about a $\pm 40\%$ uncertainty, primarily because ρ and η are only constrained to lie in the (correlated) range [9]

$$-0.37 \lesssim \rho \lesssim 0.29, \tag{11}$$

$$0.22 \lesssim \eta \lesssim 0.45. \tag{12}$$

Testing the standard model prediction in (1) is extremely important. Doing so confronts the underlying structure of quark mixing and CP violation. A deviation from expectations would signal the presence of "new physics" [10]. In that regard, "redundant" checks are very useful, since different processes may depend on the same standard model parameters, but exhibit different sensitivity to the "new physics."

The two best known predictions which follow from (1) are the branching ratios for $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$. Both of those semileptonic decays are very clean theoretically because they involve the hadronic vector current in (1) which is conserved in the limit of zero quark masses. Renormalization due to strong interactions is therefore of second order in SU(3) breaking and thus small. In addition, those decays can be related by isospin to the well measured K_{e3} decay rates; a prescription generally followed in the literature. However, as we shall show, isospin violating quark mass effects and electroweak radiative corrections must be included for precise predictions.

From the interaction in (1), the predicted branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ is given by

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = \frac{G_\mu^2}{192\pi^3} \frac{\alpha^2(m_Z)}{4\pi^2 \sin^4\theta_W} |f_+^{K^+\pi^+}(0)|^2 I(m_{K^+}, m_{\pi^+}) \tau_{K^+} \left(\sum_{i=e,\mu,\tau} |C_i|^2 \right), \tag{13}$$

where

$$I(m_K, m_\pi) = (m_K - m_\pi)^5 \left(1 + \frac{m_\pi}{m_K}\right)^3 \int_0^1 dx (1-x)^{3/2} (1-ax)^{3/2} (1+0.03bx)^2, \tag{14}$$

$$a = \left(\frac{m_K - m_\pi}{m_K + m_\pi}\right)^2, \quad b = \left(\frac{m_K - m_\pi}{m_\pi^+}\right)^2, \tag{15}$$

is a phase-space integral and

$$f_+^{K^+\pi^+}(0) = 0.96, \tag{16}$$

$$\tau_{K^+} = 1.8795\times 10^{13}\ \mathrm{MeV}^{-1}, \tag{17}$$

$$m_{K^+} = 493.65\ \mathrm{MeV}, \tag{18}$$

$$m_{\pi^+} = 139.57\ \mathrm{MeV}. \tag{19}$$

The deviation of the vector form factor $f_+^{K^+\pi^+}(0)$ from 1 is due to SU(3)-breaking quark mass effects. We have taken the specific value in (16) from a study by Leutwyler and Roos [11], assuming $f_+^{K^+\pi^+}(0) \simeq f_+^{K^0\pi^+}(0)$. Deviations from that equality are expected to be small, but we have not made a quantitative study. That situation is to be contrasted with $f_+^{K^+\pi^0}(0)$ which is larger by about 2.2% due to $\pi^0 - \eta$ mixing via isospin violation [11,12]. We note that our use of $f_+^{K\pi}(0)$ from Leutwyler and Roos [11] is consistent with the employment of $\lambda = 0.22$ which is also taken from their analysis. We also note that there are no short-distance electroweak radiative corrections of the form $[1 + c(\alpha/\pi)\ln(m_Z/m_p)]$ to the interaction in (1). That situation is to be contrasted with K_{e3} decays which have a $[1 + 2(\alpha/\pi)\ln(m_Z/m_p)]$ short-distance enhancement due to electroweak radiative corrections [13].

From the formulas and parameter values in (13)–(19), one finds

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = 5.23\times 10^{-5} \left(\sum_{i=e,\mu,\tau} |C_i|^2 \right), \tag{20}$$

where the 5.23×10^{-5} coefficient has about a $\pm 3\%$ uncertainty [11] due to neglected electroweak corrections (nonleading logs), the uncertainty in $f_+^{K^+\pi^+}(0)$ and other small effects. Modulo uncertainties in the C_i from the Wolfenstein parameters, (20) represents a very precise prediction.

