Anomalous Ward identities in *D*-dimensional conformal theory

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Anomalous Ward identities for four-point Green functions containing conserved operators (currents and the energy-momentum tensor) in D-dimensional conformal field theory are obtained. They contain operator anomalous terms generalizing the central charge in two dimensions.

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In Refs. [1,2] the principal conditions which define the class of exactly solvable models in D-dimensional space were formulated. The solutions to some of these models were found in [2]. The defining conditions are represented by the following two model-independent statements.

(1) The following conformal field algebra exists:

$$\Phi_i\Phi_k=\sum_m [\Phi_m], \tag{1}$$

where $[\Phi_m]$ is an infinite set of conformal fields.

(2) Locally conserved fields exist, namely, stress-energy tensor and currents, which define an internal symmetry of the theory:

$$T_{\mu\nu}(x), \quad j_\mu(x). \tag{2}$$

These fields also belong to the algebra (1).

As shown in [2], under these assumptions the Hilbert space of the theory represents an infinite direct sum of mutually orthogonal subspaces, each space being generated by one of the fields from the algebra (1).

In a Hilbert space there exists a special subspace $H_0\subset H$ generated by the fields (2). This subspace is also a direct sum of subspaces related to a special set of fields P_S. The latter [1,3] have transformation properties similar to those of secondary fields in two-dimensional conformal theory. The subspace H_0 is called below the dynamical sector of a Hilbert space H. Each dynamical model is defined by a set of consistent (regarding a given system of Ward identities) constraints [1,2] on the states of the dynamical sector. In works [4–6] these constraints were formulated for the simplest two- and four-dimensional models. In a $D=2$ case they reduce [7–9] to the requirements of vanishing of the null vectors which define the known exactly solvable models [10–12]. In recent work [2] the solutions of nontrivial three- and four-dimensional models were discussed. This paper is aimed at the calculation of anomalous terms in Ward identities which define the dynamical sector.

In work [1] it was shown that the scalar fields

$$P_j(x) \text{ and } P_T(x) \tag{3}$$

with dimensions

$$d_{P_j}=d_{P_T}=D-2$$

are the analogues of a central charge in D dimensions. One of these fields, P_j, contributes to the Ward identity for the Green functions which include two or more conserved currents, while the other, P_T, contributes to analogous identities for the stress-energy tensor. When $D=2$, these fields become constants:

$$P_j|_{D=2}=\text{const.} \quad P_T|_{D=2}=\text{ const.} \tag{4}$$

Depending on the choice of anomalous terms in the Ward identities (for $D>2$), different realizations of the dynamical sector of a Hilbert space arise, thus leading to different definitions of the classes of exactly solvable models. In other words, every class of exactly solvable models correspond to a definite set of anomalous terms in the Ward identities. Here we restrict ourselves to the calculation of C-number anomalous terms and anomalous corrections to a stress-energy tensor.

For the sake of simplicity we examine the theory of a charged scalar field $\varphi(x)$ with a scale dimension d. Internal symmetry (for the same reason) is assumed to be Abelian.

The below results are based on an assumption that the operator product expansions for the fields (2) have the forms.

$$j_\mu(x_1)j_\nu(x_2)=[C_j]+[P_j]+[T_{\mu\nu}]+\cdots, \tag{5}$$

$$T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)=[C_T]+[P_T]+[T_{\mu\nu}]+\cdots, \tag{6}$$

$$j_\mu(x_1)T_{\rho\sigma}(x_2)=[j_\nu]+[P]+\cdots, \tag{7}$$

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where $[C_j]$ and $[C_T]$ stand for C-number contributions to the operator products. Constants C_j and C_T define normalizations for the $\langle j_\mu(x_1) j_\nu(x_2)\rangle$ and $\langle T_{\mu\nu}(x_1) T_{\rho\sigma}(x_2)\rangle$ propagators. As it is shown below, C-number terms contribute to the Ward identities in even-dimension spaces only.

