New constraints on gravity-induced birefringence

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A wide class of gravitation theories predicts gravity-induced birefringence. For Moffat's NGT, the prototypical theory of this type, Gabriel, Haugan, Mann, and Palmer used the predicted gravitational birefringence and observations of solar polarization to constrain the Sun's nonsymmetric charge $l_\odot$. We improve on this constraint by making use of improved knowledge of the solar source of polarization and of a refined analysis procedure. We obtain $l_\odot^2 < (305 \text{ km})^2$.

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Two recent papers by Gabriel *et al.* [1] draw attention to the fact that nonsymmetric gravitation theories (NGT's), such as Moffat's [2], predict a striking violation of the Einstein equivalence principle. A nonsymmetric gravitational field can single out an orthogonal pair of polarization states of light which propagate with different phase velocities. One consequence of such gravity-induced birefringence is that propagation through a gravitational field can alter light's polarization. Gabriel and his collaborators exploit this fact to constrain the degree of birefringence that the Sun's gravitational field could induce. From measurements of the polarization of light received from a magnetic feature near the Sun's limb, they infer that the Sun's NGT charge $l_\odot^2$ must be less than $(535 \text{ km})^2$. This constraint is, however, based on extremely conservative assumptions regarding the polarization of light emitted by the observed solar feature and on an outdated determination of its magnetic filling factor.

In this paper we use improved models of solar magnetic features to extract more realistic values of the parameters that characterize them and the light which they emit. Consequently, we are able to extract a sharper limit on $l_\odot^2$ from the data analyzed by Gabriel and his collaborators. In addition, we describe a new way of using the symmetry properties of Zeeman-split Stokes profiles of spectral lines to extract still sharper limits on the magnitude of gravity-induced birefringence from new observations of solar magnetic features.

While sharper constraints on nonsymmetric theories like NGT are important in their own right, we emphasize that part of the motivation for our work stems from the fact that gravity-induced birefringence is predicted by a far wider class of gravitation theories. Indeed, Ni [3] and Haugan and Kauffmann [4] show that 10 of the 21 degrees of freedom characterizing nonmetric theories of gravity in the χg formalism induce birefringence. Consequently, experiments and observations that constrain gravity-induced birefringence provide tests of the Einstein equivalence principle that have a generality and significance comparable to those of more familiar atomic anisotropy tests [5] and Eötvös and gravitational redshift tests [6].

Since the Sun's gravitational field is essentially static and spherically symmetric, a light ray that threads its way through the Sun's field is, very nearly, confined to a plane. NGT predicts that light polarized with its magnetic field perpendicular to this plane propagates with a different phase velocity than light polarized with its magnetic field lying in the plane. Gabriel *et al.* [1] show that the phase shift which accumulates between these polarization components as light propagates from a point on the solar surface to the observer is

$$\Delta\Phi(\mu) = \frac{\pi l_\odot^4}{\lambda R^3}\left\{\frac{3\pi}{16(1-\mu^2)^{3/2}} - \frac{\mu}{4} - \frac{3\mu}{8(1-\mu^2)} - \frac{3}{8(1-\mu^2)^{3/2}}\arcsin\mu\right\}, \quad (1)$$

where $\mu = \cos\theta$ denotes the cosine of the source's heliocentric angle, λ is the light's wavelength, R is the solar radius, and $l_\odot^2$ is the Sun's NGT charge. This phase shift produces cross talk between linearly and circularly polarized light. Introducing natural Stokes parameters based on the linear polarization states singled out by the NGT field, we find that the cross talk is between Stokes U and V. Since $\Delta\Phi(\mu)$ vanishes at disk center, $\mu = 1$, and increases toward the solar limb, $\mu \to 0$, the values U_{obs} and V_{obs} that an observer measures for a source on the solar surface depends on μ as well as on the intrinsic values U_{src} and V_{src} that one would measure at the source.

The way in which U_{obs} and V_{obs} differ from U_{src} and V_{src} depends on whether a source is pointlike or extended because of the finite spatial resolution of observations. Were a source pointlike, all light received from it would suffer the same phase shift $\Delta\Phi$, so the composite degrees of polarization $(U^2_{\rm obs}+V^2_{\rm obs})^{1/2}$ and $(U^2_{\rm src}+V^2_{\rm src})^{1/2}$ would be equal even though $U_{\rm obs}$ and $V_{\rm obs}$ differ from $U_{\rm src}$ and $V_{\rm src}$. On the other hand, light received from an extended source is emitted from points covering a range of μ values and, so, is an incoherent superposition of light that has suffered different phase shifts, $\Delta\Phi(\mu)$. The additive property of Stokes parameters then implies $(U^2_{\rm obs}+V^2_{\rm obs})^{1/2} < (U^2_{\rm src}+V^2_{\rm src})^{1/2}$. Since solar magnetic features are extended sources, an observation which limits the extent of such gravity-induced depolarization imposes a limit on the degree of birefringence that the Sun's gravitational field could induce.

