Warp drive and causality

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Alcubierre recently exhibited a spacetime which, within the framework of general relativity, allows travel at superluminal speeds if matter with a negative energy density can exist, and conjectured that it should be possible to use similar techniques to construct a theory containing closed causal loops and, thus, travel backwards in time. We verify this conjecture by exhibiting a simple modification of Alcubierre's model, requiring no additional assumptions, in which causal loops are possible. We also note that this mechanism for generating causal loops differs in essential ways from that discovered by Gott involving cosmic strings. [S0556-2821(96)00412-2]

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Alcubierre in an interesting recent paper [1], hereafter referred to as MA, demonstrated, with a specific example, that it is possible ''within the framework of general relativity and without the introduction of wormholes... to modify a spacetime in a way that allows a spaceship to move with an arbitrarily large speed,'' a phenomenon often referred to as ''warp drive'' in science fiction. This possibility arises because, although in general relativity two observers at the same point cannot have a relative velocity greater than c (the speed of light, henceforth set equal to 1), the relative speed of two distant observers may be arbitrarily large, even though both are moving within their own light cones; this occurs, e.g., in a possible inflationary phase of the early Universe, where arbitrarily large speeds of separation would result from the expansion of spacetime itself.

The model in MA involves a spacetime with metric given by

$$ds^2 = -d\tau^2 = -dt^2[1-v^2 f^2(r_0)] - 2vf(r_0)\,dx\,dt + dx^2 + dy^2 + dz^2, \tag{1}$$

where

$$r_0 = [(x-x_0(t))^2 + (y-y_0)^2 + z^2]^{t/2} \tag{2}$$

and

$$v = dx_0/dt. \tag{3}$$

The function f falls off exponentially for $r_0 > R$ with some characteristic distance $\sigma \lesssim R$ and rises to 1 in a distance of order σ for $r_0 < R$, with $f(0)=1$; a suitable form of f with these properties is given in MA. The constant y_0, which we take to be appreciably greater than R, so that $f(y_0) \approx 0$, but small compared to relevant astronomical distances, is introduced in Eq. (2) for reasons of convenience which will appear later. In the limit $\sigma \to 0$, f can be thought of as a step function with

$$f(r_0)=1,\ \ r_0<R, \quad f(r_0)=0,\ \ r_0>R, \tag{4}$$

so that spacetime is flat with a Lorentzian metric outside a spherical bubble of radius R centered on the point $\vec{r}_0$ with spatial coordinates $(x_0(t), y_0, 0)$ moving with speed v parallel to the x axis. The speed v is an arbitrary function of time and need *not* satisfy $v<1$, so that the bubble may attain arbitrarily large superluminal speeds. From (1), (3), and (4) it follows that a locally inertial coordinate system at the point $\vec{r}_0$ is obtained simply by replacing x by $x - x_0(t)$. Thus an observer on a spaceship with position given by $\vec{r}(t) = \vec{r}_0(t)$ is in free fall, moving along a timelike geodesic with proper time equal to the coordinate time t; hence, along the world line of such a ship, $\int d\tau = \Delta t$, the change in the coordinate time, and physical clocks on the ship will read the coordinate time t. Since t is also the proper time of observers at rest in the region outside the bubble, where space is flat, the spaceship experiences no time dilation with respect to such observers.

The possibility of superluminal speeds raises the question of whether closed timelike curves (CTC's), in which the world line of an object returns to its starting point in space *and* time, can occur, along with their associated paradoxes. Although this does not happen with the metric (1), it is suggested in MA that, using similar ideas, it is quite likely that a spacetime containing CTC's could be constructed. We demonstrate below that this is indeed true, and that, should it be possible to attain superluminal speeds through the physical realization of a spacetime described by Eqs. (1)–(4), then the Lorentz invariance of flat space implies that spacetimes containing CTC's could also be realized.

