BFV-BRST quantization of two-dimensional supergravity

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Two-dimensional supergravity theory is quantized as an anomalous gauge theory. In the Batalin-Fradkin (BF) formalism, the anomaly-canceling super-Liouville fields are introduced to identify the original second-class constrained system with a gauge-fixed version of a first-class system. The BFV-BRST quantization applies to formulate the theory in the most general class of gauges. A local effective action constructed in the configuration space contains two super-Liouville actions; one is a noncovariant but local functional written only in terms of two-dimensional supergravity fields, and the other contains the super-Liouville fields canceling the super-Weyl anomaly. Auxiliary fields for the Liouville and the gravity supermultiplets are introduced to make the BRST algebra close off-shell. Inclusion of them turns out to be essentially important especially in the super-light-cone gauge fixing, where the supercurvature equations ($\partial_-^3 g_{++} = \partial_-^2 \chi_{++} = 0$) are obtained as a result of BRST invariance of the theory. Our approach reveals the origin of the OSp(1,2) current algebra symmetry in a transparent manner.

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I. INTRODUCTION

During the last decade there has been remarkable progress in our understanding of noncritical string theories [1–3]. The first key observation of Polyakov [1] was that the conformal mode of the metric variables does not decouple from the theory at noncritical dimensions. Along this line of thought, the noncritical string was investigated in the light-cone gauge [4, 5]. Noting the SL$(2, R)$ Kac-Moody symmetry, Knizhnik, Polyakov, and Zamolodchikov (KPZ) have succeeded in deriving gravitational scaling dimensions for conformal matter interacting with two-dimensional (2D) gravity on the world sheet. Furthermore, David [6], and Distler and Kawai (DDK) [7] showed that the Polyakov path integral formulation [1] reproduces the KPZ results also in the conformal gauge. It was based on the assumption that the Jacobian associated with changing the functional measure from that for the conformal mode defined in [1] to a translational invariant one generates a Liouville-type action. This DDK ansatz for the functional measure has been examined using the heat kernel method [8]. The relation between the conformal gauge and the light-cone gauge [9, 10], has also been discussed, and analyses based on the Becchi-Rouet-Stura-Tyutin (BRST) formalism have been carried out by several authors [11, 12]. Furthermore, the analysis in the light-cone gauge and conformal gauge have been extended to the supersymmetric case in [13–15] and in [7, 16], respectively.

In our previous paper [17], we gave a systematic canonical formulation of the Polyakov string at noncritical dimensions by applying the idea developed for anomalous gauge theory [18]. It provides a general approach to noncritical strings. The BRST anomalies of the Polyakov string theory at noncritical dimensions [19–21] can be compensated by introducing new degrees of freedom and, thereby, the theory can be made gauge symmetric, i.e., invariant under Weyl rescalings of the metric variables as well as world-sheet reparametrizations. The BRST gauge-fixed action turned out to contain two Liouville type actions, one being written only in the world-sheet metric and the other containing the new degree as the Liouville field. In the conformal gauge, this reduces to the effective action of DDK, giving a justification of their functional measure ansatz from a canonical viewpoint. We further gave a systematic description of the theory in the light-cone gauge, clarifying the relation between the BRST invariance and SL$(2, R)$ Kac-Moody symmetry.

In this paper we will investigate the extension of this work to a Neveu-Schwarz-Ramond superstring [22, 23]. The locally supersymmetric action [24] can be regarded as $N=1$ 2D supergravity (SUGRA) coupled with string variables as superconformal matter. The basic strategy parallels the bosonic string case. Our starting point is the most general form of the BRST anomalies in 2D SUGRA [25] in the extended phase space (EPS) of Batalin, Fradkin, and Vilkovisky (BFV) [26]. The anomaly appeared there as anomalous Schwinger terms which destroy the first-class nature of the super-Virasoro constraints. The quantum system is described by the second-class constraints. In general, systems with second-class constraints can be regarded as gauge-fixed systems of underlying symmetric theories. Actually, one can rewrite the system with second-class constraints into a gauge symmetric form by introducing compensating fields in the EPS. Batalin and Fradkin (BF) [27] developed the

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general idea of converting systems with second-class constraints into gauge symmetric ones in a general and systematic way. Applying their method to the present case and carrying out the BFV-BRST quantization [26], one obtains a BRST gauge-fixed effective action. The construction of the action given here completely parallels that in the anomalous chiral gauge theories [18, 28]. The resulting effective action contains two super-Liouville actions. One just coincides with the local but noncovariant counterterm found in [25], and acts as the Wess-Zumino-Witten term to shift the super-Virasoro anomaly to the super-Weyl anomaly. The other with the BF variables as super-Liouville fields cancels this super-Weyl anomaly. This can be shown without invoking a particular gauge. The fact that our action reduces exactly to the one suggested in [7, 16] for the superconformal gauge implies that our formulation provides a justification of their functional measure ansatz from the canonical viewpoint.

Although our construction of the effective action almost parallels that for the bosonic theory, some new issues arise in the quantization of the fermionic theory. In the EPS, the BRST transformation incorporates supersymmetry transformation, and closes off shell by construction. One may go to the configuration space by eliminating the momentum variables. In general, the BRST transformation in the configuration space thus obtained closes only on shell. In the superconformal gauge, the on-shell closure of the algebra is enough to quantize the theory. This is not the case, however, in the supersymmetric light-cone gauge. Inclusion of auxiliary fields for the supermultiplets of the gravity and the Liouville sectors needed to close the algebra off shell turns out to be essentially important for quantization. Since a systematic way of introducing such fields in the EPS is not known, we shall discuss the auxiliary field as well as the complete form of the BRST transformation after passing to the configuration space. It should be noted in our general construction that the auxiliary field for the Liouville sector and that for the gravity sector couple nontrivially, generating a new local symmetry incorporated in the final form of the BRST transformation.

This paper is organized as follows. In Sec. II, we briefly outline the most general form of the BRST anomaly in 2D SUGRA. The BF algorithm is applied to cancel the super-Virasoro anomaly. We formulate in Sec. III the BRST gauge-fixed effective action in the EPS and describe covariantization of the action, where auxiliary fields are introduced to obtain the off-shell nilpotent BRST transformations in the configuration space. The superconformal gauge is discussed in Sec. IV. Section V is devoted to supersymmetric light-cone gauge fixing. OSp(1,2) Kac-Moody symmetry is obtained in a systematic manner based on the BRST invariance. A summary and discussion are given in Sec. VI. In the Appendix, we summarize the BRST transformation in the configuration space.

II. BRST ANOMALY IN THE FERMIONIC STRING THEORY

In this section we will briefly review the BFV formalism for fermionic string theory and the BRST anomaly along the lines of Ref. [25]. The fermionic string can be formulated as $N = 1$ 2D SUGRA coupled to string coordinates and is described by the action [24]

$$S_X = -\int d^2x e\left[\frac{1}{2}(g^{\alpha\beta}\partial_\alpha X\partial_\beta X - i\bar{\psi}\rho^\alpha\nabla_\alpha\psi) + \bar{\chi}_\alpha\rho^\beta\rho^\alpha\psi\partial_\beta X + \frac{1}{4}\bar{\psi}\psi\bar{\chi}_\alpha\rho^\beta\rho^\alpha\chi_\beta\right], \qquad (2.1)$$

where X^μ and ψ^μ ($\mu = 0,\ldots,D-1$) are, respectively, the bosonic and fermionic string variables,[1] and we have suppressed the space-time indices. The zweibein and gravitino are denoted by $e_\alpha{}^a$ and χ_α, respectively.

The action (2.1) possesses invariances under the world-sheet reparametrizations, local Lorentz rotations, local supersymmetry, local Weyl rescalings, and fermionic symmetry [24]. This suggests a convenient choice of parametrizations [25] for the zweibein and the gravitino as

$$\lambda^\pm = \pm\frac{e_0{}^\pm}{e_1{}^\pm} = \frac{\sqrt{-g}\pm g_{01}}{g_{11}}\ , \quad \xi = \ln(-e_1{}^+e_1{}^-) = \ln g_{11}\ ,$$
$$\varepsilon = \frac{1}{2}\ln\left(-\frac{e_1{}^+}{e_1{}^-}\right),$$
$$\nu_\pm = \frac{(\chi_0\pm\lambda^\mp\chi_1)_\pm}{\sqrt{\mp e_1{}^\mp}}\ , \Lambda_\pm = \frac{4\chi_{1\mp}}{\sqrt{\pm e_1{}^\pm}}\ , \qquad (2.2)$$

where $\chi_{\alpha\mp}$ stands for the upper and lower components of χ_α, and $e_\alpha{}^\pm = e_\alpha{}^0 \pm e_\alpha{}^1$. We also use the rescaled components $\psi_\pm$ defined by

$$\psi = \begin{pmatrix} e^{\frac{1}{2}(\varepsilon-\frac{\xi}{2})}\psi_- \\ e^{-\frac{1}{2}(\varepsilon+\frac{\xi}{2})}\psi_+ \end{pmatrix} \qquad (2.3)$$

for the fermionic string variables. In this parametrization ξ, ε, and $\Lambda_\pm$ are the only variables that change under the Weyl rescalings, the Lorentz rotations, and the fermionic symmetry, respectively. The other variables are all invariant under these symmetries, and the action (2.1) can be written only in terms of these variables as

$$S_X = \int d^2x\left[\frac{1}{\lambda^+ + \lambda^-}(\dot{X}-\lambda^+X')(\dot{X}+\lambda^-X') + \frac{i}{2}\psi_+(\dot{\psi}_+ - \lambda^+\psi'_+) + \frac{i}{2}\psi_-(\dot{\psi}_- + \lambda^-\psi'_-) + \frac{2}{\lambda^+ + \lambda^-}\{i(\dot{X}-\lambda^+X')\psi_-\nu_+ - i(\dot{X} + \lambda^-X')\psi_+\nu_-\} + \frac{2}{\lambda^+ + \lambda^-}\psi_+\psi_-\nu_+\nu_-\right]. \qquad (2.4)$$

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[1] We choose $\eta^{ab} = \text{diag}(-1,1)$ and $\eta^{\mu\nu} = \text{diag}(-1,1,\ldots,1)$ for flat metrics and $\epsilon^{ab} = -\epsilon^{ba}$ with $\epsilon^{01} = 1$ for the Levi-Cività symbol. The world-sheet coordinates are denoted by $x^\alpha = (\tau,\sigma)$ for $\alpha = 0,1$ and are assumed to take $-\infty < \sigma < +\infty$. It is straightforward to make the analysis on a finite interval of σ so as to impose the Neveu-Schwarz or Ramond boundary conditions. We will use the notation $\dot{A} = \partial_\tau A$ and $A' = \partial_\sigma A$ for derivatives. Dirac matrices ρ^a ($a = 0,1$) are chosen to be $\rho^0 = \sigma_2$, $\rho^1 = i\sigma_1$, and $\rho^5 \equiv \rho^0\rho^1 = \sigma_3$, where σ_k ($k = 1,2,3$) are Pauli matrices.

In the canonical description the local symmetries manifest themselves as first-class constraints according to Dirac's classification [29]. Denoting the canonical momenta for X^μ, $\lambda^\pm$, ξ, ε, $\nu_\pm$, and $\Lambda_\pm$ by P_μ, $\pi^\lambda_\pm$, π_ξ, π_ε, $\pi^\pm_\nu$, and $\pi^\pm_\Lambda$, respectively, the canonical theory of this system is characterized by the following set of first-class constraints:

$$\begin{aligned} &\varphi_{\hat{A}} \equiv \pi_{\hat{A}} \approx 0 \qquad \text{for } \hat{A} = \lambda^\pm, \xi, \varepsilon \ , \\ &\mathcal{J}^{\hat{z}} \equiv \pi^{\hat{z}} \approx 0 \qquad \text{for } \hat{z} = \nu_\pm, \Lambda_\pm \ , \\ &\varphi^X_\pm \equiv \frac{1}{4}(P \pm X')^2 \pm \frac{i}{2}\psi_\pm\psi'_\pm \approx 0 \ , \\ &\mathcal{J}^X_\pm \equiv \psi_\pm(P \pm X') \approx 0 \ , \end{aligned} \tag{2.5}$$

where $\varphi^X_\pm$ and $\mathcal{J}^X_\pm$ are the super-Virasoro constraints, and satisfy the classical super-Virasoro algebra under the equal-τ super-Poisson brackets [26]

$$\{\varphi^X_\pm(\sigma), \varphi^X_\pm(\sigma')\} = \pm[\varphi^X(\sigma) + \varphi^X(\sigma')]\partial_\sigma\delta(\sigma - \sigma') \ ,$$

$$\{\mathcal{J}^X_\pm(\sigma), \varphi^X_\pm(\sigma')\} = \pm\frac{3}{2}\mathcal{J}^X_\pm(\sigma)\partial_\sigma\delta(\sigma - \sigma') \pm \partial_\sigma\mathcal{J}^X_\pm\delta(\sigma - \sigma') \ ,$$

$$\{\mathcal{J}^X_\pm(\sigma), \mathcal{J}^X_\pm(\sigma')\} = -4i\varphi^X_\pm(\sigma)\delta(\sigma - \sigma') \ ;$$

all other super-Poisson brackets vanish. (2.6)

To set up the BFV-BRST formalism [26], we introduce the EPS by adding to the classical phase space the ghost and auxiliary field sector for each constraint as

$$\begin{aligned} &\varphi_{\rm A} : (\mathcal{C}^{\rm A}, \overline{\mathcal{P}}_{\rm A}), \quad (\mathcal{P}^{\rm A}, \overline{\mathcal{C}}_{\rm A}), \quad (N^{\rm A}, B_{\rm A}) \ , \\ &\mathcal{J}^z : (\gamma^z, \overline{\beta}_z), \quad (\beta^z, \overline{\gamma}_z), \quad (M^z, A_z) \ , \end{aligned} \tag{2.7}$$

where $A = \lambda^\pm, \xi, \varepsilon, X\pm$ and $z = \nu_\pm, \Lambda_\pm, X\pm$, respectively, label the bosonic and fermionic constraints given in (2.5). The classical BRST charge can then be constructed directly from (2.5) and (2.6) without recourse to gauge fixing as

$$\begin{aligned} Q = \int d\sigma \Big[& \mathcal{C}^{\rm A}\varphi_{\rm A} + \gamma^z\mathcal{J}_z + \mathcal{P}^{\rm A}B_{\rm A} + \beta^z A_z \\ & + \mathcal{C}^+(\overline{\mathcal{P}}_+\mathcal{C}^{+\prime} + \overline{\beta}_+\gamma^{+\prime}) + \gamma^+\left(2i\overline{\mathcal{P}}_+\gamma^+ - \frac{1}{2}\overline{\beta}_+\mathcal{C}^{+\prime}\right) \\ & - \mathcal{C}^-(\overline{\mathcal{P}}_-\mathcal{C}^{-\prime} + \overline{\beta}_-\gamma^{-\prime}) \\ & + \gamma^-\left(2i\overline{\mathcal{P}}_-\gamma^- + \frac{1}{2}\overline{\beta}_-\mathcal{C}^{-\prime}\right) \Big] \ , \end{aligned} \tag{2.8}$$

where A and z run over all constraint labels. The ghost trilinear terms are determined so that Q satisfies the nilpotency condition under the super-Poisson brackets as

$$\{Q \ , \ Q\} = 0 \ . \tag{2.9}$$

The dynamics are controlled by the BRST-invariant total Hamiltonian H_T, which consists of the canonical Hamiltonian and the gauge-fixing term. In the present case the canonical part vanishes identically, and H_T is given by

$$H_T = \{Q, \Psi\}, \tag{2.10}$$

where Ψ is a gauge fermion. We shall use the standard form of Ψ given by

$$\Psi = \int d\sigma[\ \overline{\mathcal{C}}_{\rm A}\chi^{\rm A} + \overline{\gamma}^z\chi_z + \overline{\mathcal{P}}_{\rm A}N^{\rm A} + \overline{\beta}^z M_z\ ] \ , \tag{2.11}$$

where $\chi^{\rm A}$ and χ_z denote the gauge conditions imposed on dynamical variables. The BRST invariance of H_T can be stated as

$$\{Q, H_T\} = 0 \ , \tag{2.12}$$

which is automatically satisfied by (2.9).