An alternate prescription generally employed in place of (13) is to use the measured value of $B(K^+ \to \pi^0 e^+ \nu_e)$ as input, since it is related to $K^+ \to \pi^+ \nu\bar{\nu}$ by isospin. That procedure gives

$$B(K^+ \to \pi^+ \nu\bar{\nu}) \simeq \frac{\alpha^2(m_Z)}{2\pi^2 \sin^4\theta_W} \frac{B(K^+ \to \pi^0 e^+ \nu_e)}{\lambda^2} \left(\sum_{i=e,\mu,\tau} |C_i|^2 \right) \tag{21}$$

TABLE I. Relative size of the phase space integral $I(m_K, m_\pi)$, $|f_+(0)|^2$, and short-distance electroweak radiative correction for $K \to \pi\nu\bar{\nu}$ decays in comparison with $K^+ \to \pi^0 e^+ \nu_e$. In the case of K^+_{e3}, the effect of the electron mass is included.

Decay mode	$I(m_K, m_\pi)/I(m_{K^+}, m_{\pi^0})$	$\lvert f_+^{K\pi}(0)\rvert^2/\lvert f_+^{K^+\pi^0}(0)\rvert^2$	EW radiative correction $\left[1+\frac{2\alpha}{\pi}\ln(m_Z/m_p)\right]^{-1}$
$K^+ \to \pi^+ \nu\bar{\nu}$	0.9614	0.9574	0.979
$K_L \to \pi^0 \nu\bar{\nu}$	1.0522	0.9166	0.979

up to isospin violating quark mass effects and electroweak radiative corrections. Employing $B(K^+ \to \pi^0 e^+ \nu_e) = 0.0482$ leads to a coefficient of 5.8×10^{-5} in (20) rather than 5.23×10^{-5}. They differ by 10%. That difference can be traced to three isospin breaking effects all of which reduce $B(K^+ \to \pi^+ \nu\bar{\nu})$ relative to $B(K^+ \to \pi^0 e^+ \nu_e)$. Those effects are: (1) A factor of 0.9614 phase-space reduction factor due to the $\pi^+ - \pi^0$ mass difference. (2) A factor of $0.957 = |f_+^{K^+\pi^+}(0)/f_+^{K^+\pi^0}(0)|^2$ which comes primarily from $\pi^0 - \eta$ mixing and is first order in $m_d - m_u$ [11]. (Isospin violation may also reduce slightly the 0.03 slope factor in (14), but that effect is expected to be $\sim 0.1\%$ at the branching ratio level and therefore neglected.) (3) A relative reduction factor of 0.979 because there are no leading log short-distance electroweak radiative corrections to (1), while the $\bar{s}_L \gamma_\mu u_L \bar{\nu}_L \gamma^\mu e_L$ amplitude which contributes to $K^+ \to \pi^0 e^+ \nu_e$ is enhanced by a factor $[1 + 2(\alpha/\pi)\ln(m_Z/m_p)] \simeq 1.02$. Those correction factors are illustrated in Table I.

Long distance electroweak radiative corrections to $K^+ \to \pi^0 e^+ \nu_e$ are assumed to be applied to the experimental data [11]. We will not address that issue here, but note that it should be revisited. Indeed, given the important role of K_{e3} decays in determining $|V_{us}| = \lambda$, the electroweak radiative corrections and precise values of $|f_+^{K\pi}(0)|$ should be carefully scrutinized. In addition, new experimental measurements of both K^+_{e3} and K^0_{e3} are warranted. Together, they can better determine the λ and $|f_+^{K\pi}(0)|$ values. In any event, (20) rather than (21) clearly provides a better prediction for $B(K^+ \to \pi^+ \nu\bar{\nu})$.

In the case of $K_L \to \pi^0 \nu\bar{\nu}$, the branching ratio formula in (13) must be modified by replacing $|C_i|$ by $|\mathrm{Im} C_i|$, τ_{K^+} by $\tau_{K_L} = 4.18\tau_{K^+}$, and $f_+^{K^+\pi^+}(0)$ by $f_+^{K^0\pi^0}(0) \simeq 0.978 f_+^{K^+\pi^+}(0)$ (due to $\pi^0 - \eta$ mixing). Also, the phase-space integral must be evaluated using

$$m_{K_L} = 497.67 \text{ MeV}, \tag{22}$$

$$m_{\pi_0} = 134.97 \text{ MeV}. \tag{23}$$

In that way, we find

$$B(K_L \to \pi^0 \nu\bar{\nu}) = 2.29\times 10^{-4} \left(\sum_{i=e,\mu,\tau} |\mathrm{Im} C_i|^2 \right), \tag{24}$$