Based on these assumptions we consider anomalous Ward identities for the following four-point Green functions:

$$\langle j_\mu(x_1) j_\nu(x_2) \varphi(x_3) \varphi^\dagger(x_4)\rangle, \tag{8}$$

$$\langle T_{\mu\nu}(x_1) T_{\rho\sigma}(x_2) \varphi(x_3) \varphi^\dagger(x_4)\rangle, \tag{9}$$

$$\langle j_\mu(x_1) T_{\rho\sigma}(x_2) \varphi(x_3) \varphi^\dagger(x_4)\rangle. \tag{10}$$

It is known that the form of Ward identities for the Green functions containing a conserved operator (such as the current or stress-energy tensor) is ruled by the form of equal-time commutation relations between the time component of this operator and the other operators entering the Green function. These commutators include two types of terms. The terms of the first kind are fixed by the property that the time components of conserved operators represent the densities of corresponding generators (charge, translation, Lorentz boosts, etc.). The terms of the second kind are the Schwinger terms; both the C-number and the operator quantities may occur in their expressions. What does actually define the model is the choice of the specific Schwinger terms. Since at the start the only functions under consideration are the covariant Green functions, the contributions of both kinds of terms into the Ward identity should be put into covariant form. Let us call the covariant contributions due to the terms of the first kind the *ordinary* contributions, while those caused by the terms of the second kind–the *anomalous* ones. We discuss the Schwinger terms resulting from the C-number and the operator contributions into the expansions (5)–(7).

A regular way to derive conformally invariant Ward identities for functions (8)–(10) is, at first, to write down all the possible Lorentz-invariant structures with suitable scale parameters, and then, using the requirements of the invariance under special conformal transformations, to find out the relations between the coefficients before these structures. In the Appendix we present an application of the above procedure to the derivation of a Ward identity for the function (10). The goal of the paper will be achieved after one manages to make use of this method for the other functions concerned. The final result of such analysis for functions (8)–(10) are given below

The Ward identity for the Green function $\langle j_\mu(x_1) j_\nu(x_2) \varphi(x_3) \varphi^\dagger(x_4)\rangle$ reads

$$\begin{aligned}\partial_\mu^{x_1}\langle j_\mu(x_1) j_\nu(x_2) \varphi(x_3) \varphi^\dagger(x_4)\rangle &= -e[\delta(x_1-x_3)-\delta(x_1-x_4)]\langle j_\nu(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle \\ &\quad + c_j \partial_\nu^{x_1}\Box^{(D-2)/2}\delta(x_1-x_2)\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle \\ &\quad + \partial_\nu^{x_1}\delta(x_1-x_2)\langle P_j^{D-2}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle, \end{aligned}\tag{11}$$

where e is the charge of a scalar field $\varphi(x)$,

$$\langle j_\mu(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle = e\Gamma\left(\frac{D}{2}\right)/2\pi\frac{D}{2}\lambda_\mu^{x_2}(x_3x_4)\left(\frac{x_{34}^2}{x_{23}^2x_{24}^2}\right)^{(D-2)/2}\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle, \tag{12}$$

where $\lambda_\mu^{x_2}(x_3x_4) = \frac{(x_3-x_2)_\mu}{x_{32}^2} - \frac{(x_4-x_2)_\mu}{x_{42}^2}$,

$$\lambda_{\rho\sigma}^{x_2}(x_3x_4) = \lambda_\rho^{x_2}(x_3x_4)\lambda_\sigma^{x_2}(x_3x_4) - \frac{1}{D}\delta_{\rho\sigma}\frac{x_{34}^2}{x_{23}^2x_{24}^2},$$

$$\langle P_j^{D-2}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle = g_j^P\left(\frac{x_{34}^2}{x_{23}^2x_{24}^2}\right)^{(D-2)/2} \times\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle \tag{13}$$

and $\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle = (x_{34}^2)^{-d}$ is the scalar field propagator.

Anomalous terms in this identity contain two free parameters

$$c_j \text{ and } g_j^P. \tag{14}$$

For odd D's one should set

$$c_j = 0, \quad D \text{ odd}.$$

The Ward identity for the Green function $\langle T_{\mu\nu}(x_1) T_{\rho\sigma}(x_2) \varphi(x_3) \varphi^\dagger(x_4)\rangle$ reads