So far our arguments are restricted within classical theory. In quantum theory, the operators Q and H_T must be suitably regularized, and the quantum version of the BRST invariance (2.9) and (2.12) may fail to be valid due to anomalies. Assuming that the anomalous commutators[2] can be expanded in $\hbar$ as

$$\begin{aligned} &[Q, Q] \equiv i\hbar^2\Omega + O(\hbar^3) \ , \\ &[Q, H_T] \equiv \frac{i}{2}\hbar^2\Gamma + O(\hbar^3), \end{aligned} \tag{2.13}$$

we obtain the algebraic consistency condition for Ω in the lowest order of $\hbar$ as

$$\delta\Omega = 0 \ , \tag{2.14}$$

where δ is the classical BRST transformation given by the super-Poisson bracket $\delta F = -\{Q, F\}$ for any F. On the other hand, Γ can be related to Ω by

$$\Gamma = \{\Omega, \Psi\} \ . \tag{2.15}$$

Hence it is sufficient to cancel the anomaly in Q^2 to retain BRST invariance.

The consistency condition (2.14) has been solved in the EPS [25], and the nontrivial (matter independent) solution is found to be

$$\begin{aligned} \Omega = K \int d\sigma[& (\mathcal{C}^+\partial^3_\sigma\mathcal{C}^+ - 8i\gamma^+\partial^2_\sigma\gamma^+) \\ & -(\mathcal{C}^-\partial^3_\sigma\mathcal{C}^- + 8i\gamma^-\partial^2_\sigma\gamma^-)] \ . \end{aligned} \tag{2.16}$$

The cohomology class to which this solution belongs is uniquely fixed and independent of the choices of regularizations and gauge fixing.

The overall coefficient K, however, remains undetermined in this algebraic method. To fix K, we must define operator products by some ordering prescription, and then examine the nilpotency of the BRST charge. For the purpose of defining operator ordering, we decompose operators into two parts by

[2] A careful analysis of the equal-time commutators appearing in the BRST algebra is given in [30].

$$A^{(\pm)}(\sigma) = \int d\sigma \delta^{(\mp)}(\sigma - \sigma') A(\sigma')$$

$$\text{for } A = P + X', \psi_+, \mathcal{C}^+, \overline{\mathcal{P}}_+,$$

$$A^{(\pm)}(\sigma) = \int d\sigma \delta^{(\pm)}(\sigma - \sigma') A(\sigma')$$

$$\text{for } A = P - X', \psi_-, \mathcal{C}^-, \overline{\mathcal{P}}_-, \quad (2.17)$$

with $\delta^{(\pm)}(\sigma) = \frac{1}{2\pi} \frac{\pm i}{\sigma \pm \epsilon}$. The $A^{(+)}$ and $A^{(-)}$ thus defined reduce, respectively, to positive- and negative-frequency part in the superorthonormal gauge. By putting $A^{(+)}$'s to the right of $A^{(-)}$'s, we define operator ordering. We thus obtain

$$K = \frac{10 - D}{16\pi} \quad (2.18)$$

for the anomaly coefficient. This coincides with the result of superconformal gauge fixing [23]. If we change the ordering prescription, the value of K also changes. Which ordering should be employed depends on the gauge chosen. Because of this, the operator ordering introduced above should be understood as temporal. We will come back to this point later.

The Q^2 anomaly is a direct consequence of the super-Virasoro anomaly of the generalized super-Virasoro constraints defined by

$$\Phi_\pm \equiv \varphi^X_\pm \pm 2\overline{\mathcal{P}}_\pm \mathcal{C}^{\pm\prime} \pm \overline{\mathcal{P}}'_\pm \mathcal{C}^\pm \pm \frac{3}{2}\overline{\beta}_\pm \gamma^{\pm\prime} \pm \frac{1}{2}\overline{\beta}'_\pm \gamma^\pm ,$$

$$I_\pm \equiv \mathcal{J}^X_\pm \mp \frac{3}{2}\overline{\beta}_\pm \mathcal{C}^{\pm\prime} \mp \overline{\beta}'_\pm \mathcal{C}^{\pm\prime} + 4i\overline{\mathcal{P}}_\pm \gamma^\pm , \quad (2.19)$$

where operator ordering is implicitly assumed. Suppressing $\hbar$ henceforth, we can easily show that these satisfy

$$[\Phi_\pm(\sigma), \Phi_\pm(\sigma')] = \pm i[\Phi(\sigma) + \Phi(\sigma')]\partial_\sigma \delta(\sigma - \sigma') \pm iK\partial^3_\sigma \delta(\sigma - \sigma') ,$$

$$[I_\pm(\sigma), \Phi_\pm(\sigma')]$$

$$= \pm i\frac{3}{2} I_\pm(\sigma)\partial_\sigma \delta(\sigma - \sigma') \pm iI'_\pm(\sigma) \cdot \delta(\sigma - \sigma'),$$

$$[I_\pm(\sigma), I_\pm(\sigma')] = 4\Phi_\pm(\sigma)\delta(\sigma - \sigma') + 8K\partial^2_\sigma \delta(\sigma - \sigma');$$

all other supercommutators vanish. (2.20)

One finds that, due to the appearance of the anomalous Schwinger term, the super-Virasoro constraints become second-class ones.

The result (2.16) or (2.20) with K given by (2.18) does not rely on $\hbar$ expansion and is exact as far as the Q^2 anomaly is concerned. Strictly speaking, the BRST anomaly must be canceled in order for the higher order corrections to be meaningful. The critical string with $D = 10$ is such a case, where the super-Liouville mode of the 2D SUGRA multiplet is decoupled from the theory. In noncritical strings, however, we must take account of the super-Liouville mode as a dynamical variable, which also contributes to the BRST anomaly.

Instead of taking account of the super-Liouville mode which is expected to become dynamical in quantum theory, we modify the theory to recover all the classical local symmetries violated by anomalies. This can be carried out, without affecting the physical content of the original theory, by introducing extra degrees of freedom which can be formally gauged away by the recovered local symmetry.

Following the general idea of BF [27], we introduce a canonical pair of bosonic fields (θ, π_θ) and a Majorana field $\zeta_\pm$, which we will refer to as BF fields henceforth. They are assumed to satisfy the same type of canonical supercommutation relations as string variables. We then modify the constraints (2.19) by adding to $\Phi_\pm$ and $I_\pm$ an appropriate terms containing BF fields to cancel the super-Virasoro anomaly in (2.20). Let us denote the modified super-Virasoro operators by $\tilde{\Phi}_\pm$ and $\tilde{I}_\pm$; then they are given by

$$\tilde{\Phi}_\pm \equiv \Phi_\pm + \frac{1}{4}\Theta^2_\pm - \gamma\Theta'_\pm \pm \frac{i}{2}\zeta_\pm \zeta'_\pm + \gamma^2\mu^2 e^{\frac{\theta}{\gamma}} \mp \frac{i\mu}{2} e^{\frac{\theta}{2\gamma}} \zeta_\mp \zeta_\pm ,$$

$$\tilde{I}_\pm \equiv I_\pm \pm \zeta_\pm \Theta_\pm \mp 4\gamma\zeta'_\pm \mp 2\gamma\mu e^{\frac{\theta}{2\gamma}} \zeta_\mp , \quad (2.21)$$

where we have defined $\Theta_\pm \equiv \theta' \pm \pi_\theta$. γ is a free parameter to be fixed to cancel the super-Virasoro anomaly. In fact, the super-Virasoro constraints of the BF sector satisfy the super-Virasoro algebra with the anomaly coefficient

$$\kappa \equiv 2\gamma^2 \quad (2.22)$$

under classical super-Poisson brackets. Since the BF fields also contribute to the super-Virasoro anomaly as a single superconformal matter multiplet in quantum theory, the BRST anomaly can be canceled if κ satisfies

$$\frac{D - 10 + 1}{16\pi} + \kappa = 0 . \quad (2.23)$$

For $D \leq 25$ we find the parameter γ to be real, the only case which we will consider in the following sections. In order to retain the reality of γ we must choose the BF fields to generate negative metric states for $D > 25$.

In (2.21), the term containing the mass parameter μ^2 is not necessary for the purpose of canceling the BRST anomaly but it turns out to be related to the cosmological terms in the covariant effective action as we shall see in the next section. In quantum theory, the exponential operators $\exp(\theta/2\gamma)$ will be modified by the gravitational dressing effect [2, 6, 7]. The full quantum mechanical treatment of them will be discussed in Sec. IV.

As we mentioned above, a different choice of operator ordering gives rise to a different anomaly coefficient. For the superconformal gauge fixing, the ordering prescription introduced above can be used and (2.23) turns out to be correct. In the light-cone gauge, we will adopt different ordering and arrive at different conditions for κ as we will see in Sec. V.

III. EFFECTIVE ACTION AND GEOMETRIZATION

In the previous section we introduced the BF fields, and modified the super-Virasoro constraints so as to cancel the BRST anomaly. In this section we will apply the BFV algorithm to the gauge symmetrized system and investigate the BRST-invariant effective action.

Let us denote the BRST charge modified by the BF fields by $\tilde{Q}$, then it generates the BRST transformation of any variable F by

$$\delta F = i[\tilde{Q}, F] \ . \tag{3.1}$$

We thus obtain the following set of BRST transformations in the EPS:

$$\begin{aligned}
\delta X &= \frac{1}{2}\{(\mathcal{C}^+ - \mathcal{C}^-)X' + (\mathcal{C}^+ + \mathcal{C}^-)P\} + \gamma^+\psi_+ \\
&\quad +\gamma^-\psi_-, \\
\delta P &= \left(\frac{1}{2}\{(\mathcal{C}^+ - \mathcal{C}^-)P + (\mathcal{C}^+ + \mathcal{C}^-)X'\} \right. \\
&\quad \left. +\gamma^+\psi_+ - \gamma^-\psi_-\right)', \\
\delta\psi_\pm &= \pm\frac{1}{2}\mathcal{C}^{\pm\prime}\psi_\pm \pm \mathcal{C}^\pm\psi'_\pm + i\gamma^\pm(P \pm X'), \\
\delta\theta &= \frac{1}{2}(\ \mathcal{C}^+\Theta_+ - \mathcal{C}^-\Theta_-\ ) - \gamma(\mathcal{C}^+ - \mathcal{C}^-)' \\
&\quad +\gamma^+\zeta_+ + \gamma^-\zeta_-\ , \\
\delta\pi_\theta &= \left(\ \frac{1}{2}(\mathcal{C}^+\Theta_+ - \mathcal{C}^-\Theta_-) - \gamma(\mathcal{C}^+ + \mathcal{C}^-)' \right. \\
&\quad \left. +\gamma^+\zeta_+ - \gamma^-\zeta_-\right)', \\
\delta\zeta_\pm &= \pm\mathcal{C}^\pm\zeta'_\pm \pm \frac{1}{2}\mathcal{C}^{\pm\prime}\zeta_\pm \pm i\gamma^\pm\Theta_\pm \mp 4i\gamma\gamma^{\pm\prime}, \\
\delta\lambda^\pm &= \mathcal{C}^\pm_\lambda,\quad \delta\xi = \mathcal{C}^\xi, \delta\varepsilon = \mathcal{C}^\varepsilon, \\
\delta\mathcal{C}^\pm_\lambda &= 0,\quad \delta\mathcal{C}^\xi = 0,\quad \delta\mathcal{C}^\varepsilon = 0, \\
\delta\nu_\pm &= -\gamma^\nu_\pm,\quad \delta\Lambda_\pm = -\gamma^\Lambda_\pm,\quad \delta\gamma^\nu_\pm = 0,\quad \delta\gamma^\Lambda_\pm = 0, \\
\delta N^A &= \mathcal{P}^A,\quad \delta M_z = -\beta_z,\quad \delta\mathcal{P}^A = 0,\quad \delta\beta^z = 0\ , \\
\delta\overline{\mathcal{C}}_A &= -B_A,\quad \delta\overline{\gamma}^z = -A^z,\quad \delta B_A = 0,\quad \delta A^z = 0, \\
\delta\overline{\mathcal{P}}_{\hat{A}} &= -\varphi_{\hat{A}},\quad \delta\overline{\beta}^{\hat{z}} = -\mathcal{J}^{\hat{z}},\quad \delta\overline{\mathcal{P}}_\pm = -\tilde{\Phi}_\pm,\quad \delta\overline{\beta}_\pm = -\tilde{I}_\pm, \\
\delta\mathcal{C}^\pm &= \pm\mathcal{C}^\pm\mathcal{C}^{\pm\prime} - 2i(\gamma^\pm)^2,\quad \delta\gamma^\pm = \mp\frac{1}{2}\mathcal{C}^{\pm\prime}\gamma^\pm \pm \mathcal{C}^\pm\gamma^{\pm\prime}\ ,
\end{aligned} \tag{3.2}$$

where the constraint labels $\hat{A}$ and $\hat{z}$ run only through primary constraints, while A and z are taken over all the bosonic and fermionic constraints.