As pointed out in MA, it is not at all certain that a space described by the metric (1) is physically realizable. Of course, in general relativity any metric is possible, given that the energy-momentum tensor is related to the metric by the Einstein field equations. However, the energy-momentum tensor satisfying the field equations with the metric given by (1) implies that the energy density within the bubble, as measured by some observers, will be negative, a requirement reminiscent of the case of possible stable Lorentzian wormholes. It is well known that the existence of regions with negative-energy density is allowed in quantum mechanics [2]. However, it is by no means clear that a negative-energy density can persist over macroscopic regions of spacetime [3]. In addition, the Hawking ''chronology protection conjecture,'' if correct, implies that quantum effects conspire to prevent the formation of CTC's [4]. We shall have nothing to say about these problems, but simply imagine that it is possible to create a spacetime described by Eq. (1), and explore one of the consequences.

Consider two stars S_1 and S_2 at rest in the coordinate system of Eq. (1) and located on the x axis at $x=0$ and $x=D$, respectively, with $D \gg y_0$. On the x axis the metric is Minkowskian because $y_0 \gg R$; thus a light signal emitted along the x axis at $t=0$ from S_1 will move so that $dx/dt=1$ and arrive at S_2 at $t=D$. On the other hand, consider a spaceship moving at the center of the bubble, and suppose the bubble has $v=0$ initially and undergoes, e.g., uniform acceleration a for $0<x<D/2$, and $-a$ for $D/2<x<D$. Neglecting the time required to cover the small distance from $y=0$ to $y=y_0$ and back, the spaceship will arrive at S_2 at the spacetime point with coordinates $x=D$ and $t=2\sqrt{D/a}\equiv T$. Thus if

$$a>4/D, \tag{5}$$

$D^2-T^2=D^2[1-4/(aD)]>0$, and the spaceship arrives at S_2 before a light signal emitted at the same time and moving in a straight line. Inertial observers at rest outside the bubble on S_1 and S_2 will see the motion of the bubble with the spaceship at its center as superluminal, since it covers a distance D in a time interval $T<D$. This is true even though the spaceship moves at all times within its forward light cone; setting $d\tau^2=0$ at $\vec{r}=\vec{r}_0$ in Eq. (1) one finds immediately that the two branches of the ship's forward light cone for light moving along the x axis are given by $x=(v\pm 1)t$, while the spaceship's world line is $x=vt$. The spaceship beats the light signal to S_2 not because its motion is spacelike but because, in effect, the bubble acts like a wormhole and provides a shortcut from S_1 to S_2.

Since a four-vector with components $(T,0,0,D)$ is spacelike, the temporal order of the spaceship's arrival and departure is not well defined; if we introduce a new set of primed coordinates related to the coordinates of Eq. (1) by a Lorentz boost along the x axis with velocity parameter β, the spacetime coordinates in the primed system of the spaceship's arrival at S_2 will be $x'=X'$ and $t'=T'$, where

$$X'=\gamma(D-\beta 2\sqrt{D/a}), \quad T'=\gamma(2\sqrt{D/a}-\beta D) \tag{6}$$

with $\gamma=1/(1-\beta^2)^{t/2}$. From Eq. (6) one sees that $T'<0$ provided

$$a>4/(D\beta^2). \tag{7}$$

Note that, provided (5) holds, (7) is compatible with $\beta<1$ so that outside the bubble the new coordinates are just those in a physically realizable Lorentz frame. Of course the image of the bubble will not be described by the metric of Eq. (1) in the primed coordinates; this need not however concern us since the transformation to the primed coordinates is a perfectly well defined, and in fact simply a linear, coordinate transformation. The Lorentz velocity transformation equations give