where the coefficient again has about $\pm 3\%$ uncertainty. That prediction is about 5.6% smaller than the result usually quoted using $B(K^+ \to \pi^0 e^+ \nu_e)$ as input and assuming isospin. In this case, phase-space actually gives a 5.22% enhancement of $B(K_L \to \pi^0 \nu\bar{\nu})$ relative to $B(K^+ \to \pi^0 e^+ \nu_e)$; however, it is more than offset by a -8.34% reduction due to a significant deviation in $|f_+^{K^0\pi^0}(0)|^2/|f_+^{K^+\pi^0}(0)|^2$ from 1 due to $\pi^0 - \eta$ mixing. Indeed they are each shifted away from $f_+^{K^0\pi^+}(0) \simeq f_+^{K^+\pi^+}(0)$ by equal but opposite amounts. Finally, the short-distance leading log radiative correction to $K^+ \to \pi^0 e^+ \nu_e$ effectively suppresses $B(K_L \to \pi^0 \nu\bar{\nu})$ by a 0.979 factor. All those isospin violating factors are illustrated in Table I.

Employing the central C_i values in (9,10), one finds, from (20) and (21),

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = 0.96\times 10^{-10}, \tag{25}$$

$$B(K_L \to \pi^0 \nu\bar{\nu}) = 2.78\times 10^{-11}. \tag{26}$$

Those predictions currently carry about a factor of 2 uncertainty mainly from the uncertainty in ρ and η. There is currently also about a $\pm 30\%$ error due to correlated uncertainties in λ and A, that will hopefully be reduced by better determinations of $|V_{cb}|$. $B(K^+ \to \pi^+ \nu\bar{\nu})$ has an additional $\pm 12\%$ uncertainty due to its dependence on the charm quark mass and perturbative QCD.

An ongoing experiment (E787) at Brookhaven National Laboratory has lowered the bound [14] on $K^+ \to \pi^+ \nu\bar{\nu}$ to $B(K^+ \to \pi^+ \nu\bar{\nu}) < 2.4\times 10^{-9}$ and expects to eventually observe that rare decay if it actually occurs at the 10^{-10} level. Their next step would be a measurement of the branching ratio to about $\pm 20\%$, thereby determining $|C_i|$ and $|V_{td}|$ to an experimental accuracy of about $\pm 10\%$. The underlying theoretical uncertainties mentioned above are well matched to such a measurement. When completed, such a program would cleanly determine ρ to roughly ± 0.10 (assuming $|V_{cb}|$ is better known).

Recently, there has been growing interest in trying to measure $B(K_L \to \pi^0 \nu\bar{\nu})$ because it probes direct CP violation unambiguously [15]. In addition, it is even cleaner theoretically than $K^+ \to \pi^+ \nu\bar{\nu}$, since it exhibits almost no dependence on the charm quark mass or QCD uncertainties. Thus, it could be used to determine η. A bound of $B(K_L \to \pi^0 \nu\bar{\nu}) < 5.8\times 10^{-5}$ has been set [16] and efforts are underway to mount a dedicated experiment capable of reaching the prediction in (26). Such a measurement is even more challenging than $K^+ \to \pi^+ \nu\bar{\nu}$ because K_L decays in flight and the final state kinematics are, therefore, more uncertain.

By-products of searches for $K \to \pi +$ "*missing energy*" are constraints on nonstandard new interactions which might be

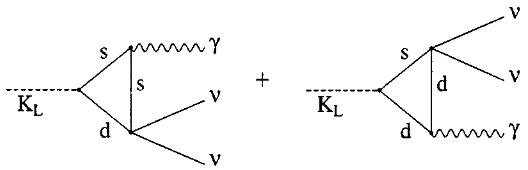

FIG. 1. Structure dependent contributions to the decay $K_L \to \nu\bar{\nu}\gamma$.