The change in the BRST charge is reflected in the dynamics through the total Hamiltonian

$$\tilde{H}_T = \frac{1}{i}[\tilde{Q}\ ,\ \Psi]\ . \tag{3.3}$$

This is BRST invariant if the $\tilde{Q}^2$ anomaly is absent. To construct the effective action we choose the standard form of gauge fermion (2.11) shifted by $\Psi \to \int d\sigma[\overline{\mathcal{C}}_A\dot{N}^A + \overline{\gamma}^z\dot{M}_z]$. This just cancels the Legendre terms $\int d^2x[\overline{\mathcal{C}}_A\dot{\mathcal{P}}^A + \overline{\gamma}^z\dot{\beta}_z + B_A\dot{N}^A + A^z\dot{M}_z]$ in constructing the effective action. The BRST-invariant effective action can be obtained as

$$S_{\text{eff}} = S_0 + S_{\text{gh}} + S_{\text{gf}}, \tag{3.4}$$

where

$$\begin{aligned}
S_0 = \int d^2x\Bigg[\ & P\dot{X} + \frac{i}{2}(\psi_+\dot{\psi}_+ + \psi_-\dot{\psi}_-) + \pi_\theta\dot{\theta} + \frac{i}{2}(\zeta_+\dot{\zeta}_+ + \zeta_-\dot{\zeta}_-) \\
& +\pi^\lambda_+\dot{\lambda}^+ + \pi^\lambda_-\dot{\lambda}^- + \pi_\xi\dot{\xi} + \pi_\varepsilon\dot{\varepsilon} + \pi^+_\nu\dot{\nu}_+ + \pi^-_\nu\dot{\nu}_- + \pi^-_\Lambda\dot{\Lambda}_+ + \pi^-_\Lambda\dot{\Lambda}_- \\
& -N^{\hat{A}}\varphi_{\hat{A}} - N^+\tilde{\varphi}_+ - N^-\tilde{\varphi}_- + M_{\hat{z}}\mathcal{J}^{\hat{z}} + M^+\tilde{\mathcal{J}}_+ + M^-\tilde{\mathcal{J}}_-\Bigg]\ ;
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
S_{\text{gh}} = \int d^2x\Bigg[& \overline{\mathcal{P}}_A\dot{\mathcal{C}}^A + \overline{\beta}^z\dot{\gamma}_z - \overline{\mathcal{C}}_A\delta\chi^A + \overline{\gamma}^z\delta\chi_z - \overline{\mathcal{P}}_A\mathcal{P}^A - \overline{\beta}^z\beta_z \\
& -N_+\left(2\overline{\mathcal{P}}_+\mathcal{C}^{+\prime} + \overline{\mathcal{P}}'_+\mathcal{C}^+ + \frac{3}{2}\overline{\beta}_+\gamma^{+\prime} + \frac{1}{2}\overline{\beta}'_+\gamma^+\right) + N_-\left(2\overline{\mathcal{P}}_-\mathcal{C}^{-\prime} + \overline{\mathcal{P}}'_-\mathcal{C}^- + \frac{3}{2}\overline{\beta}_-\gamma^{-\prime} + \frac{1}{2}\overline{\beta}'_-\gamma^-\right) \\
& -M^+\left(\frac{3}{2}\overline{\beta}_+\mathcal{C}^{+\prime} + \overline{\beta}'_+\mathcal{C}^+ - 4i\overline{\mathcal{P}}_+\gamma^+\right) + M^-\left(\frac{3}{2}\overline{\beta}_-\mathcal{C}^{-\prime} + \overline{\beta}'_-\mathcal{C}^- + 4i\overline{\mathcal{P}}_-\gamma^-\right)\Bigg]\ ,
\end{aligned} \tag{3.6}$$

$$S_{\text{gf}} = \int d^2x[-B_A\chi^A - A^z\chi_z]\ . \tag{3.7}$$

In (3.5) $\tilde{\varphi}_\pm$ and $\tilde{\mathcal{J}}_\pm$, respectively, are obtained from the super-Virasoro constraints $\tilde{\Phi}_\pm$ and $\tilde{\mathcal{J}}_\pm$ given by (2.21) with all the ghost contributions removed. The χ^A and χ_z stand for the gauge fixing conditions. We have used the BRST transformations (3.2) in deriving these actions.

For a wide class of gauge conditions the effective action provides us with a starting point to analyze the quantum theory of a fermionic string as 2D SUGRA. It is, however, written in terms of the EPS variables, which have no direct geometrical interpretation unless we explicitly specify the gauge conditions. In order to see the physical significance of the effective action, we relate the EPS variables to the configuration space variables to geometrize the effective action. The geometrization of the EPS variables has been developed in Refs. [21, 25]. No essential change will appear in relating the ghost variables in the EPS to those of configuration space in the presence of the BF variables.

Since the auxiliary fields $N^\pm$ and $M^\pm$, respectively, play the roles of $\lambda^\pm$ and $\pm i\nu_\mp$ in (3.5), we identify them by imposing the gauge conditions

$$\chi_\lambda^\pm = \lambda^\pm - N^\pm, \qquad \chi_\pm^\nu = \nu_\pm \mp iM^\mp . \tag{3.8}$$

As for the rest of the gauge conditions, we assume that they and their BRST transforms are independent of P, $\psi_\pm$, π_θ, $\zeta_\pm$, $\pi_{\hat{A}}$, $\pi^{\hat{z}}$, $\overline{\mathcal{P}}_A$, and $\overline{\beta}^z$. They are otherwise arbitrary. This is sufficient for the purpose of geometrization.

Due to the assumptions on the gauge conditions, we can derive equations of motion for P, $\psi_\pm$, π_θ, $\zeta_\pm$, $\pi_{\hat{A}}$, $\pi^{\hat{z}}$, $\overline{\mathcal{P}}_A$, and $\overline{\beta}^z$ by taking the variation of (3.4) with respect to these variables as

$$\begin{aligned}
&\dot{X} - \frac{1}{2}\{(N^+ + N^-)P + (N^+ - N^-)X'\} \\
&\qquad\qquad + M^+\psi_+ + M^-\psi_- = 0, \\
&\dot{\theta} - \frac{1}{2}\{(N^+ + N^-)\pi_\theta + (N^+ - N^-)\theta'\} \\
&\qquad\qquad -\gamma(N^+ - N^-)' + M^+\zeta_+ + M^-\zeta_- = 0, \\
&\dot{\psi}_\pm \mp N^\pm\psi_\pm' \mp \frac{1}{2}N^{\pm\prime}\psi_\pm + iM^\pm(P \pm X') = 0, \\
&\dot{\zeta}_\pm \mp N^\pm\zeta_\pm' \mp \frac{1}{2}N^{\pm\prime}\zeta_\pm + iM^\pm(\pi_\theta \pm \theta') \mp 4i\gamma M^{\pm\prime} = 0, \\
&\dot{\lambda}^\pm = N_\lambda^\pm, \;\; \dot{\xi} = N^\xi, \;\; \dot{\varepsilon} = N^\varepsilon, \;\; \dot{\nu}_\pm = M_\pm^\nu, \;\; \dot{\Lambda}_\pm = M_\pm^\Lambda, \\
&\dot{\mathcal{C}}_\lambda^\pm = \mathcal{P}_\lambda^\pm, \;\; \dot{\mathcal{C}}^\xi = \mathcal{P}^\xi, \;\; \dot{\mathcal{C}}^\varepsilon = \mathcal{P}^\varepsilon, \;\; \dot{\gamma}_\pm^\nu = \beta_\pm^\nu, \;\; \dot{\gamma}_\pm^\Lambda = \beta_\pm^\Lambda, \\
&\mathcal{P}^\pm = \dot{\mathcal{C}}^\pm \pm \mathcal{C}^\pm N^{\pm\prime} \mp \mathcal{C}^{\pm\prime}N^\pm - 4i\gamma^\pm M^\pm, \\
&\beta^\pm = \dot{\gamma}^\pm \pm \frac{1}{2}\gamma^\pm N^{\pm\prime} \mp \gamma^{\pm\prime}N^\pm \mp \mathcal{C}^\pm M^{\pm\prime} \pm \frac{1}{2}\mathcal{C}^{\pm\prime}M^\pm .
\end{aligned} \tag{3.9}$$

We require that, if we use the equations of motion (3.9) to eliminate P, $\mathcal{P}^\pm$, and $\beta^\pm$, the BRST transformations (3.2) in the EPS reduce to those in the configuration space given in the Appendix. For instance, the covariant reparametrization ghosts C^α and the supersymmetry ghosts ω can be identified by comparing the BRST transformations of X in the EPS and in the configuration space in (A2). They are given by

$$\begin{aligned}
&C^0 = \frac{\mathcal{C}^+ + \mathcal{C}^-}{N^+ + N^-}, \qquad C^1 = \frac{N^-\mathcal{C}^+ - N^+\mathcal{C}^-}{N^+ + N^-}, \\
&\omega = \begin{pmatrix} e^{\frac{1}{2}(\varepsilon + \frac{\xi}{2})}\omega_- \\ e^{\frac{1}{2}(-\varepsilon + \frac{\xi}{2})}\omega_+ \end{pmatrix} \\
&\qquad \text{with } \omega_\pm = \mp i\left(\gamma^\mp + \frac{\mathcal{C}^+ + \mathcal{C}^-}{N^+ + N^-}M^\mp\right) .
\end{aligned} \tag{3.10}$$

The Weyl ghost C_W and the local Lorentz ghost C_L can be found from the BRST transformation properties of ξ and ε to be

$$\begin{aligned}
C_W &= \mathcal{C}^\xi - V_\mathcal{C}^+ + V_\mathcal{C}^- , \\
C_L &= \mathcal{C}^\varepsilon - Z_\mathcal{C}^+ + Z_\mathcal{C}^- ,
\end{aligned} \tag{3.11}$$

where $V_\mathcal{C}^\pm$ and $Z_\mathcal{C}^\pm$ are defined by

$$\begin{aligned}
V_\mathcal{C}^\pm &= \frac{1}{2}G_\pm\mathcal{C}^\pm \pm \Lambda_\pm\gamma^\pm + \mathcal{C}^{\pm\prime} , \\
Z_\mathcal{C}^\pm &= \pm\frac{1}{2}\mathcal{C}^\pm L_\pm + \frac{1}{2}\gamma^\pm\Lambda_\pm \pm \frac{1}{2}\mathcal{C}^{\pm\prime}
\end{aligned} \tag{3.12}$$

with

$$\begin{aligned}
G_\pm &= \frac{2}{N^+ + N^-}[\pm N^\xi + N^\mp\xi' \mp (N^+ - N^-)' \\
&\qquad \mp(\Lambda_+M^+ + \Lambda_-M^-)] , \\
L_\pm &= \frac{2}{N^+ + N^-}[\pm N^\varepsilon + N^\mp\varepsilon' \mp (N^+ + N^-)' \\
&\qquad \mp(\Lambda_+M^+ - \Lambda_-M^-)] .
\end{aligned} \tag{3.13}$$

We finally obtain the super-Weyl ghost from the BRST transformations of $\Lambda_\pm$ as

$$\begin{aligned}
&\eta_W = \begin{pmatrix} e^{\frac{1}{2}(\varepsilon - \frac{\xi}{2})}\eta_{W-} \\ e^{-\frac{1}{2}(\varepsilon + \frac{\xi}{2})}\eta_{W+} \end{pmatrix} \\
&\qquad \text{with } \eta_{W\pm} = W_\mathcal{C}^\pm + \frac{\mathcal{C}^+ + \mathcal{C}^-}{N^+ + N^-}W_N^\pm ,
\end{aligned} \tag{3.14}$$

where $W_N^\pm$ and $W_\mathcal{C}^\pm$ are defined by

$$\begin{aligned}
W_N^\pm &= M_\pm^\Lambda \mp \Lambda_\pm' N^\pm \mp \frac{1}{2}\Lambda_\pm N^{\pm\prime} \\
&\qquad \pm i(G_\pm M^\pm + 4M^{\pm\prime}) , \\
W_\mathcal{C}^\pm &= \gamma_\pm^\Lambda \mp \Lambda_\pm'\mathcal{C}^\pm \mp \frac{1}{2}\Lambda_\pm\mathcal{C}^{\pm\prime} \\
&\qquad \pm i(G_\pm\gamma^\pm + 4\gamma^{\pm\prime}) .
\end{aligned} \tag{3.15}$$

The equations (3.10), (3.11), and (3.14) fix the ghost relations between the BFV basis and the covariant one.

Except for the BF fields, which we will shortly discuss, it is straightforward to convert the BRST transformations of the remaining variables in terms of configuration space variables. Furthermore, the BRST transformations of the covariant ghost variables can be easily obtained from (3.2) and (3.9). We shall introduce here auxiliary fields, F_X^μ, F_G, F_L, and F_W for the supermultiplets corresponding to the string variables, the gravity, the Liouville fields and the super-Weyl ghost, respectively. These are needed to close the algebra off-shell. The BRST transformations constructed from (3.2) by using (3.9) and the ghost relations given above coincide with those of the Appendix up to the terms containing auxiliary fields, being only on-shell-nilpotent. The original BRST transformations (3.2) in the EPS satisfy the off-shell nilpotency by construction. There, the number of bosonic fields balances that of the fermionic fields. Eliminating the momentum variables, however, destroys the balance, and the off-shell nilpotency is no longer satisfied. Incidentally, the off-shell nilpotency of these two BRST transformations is realized in a quite different manner. In the configuration space, the covariance is manifestly maintained and the nilpotency is consistent with the covariance. On the other hand, the covariance is only manifest on the mass shell in the EPS and the off-shell nilpotency is realized by sacrificing the manifest covariance. Therefore, when passing to the configuration space, we include auxiliary fields to retain the off-shell nilpotency needed to investigate the theory in arbitrary gauges.