The depolarization suffered by light received from a solar feature depends on the magnitude of the Sun's NGT charge $l^2_\odot$, on the feature's location and extent and on the observation's spatial resolution. The NGT charge sets the scale of the phase shift (1). The feature's location and extent then determine the range of phase shifts suffered by light from its (the feature's) different parts. In the event that the feature is resolved the range of phase shifts suffered by observed light is restricted. In any event, this range depends on $Af_{\rm src}$, where A is the area of the (circular) entrance aperture of the observer's Fourier transform spectrograph (FTS) and where $f_{\rm src}$ is the feature's magnetic filling factor, the fraction of the aperture covered by magnetic field. Other things being equal, depolarization will be most pronounced for features near the solar limb since the phase shift (1) varies most rapidly there.

The data analyzed by Gabriel *et al.* are FTS spectra of an active region at $\mu = 0.1$ [7]. Like Gabriel and his collaborators, we use the Stokes V profile of 525.02 nm, with an amplitude of 4.3%, to impose a limit on $l^2_\odot$. A survey between 480 nm and 600 nm showed that the largest Stokes V signal near the solar limb is exhibited by this spectral line, in contrast to a survey at disk center [8]. The diameter of the spectrograph's circular entrance aperture corresponded to 3600 km measured parallel to the limb on the Sun's surface.

The following analysis differs from that of Gabriel *et al.* in two respects. We do not assume that the light emitted in the solar atmosphere at the wavelength of the V maximum of 525.02 nm, $V_{\rm src}$ is 100% circularly polarized and we make a self-consistent determination of the feature's filling factor allowing for any gravity-induced depolarization. To be conservative, we do follow Gabriel *et al.* in assuming that the observed feature is a single, circular magnetic region within the FTS resolution element.

The Zeeman effect simply cannot completely circularly polarize the light in an absorption line with finite rest intensity, like 525.02 nm. The polarization is even lower when the Zeeman splitting is incomplete, i.e., the Zeeman splitting is smaller than the thermal Doppler width of the spectral line. To determine the maximum possible polarization produced within the solar atmosphere $V_{\rm max}$, we fit profiles predicted by numerous models of solar magnetic features to the observed profile. We make the conservative assumption that when light reaching the observer was emitted its circular polarization was $V_{\rm src} = f_{\rm src}V_{\rm max}$. That is, we assume the solar feature to be as small as possible, within the constraints imposed by the radiative properties of the solar atmosphere.

The normal procedure for determining a solar feature's filling factor from observed spectra assumes that the light's polarization does not change once it leaves the solar surface, which is to say $V_{\rm obs} = V_{\rm src}$. This assumption does not allow for the possibility of gravity-induced depolarization and leads to an estimated filling factor we call $f_{\rm obs}$ that satisfies $V_{\rm obs} = f_{\rm obs}V_{\rm max}$. Any gravity-induced depolarization leads to $V_{\rm obs} < V_{\rm src}$ and, so, to the conclusion that the true filling factor $f_{\rm src}$ is larger than $f_{\rm obs}$. Indeed, $f_{\rm src}/f_{\rm obs} = V_{\rm src}/V_{\rm obs}$. Since Gabriel *et al.* implicitly assume $f_{\rm obs} = f_{\rm src}$, they underestimate the size of the feature they analyze and the strength of the resulting constraint on $l^2_\odot$. In effect, our analysis self-consistently determines and uses the feature's true filling factor $f_{\rm src}$.

Our analysis proceeds as follows. We determine $V_{\rm max}$ and $f_{\rm obs}$ from the observed spectra (see details below). For given $l^2_\odot$ we then use the phase shift (1) to predict values for the observed polarization $\tilde{V}_{\rm obs}$, given $V_{\rm src}$ values in the range $V_{\rm obs}$ to $V_{\rm max}$ and $f_{\rm src}$ values in the range $f_{\rm obs}$ to unity. Finally, we search for a critical value of $l^2_\odot$ defined by the fact that $\tilde{V}_{\rm obs} < V_{\rm obs}$ for all values of $V_{\rm src}$ and $f_{\rm src}$ in the allowed ranges. We conclude that the Sun's NGT charge $l^2_\odot$ must be less than this critical value.