$$\frac{dx'}{dt'}=\frac{dx/dt-\beta}{1-\beta dx/dt}. \tag{8}$$

From this it follows that when v reaches the value $(1/\beta)-1$, the branch of the light cone at $\vec{r}_0$ on which $x=(v+1)t$ crosses the x' axis in the primed system, and a light signal emitted in the positive x direction from the spaceship will travel in the negative t' direction as t or x' increases. Let t_1' and t_2' be the values of t' labeling the events at which the speed of the spaceship increases and decreases, respectively, through $v=1/\beta$. [This will always occur provided (5) is satisfied, since then v reaches a maximum value $2/\beta$ at $x=D/2$.] In the time interval between t_1' and t_2' the rocketship itself travels in the negative t' direction as t and x' increase; i.e., $t_2'<t_1'$. For observers at rest outside the bubble in the primed system, $d\tau=dt'$ and hence the reading of physical clocks, including, presumably, the amount of information stored in a human memory, will increase with increasing coordinate time t'. To such observers, ''living forward in t','' it will actually appear that two spaceships, moving initially at infinite speed [from the denominator in Eq. (8)] in opposite directions along the x' axis appear suddenly at $t'=t_2'$. The one moving in the positive direction will eventually reach S_2 at $x'=X'$; the one moving in the negative direction will appear to merge at $t'=t_1'$ with a spaceship originating at S_1 at $x=0$ and disappear. Since $d\tau=dt$ on the spaceship, the reading of physical clocks at rest on the spaceship at the center of the bubble, will increase with increasing t and hence decrease with increasing t'. (The foregoing discussion is reminiscent of the ''reinterpretation principle'' introduced a number of years ago by Bilaniuk, Deshpande, and Sudarshan [5] which played an important role in discussions of the physics of tachyons. To observers outside the bubbles a theory in which bubbles of curved space moving with superluminal velocity exist would appear very similar to a theory containing physical tachyons, i.e., physical particles whose speeds are always superluminal; such theories are well known to involve causal loops [6], the closed loops being tachyon world lines which are spacelike. The theory in MA differs, however, from one with tachyons in that the world lines of all objects are timelike, even within a bubble with $v>1$, since the local speed of the objects is less than c; hence causal loops can occur only if there are CTC's.)

The fact that the spaceship arrives at S_2 at $t'<0$ does not by itself lead to the possibility of closed causal loops. These will not occur in the space described by Eq. (1), since for that space the forward light cone of an event at $t=t_0$ includes only events with $t>t_0$; signals can only be sent in one direction in t. However, as we have seen, one can find a solution in which signals can be sent backward in t by taking a solution of the form of Eq. (1) in a coordinate system related to the original one by Lorentz transformation. In particular, consider a metric obtained from that in Eq. (1) by replacing x and t by $\Delta x'=x'-X'$ and $\Delta t'=t'-T'$, respectively, $v(t)$ by $-v'(t')$, a by $-a'$, and y_0 by $-y_0$. This metric will thus describe a spacetime in which a bubble of curved space originates at $t'=T'$, and moves along the line $y=-y_0$, $z=0$ from S_2 to S_1 with speed $v'(t')$ which increases uniformly during the first half of the trip and decreases uniformly during the second half with acceleration of magnitude a'; to observers outside the bubble and at rest in the primed coordinate system this bubble, which we will refer to as the primed bubble, will appear identical to the bubble of Eq. (1)

as seen in the original coordinate system, apart from the change of origin and direction of motion and possibly a different choice of the acceleration. Since the two coordinate systems are equivalent by Lorentz invariance (apart from the presumably negligible perturbation arising from the stars' being at rest in one system and moving with speed β in the negative x direction in the other), if the spacetime of Eq. (1) is physically realizable, so is the corresponding spacetime containing the primed bubble. In the new metric we will have, by analogy with our previous discussion, $d\tau = dt'$; physical clocks on a spaceship at the center of the primed bubble will read the coordinate time t', and the spaceship will arrive at S_1 after temporal and spatial intervals $\Delta t' > 0$ and $\Delta x' < 0$, respectively, in the primed system. However, by an argument precisely analogous to that given above, the sign of the time interval between the departure of the spaceship from S_2 and its arrival at S_1 will not be invariant, provided a' is large enough to satisfy the analog of (5). In fact, as we proceed to show, as x and x' decrease and t' increases, t decreases, and the spaceship will arrive at S_1 at $t < T$, and, in fact, can arrive at $t < 0$.