With these preparations, we turn to the geometrization of the BF fields. In terms of covariant ghost variables the BRST transformations of the BF fields are given by

$$\begin{aligned}\delta\theta &= C^\alpha\partial_\alpha\theta - i(\omega_-\zeta_+ - \omega_+\zeta_-) \\ &\quad -\gamma[2C^{1\prime} + C^{0\prime}(\lambda^+ - \lambda^-)]\,, \\ \delta\zeta_\pm &= C^\alpha\partial_\alpha\zeta_\pm + \frac{1}{2}(C^{1\prime} \pm \lambda^\pm C^{0\prime})\zeta_\pm - 4\gamma C^{0\prime}\nu_\mp - 4\gamma\omega'_\mp \\ &\quad \pm\frac{2\omega_\mp}{\lambda^+ + \lambda^-}[\dot\theta \pm \lambda^\mp\theta' + \gamma(\lambda^+ - \lambda^-)' \\ &\quad +i(\nu_-\zeta_+ - \nu_+\zeta_-)]\,. \end{aligned} \tag{3.16}$$

The presence of the terms proportional to γ implies that θ and $\zeta_\pm$ have no simple transformation properties as scalar and spinor components of a scalar supermultiplet for the string variables. Rather, the BF fields possess transformation properties similar to those of ξ and $\Lambda_\pm$ as is easily seen from (A2). In fact we can construct covariants by taking the combinations

$$\begin{aligned}\phi &= \xi - \frac{1}{\gamma}\theta, \\ \eta &= \begin{pmatrix} e^{\frac{1}{2}(\epsilon-\frac{\xi}{2})}\eta_- \\ e^{-\frac{1}{2}(\epsilon+\frac{\xi}{2})}\eta_+ \end{pmatrix} \quad \text{with} \quad \eta_\pm = \Lambda_\pm - \frac{1}{\gamma}\zeta_\pm\,. \end{aligned} \tag{3.17}$$

The covariant BRST transformations of these variables coincide with those of the super-Liouville multiplet given in the Appendix, where we have included the auxiliary fields F_L as mentioned above. We see from this that (ϕ,η,F_L) can be regarded as a super-Liouville multiplet. They not only transform as a scalar multiplet under reparametrizations and supersymmetry but also change under super-Weyl transformations.

The master action (3.4) contains many nonpropagating degrees, i.e., P, π_θ, $\mathcal{P}^A$, $\overline{\mathcal{P}}_A$, β_z, $\overline{\beta}^z$, N^A, and M_z, which can be eliminated by virtue of the equations of motion (3.9). After eliminating these variables from the master action and rewriting it in terms of the covariant variables defined above, we arrive at the gauge-fixed covariant action

$$S_{\rm eff} = S_X + S_\phi + S_g + S_{\rm cosm} + S_{\rm aux} + S_{\rm gh} + S_{\rm gf} \tag{3.18}$$

with

$$\begin{aligned}S_\phi &= \frac{\kappa}{2}S_L^{\phi,\eta}, \\ S_g &= -\frac{\kappa}{2}\left[S_L^{\xi,\Lambda} + \int d^2x e g^{00}\left\{\left(\frac{g_{01}}{g_{11}}\right)'\right\}^2\right], \\ S_{\rm cosm} &= -\frac{\kappa}{2}\mu\int d^2x e e^{-\frac{\phi}{2}}\Big[-2(F_G - F_L) + \frac{1}{2}\overline{\eta}\eta \\ &\quad -2i\overline{\chi}_\alpha\rho^\alpha\eta + 4\epsilon^{\alpha\beta}\overline{\chi}_\alpha\rho_5\chi_\beta\Big], \\ S_{\rm aux} &= \frac{1}{2}\int d^2x e F_X^2 + \frac{\kappa}{4}\int d^2x e(F_G - F_L)^2, \\ S_{\rm gh} &= \int d^2x[-\overline{C}_+\delta\chi_\varphi^+ - \overline{C}_-\delta\chi_\varphi^- - \overline{C}_\xi\delta\chi^\xi - \overline{C}_\epsilon\delta\chi^\epsilon \\ &\quad -\overline{C}_f\delta\chi^f + \overline{\gamma}_+\delta\chi_{\mathcal{J}}^+ + \overline{\gamma}_-\delta\chi_{\mathcal{J}}^- + \overline{\gamma}_\Lambda^+\delta\chi_+^\Lambda + \overline{\gamma}_\Lambda^-\delta\chi_-^\Lambda]\,, \\ S_{\rm gf} &= \int d^2x[-B_+\chi_\varphi^+ - B_-\chi_\varphi^- - B_\xi\chi^\xi - B_\epsilon\chi^\epsilon - B_f\chi^f \\ &\quad -A_+\chi_{\mathcal{J}}^+ - A_-\chi_{\mathcal{J}}^- - A_\Lambda^+\chi_+^\Lambda - A_\Lambda^-\chi_-^\Lambda], \end{aligned} \tag{3.19}$$

where $S_L^{\phi,\eta}$ is the supersymmetric Liouville action given by

$$\begin{aligned}S_L^{\phi,\eta} = \int d^2x\, e\Big[&-\frac{1}{2}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - i\overline{\eta}\rho^\alpha\nabla_\alpha\eta) \\ &-\overline{\chi}_\alpha\rho^\beta\rho^\alpha\eta\partial_\beta\phi - \frac{1}{4}\overline{\eta}\eta\overline{\chi}_\alpha\rho^\beta\rho^\alpha\chi_\beta \\ &+R\phi + 4i\epsilon^{\alpha\beta}\overline{\chi}_\alpha\rho_5\rho^\gamma\chi_\gamma\partial_\beta\phi + 4\epsilon^{\alpha\beta}\overline{\chi}_\alpha\rho_5\nabla_\beta\eta\Big] \end{aligned} \tag{3.20}$$

with R being the scalar curvature of the metric $g_{\alpha\beta}$. The $S_L^{\xi,\Lambda}$ can be obtained from (3.20) by the replacement $\phi\to\xi$ and $\eta_\pm\to\Lambda_\pm$. This is a local functional of 2D SUGRA fields and can be considered a super-Liouville action with

$$\xi = \ln g_{11}, \qquad \Lambda = \begin{pmatrix} 4e^{\epsilon-\frac{\xi}{2}}\chi_{1+} \\ 4e^{-\epsilon-\frac{\xi}{2}}\chi_{1-} \end{pmatrix} \tag{3.21}$$

as the super-Liouville fields.

In (3.18) we have also included the auxiliary fields to retain the BRST invariance in the configuration space given in the Appendix. In this connection, we notice here the appearance of a new local symmetry associated with $F_G(x)\to F_G(x)+\lambda(x)$ and $F_L(x)\to F_L(x)+\lambda(x)$ by an arbitrary function $\lambda(x)$. To gauge-fix the local symmetry, we have added a new gauge condition χ^f and an antighost $\overline{C}_f$ to (3.18). The F_W required by the off-shell nilpotency of the BRST transformation in the super-Weyl ghost sector can be considered as the ghost corresponding to this symmetry, as can be seen from the BRST transformations of F_G and F_L given in (A1).

It is very interesting to see the properties of (3.18) under the classical local symmetries of (2.1). As was discussed in Ref. [25], S_g is not invariant under the classical symmetries (except for the local Lorentz invariance) and produces both super-Virasoro and super-Weyl anomalies even in the classical theory. The quantization of the string variables breaks the reparametrization invariance and the local supersymmetry, and leads to the super-Virasoro anomaly. This anomaly together with the contributions from other sectors including supergravity itself is canceled by the classical super-Virasoro anomaly produced by S_g. The super-Weyl anomaly of S_g is canceled by that of S_ϕ. This is rather obvious since the combination $S_\phi + S_g$ depends on ξ, ϕ, $\Lambda_\pm$, and $\eta_\pm$ only through the BF fields which are invariant under super-Weyl transformations. Furthermore, the cosmological term can be obtained by eliminating the auxiliary by the equation of motion $F_G - F_L = -2\mu e^{-\frac{\phi}{2}}$ as

$$-\frac{\kappa}{2}\int d^2xe\Big[2\mu^2e^{-\phi} + \frac{\mu}{2}\overline{\eta}\eta e^{-\frac{\phi}{2}} -2i\mu\overline{\chi}_\alpha\rho^\alpha\eta e^{-\frac{\phi}{2}} + 4\mu\epsilon^{\alpha\beta}\overline{\chi}_\alpha\rho_5\chi_\beta e^{-\frac{\phi}{2}}\Big] . \quad (3.22)$$

It is easy to show that this is invariant under super-Weyl transformations.

We now come back to the invariance of the effective action (3.18) under the BRST transformations given in the Appendix. From the very construction of our symmetrization procedure, it is invariant under the transformations without the auxiliary fields. When they are included, the BRST transformations of Majorana fields are modified by

$$\delta_F\psi = \omega F_X , \qquad \delta_F\chi_\alpha = -\frac{i}{4}\rho_\alpha\omega F_G , \qquad \delta_F\eta = \omega F_L . \quad (3.23)$$

It is well known that the noninvariance of S_X is canceled by the variation of the F_X^2 term in $S_{\rm aux}$. Noting that the modifications due to F_G can be regarded as a fermionic transformation, we easily find

$$\delta_F S_\phi = \frac{\kappa}{2}\int d^2xe[i\overline{\omega}\rho^\alpha\{\nabla_\alpha\eta + i\rho^\beta(\partial_\beta\phi - \overline{\chi}_\beta\eta)\chi_\alpha\} \times(F_L - F_G) - 4\epsilon^{\alpha\beta}\overline{\omega}\rho_5\nabla_\alpha\chi_\beta F_L] ,$$

$$\delta_F S_g = 2\kappa\int d^2xe\epsilon^{\alpha\beta}\overline{\omega}\rho_5\nabla_\alpha\chi_\beta F_G . \quad (3.24)$$

These can be shown to cancel exactly the BRST transformation of $S_{\rm aux}$. The fact that only the difference $F_G - F_L$ appears in the action is related to the extra symmetry mentioned above. The action (2.1) supplemented by the auxiliary field F_X by itself is classically invariant under the full BRST transformations containing F_G. The auxiliary field F_G, however, enters into the action only through quantum effects, i.e., the super-Weyl anomaly [31].

We emphasize that, though the effective action (3.18) has been derived by assuming the restricted class of gauge conditions stated below (3.8), it is considered to be valid for arbitrary gauge conditions. This is because the effective action is invariant under the BRST transformations maintaining the off-shell nilpotency. If we restrict ourselves to the gauge conditions satisfying the assumption stated below (3.8), we need not introduce the auxiliary fields. In such a case, the BRST transformations are only on-shell nilpotent, but the equations of motion can be consistent with the BRST symmetry. Actually this happens in the superconformal gauge. The restriction on the gauge conditions, however, is too strong to allow interesting gauges such as the light-cone gauge. Let us discuss now these specific gauge fixings.

IV. SUPERCONFORMAL GAUGE FIXING

In the previous section we formulated the BRST-invariant effective action. So far our argument does not rely on particular gauge conditions. The effective action (3.4) or its covariantized version (3.18) can be applicable to any gauge fixing. It is, however, instructive to describe explicit calculations and illustrate some issues which arise in quantizing the effective action. In this section we will discuss superconformal gauge fixing.

Since the effective action (3.18) possesses all the classical local symmetries, we can fix the zweibeins and the gravitinos to arbitrary background fields. The gauge-fixed action can be easily constructed in this general case. In particular we can choose the background to be flat Minkowskian superspace. Here we shall discuss this simplest case, i.e., $e_\alpha^a = \delta_\alpha^a$, $\chi_\alpha = 0$. In terms of (2.2) the flat superorthonormal gauge can be implemented by the following set of gauge conditions:

$$\begin{aligned} &\chi_\varphi^\pm = N^\pm - 1, \qquad \chi^\varepsilon = \varepsilon, \qquad \chi^\xi = \xi, \\ &\chi_{\mathcal{J}}^\pm = \mp iM^\pm, \qquad \chi_\pm^\Lambda = \Lambda_\pm , \qquad \chi^f = f_G , \end{aligned} \quad (4.1)$$

where $\chi_\varphi^\pm$ and $\chi_{\mathcal{J}}^\pm$ denote the gauge conditions for the super-Virasoro constraints. The last gauge condition in (4.1) is to fix the local symmetry associated with the presence of the auxiliary fields discussed in the previous section. In this gauge the Lorentz ghost, the super-Weyl ghosts can be related to the super-reparametrization ghosts as

$$\begin{aligned} &C_L = -\frac{1}{2}\epsilon^{\alpha\beta}\partial_\alpha C_\beta , \qquad C_W = -\partial_\alpha C^\alpha , \\ &F_W = 0 , \qquad \eta_W = -2i\rho^\alpha\partial_\alpha\omega , \end{aligned} \quad (4.2)$$

and their antighosts vanish. Integrating out the multipliers B_A, A^z, the auxiliary fields $F_{X,L}$, and eliminating nonpropagating variables by the equations of motion, we can reduce the effective action (3.18) to the following expression:[3]

[3]The light-cone coordinates are denoted by $x^\pm = x^0 \pm x^1$. Correspondingly, the flat metric is given by $\eta_{++} = \eta_{--} = 0$, $\eta_{+-} = \eta_{-+} = -\frac{1}{2}$. We exceptionally define the derivatives by $\partial_\pm = \partial_0 \pm \partial_1$; hence $\partial_\pm x^\pm = 2$.

$$S_{\rm eff} = \int d^2x\Big[-\frac{1}{2}\partial_+X\partial_-X + \frac{i}{2}\psi_+\partial_-\psi_+ + \frac{i}{2}\psi_-\partial_+\psi_-\Big]$$
$$+\frac{\kappa}{2}\int d^2x\Big[\frac{1}{2}\partial_+\phi\partial_-\phi + \frac{i}{2}\eta_+\partial_-\eta_+ + \frac{i}{2}\eta_-\partial_+\eta_- - 2\mu^2e^{-\phi} - i\mu\eta_-\eta_+e^{-\frac{\phi}{2}}\Big]$$
$$+\int d^2x[-b_{++}\partial_-C^+ - b_{--}\partial_+C^- + \beta_{++}\partial_-\omega_- + \beta_{--}\partial_+\omega_+]\ , \tag{4.3}$$

where the antighosts $\overline{C}_\pm$ and $\overline{\gamma}_\pm$ are, respectively, denoted by $b_{\pm\pm}$ and $\beta_{\pm\pm}$. Then the string variables and the ghosts become free fields, and the BF fields satisfy the supersymmetric Liouville equations, i.e.,

$$\begin{aligned}&\partial_+\partial_-X = \partial_\pm\psi_\mp = 0\ ,\\ &\partial_\mp C^\pm = \partial_\mp\omega_\mp = \partial_\mp b_{\pm\pm} = \partial_\mp\beta_{\pm\pm} = 0\ ,\\ &\partial_+\partial_-\phi - 2\mu^2e^{-\phi} + \frac{i\mu}{2}\eta_-\eta_+e^{-\frac{\phi}{2}} = 0\ ,\\ &\partial_\pm\eta_\mp \pm \eta_\pm e^{-\frac{\phi}{2}} = 0\ .\end{aligned} \tag{4.4}$$