To determine a realistic estimate of $V_{\rm max}$ we performed automated least-squares fits to the observed profile with synthetic profiles calculated by numerically solving the Unno-Rachkovsky equations for polarized radiative transfer in the best currently available atmospheric models of solar magnetic features. The code used has been described by Solanki *et al.* [9]. It looks for a χ^2 minimum in the parameter space spanned by all the free parameters. To be conservative we have assumed that the solar magnetic field is aligned with the line of sight. Any other inclination gives smaller $V_{\rm max}$ values and, therefore, tighter limits on $l^2_\odot$.

We fit four iron spectral lines: Fe I 525.02, 524.71, and 525.06 nm, and Fe II 523.5 nm. This set of lines allows the field strength, temperature, turbulence velocity, and filling factor to be estimated. From these $V_{\rm max}$ follows. The present determinations of $V_{\rm max}$ and $f_{\rm obs}$ must be considered more reliable than those made by Solanki *et al.* [10] and used by Gabriel *et al.* [1] because of the superiority of our automated procedure over manual fitting, the use of four carefully chosen spectral lines instead of two, and our use of improved atmospheric models. The only major uncertainty remaining in a given fit is due to the feature's temperature gradient which is not very well constrained. To allow for this we do fits for a wide variety of models of magnetic features and use the fit giving the largest $V_{\rm max}$ and, thus, the smallest $f_{\rm obs}$ from among the 32 inversions performed. This turns out to be not the fit with the smallest χ^2. That fit gives a sharper constraint on $l^2_\odot$.

The model atmospheres we have considered describe umbral [11], penumbral [12] and quiet Sun temperature stratifications [12,13], flux-constant models with a range of effective temperatures [14], and plage and network flux-tube models [15]. We have also allowed the temperature stratification to be interpolated between two models. The largest $V_{\max}$ we obtain is 0.187, corresponding to $f_{\rm src} = 0.23$, the smallest possible value. The magnetic field strength is 1420 G for this model.

The most conservative $\bar{V}_{\rm obs}$ value that we predict for the feature studied by Gabriel *et al.* is plotted in Fig. 1 as a function of $|l_\odot|$. Tests reveal that this curve does not depend significantly on the initial difference between the phases of the polarization components singled out by NGT, i.e., on the strength of $U_{\rm src}$ relative to $V_{\rm src}$. The horizontal line in Fig. 1 represents the observed circular polarization less one sigma. Clearly, one must have $l_\odot^2 < (305\text{ km})^2$ to keep gravity-induced depolarization small enough to be compatible with observation. Since the physical consequences of the antisymmetric part of the Sun's NGT field are all proportional to $l_\odot^4$, our new constraint on the magnitude of the Sun's NGT charge forces all such effects to be nearly an order of magnitude smaller than those consistent with the constraint imposed by Gabriel *et al.* and 4 orders of magnitude smaller than those predicted by the $l_\odot^2$ value recently favored by Moffat [2].

To impose still tighter constraints on the degree of birefringence that the Sun's gravitational field could induce we propose to use the symmetry properties of Stokes spectral profiles produced by the weak-field Zeeman effect in the solar astmosphere. The case of a static atmosphere provides a clean and simple example of such symmetry properties and a natural setting in which to outline our proposal. After discussing this case, we briefly consider more realistic ones and the potential for sharp new empirical constraints on gravity-induced birefringence.

Recall that for a static atmosphere, the $V_{\rm src}$ profile of a spectral line split by the Zeeman effect is antisymmetric about the line's central wavelength while the $U_{\rm src}$ profile is symmetric [16]. Gravity-induced birefringence alters the polarization of these components of emitted light as they propagate to a distant observer. We note, however, that since the phase shift (1) is essentially constant across the narrow wavelength range of a solar spectral line the symmetry properties of these components are not altered by propagation. Thus, for example, initially circularly polarized light within an antisymmetric $V_{\rm src}$ profile contributes to both $V_{\rm obs}$ and $U_{\rm obs}$ but both contributions have the initial antisymmetric profile. Similarly, initially linearly polarized light in the corresponding symmetric $U_{\rm src}$ profile contributes to both $V_{\rm obs}$ and $U_{\rm obs}$ but both contributions have the initial symmetric profile. Consequently, gravity-induced birefringence causes the symmetry properties of the full $U_{\rm obs}$ and $V_{\rm obs}$ profiles to differ from those of the $U_{\rm src}$ and $V_{\rm src}$ profiles. While $U_{\rm src}$ is symmetric and $V_{\rm src}$ is antisymmetric, $U_{\rm obs}$ and $V_{\rm obs}$ possess both symmetric and antisymmetric components, which we denote by $U_{s,\rm obs}$, and $V_{s,\rm obs}$ (symmetric) and $U_{a,\rm obs}$ and $V_{a,obs}$ (antisymmetric), respectively.