To simplify the calculation, consider an approximation in which a and a', and consequently the average values of v and v', are very large, in the sense that $T \ll D$ and $\Delta t' \ll -\Delta x'$; since we wish only to demonstrate that CTC's are in principle possible in the model, it suffices to consider this limit, since if $v > 1$ is realizable at all, one expects that there are no limits of principle on the possible values of v or a. From Eq. (6) one sees that in this limit $T \approx 0$, i.e., the travel of the bubble from S_1 to S_2 is approximately instantaneous, and $X' \approx \gamma D$, $T' \approx -\gamma\beta D$. We wish to find the value of t, which we denote by T_1, when the spaceship at rest in the primed bubble arrives at S_1, i.e., the value of t when $x = 0$ for the primed bubble. Again using the assumption that the accelerations are large, so that $\Delta t' \approx 0$ and $\Delta t = T_1 - T \approx T_1$, we have $\Delta x = -D \approx \gamma \Delta x'$ and $\Delta t \approx \gamma\beta\Delta x'$ from which

$$T_1 \approx -\beta D < 0 \tag{9}$$

so that T_1 can be negative and large. Note that in the limit that a and a' are both large the result is independent of their values.

Finally, imagine a spacetime with metric containing both the bubble of Eq. (1) moving forward in t but backward in t' from S_1 to S_2 along the line $y = y_0$, and the primed bubble, moving forward in t' but backward in t from S_2 to S_1 along $y = -y_0$. Explicitly, if we write the metric of Eq. (1) as $g_{\mu\nu} \equiv \eta_{\mu\nu} + \hbar_{\mu\nu}$ and the metric, in the primed coordinates, of the space containing only the primed bubble as $g'_{\mu\nu} \equiv \eta'_{\mu\nu} + k'_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Lorentz metric, then we take the metric, in the unprimed coordinates, of the space with both bubbles to be given by $G_{\mu\nu}$, where

$$G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + k_{\mu\nu}. \tag{10}$$

The deviations of $G_{\mu\nu}$ from Lorentzian form are the sum of those due to the unprimed and primed bubbles, and $G_{\mu\nu}$ manifestly varies smoothly across the x axis. Since $y_0 \gg R$ and $f(r)$ can be chosen to vanish arbitrarily rapidly for $r > R$, the overlap of the two bubbles can be made as small as desired, even when they pass each other, and hence the energy-momentum tensor obtained from $G_{\mu\nu}$, corresponding to the existence of the two bubbles, can be made arbitrarily close to the sum of the energy-momentum tensors calculated from the metrics of the two bubbles separately. Hence if a spacetime of the type discussed in MA can be realized, it appears there should be no additional questions of principle in creating the spacetime under discussion here containing the primed bubble in addition.