The gauge-fixed action (4.3) is invariant under the BRST transformations

$$\begin{aligned}\delta X &= \frac{1}{2}C^+\partial_+X + \frac{1}{2}C^-\partial_-X - i(\omega_-\psi_+ - \omega_+\psi_-)\ ,\\ \delta\psi_\pm &= \frac{1}{2}C^\pm\partial_\pm\psi_\pm + \frac{1}{4}\partial_\pm C^\pm\psi_\pm \pm \omega_\mp\partial_\pm X\ ,\\ \delta\phi &= -\frac{1}{2}\partial_+C^+ - \frac{1}{2}\partial_-C^-\\ &\quad+\frac{1}{2}C^+\partial_+\phi + \frac{1}{2}C^-\partial_-\phi - i(\omega_-\eta_+ - \omega_+\eta_-)\ ,\\ \delta\eta_\pm &= \mp2\partial_\pm\omega_\mp + \frac{1}{2}C^\pm\partial_\pm\eta_\pm + \frac{1}{4}\partial_\pm C^\pm\eta_\pm\\ &\quad\pm\omega_\mp\partial_\pm\phi \pm \frac{\mu}{2}C^\mp\eta_\mp e^{-\frac{\phi}{2}} + 2\mu\omega_\pm e^{-\frac{\phi}{2}}\ ,\\ \delta C^\pm &= \frac{1}{2}C^\pm\partial_\pm C^\pm + 2i\omega_\mp^2\ ,\\ \delta\omega_\pm &= \frac{1}{2}C^\mp\partial_\mp\omega_\pm - \frac{1}{4}\partial_\mp C^\mp\omega_\pm\ ,\\ \delta b_{\pm\pm} &= T^X_{\pm\pm} + T^L_{\pm\pm} + T^{gh(2)}_{\pm\pm} + T^{gh(3/2)}_{\pm\pm}\ ,\\ \delta\beta_{\pm\pm} &= \pm i(J^X_{\pm\pm} + J^L_{\pm\pm} + J^{gh}_{\pm\pm})\ ,\end{aligned} \tag{4.5}$$

where $T^{X,L,gh(2,3/2)}_{\pm\pm}$ and $J^{X,L,gh}_{\pm\pm}$ are the components of the stress tensors and the supercurrents of the string, Liouville, and ghost sectors. They are given by

$$\begin{aligned}T^X_{\pm\pm} &\equiv \varphi^X_\pm = \frac{1}{4}(\partial_\pm X)^2 + \frac{i}{4}\psi_\pm\partial_\pm\psi_\pm\ ,\\ T^L_{\pm\pm} &\equiv \frac{\kappa}{2}\left[\frac{1}{4}\left(\phi' \pm \frac{2}{\kappa}\pi_\phi\right)^2 - \left(\phi' \pm \frac{2}{\kappa}\pi_\phi\right)' \pm \frac{i}{2}\eta_\pm\eta'_\pm\right.\\ &\quad\left.+\mu^2e^{-\phi} \mp \frac{i\mu}{2}e^{-\frac{\phi}{2}}\eta_\mp\eta_\pm\right]\\ &= \frac{\kappa}{2}\Big[\frac{1}{4}(\partial_\pm\phi)^2 - \frac{1}{2}\partial_\pm^2\phi + \frac{i}{4}\eta_\pm\partial_\pm\eta_\pm\Big]\ ,\\ T^{gh(2)}_{\pm\pm} &\equiv -b_{\pm\pm}\partial_\pm C^\pm - \frac{1}{2}\partial_\pm b_{\pm\pm}C^\pm\ ,\\ T^{gh(3/2)}_{\pm\pm} &\equiv \frac{3}{4}\beta_{\pm\pm}\partial_\pm\omega_\mp + \frac{1}{4}\partial_\pm\beta_{\pm\pm}\omega_\mp\ ,\\ J^X_{\pm\pm} &\equiv \mathcal{J}^X_\pm = \psi_\pm\partial_\pm X\ ,\\ J^L_{\pm\pm} &\equiv \pm\frac{\kappa}{2}\left[\eta_\pm\left(\phi' \pm \frac{2}{\kappa}\pi_\phi\right) - 4\eta'_\pm - 2\mu e^{-\frac{\phi}{2}}\eta_\mp\right]\\ &= \frac{\kappa}{2}(-\eta_\pm\partial_\pm\phi + 2\partial_\pm\eta_\pm)\ ,\\ J^{\rm gh}_{\pm\pm} &\equiv \mp i\Big(\frac{3}{4}\beta_{\pm\pm}\partial_\pm C^\pm + \frac{1}{2}\partial_\pm\beta_{\pm\pm}C^\pm - 4ib_{\pm\pm}\omega_\mp\Big)\ ,\end{aligned} \tag{4.6}$$

where $\pi_\phi \equiv (\kappa/2)\dot{\phi}$ is the canonical momentum conjugate to ϕ. These results can be obtained directly from (3.2) by using the equations of motion.

We now turn to the BRST charge $\tilde{Q}$ in the superconformal gauge and examine the nilpotency. It is given by (2.8) with the contribution of the BF fields included. Since $\mathcal{P}^A = \beta_z = 0$ in this gauge, we obtain

$$\begin{aligned}\tilde{Q} = \int d\sigma\Big[&C^+(T^X_{++} + T^L_{++} + \tfrac{1}{2}T^{gh(2)}_{++} + T^{gh(3/2)}_{++})\\ &+C^-(T^X_{--} + T^L_{--} + \tfrac{1}{2}T^{gh(2)}_{--} + T^{gh(3/2)}_{--})\\ &-i\omega_-(J^X_{++} + J^L_{++}) + i\omega_+(J^X_{--} + J^L_{--})\\ &+2ib_{++}\omega_-^2 + 2ib_{--}\omega_+^2\Big]\ .\end{aligned} \tag{4.7}$$

As was mentioned in Sec. III, the operator products in the right-hand side (RHS) of (4.7) are defined by the ordering prescription. For the string variables and the ghosts, it coincides with the free field normal ordering. We do not know, however, what operator ordering should be chosen for the BF fields. Since they are not free fields, the ordering prescription introduced in Sec. III would not be completely legitimate. But we shall continue to use the free field ordering prescription in this section.

To satisfy the nilpotency of $\tilde{Q}$ it is necessary for the total stress tensor and the total supercurrents defined by

$$\begin{aligned}T^{\rm tot}_{\pm\pm} &= T^X_{\pm\pm} + T^L_{\pm\pm} + T^{gh(2)}_{\pm\pm} + T^{gh(3/2)}_{\pm\pm}\ ,\\ J^{\rm tot}_{\pm\pm} &= J^X_{\pm\pm} + J^L_{\pm\pm} + J^{gh}_{\pm\pm}\end{aligned} \tag{4.8}$$

to satisfy super-Virasoro algebra with the total central charge being canceled. Except for the BF sector, (4.6) satisfies the super-Virasoro algebra with central charges $3D/2$, -26, and 11 for the string, the reparametrization ghost, and the bosonic superghost sectors, respectively. On the other hand the $T^L_{\pm\pm}$ and $J^L_{\pm\pm}$ given in (4.6) do not form super-Virasoro commutation relations for nonvanishing cosmological terms. To recover the super-Virasoro algebra, let us modify these operators by

$$T^L_{\pm\pm} = \frac{\kappa}{2}\Big[\frac{1}{4}\left(\phi' \pm \frac{2}{\kappa}\pi_\phi\right)^2 - \left(\phi' \pm \frac{2}{\kappa}\pi_\phi\right)' \pm \frac{i}{2}\eta_\pm\eta'_\pm + \mu^2 V_\alpha^2 \mp i\alpha\mu V_\alpha \eta_\mp \eta_\pm\Big] ,$$
$$J^L_{\pm\pm} = \pm\frac{\kappa}{2}\left[\eta_\pm\left(\phi' \pm \frac{2}{\kappa}\pi_\phi\right) - 4\eta'_\pm - 2\mu V_\alpha \eta_\mp\right] , \quad (4.9)$$

where α is a free parameter and V_α is defined by

$$V_\alpha \equiv :\exp(-\alpha\phi): . \quad (4.10)$$

For $\alpha = 1/2$, (4.9) reduces to the classical expression given in (4.6). The operator products in the RHS of (4.9) are defined by the free field ordering prescription except for the V_α^2 term, while the operator V_α^2 is assumed to be the square of the normal ordered operator (4.10); hence it is not well defined. But as far as we know it is not possible to recover the super-Virasoro algebra by the free field ordering prescription for a nonvanishing cosmological term. Besides this shortcoming of our argument, (4.9) satisfies the super-Virasoro algebra with central charge $3/2 + 24\pi\kappa$ for

$$\alpha\gamma - \frac{\alpha^2}{8\pi} = \frac{1}{2} . \quad (4.11)$$

This is just an extension of the canonical analysis of Ref. [2] to a superstring. The deviation of α from the classical value can be interpreted as gravitational self-dressing. The condition (4.11) corresponds to the requirement that the vertex operator (4.10) must be a conformal field with conformal weight $(\frac{1}{2}, \frac{1}{2})$ as discussed in Ref. [7].

These arguments show that the BRST charge (4.7) satisfies the nilpotency for κ and α satisfying (2.23) and (4.11), i.e.,

$$\alpha = \sqrt{\frac{\pi}{2}}(\sqrt{9-D} \pm \sqrt{1-D}) . \quad (4.12)$$

V. SUPERSYMMETRIC LIGHT-CONE GAUGE FIXING

In this section we will investigate the supersymmetric extension of the light-cone gauge [13–15]. It is defined by the following set of gauge conditions:

$$e_+{}^+ = e_-{}^- = 1 , \qquad e_-{}^+ = 0 , \qquad \chi_{--} = \chi_{-+} = 0 , \quad (5.1)$$

on the zweibeins and gravitinos. These fix the gauges of reparametrizations, local Lorentz, and local supersymmetry. To specify the gauge for the super-Weyl symmetry, we impose additional gauge conditions on the super-Liouville fields as

$$\phi = \eta = F_L = 0 . \quad (5.2)$$

In this gauge the only independent components of the gravity multiplet are $e_+{}^- \equiv -g_{++}$, χ_{++}, χ_{+-}, and F_G, which behave as the analog of the super-Liouville fields.

The gauge conditions (5.1) and (5.2) can be implemented by choosing

$$\chi^+_\varphi = N^+ - 1 , \quad \chi^-_\varphi = (N^- + 1)e^\xi - 2 ,$$
$$\chi^\varepsilon = \varepsilon , \quad \chi^\xi = \phi , \quad \chi^f = f_L ,$$
$$\chi^+_{\mathcal{J}} = -iM^+, \quad \chi^-_{\mathcal{J}} = iM^- - \frac{1}{4}(N^- + 1)\Lambda_-, \quad \chi^\pm_\Lambda = \eta_\pm . \quad (5.3)$$

It can be easily seen that these together with $N^\pm = \lambda^\pm$ and $M^\pm = \pm i\nu_\mp$ given by (3.8) indeed lead to (5.1) and (5.2).

The gauge conditions (5.3) do not satisfy the assumption given below (3.8) since their BRST transforms contain the canonical momentum π_θ and $\zeta_\pm$. Then the master action (3.4) formulated in the EPS leads to erroneous results when (5.3) are imposed. This is because (3.17) cannot be identified with the covariant super-Liouville fields. It seems rather difficult to find covariant super-Liouville multiplets without the assumption on the gauge conditions. To avoid this difficulty, we apply (5.3) to (3.18) instead of (3.4) and use the covariant BRST transformations (A2) rather than (3.2). This should be compared with the superconformal gauge fixing where the gauge conditions (4.1) satisfy the assumption given below (3.8),[4] and we can work with (3.4) from the beginning as well.

Substituting (5.3) into (3.19), we obtain the effective action (3.18) in the light-cone gauge.[5] As in the superconformal gauge fixing, C_L, C_W, η_W, and F_W in the ghost sector become nonpropagating, and can be eliminated via equations of motion. Taking variations of $S_{\rm eff}$ with respect to $N^\pm = \lambda^\pm$ and $M^\pm = \pm i\nu_\mu$, we obtain the multipliers $B_\pm$ and $A^\pm$ as

$$B_+ = \varphi^X_+ + \varphi^g_+ + \varphi^{gh}_+ ,$$
$$g_{11}B_- = \varphi^X_- + \varphi^g_- + \varphi^{gh}_- + \frac{i}{4}(\mathcal{J}^X_- + \mathcal{J}^g_- + \mathcal{J}^{gh}_-)\Lambda_- ,$$
$$A^\pm = \pm i(\mathcal{J}^X_\mp + \mathcal{J}^g_\mp + \mathcal{J}^{gh}_\mp) , \quad (5.4)$$

where $\varphi^{X,g,gh}_\pm$ and $\mathcal{J}^{X,g,gh}_\pm$ are the super-Virasoro constraints given by

$$\varphi^{X,g,gh}_\pm \equiv -\frac{\delta S_{X,g,gh}}{\delta\lambda^\pm} , \quad \mathcal{J}^{X,g,gh}_\pm \equiv \mp i\frac{\delta S_{X,g,gh}}{\delta\nu_\mp} . \quad (5.5)$$

We can relate the BRST transformations of the antighosts to the super-Virasoro operators via (5.4).

In the light-cone gauge, the ghost action takes the following form:

[4] In the case of the EPS the gauge condition corresponding to $f_L = 0$ is inherently absent.