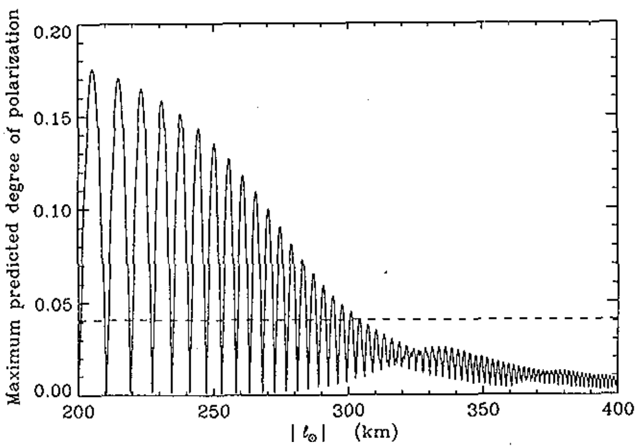

FIG. 1. The maximum predicted value of the degree of circular polarization of 525.0 nm light from a region located near the solar limb, plotted as a function of $|l_\odot|$. The observed degree of polarization from the relevant solar feature less one standard deviation is denoted by the horizontal line. Note that for $l_\odot^2 > (305\,\text{km})^2$ all predicted values lie below the observations.

Even when $U_{\rm src}$ and $V_{\rm src}$ possess both symmetric and antisymmetric components, which they can because of velocity gradients in the solar atmosphere [17], the degree to which their symmetry properties are changed by propagation through the Sun's gravitational field is a direct measure of the strength of gravity-induced birefringence. Quantitatively, the relationship between the symmetric and antisymmetric components of $V_{\rm obs}$ and $U_{\rm obs}$ and those of $V_{\rm src}$ and $U_{\rm src}$ is

$$\begin{aligned}
V_{a,\rm obs} &= V_{a,\rm src}\cos\Delta\Phi + U_{a,\rm src}\sin\Delta\Phi,\\
U_{a,\rm obs} &= U_{a,\rm src}\cos\Delta\Phi + V_{a,\rm src}\sin\Delta\Phi,\\
V_{s,\rm obs} &= V_{s,\rm src}\cos\Delta\Phi + U_{s,\rm src}\sin\Delta\Phi,\\
U_{s,\rm obs} &= U_{s,\rm src}\cos\Delta\Phi + V_{s,\rm src}\sin\Delta\Phi,
\end{aligned} \tag{2}$$

where $\Delta\Phi$ is the line-center phase shift (1). From this we derive

$$\frac{U_{a,\rm src}}{U_{s,\rm src}} = \frac{V_{a,\rm obs}\sin\Delta\Phi + U_{a,\rm obs}\cos\Delta\Phi}{V_{s,\rm obs}\sin\Delta\Phi + U_{s,\rm obs}\cos\Delta\Phi}, \tag{3}$$

$$\frac{V_{s,\rm src}}{V_{a,\rm src}} = \frac{V_{s,\rm obs}\cos\Delta\Phi + U_{s,\rm obs}\sin\Delta\Phi}{V_{a,\rm obs}\cos\Delta\Phi + U_{a,\rm obs}\sin\Delta\Phi}. \tag{4}$$

If we observe the symmetric and antisymmetric fractions of U and V over a range of μ values, then for a given $l_\odot$, Eqs. (3) and (4) predict, respectively, the ratios $U_{a,\rm src}/U_{s,\rm src}$ and $V_{s,\rm src}/V_{a,\rm src}$ at these μ. We can then use our knowledge of solar magnetic features to set upper limits on $U_{a,\rm src}/U_{s,\rm src}$ and $V_{s,\rm src}/V_{a,\rm src}$ and hence rule out those $l_\odot$ which give rise to ratios exceeding these limits (at any μ). Test calculations show that above a certain critical $l_\odot$, every value of $l_\odot$ produces ratios larger than these limits. Thus by observing V and U on a fine μ grid we estimate that $l_\odot^2$ can be constrained to values smaller than $(100\text{ km})^2$ in this manner.

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