$$\begin{aligned}\partial_\mu^{x_1}\langle T_{\mu\nu}(x_1) T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle = &-\left\{\delta(x_1-x_3)\partial_\nu^{x_3} - \frac{d}{D}\partial_\nu^{x_1}\delta(x_1-x_3) + \delta(x_1-x_4)\partial_\nu^{x_4} - \frac{d}{D}\partial_\nu^{x_1}\delta(x_1-x_4)\right\} \\ &\times\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle - \left\{\delta(x_1-x_2)\partial_\nu^{x_2} - \left(\frac{D-2}{D} - \frac{8}{D}a\right)\partial_\nu^{x_1}\delta(x_1-x_2)\right\} \\ &\times\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle + (1+2a)\left\{\delta_{\tau\sigma}\partial_\rho^{x_1}\delta(x_1-x_2) + \delta_{\tau\rho}\partial_\sigma^{x_1}\delta(x_1-x_2)\right. \\ &\left. - \frac{2}{D}\delta_{\rho\sigma}\partial_\tau^{x_1}\delta(x_1-x_2)\right\}\langle T_{\nu\tau}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle + a\,\partial_\tau^{x_1}\delta(x_1-x_2) \\ &\times\left\{\delta_{\nu\rho}\delta_{\lambda\sigma} + \delta_{\nu\sigma}\delta_{\lambda\rho} - \frac{2}{D}\delta_{\rho\sigma}\delta_{\nu\lambda}\right\}\langle T_{\tau\lambda}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle \end{aligned}\tag{15}$$

$$+c_T\Bigg[\partial_\nu^{x_1}\partial_\rho^{x_1}\partial_\sigma^{x_1}-\frac{D-1}{2D}(\delta_{\nu\rho}\partial_\sigma^{x_1}+\delta_{\nu\sigma}\partial_\rho^{x_1})\Box^{x_1}$$
$$-\frac{1}{D^2}\delta_{\rho\sigma}\partial_\nu^{x_1}\Box^{x_1}\Bigg]\Box^{(D-2)/2}\delta(x_1-x_2)\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle$$
$$+P_{\nu;\rho\sigma}(\delta(x_1-x_2);\partial^{x_2})\langle P_T^{D-2}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle,$$

where

$$\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle=-\frac{D}{D-1}d\frac{\Gamma\left(\frac{D}{2}\right)}{2\pi^{D/2}}\lambda_{\rho\sigma}^{x_2}(x_3x_4)\left(\frac{x_{34}^2}{x_{23}^2x_{24}^2}\right)^{(D-2)/2}\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle, \tag{16}$$

$$\langle P_T^{D-2}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle=g_T^P\left(\frac{x_{34}^2}{x_{23}^2x_{24}^2}\right)^{(D-2)/2}\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle. \tag{17}$$

The $P_{\nu;\rho\sigma}(\delta(x_1-x_2);\partial^{x_2})$ operator has the form

$$P_{\nu;\rho\sigma}(\delta(x_1-x_2);\partial^{x_2})=-\Bigg\{(D-2)(1+b)\partial_\rho^{x_1}\partial_\sigma^{x_1}\partial_\nu^{x_1}\delta(x_1-x_2)+(b+2)(\delta_{\rho\tau}\partial_\sigma^{x_1}+\delta_{\sigma\tau}\partial_\rho^{x_1})\partial_\nu^{x_1}(x_1-x_2)\partial_\tau^{x_2}$$
$$-(bD+D+2)\partial_\rho^{x_1}\partial_\sigma^{x_1}\delta(x_1-x_2)\partial_\nu^{x_2}-\frac{1}{D-2}\left(Db+\frac{D^2+2D-2}{D-1}\right)(\delta_{\nu\rho}\partial_\sigma^{x_1}+\delta_{\nu\sigma}\partial_\rho^{x_1})\delta(x_1-x_2)\Box^{x_1}$$
$$+\frac{D-2}{2}(\delta_{\nu\rho}\partial_\sigma^{x_1}+\delta_{\nu\sigma}\partial_\rho^{x_1})\Box^{x_1}\delta(x_1-x_2)+\Box^{x_1}\delta(x_1-x_2)(\delta_{\nu\rho}\partial_\sigma^{x_2}+\delta_{\nu\sigma}\partial_\rho^{x_2})$$
$$-\frac{D}{2}(\delta_{\nu\sigma}\partial_\rho^{x_1}+\delta_{\nu\rho}\partial_\sigma^{x_1})\partial_\tau^{x_1}\delta(x_1-x_2)\partial_\tau^{x_2}-\frac{D}{2(D-1)}(\delta_{\rho\tau}\partial_\sigma^{x_1}+\delta_{\sigma\tau}\partial_\rho^{x_1})\delta(x_1-x_2)\partial_\nu^{x_2}\partial_\tau^{x_2}$$
$$+\frac{1}{D-1}\partial_\nu^{x_1}\delta(x_1-x_2)\partial_\rho^{x_2}\partial_\sigma^{x_2}-\frac{D}{2(D-1)}\partial_\tau^{x_1}\delta(x_1-x_2)(\delta_{\nu\rho}\partial_\sigma^{x_2}+\delta_{\nu\sigma}\partial_\rho^{x_2})\partial_\tau^{x_2}-\text{traces in } r,\sigma.\Bigg\} \tag{18}$$