[5] In this section we consider the case $\mu^2 = 0$ for simplicity.

$$S_{\rm gh} = \int d^2\sigma \Big[-\overline{\mathcal{C}}_+ \partial_- C^+ - \overline{\mathcal{C}}_- (\partial_+ C^+ + \partial_- C^- - g_{11}\partial_- C^+ - 2i\omega_- \Lambda_+) + \overline{\gamma}_+ \partial_- \omega_- + \overline{\gamma}_- \left(\partial_- \omega_+ + \frac{1}{2}\omega_+ \partial_- \xi + \frac{1}{4}\partial_- C^+ \Lambda_- - \frac{1}{2\sqrt{g_{11}}} f_G \omega_- \right) \Big]. \tag{5.6}$$

The presence of the auxiliary field f_G turns out to be essential in order for the BRST transformations to be consistent with the equations of motion. Eliminating f_G via the equation of motion $\kappa\sqrt{g_{11}} f_G = \overline{\gamma}_- \omega_-$, we see that a quartic term appears in the bosonic ghosts and antighosts. $S_{\rm gh}$ also contains interaction terms between the supergravity sector and the ghost sector. Remarkably enough, all the ghost variables and Λ_+ can be made free fields by the following field redefinitions:

$$\begin{aligned} c^+ &\equiv C^+, \\ c_+ &\equiv C^- + \frac{x^-}{2}\partial_+ C^+ - ix^- \omega_- \Lambda_+ + \frac{i}{\kappa}(x^-)^2 \overline{\mathcal{C}}_- (\omega_-)^2, \\ \gamma_- &\equiv \omega_-, \\ \gamma_+ &\equiv \sqrt{g_{11}}\omega_+ - \frac{x^-}{4\kappa}\frac{\overline{\gamma}_-}{\sqrt{g_{11}}}(\omega_-)^2, \\ b &\equiv \overline{\mathcal{C}}_-, \\ b_{++} &\equiv \overline{\mathcal{C}}_+ - g_{11}\overline{\mathcal{C}}_- + \frac{1}{4}\overline{\gamma}_- \Lambda_- + \frac{x^-}{2}\partial_+ \overline{\mathcal{C}}_-, \\ \beta_+ &\equiv \frac{\overline{\gamma}_-}{\sqrt{g_{11}}}, \\ \beta_{++} &\equiv \overline{\gamma}_+ - ix^- \overline{\mathcal{C}}_- \Lambda_+ + \frac{x^-}{4\kappa}\left(\frac{\overline{\gamma}_-}{\sqrt{g_{11}}}\right)^2 \omega_- , \\ \chi_+ &\equiv \Lambda_+ - \frac{2}{\kappa}x^- \overline{\mathcal{C}}_- \omega_- . \end{aligned} \tag{5.7}$$

In terms of these variables the gauge-fixed action then takes the form

$$\begin{aligned} S_{\rm eff} = &\int d^2x \Big[\frac{1}{2}\partial_- X(\partial_+ X + g_{++}\partial_- X) + \frac{i}{2}\psi_+ \partial_- \psi_+ + \frac{i}{2}\psi_-(\partial_+\psi_- + g_{++}\partial_-\psi_-) - 2i\chi_{++}\psi_-\partial_- X \Big] \\ &+ \frac{\kappa}{2}\int d^2x \Big[\frac{1}{2g_{11}}\{(\partial_- g_{11})^2 - 2\partial_- g_{11}(\ln g_{11})' + 4(\ln g_{11})''\} + \frac{8i}{g_{11}}\chi_{++}\left(\partial_-\chi_{++} - \frac{2}{g_{11}}\chi'_{++}\right) + \frac{i}{2}\chi_+ \partial_- \chi_+ \Big] \\ &+ \int d^2x(-b_{++}\partial_- c^+ - b\partial_- c_+ + \beta_{++}\partial_-\gamma_- + \beta_+\partial_-\gamma_+) , \end{aligned} \tag{5.8}$$

where we have used the relations $g_{11} = 1 + g_{++}$ and $\sqrt{g_{11}}\Lambda_- = 4\chi_{++}$, and $\psi_+/\sqrt{g_{11}}$, the lower component of the fermionic string coordinates ψ in our representation, has been newly denoted by ψ_+. $S_{\rm eff}$ is the supersymmetric extension of 2D gravity action discussed in [17]. As in the bosonic string case, it leads to the free ghost equations

$$\begin{aligned} &\partial_- c^+ = \partial_- c_- = \partial_- \gamma_- = \partial_- \gamma_+ = 0 , \\ &\partial_- b_{++} = \partial_- b = \partial_- \beta_{++} = \partial_- \beta_+ = 0 , \end{aligned} \tag{5.9}$$

and the canonical supercommutation relations among ghost variables as

$$\begin{aligned} &[c^+(\sigma), b_{++}(\sigma')] = [c_+(\sigma), b(\sigma')] = -i\delta(\sigma - \sigma') , \\ &[\gamma_-(\sigma), \beta_{++}(\sigma')] = [\gamma_+(\sigma), \beta_+(\sigma')] = i\delta(\sigma - \sigma') , \\ &\text{all other supercommutators vanish} . \end{aligned} \tag{5.10}$$

By taking variations of (5.8) with respect to g_{++} and χ_{++}, we obtain the equations of motion for the graviton and gravitino as

$$\begin{aligned} \varphi^X_- + \varphi^g_- &= \frac{\kappa}{4} g_{11}\partial_-^2 g_{11} - 4i\kappa\chi_{++}\partial_-\chi_{++} , \\ \mathcal{J}^X_- + \mathcal{J}^g_- &= 4\kappa\sqrt{g_{11}}\partial_-\chi_{++} , \end{aligned} \tag{5.11}$$

where we have used (5.5).

The BRST transformations of the variables appearing in (5.8) can be found from (5.7) and (A2), by using the gauge conditions (5.1) and the equations of motion for ghosts (5.9) as follows:

$$\begin{aligned}
\delta g_{++} =& \frac{1}{2}c^{+}\partial_{+}g_{++} + g_{++}\partial_{+}c^{+} + \frac{1}{2}\Big(c_{+} - \frac{x^{-}}{2}\partial_{+}c^{+}\Big)\partial_{-}g_{++} - \frac{1}{2}\partial_{+}\Big(c_{+} - \frac{x^{-}}{2}\partial_{+}c^{+}\Big) \\
&+4i\gamma_{+}\chi_{++} - i\gamma_{-}\chi_{+}\Big(1 - \frac{x^{-}}{2}\partial_{-}\Big)g_{++} - \frac{i}{2}x^{-}\partial_{+}(\gamma_{-}\chi_{+}) \\
&+\frac{i}{\kappa}x^{-}\beta_{-}\gamma_{-}^{2}\chi_{++} - \frac{2i}{\kappa}b\gamma_{-}^{2}\Big(x^{-} - \frac{(x^{-})^{2}}{4}\Big)g_{++} - \frac{i}{2\kappa}(x^{-})^{2}\partial_{+}(b\gamma_{-}^{2}) , \\
\delta\chi_{++} =& \frac{1}{2}c^{+}\partial_{+}\chi_{++} + \frac{3}{4}\partial_{+}c^{+}\chi_{++} + \frac{1}{2}\Big(c_{+} - \frac{x^{-}}{2}\partial_{+}\Big)\partial_{-}\chi_{++} \\
&-\frac{i}{2}\gamma_{-}\chi_{+}(1 - x^{-}\partial_{-})\chi_{++} - \frac{i}{\kappa}b\gamma_{-}^{2}\Big\{x^{-} - \frac{(x^{-})^{2}}{2}\partial_{-}\Big\}\chi_{++} \\
&-\frac{1}{4}\gamma_{+}\partial_{-}g_{++} + \frac{1}{4\kappa}\beta_{+}\gamma_{-}^{2}\Big(1 - \frac{x^{-}}{4}\partial_{-}\Big)g_{++} + \frac{x^{-}}{8\kappa}\partial_{+}(\beta_{+}\gamma_{-}^{2}) , \\
\delta\chi_{+} =& \frac{1}{2}c^{+}\partial_{+}\chi_{+} - \frac{1}{4}\partial_{+}c^{+}\chi_{+} + 2\partial_{+}\gamma_{-} + \gamma_{-}\Big(1 - \frac{x^{-}}{2}\partial_{-}\Big)\partial_{-}g_{++} - \frac{2}{\kappa}\gamma_{-}\Big(bc_{+} - \frac{1}{2}\beta_{+}\gamma_{+}\Big) , \\
\delta c^{+} =& \frac{1}{2}c^{+}\partial_{+}c^{+} + 2i\gamma_{-}^{2} , \\
\delta c_{+} =& \frac{1}{2}c^{+}\partial_{+}c_{+} + \frac{1}{2}\partial_{+}c^{+}c_{+} + 2i\gamma_{+}^{2} , \\
\delta\gamma_{-} =& \frac{1}{2}c^{+}\partial_{+}\gamma_{-} - \frac{1}{4}\partial_{+}c^{+}\gamma_{-} , \\
\delta\gamma_{+} =& \frac{1}{2}c^{+}\partial_{+}\gamma_{+} + \frac{1}{4}\partial_{+}c^{+}\gamma_{+} - \frac{i}{2}\gamma_{+}\gamma_{-}\chi_{+} - 2i\gamma_{-}^{2}\Big(1 - \frac{x^{-}}{2}\partial_{-}\Big)\chi_{++} + \frac{1}{4\kappa}c_{+}\beta_{+}\gamma_{-}^{2}, \\
\delta b_{++} =& T_{++}^{X} + T_{++}^{g} + \frac{1}{2}T_{++}^{gh(2)} + T_{++}^{gh(0)} + T_{++}^{gh(3/2)} + T_{++}^{gh(1/2)} , \\
\delta b =& \frac{\kappa}{4}\partial_{-}^{2}g_{++} - \frac{1}{2}\partial_{+}bc^{+} - ib\gamma_{-}\chi_{+} + \frac{1}{8\kappa}(\beta_{+}\gamma_{-})^{2} , \\
\delta\beta_{++} =& iJ_{++}^{X} + \frac{i\kappa}{2}\chi_{+}\Big(1 - \frac{x^{-}}{2}\partial_{-}\Big)\partial_{-}g_{++} - i\kappa\partial_{+}\chi_{+} + \frac{1}{2}\partial_{+}\beta_{++}c^{+} + \frac{3}{4}\beta_{++}\partial_{+}c^{+} - 4ib_{++}\gamma_{-} \\
&-4ib\gamma_{-}\Big\{1 - \frac{x^{-}}{2}\partial_{-} + \frac{(x^{-})^{2}}{8}\partial_{-}^{2}\Big\}g_{++} + 4i\beta_{+}\gamma_{-}\Big(1 - \frac{x^{-}}{2}\partial_{-}\Big)\chi_{++} \\
&-i\big(bc_{+} - \frac{1}{2}\beta_{+}\gamma_{+}\big)\chi_{+} - \frac{1}{4\kappa}\beta_{+}^{2}\gamma_{-}c_{+} , \\
\delta\beta_{+} =& -4i\kappa\partial_{-}\chi_{++} + \frac{1}{2}\partial_{+}\beta c^{+} + \frac{1}{4}\beta_{+}\partial_{+}c^{+} - 4ib\gamma_{+} + \frac{i}{2}\beta_{+}\gamma_{-}\chi_{+} ,
\end{aligned} \tag{5.12}$$

where we have omitted the transformations of string variables. In deriving the BRST transformations of antighosts, we have used (5.4) and (5.11). The stress tensors and the supercurrents are defined by

$$\begin{aligned}
T_{++}^{X} \equiv& \varphi_{+}^{X} = \frac{1}{4}[(\partial_{+}X + g_{++}\partial_{-}X)^{2} + i\psi_{+}\partial_{+}\psi_{+}] , \\
J_{++}^{X} \equiv& \mathcal{J}_{+}^{X} = \psi_{+}(\partial_{+}X + g_{++}\partial_{-}X) , \\
T_{++}^{g} \equiv& \frac{\kappa}{2}\Big[\frac{1}{4}(\partial_{-}g_{++})^{2} - \frac{1}{2}g_{++}\partial_{-}^{2}g_{++} \\
&-\frac{1}{2}\Big(\partial_{-} - \frac{x^{-}}{2}\partial_{-}\partial_{+}\Big)\partial_{-}g_{++}\Big] \\
&+4i\kappa\chi_{++}\partial_{-}\chi_{++} + \frac{i\kappa}{8}\chi_{+}\partial_{+}\chi_{+} , \\
T_{++}^{gh(2)} \equiv& -\frac{1}{2}\partial_{+}b_{++} - b_{++}\partial_{+}c^{+} , \\
T_{++}^{gh(0)} \equiv& \frac{1}{2}\partial_{+}bc^{+} , \\
T_{++}^{gh(3/2)} \equiv& \frac{3}{4}\beta_{++}\partial_{+}\gamma_{-} + \frac{1}{4}\partial_{+}\beta_{++}\gamma_{-} , \\
T_{++}^{gh(1/2)} \equiv& \frac{1}{4}\beta_{+}\partial_{+}\gamma_{+} - \frac{1}{4}\partial_{+}\beta_{+}\gamma_{+} .
\end{aligned} \tag{5.13}$$

Except for the gravitational stress tensor T_{++}^{g}, these satisfy conservation laws. The index $j = 2, 0, 3/2, 1/2$ of the stress tensors for the ghost sector labels the canonical pairs of ghost and antighost with conformal weights $1 - j$ and j [32]. The $T_{++}^{gh(j)}$ defined by the usual normal ordering satisfies the Virasoro algebra with central charge

$$2\epsilon(6j^{2} - 6j + 1) , \tag{5.14}$$

where ϵ stands for the Grassmannian parity of the ghost pair. The total central charge of the ghost sectors is then given by

$$c_{\rm gh} = -18 . \tag{5.15}$$

Using (5.10) and the BRST transformations for the ghosts, we can construct the BRST charge generating (5.12) as

$$\tilde{Q} = \int d\sigma \Bigg\{ c^+ \left(T^X_{++} + T^g_{++} + \frac{1}{2} T^{gh(2)}_{++} + T^{gh(0)}_{++} + T^{gh(3/2)}_{++} + T^{gh(1/2)}_{++} \right)$$
$$+ c_+ \left(\frac{\kappa}{4} \partial_-^2 g_{++} - ib\gamma_- \chi_+ + \frac{1}{8\kappa} (\beta_+ \gamma_-)^2 \right)$$
$$+ \gamma_- \Bigg[-iJ^X_{++} - \frac{i\kappa}{2} \chi_+ \left(1 - \frac{x^-}{2} \partial_- \right) \partial_- g_{++} + i\kappa \partial_+ \chi_+ + 2ib_{++} \gamma_-$$
$$+ 2ib\gamma_- \left(1 - \frac{x^-}{2} \partial_- + \frac{(x^-)^2}{8} \partial_-^2 \right) g_{++} - 2i\beta_+ \gamma_- \left(1 - \frac{x^-}{2} \partial_- \right) \chi_{++} \Bigg]$$
$$+ \gamma_+ \left(4i\kappa \partial_- \chi_{++} + 2ib\gamma_+ - \frac{i}{2} \beta_+ \gamma_- \chi_+ \right) \Bigg\} , \tag{5.16}$$

where normal products among the ghost variables are implicitly assumed.