induced by horizontal gauge bosons, leptoquarks [17], extended technicolor, etc. Such interactions could give rise to an amplitude

$$2\frac{G_F}{\sqrt{2}}\frac{m_W^2}{\Lambda^2}\bar{s}\gamma_\mu(a+b\gamma_5)d\bar{\nu}_L\gamma^\mu\nu_L', \tag{27}$$

where ν and ν' may or may not be the same flavor and Λ is the scale of the new physics. If a is of order 1, then a measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ at the 10^{-10} level probes $\Lambda \simeq 30$ TeV and a measurement of $B(K_L \to \pi^0\nu\bar{\nu})$ at 3×10^{-11} probes $\Lambda \simeq 57\sqrt{|\mathrm{Im}a|}$TeV. "Missing energy" searches can also be used to constrain [5,10,18] K decays into neutral weakly interacting particles other than neutrinos, such as $K \to \pi +$ axion, familon or photinos $(\tilde{\gamma}\tilde{\gamma})$.

Next, we would like to discuss two not so well-known decays of the K_L (or K_S at an even more suppressed level) that follow from (1), $K_L \to \nu\bar{\nu}\gamma$ and $K_L \to \nu\bar{\nu}$ (if neutrinos have mass). Both are likely to only be of academic interest, since they would be even harder to detect than $K \to \pi\nu\bar{\nu}$. Nevertheless, we feel that a discussion is useful for completeness as well as the off chance that it may inspire experimental creativity. Also, searches for those modes may prove useful for uncovering or bounding other "new physics" scenarios.

We first consider the radiative decay $K_L \to \nu\bar{\nu}\gamma$. An earlier study [19] of that decay considered only doubly weak amplitudes corresponding to the decay chain $K_L \to \gamma Z_{\mathrm{virtual}} \to \gamma\nu\bar{\nu}$ and found an extremely small branching ratio of $\sim 10^{-18}$. That contribution is, however, not the leading effect. The diagrams in Fig. 1, which follow from the interaction in (1), are in fact dominant. Those amplitudes are analogous to what are called "structure-dependent" (SD) amplitudes in $K^+ \to e^+\nu_e\gamma$ decay [20]. Both proceed through a chain $K \to \gamma K^*_{\mathrm{virtual}}$ followed by K^*_{virtual} decay to lepton pairs and are therefore not helicity suppressed. We can bypass hadronic form factors by relating $K_L \to \nu\bar{\nu}\gamma$ to the well studied SD part of $K^+ \to e^+\nu_e\gamma$. The hadronic vector current in (1) dominates. It is approximately related to the structure dependent vector (SDV) current contribution to $K^+ \to e^+\nu_e\gamma$ via

$$\Gamma(K_L \to \nu\bar{\nu}\gamma)_{\mathrm{SDV}} \simeq \frac{1}{\lambda^2}\left(\frac{\alpha(m_Z)}{\pi\sin^2\theta_W}\right)^2\left(\sum_{i=e,\mu,\tau}|\mathrm{Re}C_i|^2\right) \times\Gamma(K^+ \to e^+\nu\gamma)_{\mathrm{SDV}}. \tag{28}$$

Using $B(K^+ \to e^+\nu_e\gamma)_{\mathrm{SDV}} \simeq 1.2\times 10^{-5}$, $\tau_{K_L}/\tau_{K^+} \simeq 4.18$, and the values of $\mathrm{Re}C_i$ in (9) then implies

$$B(K_L \to \nu\bar{\nu}\gamma)_{\mathrm{SDV}} \simeq 2\times 10^{-13}. \tag{29}$$

For completeness, we also give the approximate photon energy spectrum (neglecting form factor attenuation)

$$\frac{1}{\Gamma(K_L \to \nu\bar{\nu}\gamma)_{\mathrm{SDV}}}\frac{d\Gamma(K_L \to \nu\bar{\nu}\gamma)_{\mathrm{SDV}}}{dx} = 20(x^3 - x^4), \tag{30}$$

where

$$x = 2E_\gamma/m_{K_L}. \tag{31}$$

The structure dependent axial-vector SDA contribution to $K_L \to \nu\bar{\nu}\gamma$ goes through direct CP violation in (1). That contribution is suppressed by $1/4|\mathrm{Im}C_i|^2/|\mathrm{Re}C_i|^2 \simeq 1/64$ relative to SDV.

Our result in (29) is five orders of magnitude larger than the previous estimate [19]. It is, however, still outside the realm of experimental accessibility. Nevertheless, it might be useful to search for that decay or place a bound on its occurrence. It is possible that such a decay might be enhanced in some models. For example, a CP-violating axial vector hadronic amplitude would not contribute to $K \to \pi\nu\bar{\nu}$ decays but could give rise to $\Gamma(K_L \to \nu\bar{\nu}\gamma)_{\mathrm{SDA}}$.