Anomalous terms in the Ward identity (15) contain four free parameters:

$$c_T,a,b,g_T^P. \tag{19}$$

For odd D's one should set

$$c_T=0,\quad D\ \text{odd}.$$

The Ward identities for the Green function $\langle j_\mu(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle$ reads

$$\partial_\mu^{x_1}\langle j_\mu(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle=-e[\delta(x_1-x_3)-\delta(x_1-x_4)]\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle+(1+f)\Bigg\{\delta_{\sigma\tau}\partial_\rho^{x_1}\delta(x_1-x_2)$$
$$+\delta_{\rho\tau}\partial_\sigma^{x_1}\delta(x_1-x_2)-\frac{2}{D}\delta_{\rho\sigma}\partial_\tau^{x_1}\delta(x_1-x_2)\Bigg\}\langle j_\tau(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle$$
$$+2\Bigg\{\left(\partial_\rho^{x_1}\partial_\sigma^{x_1}-\frac{1}{D}\delta_{\rho\sigma}\Box^{x_1}\right)\delta(x_1-x_2)+\frac{1}{D-2}\Bigg(\partial_r^{x_1}\delta(x_1-x_2)\partial_\sigma^{x_2}+\partial_\sigma^{x_1}\delta(x_1-x_2)\partial_\rho^{x_2}$$
$$-\frac{2}{D}\delta_{\rho\sigma}\partial_\tau^{x_1}\delta(x_1-x_2)\partial_\tau^{x_2}\Bigg)\Bigg\}\langle P^{D-2}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle, \tag{20}$$

$$\partial_\rho^{x_2}\langle j_\mu(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi^\dagger(x_4)\rangle$$

$$= -\left\{\delta(x_2-x_3)\partial_\sigma^{x_3} - \frac{d}{D}\partial_\sigma^{x_2}\delta(x_2-x_3) + \delta(x_2-x_4)\partial_\sigma^{x_4} - \frac{d}{D}\partial_\sigma^{x_2}\delta(x_2-x_4)\right\}\langle j_\mu(x_1)\varphi(x_3)\varphi^\dagger(x_4)\rangle$$
$$-\left\{\delta(x_2-x_1)\partial_\sigma^{x_1} - \left(\frac{D-2}{D} - \frac{2}{D}f\right)\partial_\sigma^{x_2}\delta(x_2-x_1)\right\}\langle j_\mu(x_1)\varphi(x_3)\varphi^\dagger(x_4)\rangle + (1+f)\partial_\mu^{x_2}\delta(x_2-x_1)\langle j_\sigma(x_1)\varphi(x_3)\varphi^\dagger(x_4)\rangle$$
$$+ f\delta_{\mu\sigma}\partial_\tau^{x_2}\delta(x_2-x_1)\langle j_\tau(x_1)\varphi(x_3)\varphi^\dagger(x_4)\rangle + \left\{-\frac{D-2}{D}\partial_\mu^{x_2}\partial_\sigma^{x_2}\delta(x_2-x_1) - \delta_{\mu\sigma}\Box^{x_2}\delta(x_2-x_1) + \partial_\mu^{x_2}\delta(x_2-x_1)\partial_\sigma^{x_1}\right.$$
$$\left. - \frac{2}{D-2}\partial_\sigma^{x_2}\delta(x_2-x_1)\partial_\mu^{x_1} + \frac{D}{D-2}\delta_{\mu\sigma}\partial_\tau^{x_2}\delta(x_2-x_1)\partial_\tau^{x_1}\right\}\langle P^{D-2}(x_1)\varphi(x_3)\varphi^\dagger(x_4)\rangle, \tag{21}$$

where

$$\langle P^{D-2}(x_1)\varphi(x_3)\varphi^\dagger(x_4)\rangle = g^P\left(\frac{x_{34}^2}{x_{13}^2 x_{14}^2}\right)^{(D-2)/2}\langle\varphi(x_3)\varphi^\dagger(x_4)\rangle. \tag{22}$$

Anomalous terms in the Ward identities (20) and (21) contain two free parameters:

$$f, g^P. \tag{23}$$

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APPENDIX

As already noted above, a regular way to derive conformally invariant Ward identities consists in the following: at first, all the possible Lorentz-invariant structures with suitable scale parameters are written down, and then, from the requirements of the invariance under special conformal transformations, the relations between the coefficients before these structures are obtained. However, in a number of cases, instead of requirement of the invariance under special conformal transformations, it proves useful to require the invariance under inversion transformations R, since

$$K_\mu = RP_\mu R, \quad Rx_\mu = \frac{x_\mu}{x^2},$$

where P_μ and K_μ are the generators of translations and special conformal transformations.