The BRST invariance of (5.8) implies that (5.12) must be consistent with the equations of motion (5.9). We thus obtain the supercurvature equations

$$\partial_-^3 g_{++} = 0 , \qquad \partial_-^2 \chi_{++} = 0 , \qquad \partial_- \chi_+ = 0 . \tag{5.17}$$

Then the gravitational stress tensor T^g_{++} in (5.13) also turns out to be conserved, i.e.,

$$\partial_- T^g_{++} = 0 . \tag{5.18}$$

By virtue of (5.9) and (5.17) and the conservation of T^X_{++} and J^X_{++},[6] we can show that the BRST charge (5.16) is a constant of motion.

It remains to show the nilpotency of $\tilde{Q}$. Before turning to this issue, we must fix the commutation relations for the 2D supergravity sector. This can be done by comparing the BRST transformations (5.12) and (3.1) for g_{++} and χ_{++}. We first expand these operators in terms of conserved currents by noting (5.17) as

$$g_{++} = -\frac{1}{2\kappa} [J^+(x^+) - 2x^- J^0(x^+) + (x^-)^2 J^-(x^+)] ,$$
$$\chi_{++} = -\frac{1}{2\kappa} [\Psi^{-1/2}(x^+) + x^- \Psi^{1/2}(x^+)] . \tag{5.19}$$

The BRST transformations of g_{++} and χ_{++} given in (5.12) can be transcribed into the transformations of these currents as

$$\delta J^+ = \frac{1}{2} c^+ \partial_+ J^+ + \partial_+ c^+ J^+ - 2c_+ J^0$$
$$+ \kappa \partial_+ c_+ + 4i\gamma_+ \Psi^{-1/2} - i\gamma_- \chi_+ J^+ ,$$

$$\delta J^0 = \frac{1}{2} c^+ \partial_+ J^0 + \frac{1}{2} \partial_+ c^+ J^0 - c_+ J^-$$
$$+ \frac{\kappa}{4} \partial_+^2 c^+ - 2i\gamma_+ \Psi^{1/2}$$
$$- \frac{i\kappa}{2} \partial_+ (\gamma_- \chi_+) + \frac{i}{\kappa} (\gamma_-)^2 \left(bJ^+ - \frac{1}{2} \beta_+ \Psi^{-1/2} \right) ,$$

$$\delta J^- = \frac{1}{2} c^+ \partial_+ J^- + i\gamma_- \chi_+ J^-$$
$$+ \frac{2i}{\kappa} (\gamma_-)^2 \left(bJ^0 + \frac{1}{2} \beta_+ \Psi^{1/2} \right) + i\partial_+ [b(\gamma_-)^2],$$

$$\delta \Psi^{-1/2} = \frac{1}{2} c^+ \partial_+ \Psi^{-1/2} + \frac{3}{4} \partial_+ c^+ \Psi^{-1/2} + c_+ \Psi^{1/2} \tag{5.20}$$
$$- \kappa \partial_+ \gamma_+ + \gamma_+ J^0 - \frac{i}{2} \gamma_- \chi_+ \Psi^{-1/2}$$
$$+ \frac{1}{4\kappa} \beta_+ (\gamma_-)^2 J^+ ,$$

$$\delta \Psi^{1/2} = \frac{1}{2} c^+ \partial_+ \Psi^{1/2} + \frac{1}{4} \partial_+ c^+ \Psi^{1/2}$$
$$- \gamma_+ J^- + \frac{i}{2} \gamma_- \chi_+ \Psi^{1/2}$$
$$- \frac{1}{4\kappa} (\gamma_-)^2 \left(\beta_+ J^0 + 4ib\Psi^{-1/2} \right) - \frac{1}{4} \partial_+ [\beta_+ (\gamma_-)^2],$$

$$\delta \chi_+ = \frac{1}{2} c^+ \partial_+ \chi_+ + \frac{1}{4} \partial_+ c^+ \chi_+ + 2\partial_+ \gamma_-$$
$$+ \frac{2}{\kappa} \gamma_- \left(J^0 - bc_+ + \frac{1}{2} \beta_+ \gamma_+ \right) .$$

Then J^a and Ψ^r can be shown to satisfy OSp(1,2) Kac-Moody current algebra

$$[\ J^a(\sigma)\ ,\ J^b(\sigma')\ ]$$
$$= if^{ab}{}_c\, J^c(\sigma)\, \delta(\sigma - \sigma') - i\kappa \eta^{ab} \partial_\sigma \delta(\sigma - \sigma'),$$
$$[\ J^a(\sigma)\ ,\ \Psi^r(\sigma')\ ] = if^{ar}{}_s\, \Psi^s(\sigma)\, \delta(\sigma - \sigma') , \tag{5.21}$$
$$[\ \Psi^r(\sigma)\ ,\ \Psi^s(\sigma')\ ]$$
$$= f^{rs}{}_a\, J^a(\sigma)\, \delta(\sigma - \sigma') - \kappa \eta^{rs} \partial_\sigma \delta(\sigma - \sigma') ,$$

[6]The possible anomalies in these currents due to the super-Virasoro anomaly indeed vanish in the light-cone gauge where $\lambda^+ = 1$ and $\nu_- = 0$ [33].

where the f's and η's are, respectively, the structure constants and the Killing metric for OSp(1,2) algebra. They satisfy $f^{ab}{}_c = -f^{ba}{}_c$, $f^{rs}{}_a = f^{sr}{}_a$, $\eta^{ab} = \eta^{ba}$, and $\eta^{rs} = -\eta^{sr}$ with nonvanishing components $f^{\pm 0}{}_{\pm} = \pm 1$, $f^{+-}{}_0 = 2$, $f^{+\,1/2}{}_{-1/2} = -f^{-\,-1/2}{}_{1/2} = -1$, $f^{0\,1/2}{}_{1/2} = -f^{0\,-1/2}{}_{-1/2} = 1/2$, $f^{-1/2\,-1/2}{}_{+} = -f^{-1/2\,1/2}{}_0 = f^{1/2\,1/2}{}_{-} = -1/4$, $\eta^{+-} = -2\eta^{00} = 2$, and $\eta^{-1/2\,1/2} = 1/2$.

It is natural to redefine operator ordering for the gravitational sector to ensure the symmetry associated with the OSp(1,2) current algebra. To this end we decompose $J^a(x^+)$ into positive and negative frequency parts by

$$J^{a(\pm)}(x^+) = \int dy^+ \delta^{(\mp)}(x^+ - y^+) J^a(y^+) \ , \qquad (5.22)$$

and similarly for $\Psi^r(x^+)$. We then define operator ordering with respect to this decomposition. The gravitational stress tensor T^g_{++} given in (5.13) must be defined by the Sugawara form

$$T^g_{++} = -\frac{1}{2\kappa'} : (\eta_{ab} J^a J^b + i\eta_{rs}\Psi^r\Psi^s) : -\frac{1}{2}\partial_+ J^0 + \frac{i\kappa'}{8}\chi_+\partial_+\chi_+ \ , \qquad (5.23)$$

where η_{ab} and η_{rs} are the inverses of η^{ab} and η^{rs}. The parameter κ' is modified from its classical value κ by

$$\kappa' = \kappa - \frac{3}{8\pi} \ . \qquad (5.24)$$

The stress tensor thus defined not only ensures the BRST transformations (5.20) but also satisfies the Virasoro algebra with the central charge given by

$$c_g \equiv \frac{2k}{2k-3} + 6k + \frac{1}{2} \ , \qquad (5.25)$$

where $k \equiv 4\pi\kappa$ is the central charge of the current algebra (5.21). The last term in the RHS of (5.25) is the contributions due to χ_+.

In (5.23) we have used the χ_+ rescaled by $\sqrt{\kappa/\kappa'}\chi_+ \to \chi_+$ satisfying

$$[\chi_+(x^+), \chi_+(y^+)] = \frac{2}{\kappa'}\delta(x^+ - y^+) \ . \qquad (5.26)$$

The stress tensor (5.23) can be regarded as that given in (5.13) with all the parameters κ replaced by κ'. This enables one to interpret the result that the quantum modifications appear not in the stress tensor but in the current algebra (5.21) and hence in (5.20). Since κ is a free parameter both interpretation can be legitimized.

We are now in a position to investigate the nilpotency of the BRST charge. The $\tilde{Q}$ given by (5.16) does not satisfy the nilpotency even after the substitution of (5.23) for T^g_{++}. We must replace κ appearing in (5.16) by κ' corresponding to the change (5.24). After a rather lengthy computation, it can be shown that the BRST charge thus defined yet contains a BRST anomaly of trivial type as well as the nontrivial one [25]. The former can be removed by shifting the BRST charge by

$$\frac{i}{8\pi}\int d\sigma \gamma_- \partial_+ \chi_+ \ . \qquad (5.27)$$

The correct quantum mechanical BRST charge is finally given by

$$\begin{aligned}\tilde{Q} = \int d\sigma : \Big\{ & c^+\left(T^X_{++} + T^g_{++} + \frac{1}{2}T^{gh(2)}_{++} + T^{gh(0)}_{++} + T^{gh(3/2)}_{++} + T^{gh(1/2)}_{++}\right) \\ & + c_+\left(-J^- - ib\gamma_-\chi_+ + \frac{1}{8\kappa'}(\beta_+\gamma_-)^2\right) \\ & + \gamma_-\left[-iJ^X_{++} - i\chi_+ J^0 + i\left(\kappa' + \frac{1}{8\pi}\right)\partial_+\chi_+ + 2ib_{++}\gamma_- - \frac{i}{\kappa'}\gamma_-(bJ^+ - \beta_+\Psi^{-1/2})\right] \\ & + \gamma_+\left(-4i\Psi^{1/2} + 2ib\gamma_+ - \frac{i}{2}\beta_+\gamma_-\chi_+\right)\Big\} : . \end{aligned} \qquad (5.28)$$

The $\tilde{Q}^2$ contains only the cohomologically nontrivial anomaly of the type (2.16) given by

$$\tilde{Q}^2 = -\frac{ic_{\text{tot}}}{48\pi}\int d\sigma (c^+ c^{+\prime\prime\prime} + 8i\gamma_-\gamma_-'') \ , \qquad (5.29)$$

where $c_{\text{tot}} = c_X + c_g + c_{\text{gh}}$ is the total Virasoro central charge. We thus arrive at the KPZ condition for the nilpotent BRST charge in the $N = 1$ NSR superstring as

$$c_{\text{tot}} = \frac{3}{2}D + \frac{2k}{2k-3} + 6k + \frac{1}{2} - 18 = 0 \ , \qquad (5.30)$$

where use has been made of the results (5.15) and (5.25) as well as $c_X = \frac{3}{2}D$ for the string sector.

VI. SUMMARY AND DISCUSSION

We have investigated BRST quantization of the NSR superstring at noncritical dimensions as 2D SUGRA coupled with the string variables. It is done with special emphasis on the point that the super-Liouville mode which is decoupled from the theory at the classical level becomes dynamically active through the superconformal anomaly. At noncritical dimensions the super-Virasoro anomaly destroying the reparametrization invariance and the local supersymmetry can be canceled by introducing the BF fields. This naturally leads to a gauge symmetric extension of the original system which suffers from the super-Virasoro anomaly.

The gauge-fixed effective action thus obtained turns out to contain two actions of super-Liouville type. The one written only in terms of 2D supergravity fields can be regarded as the counterterm removing the super-Virasoro anomaly. This action turns out to reproduce the correct super-Weyl anomaly as was argued in [25]. The BF fields constitute the other one, which cancels the super-Weyl anomaly. By introducing the BF fields we have been able not only to construct an effective action possessing all the classical local symmetries but also to show within canonical formalism how the super-Liouville mode acquires dynamical behavior through the superconformal anomaly without invoking particular gauge conditions or weak field approximations.

As we have mentioned in Sec. IV, it is possible to gauge-fix the 2D supergravity fields to flat ones as in Refs. [23]. These authors ignored the superconformal anomaly, and necessarily found that the theory is only consistent at the critical dimensions [19, 23]. In the present case, the BRST invariance does not lead to any inconsistency even at noncritical dimensions but yields the vanishing condition of the total central charge and the gravitational dressing effect [7] in the superconformal gauge. The well-known barrier at $D = 1$ also arises in our approach as can be seen from (4.12), indicating the breakdown of validity of the continuum Liouville approach [34]. The superconformal mode does not decouple from the theory and is described by the supersymmetric extension of the Liouville action given by DDK [6, 7]. By simply transcribing our canonical argument into a path integral one, the functional measure for the super-Liouville mode turns out to be translational invariant. This provides a canonical verification of the functional measure ansatz of DDK. This can be understood as follows. The essential point that leads the authors of [7] to their super-Liouville action is the fake super-Weyl invariance arising in the decompositions of 2D supergravity fields into a super-Liouville mode and fiducial background fields. Requiring the symmetry not to be broken by the superconformal anomaly necessarily results in the anomaly-canceling super-Liouville action given in (4.3) up to trivial rescaling of the fields. Since canceling the BRST anomaly in our BFV-BRST approach is equivalent to eliminating superconformal anomaly, we arrive at the effective action of Ref. [7].

One of the advantageous points of our canonical approach is that the effective action (3.18) is a local functional without referring to any particular gauge and arbitrary gauges can be argued on an equal footing. In particular, we can explain the manifestation of OSp(1,2) current algebra from the BRST invariance in the light-cone gauge. This is contrasted with the approaches of Refs. [13–15, 11]. These authors started with the anomalous Ward-Takahashi identities corresponding to the supercurvature equations (5.17) and then extracted OSp(1,2) current algebra. In Refs. [11, 12], BRST analyses were carried out for the system with the stress tensor and the supercurrent obtained by applying the Sugawara construction. Although the KPZ condition (5.30) coincides with the result of Refs. [14, 11] for N=1 2D SUGRA, there are crucial differences between our method and that in Refs. [14, 11] in the ghost content and, consequently, the expression of the BRST charge (5.28). In Ref. [11], six ghost-antighost pairs were introduced corresponding to the six generators of the residual transformations leaving the light-cone gauge unchanged. In our case, there are four pairs and the rest can be eliminated by the equations of motion as multiplier fields. What yields these qualitative differences is the inclusion of the auxiliary fields to ensure the off-shell nilpotency of the BRST transformations (A1) and their appearance in the effective action (3.18) through the super-Weyl anomaly. This leads to the nontrivial redefinition of χ_{+-} as in (5.7) and the ghost higher order terms in (5.28), to which very little attention seems to have been paid so far. In this paper the inclusion of the supersymmetric auxiliary fields has been done only after passing to the configuration space. Systematic methods for including such variables in the EPS seem to be lacking yet, and it is certainly worth exploring them for the BFV-BRST formalism.