If neutrinos have mass, the interaction in (1) can give rise to $K_L \to \nu\bar{\nu}$ via the hadronic axial-current in (1). That decay amplitude is proportional to m_ν; so, only the ν_τ might give a non-negligible contribution. Its mass bound, $m_{\nu_\tau} \lesssim 24$ MeV leaves some room for $B(K_L \to \nu_\tau\bar{\nu}_\tau)$ at the 10^{-10} level, as we shall see. In addition, a relatively heavy $m_{\nu_\tau} \simeq 10$ MeV has been suggested in some cosmological scenarios [21].

To compute the rate for $K_L \to \nu\bar{\nu}$, one must distinguish Majorana and Dirac neutrino cases. In the massless limit, the two cases are the same, but when mass plays a crucial role, they will differ [22,23]. For a given neutrino mass, we find that the decay rate into Majorana neutrinos is a factor of 2 larger than the case of Dirac neutrinos. That overall factor results from a factor of 2 enhancement of the amplitude which gets squared times $\frac{1}{2}$ for identical particles in the final state.

From the axial-vector hadronic current interaction in (1), we find for Majorana neutrinos

$$B(K_L \to \nu_i\nu_i) = \frac{G_\mu^2 f_K^2\alpha^2(m_Z)}{4\pi^3\sin^4\theta_W}|\mathrm{Re}C_i|^2 m_{K_L}m_{\nu_i}^2 \times\left(1 - \frac{4m_{\nu_i}^2}{m_{K_L}^2}\right)^{1/2}\tau_{K_L}. \tag{32}$$

(For Dirac neutrinos, that quantity should be multiplied by 1/2.) From (32), we find

$$B(K_L \to \nu_i\nu_i) = 1.3\times 10^{-6}|\mathrm{Re}C_i|^2\left(\frac{m_{\nu_i}}{1\ \mathrm{MeV}}\right)^2\left(1 - \frac{4m_{\nu_i}^2}{m_{K_L}^2}\right)^{1/2} = 8\times 10^{-13}\left(\frac{m_{\nu_i}}{1\ \mathrm{MeV}}\right)^2\left(1 - \frac{4m_{\nu_i}^2}{m_{K_L}^2}\right)^{1/2}. \tag{33}$$

If the neutrino mass is $\sim$10 MeV, one expects a $B(K_L \to \nu\nu) \simeq 10^{-10}$. Trying to measure that branching ratio would be extremely difficult. First one must tag the K_L and then determine that it actually decayed into "nothing." Given the long lifetime of the K_L, such a determination is perhaps impossible at the 10^{-10} level.

For comparison, we recall the branching ratio for $\pi^0 \to \nu_i \nu_i$ in the case of massive Majorana neutrinos

$$B(\pi^0 \to \nu_i \nu_i) \simeq 3 \times 10^{-12} \left(\frac{m_{\nu_i}}{1\ \mathrm{MeV}} \right)^2 \left(1 - \frac{4 m_{\nu_i}^2}{m_\pi^2} \right)^{1/2}. \tag{34}$$

That formula is a factor of 2 larger than those in the literature [24,25] which were derived for Dirac neutrinos. Bounding that decay is easier because a π^0 can be efficiently tagged and once that has been done, the short π^0 lifetime guarantees it must decay in the detector. Nevertheless, even in that case, reaching the 10^{-10} level is extremely challenging. Current experiments probe $\sim 8.3 \times 10^{-7}$ [26].

It is possible that some new interaction contributes to $K_L \to \nu\nu$ (or $\pi^0 \to \nu\nu$) and enhances its branching ratio. If it were in the $\bar{s}d$ axial-vector current, it would allude $K \to \pi\nu\bar{\nu}$ searches. Also, there might be other weakly interacting neutral particles that could contribute to $K_L \to nothing$. For example, photinos ($\tilde{\gamma}$) with masses $\sim$100 MeV might be emitted with $\sim 10^{-8}$ branching ratio, depending on the loop structure and magnitude that induces $\bar{s}d\tilde{\gamma}\tilde{\gamma}$ interactions [27]. We note, however, that photinos in that mass range present cosmological problems.