The fields transformation law under R transformation has the form

$$\varphi(x) \xrightarrow{R} \varphi'(x) = (x^2)^{-d}\varphi(Rx) \tag{A1}$$

for the scalar field φ with the scale dimension d,

$$j_\mu(x) \xrightarrow{R} j'_\mu(x) = (x^2)^{-(D-1)}g_{\mu\nu}(x)j_\nu(Rx) \tag{A2}$$

with $g_{\mu\nu}(x) = \delta_{\mu\nu} - 2x_\mu x_\nu/x^2$ for the conserved current with the scale dimension $D-1$, and

$$T_{\mu\nu}(x) \xrightarrow{R} T'_{\mu\nu}(x) = (x^2)^{-D}g_{\mu\rho}(x)g_{\nu\sigma}(x)T_{\rho\sigma}(Rx) \tag{A3}$$

for the conserved stress-energy tensor with the scale dimension D.

The divergence of the conserved current transforms in a way similar to the transformation of the scalar field with dimension D:

$$U(R)\partial_\mu^x j_\mu(x)U^{-1}(R) = \partial_\mu^x\{(x^2)^{-(D-1)}g_{\mu\nu}(x)j_\nu(Rx)\} = (x^2)^{-(D-1)}g_{\mu\nu}(x)j_\nu(Rx) = (x^2)^{-D}\partial_\nu^{Rx}j_\nu(Rx),$$

which is proved with the help of the identities

$$\partial_\mu^x\{(x^2)^{-(D-1)}g_{\mu\nu}(x)\} = 0, \qquad \partial_\nu^{Rx} = x^2 g_{\nu\mu}(x)\partial_\mu^x. \tag{A4}$$

By analogy, one can check that the divergence of the stress-energy tensor transforms similarly to the vector field of dimension $D+1$:

$$U(R)\partial_\mu^x T_{\mu\nu}(x)U^{-1}(R)=(x^2)^{-(D+1)}g_{\nu\lambda}(x)\partial_\mu^{Rx}T_{\mu\nu}(Rx). \tag{A5}$$

As an example of action of the method let us apply it to the derivation of Eq. (15).

Taking into account Eqs. (A1), (A3), and (A5) one may find the invariance condition for the divergence of the Green function $\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$ under R transformation:

$$\partial_\mu^{x_1}\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$
$$=(x_1^2)^{-(D+1)}(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\nu\tau}(x_1)g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_3)\partial_\mu^{Rx_1}\langle T_{\mu\tau}(Rx_1)T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle. \tag{A6}$$

The most general Lorentz and scale-invariant expression for the divergence of the function $\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$ may be represented in the form