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APPENDIX: BRST TRANSFORMATIONS

In this Appendix we summarize the BRST transformations in the configuration space. The covariant ghosts are denoted by C^α, C_L, ω, C_W, and η_w for reparametrizations, local Lorentz, local supersymmetry, Weyl rescaling, and fermionic transformations, respectively. In addition to these, we introduce the auxiliary fields F_X^μ, F_G, F_L, and F_W for supermultiplets of string variables, 2D supergravity, super-Liouville fields, and Weyl ghosts, respectively, to ensure the off-shell nilpotency of the BRST transformations. The complete list of them is given by

$$\begin{aligned}\delta X &= C^\alpha\partial_\alpha X + \overline{\omega}\psi, \\ \delta\psi &= -\frac{1}{4}C_W\psi + C^\alpha\partial_\alpha\psi + \frac{1}{2}C_L\rho_5\psi \\ &\quad -i\rho^\alpha\omega(\partial_\alpha X - \overline{\chi}_\alpha\psi) + \omega F_X \ ,\end{aligned}$$

$$\begin{aligned}\delta F_X &= -\frac{1}{2}C_W F_X + C^\alpha\partial_\alpha F_X \\ &\quad -i\overline{\omega}\rho^\alpha[\nabla_\alpha\psi + i\rho^\beta(\partial_\beta X - \overline{\chi}_\beta\psi)\chi_\alpha - \chi_\alpha F_X] \ , \\ \delta\phi &= C_W + C^\alpha\partial_\alpha\phi + \overline{\omega}\eta \ , \\ \delta\eta &= -\frac{1}{4}C_W\eta - \eta_w + C^\alpha\partial_\alpha\eta + \frac{1}{2}C_L\rho_5\eta \\ &\quad -i\rho^\alpha\omega(\partial_\alpha\phi - \overline{\chi}_\alpha\eta) + \omega F_L \ ,\end{aligned}$$

$$\delta F_L = -\frac{1}{2}C_W F_L + F_W + C^\alpha \partial_\alpha F_L$$
$$-i\overline{\omega}\rho^\alpha[\nabla_\alpha \eta + i\rho^\beta(\partial_\beta\phi - \overline{\chi}_\beta\eta)\chi_\alpha - \chi_\alpha F_L] ,$$
$$\delta e_\alpha{}^a = \frac{1}{2}C_W e_\alpha{}^a + C^\beta\partial_\beta e_\alpha{}^a + \partial_\alpha C^\beta e_\beta{}^a$$
$$+\epsilon^a{}_b C_L e_\alpha{}^b - 2i\overline{\omega}\rho^a\chi_\alpha ,$$
$$\delta\chi_\alpha = \frac{1}{4}C_W\chi_\alpha + \frac{i}{4}\rho_\alpha\eta_W + C^\beta\partial_\beta\chi_\alpha + \partial_\alpha C^\beta\chi_\beta$$
$$+\frac{1}{2}C_L\rho_5\chi_\alpha + \nabla_\alpha\omega - \frac{i}{4}\rho_\alpha\omega F_G ,$$
$$\delta F_G = -\frac{1}{2}C_W F_G + F_W + C^\alpha\partial_\alpha F_G$$
$$-4\epsilon^{\alpha\beta}\overline{\omega}\rho_5\nabla_\alpha\chi_\beta + i\overline{\omega}\rho^\alpha\chi_\alpha F_G ,$$
$$\delta C^\alpha = C^\beta\partial_\beta C^\alpha + i\overline{\omega}\rho^\alpha\omega ,$$
$$\delta\omega = \frac{1}{4}C_W\omega + C^\alpha\partial_\alpha\omega + \frac{1}{2}C_L\rho_5\omega - i\overline{\rho}^\alpha\omega\chi_\alpha ,$$
$$\delta C_L = C^\alpha\partial_\alpha C_L + \frac{1}{2}F_G\overline{\omega}\rho_5\omega - \frac{1}{2}\overline{\omega}\rho_5\eta_W$$
$$-i\epsilon_{ab}e^{\beta a}\overline{\omega}\rho^\alpha\omega(\partial_\alpha e_\beta{}^b - \partial_\beta e_\alpha{}^b - i\overline{\chi}_\alpha\rho^b\chi_\beta) ,$$
$$\delta C_W = C^\alpha\partial_\alpha C_W + \overline{\omega}\eta_W ,$$
$$\delta\eta_W = -\frac{1}{4}C_W\eta_W + C^\alpha\partial_\alpha\eta_W + \frac{1}{2}C_L\rho_5\eta_W$$
$$-i\rho^\alpha\omega(\partial_\alpha C_W - \overline{\eta}_W\chi_\alpha) + \omega F_W ,$$
$$\delta F_W = C^\alpha\partial_\alpha F_W - i\overline{\omega}\rho^\alpha[\nabla_\alpha\eta_W$$
$$+i\rho^\beta\chi_\alpha(\partial_\beta C_W - \overline{\chi}_\beta\eta_W) - \chi_\alpha F_W] . \tag{A1}$$

In terms of the new variables introduced in Secs. II and III these transformations can be rewritten as

$$\delta X = C^\alpha\partial_\alpha X - i(\omega_-\psi_+ - \omega_+\psi_-) ,$$
$$\delta\psi_\pm = C^\alpha\partial_\alpha\psi_\pm + \sqrt{\frac{2}{\lambda^+ + \lambda^-}}f_X\omega_\pm + \frac{1}{2}(C^{1\prime} \pm \lambda^\pm C^{0\prime})\psi_\pm \pm \frac{2\omega_\mp}{\lambda^+ + \lambda^-}\{\dot{X} \pm \lambda^\mp X' + i(\nu_-\psi_+ - \nu_+\psi_-)\} ,$$
$$\delta f_X = C^\alpha\partial_\alpha f_X + \frac{1}{2}\partial_\alpha C^\alpha f_X - i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_+\left[\dot{\psi}_+ - \lambda^+\psi_+' - \frac{1}{2}\lambda^{+\prime}\psi_+ - \frac{2\nu_-}{\lambda^+ + \lambda^-}(\dot{X} + \lambda^- X' - i\nu_+\psi_-)\right]$$
$$-i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_-\left[\dot{\psi}_- + \lambda^-\psi_-' + \frac{1}{2}\lambda^{-\prime}\psi_- + \frac{2\nu_+}{\lambda^+ + \lambda^-}(\dot{X} - \lambda^+ X' + i\nu_-\psi_+)\right] ,$$
$$\delta\phi = C_W + C^\alpha\partial_\alpha\phi - i(\omega_-\eta_+ - \omega_+\eta_-) ,$$
$$\delta\eta_\pm = -\eta_{W\pm} + C^\alpha\partial_\alpha\eta_\pm + \sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_\pm f_L + \frac{1}{2}(C^{1\prime} \pm \lambda^\pm C^{0\prime})\eta_\pm \pm \frac{2\omega_\mp}{\lambda^+ + \lambda^-}\{\dot{\phi} \pm \lambda^\mp\phi' + i(\nu_-\eta_+ - \nu_+\eta_-)\} ,$$
$$\delta f_L = f_W + C^\alpha\partial_\alpha f_L + \frac{1}{2}\partial_\alpha C^\alpha f_L$$
$$-i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_+\left[\dot{\eta}_+ - \lambda^+\eta_+' - \frac{1}{2}\lambda^{+\prime}\eta_+ - \frac{2\nu_-}{\lambda^+ + \lambda^-}(\dot{\phi} + \lambda^-\phi' - i\nu_+\eta_-)\right]$$
$$-i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_-\left[\dot{\eta}_- + \lambda^-\eta_-' + \frac{1}{2}\lambda^{-\prime}\eta_- + \frac{2\nu_+}{\lambda^+ + \lambda^-}(\dot{\phi} - \lambda^+\phi' + i\nu_-\eta_+)\right] ,$$

$$\delta\lambda^\pm = C^\alpha\partial_\alpha\lambda^\pm \pm (\dot{C}^1 \pm \lambda^\pm\dot{C}^0) - \lambda^\pm(C^{1\prime} \pm \lambda^\pm C^{0\prime}) - 4i\omega_\mp\nu_\mp ,$$
$$\delta\xi = C_W + C^\alpha\partial_\alpha\xi + 2C^{1\prime} + C^{0\prime}(\lambda^+ - \lambda^-) - i(\omega_-\Lambda_+ - \omega_+\Lambda_-) ,$$
$$\delta\varepsilon = C_L + C^\alpha\partial_\alpha\varepsilon + \frac{1}{2}(\lambda^+ + \lambda^-)C^{0\prime} - \frac{i}{2}(\omega_-\Lambda_+ + \omega_+\Lambda_-) ,$$
$$\delta\nu_\pm = C^\alpha\partial_\alpha\nu_\pm + (\dot{C}^0 \pm \lambda^\mp C^{0\prime})\nu_\pm - \frac{1}{2}(C^{1\prime} \mp \lambda^\mp C^{0\prime})\nu_\pm + \dot{\omega}_\pm \pm \lambda^\mp\omega_\pm' \mp \frac{1}{2}\lambda^{\mp\prime}\omega_\pm ,$$
$$\delta\Lambda_\pm = -\eta_{W\pm} + C^\alpha\partial_\alpha\Lambda_\pm + \sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_\pm f_G + 4C^{0\prime}\nu_\mp + \frac{1}{2}(C^{1\prime} \pm \lambda^\pm C^{0\prime})\Lambda_\pm$$
$$\pm\frac{2\omega_\mp}{\lambda^+ + \lambda^-}\{\dot{\xi} \pm \lambda^\mp\xi' - (\lambda^+ - \lambda^-)' - i(\nu_+\Lambda_- - \nu_-\Lambda_+)\} + 4\omega_\mp' ,$$
$$\delta f_G = f_W + C^\alpha\partial_\alpha f_G + \frac{1}{2}\partial_\alpha C^\alpha f_G$$
$$-i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_+\left[\dot{\Lambda}_+ - \lambda^+\Lambda_+' - \frac{1}{2}\lambda^{+\prime}\Lambda_+ - 4\nu_-' - \frac{2\nu_-}{\lambda^+ + \lambda^-}\{\dot{\xi} + \lambda^-\xi' - (\lambda^+ - \lambda^-)' - i\nu_+\Lambda_-\}\right]$$
$$-i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_-\left[\dot{\Lambda}_- + \lambda^-\Lambda_-' + \frac{1}{2}\lambda^{-\prime}\Lambda_- - 4\nu_+' + \frac{2\nu_+}{\lambda^+ + \lambda^-}\{\dot{\xi} - \lambda^+\xi' - (\lambda^+ - \lambda^-)' + i\nu_-\Lambda_+\}\right] ,$$

$$
\begin{aligned}
\delta C^0 &= C^\alpha\partial_\alpha C^0 + \frac{2i}{\lambda^+ + \lambda^-}(\omega_+^2 + \omega_-^2)\ ,\\
\delta C^1 &= C^\alpha\partial_\alpha C^1 - \frac{2i}{\lambda^+ + \lambda^-}(\lambda^+\omega_+^2 - \lambda^-\omega_-^2)\ ,\\
\delta\omega_\pm &= C^\alpha\partial_\alpha\omega_\pm - \frac{1}{2}(C^{1\prime} \mp \lambda^\mp C^{0\prime})\omega_\pm - \frac{2i\nu_\pm}{\lambda^+ + \lambda^-}(\omega_+^2 + \omega_-^2)\ ,\\
\delta C_L &= C^\alpha\partial_\alpha C_L + i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_-\omega_+ f_G\\
&\quad + \frac{2i\omega_+^2}{\lambda^+ + \lambda^-}\left[-\dot{\varepsilon} + \lambda^+\varepsilon' + \lambda^{+\prime} - i\nu_-\Lambda_+ - \frac{1}{2}(\dot{\xi} - \lambda^+\xi')\right] - \frac{i}{2}\omega_+\eta_{W-}\\
&\quad + \frac{2i\omega_-^2}{\lambda^+ + \lambda^-}\left[-\dot{\varepsilon} - \lambda^-\varepsilon' + \lambda^{-\prime} - i\nu_+\Lambda_- + \frac{1}{2}(\dot{\xi} + \lambda^-\xi')\right] - \frac{i}{2}\omega_-\eta_{W+}\ ,\\
\delta C_W &= C^\alpha\partial_\alpha C_W - i(\omega_-\eta_{W+} - \omega_+\eta_{W-})\ ,\\
\delta\eta_{W\pm} &= C^\alpha\partial_\alpha\eta_{W\pm} + \frac{1}{2}(C^{1\prime} \pm \lambda^\pm C^{0\prime})\eta_{W\pm} + \sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_\pm f_W\\
&\quad \pm\frac{2\omega_\mp}{\lambda^+ + \lambda^-}\{\dot{C}_W \pm \lambda^\mp C_W' + i(\nu_-\eta_{W+} - \nu_+\eta_{W-})\},\\
\delta f_W &= C^\alpha\partial_\alpha f_W + \frac{1}{2}\partial_\alpha C^\alpha f_W\\
&\quad -i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_+\left[\dot{\eta}_{W+} - \lambda^+\eta_{W+}' - \frac{1}{2}\lambda^{+\prime}\eta_{W+} - \frac{2\nu_-}{\lambda^+ + \lambda^-}(\dot{C}_W + \lambda^- C_W' - i\nu_+\eta_{W-})\right]\ ,\\
&\quad -i\sqrt{\frac{2}{\lambda^+ + \lambda^-}}\omega_-\left[\dot{\eta}_{W-} + \lambda^-\eta_{W-}' + \frac{1}{2}\lambda^{-\prime}\eta_{W-} + \frac{2\nu_-}{\lambda^+ + \lambda^-}(\dot{C}_W - \lambda^+ C_W' + i\nu_-\eta_{W+})\right]\ , \qquad \text{(A2)}
\end{aligned}
$$

where $f_{X,L,G,W}$ are defined by

$$f_{X,L,G,W} = \sqrt{e}F_{X,L,G,W}\ . \qquad \text{(A3)}$$

It is straightforward to ascertain the nilpotency of the BRST transformations.

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