In the space described by Eq. (10) there is a region within one of the bubbles, both in the primed and unprimed coordinates, in which the "forward" light cone runs backward in time, and CTC's occur. Suppose a passenger leaves S_1 on a spaceship traveling in the unprimed bubble, starting at $t = t' = 0$, moving along the line $y = y_0$, and arriving at S_2 at $t \approx 0$, $t' = T' \approx -\gamma BD$, at which point the spaceship has come to rest and the bubble has disappeared. He now travels the short distance to $y = -y_0$ at subluminal speed through flat space, and accelerates to sublight velocity β, so as to be at rest in the primed coordinates. [We assume that this process requires negligible time since $y_0 \ll D$ and, from Eq. (7), β may be small when a is large.] The passenger can then board a second spaceship bubble and travels back to S_1 in the primed bubble, arriving at $t = T_1 = -\beta D$, thus arriving home before starting by a macroscopic time interval and carrying a newspaper reporting on events which have not yet occurred. This, of course, raises the problems with paradoxes always associated with closed causal loops. It would appear possible, e.g., to arrange a mechanism which ensures that a spaceship will depart from S_1 at $t = 0$ if and only if no news of such an event has arrived from S_2 at $t < 0$. This does not mean that a model of the type introduced in MA is ruled out as being logically inconsistent, but it does mean that in such a model there are restrictions placed on the initial conditions. That is, apparently if superluminal travel through some mechanism similar to that discussed in MA could actually be realized, it would imply that the laws of physics include a principle of consistency, as discussed by Friedman *et al.* [7], which constrains the initial conditions on spacelike surfaces at times subsequent to the creation of closed timelike curves, so as to ensure in some way that no contradiction arises; for example, the initial conditions might guarantee the failure of the mechanism by which the previous arrival of news of the spaceship's departure prevents its later departure from occurring. While not logically inconsistent, such theories appear to enforce correlations which are certainly counterintuitive.

Finally it may be useful to ask whether there is any relation between the possible mechanism for the production of CTC's discussed here, referred to hereafter as the warp drive (WD) mechanism, and the discovery by Gott [8] that CTC's encircle pairs of infinite, straight, parallel cosmic strings (or equivalently point masses in 2+1 dimensions) moving noncolinearly and having sufficiently large mass and relative speed. These two mechanisms are somewhat similar, in that in both cases CTC's arise from a situation in which two sources of gravitational disturbance move past one another on parallel, noncolinear paths. However, there are substantial differences between the two cases, arising from the different physics in the effective two spatial dimensions of the Gott mechanism and in the three-dimensional space in which the motion occurs in the WD case. The properties of (2+1)-dimensional gravity are reflected in the "angle deficit" around infinite strings, producing a missing wedge with edges identified, in the surrounding flat space [9]. A spaceship in effect jumps instantaneously from one edge of the wedge to the other (and can also jump forward or backward in time depending on the string's velocity relative to the spaceship in the reference frame in use); in this way a cosmic string located between S_1 and S_2 allows a spaceship leaving S_1 to reach S_2 before a light pulse emitted from S_1 at the same time and traveling to S_2 along a geodesic passing on the opposite side of the string, so that the spaceship effectively travels faster than light, the missing wedge playing the role of the Alcubierre bubble (or of a wormhole which could produce the same effect by connecting S_1 and S_2). In the Gott mechanism, however, the motion of the individual strings and of the spaceship is subluminal, in contrast with the WD bubbles. (However, because of the peculiarities of gravity in $2+1$ dimensions, the two-string system as a whole is somewhat analagous to a single tachyon [10], although the analogy is far from perfect [11].) An additional difference between the Gott and WD mechanisms is that, in the former the spatial region containing CTC's extends to infinity [12] and the boundary conditions at infinity are altered [10]. In contrast, in the three space dimensions of the WD case gravitational effects are localized and conditions at infinity unaffected; all CTC's must pass through points on the world tube swept out by each of the two bubbles and as a result it is easy to see that no CTC can pass through any point outside a spatial region with radius of order D surrounding the two world tubes. Hence the sets of CTC's arising from the WD and Gott mechanisms are quite different, indicating the two mechanisms differ fundamentally. One might try to extend the Gott mechanism to avoid the artifacts associated with two space dimensions by replacing infinite strings by parallel string loop segments long compared to their separation. In this case gravitational effects and CTC's would be confined to a finite region of space, as in the WD case. However, it seems likely that, in this case, a singularity will be formed and the region of space containing the CTC's will be hidden behind an event horizon [8,11]. If, on the other hand, no singularity is formed, the formation of CTC's is forbidden because the system would have positive-energy density, since the string energies are positive definite and momentum and energy are additive in $3+1$ dimensions, and Hawking [4] has shown that CTC's cannot be created, even classically, in an asymptotically flat and singularity-free spacetime if the weak energy condition is not violated.

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