In conclusion, we have refined the predictions for $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ in the standard model by including isospin violating quark mass effects and some electroweak loop differences with K_{e3} decays. Those interesting rare decays can serve as theoretically clean laboratories for measuring quark mixing parameters and searching for new physics. The experiments are very challenging, but within the realm of being possible and certainly worth the effort. We have also given standard model predictions for $K_L \to \nu\bar{\nu}\gamma$ and $K_L \to \nu\bar{\nu}$ (if neutrinos have mass). Those decays would be interesting to explore, but their detection looks essentially impossible. New ingenious experimental ideas are required.

The authors thank the National Institute for Nuclear Theory for its hospitality during the completion of this work. We also thank F. Botella, P. Herczeg, L. Littenberg, and S. Wasserbaech for discussions related to this work. This manuscript has been authored under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy.

[1] S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 2674 (1970).
[2] M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).
[3] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981).
[4] G. Buchalla and A. Buras, Nucl. Phys. **B412**, 106 (1994).
[5] J. Ellis and J. Hagelin, Nucl. Phys. **B217**, 189 (1983).
[6] C. Dib, I. Dunietz, and F. Gilman, Mod. Phys. Lett. A **6**, 3573 (1991).
[7] G. Buchalla and A. Buras, Phys. Lett. B **333**, 221 (1994).
[8] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[9] S. Herrlich and U. Nierste, PSI report, 1995 (unpublished).
[10] L. Littenberg and G. Valencia, Annu. Rev. Nucl. Part. Sci. **43**, 729 (1993).
[11] H. Leutwyler and M. Roos Z. Phys. C **25**, 91 (1984).
[12] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985); R. Urech, Phys. Lett. B **355**, 308 (1995). Urech has updated the Gasser and Leutwyler analysis of $m_d - m_u$ isospin violation. His result increases $\pi^0 - \eta$ mixing and $f_+^{K^+\pi^0}(0)$. However, the total effect merely reduces $B(K_L \to \pi^0 \nu\bar{\nu})$ by 0.1%; so, we do not include it here.
[13] A. Sirlin, Nucl. Phys. **B196**, 83 (1982).
[14] M. Atiya *et al.*, Phys. Rev. D **48**, 1 (1993); S. Adler *et al.*, BNL Report No. 62327, 1995 (unpublished).
[15] L. Littenberg, Phys. Rev. D **39**, 3322 (1989); A. Buras, Phys. Lett. B **333**, 476 (1994).
[16] M. Weaver *et al.*, Phys. Rev. Lett. **72**, 3758 (1994).
[17] M Leurer, Phys. Rev. Lett. **71**, 1324 (1993).
[18] F. Botella and M. Sanchis, Z. Phys. C **42**, 553 (1989).
[19] E. Ma and J. Okada, Phys. Rev. D **18**, 4219 (1978).
[20] J. Bijnens, G. Ecker, and J. Gasser, Nucl. Phys. **B396**, 81 (1993), and references therein.
[21] S. Donaldson *et al.*, Phys. Rev. Lett. **72**, 3754 (1994).
[22] J. Nieves and P. Pal, Phys. Rev. D **32**, 1849 (1985); B. Kayser, *ibid.* **26**, 1662 (1982).
[23] For a discussion of neutrino mass effects in $K \to \pi\nu\bar{\nu}$, see N. Deshpande and G. Eilam, Phys. Rev. Lett. **53**, 2289 (1984) and Nieves and Pal [22].
[24] E. Fischbach *et al.*, Phys. Rev. D **16**, 2377 (1977); A. Halprin, *ibid.* **11**, 147 (1975); M. A. B. Bég, W. Marciano, and M. Ruderman, *ibid.* **17**, 1395 (1978); T. Kalogeropoulos, J. Schechter, and J. Valle, Phys. Lett. **86B**, 72 (1979); P. Herczeg and C. Hoffman, *ibid.* **100B**, 347 (1981).
[25] L. Arnellos, W. Marciano, and Z. Parsa, Nucl. Phys. **B196**, 365 (1982).
[26] M. Atiya *et al.*, Phys. Rev. Lett. **66**, 2189 (1991).
[27] M. Gaillard *et al.*, Phys. Lett. **123B**, 241 (1983).