$$\partial_\mu^{x_1}\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$
$$=-\{\delta(x_1-x_3)\partial_\nu^{x_3}+b\partial_\nu^{x_1}\delta(x_1-x_3)+\delta(x_1-x_4)\partial_\nu^{x_4}+b\partial_\nu^{x_1}\delta(x_1-x_4)\}\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$
$$+\Bigg\{-[\delta(x_1-x_2)\partial_\nu^{x_2}-2a_1\partial_\nu^{x_1}\delta(x_1-x_2)]\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$
$$+2a_2\left(\delta_{\rho\lambda}\delta_{\sigma\tau}+\delta_{\sigma\lambda}\delta_{\rho\tau}-\frac{2}{D}\delta_{\rho\delta}\delta_{\tau\lambda}\right)\partial_\lambda^{x_1}\delta(x_1-x_2)\langle T_{\nu\tau}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$
$$+2a_3\left(\delta_{\nu\rho}\delta_{\lambda\sigma}+\delta_{\nu\sigma}\delta_{\lambda\rho}-\frac{2}{D}\delta_{\rho\sigma}\delta_{\nu\lambda}\right)\partial_\tau^{x_1}\delta(x_1-x_2)\langle T_{\lambda\tau}(x_2)\varphi(x_3)\varphi(x_4)\rangle\Bigg\}$$
$$-\Bigg\{c_1\partial_\nu^{x_1}\delta(x_1-x_2)\left(\partial_\rho^{x_2}\partial_\sigma^{x_2}-\frac{1}{D}\delta_{\rho\sigma}\Box^{x_2}\right)+c_2\partial_\lambda^{x_1}\delta(x_1-x_2)\left(\delta_{\lambda\rho}\partial_\nu^{x_2}\partial_\sigma^{x_2}+\delta_{\lambda\sigma}\partial_\nu^{x_2}\partial_\rho^{x_2}-\frac{2}{D}\delta_{\rho\sigma}\partial_\nu^{x_2}\partial_\lambda^{x_2}\right)$$
$$+c_3\partial_\lambda^{x_1}\delta(x_1-x_2)\left(\delta_{\nu\rho}\partial_\sigma^{x_2}+\delta_{\nu\sigma}\partial_\rho^{x_2}-\frac{2}{D}\delta_{\rho\sigma}\partial_\nu^{x_2}\right)\partial_\lambda^{x_2}+c_4\partial_\lambda^{x_1}\delta(x_1-x_2)\left(\delta_{\nu\rho}\delta_{\sigma\tau}+\delta_{\nu\sigma}\delta_{\rho\tau}-\frac{2}{D}\delta_{\rho\sigma}\delta_{\nu\tau}\right)\Box^{x_2}$$
$$+e_1\left(\partial_\rho^{x_1}\partial_\sigma^{x_1}-\frac{1}{D}\delta_{\rho\sigma}\Box^{x_1}\right)\delta(x_1-x_2)\partial_\nu^{x_2}+e_2\left(\delta_{\rho\lambda}\partial_\sigma^{x_1}+\delta_{\sigma\lambda}\partial_\rho^{x_1}-\frac{2}{D}\delta_{\rho\sigma}\partial_\lambda^{x_1}\right)\partial_\nu^{x_1}\delta(x_1-x_2)\partial_\lambda^{x_2}$$
$$+e_3\left(\delta_{\nu\rho}\partial_\sigma^{x_1}+\delta_{\nu\sigma}\partial_\rho^{x_1}-\frac{2}{D}\delta_{\rho\sigma}\partial_\nu^{x_1}\right)\partial_\lambda^{x_1}\delta(x_1-x_2)\partial_\lambda^{x_2}+e_4\Box^{x_1}\delta(x_1-x_2)\left(\delta_{\nu\rho}\delta_{\sigma\lambda}+\delta_{\nu\sigma}\delta_{\rho\lambda}-\frac{2}{D}\delta_{\rho\sigma}\delta_{\nu\lambda}\right)\partial_\lambda^{x_2}$$
$$+f_1\left(\partial_\rho^{x_1}\partial_\sigma^{x_1}-\frac{1}{D}\delta_{\rho\sigma}\Box^{x_1}\right)\partial_\nu^{x_1}\delta(x_1-x_2)+f_2\left(\delta_{\nu\rho}\partial_\sigma^{x_1}+\delta_{\nu\sigma}\partial_\rho^{x_1}-\frac{2}{D}\delta_{\rho\sigma}\partial_\nu^{x_1}\right)\Box^{x_1}\delta(x_1-x_2)\Bigg\}\langle P_T^{D-2}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$
$$+c_T\Bigg\{\Bigg[\left(\partial_\rho^{x_1}\partial_\sigma^{x_1}-\frac{1}{D}\delta_{\rho\sigma}\Box^{x_1}\right)\partial_\nu^{x_1}+q\left(\delta_{\nu\rho}\partial_\sigma^{x_1}+\delta_{\nu\sigma}\partial_\rho^{x_1}-\frac{2}{D}\delta_{\rho\sigma}\partial_\nu^{x_1}\right)\Box^{x_1}\Bigg]\Box^{x_1}_{(D-2)/2}\delta(x_1-x_2)\Bigg\}\langle\varphi(x_3)\varphi(x_4)\rangle. \tag{A7}$$

Here the only Schwinger terms in the commutator of the components of stress-energy tensor taken into account are the C-number and the $P_T^{D-2}(x)$-operator ones. Several groups of terms on the right-hand side (RHS) of Eq. (A7) must satisfy Eq. (A6) separately from each other: namely, terms proportional to $\delta(x_1-x_3)$ [or $\delta(x_1-x_4)$] and its derivative; terms proportional to $\delta(x_1-x_2)$, its derivatives, and to the Green function $\langle T_{\alpha\beta}(x_2)\varphi(x_3)\varphi(x_4)\rangle$; terms proportional to the Green function $\langle P_T^{D-2}(x_2)\varphi(x_3)\varphi(x_4)\rangle$, i.e., anomalous terms induced by operator Schwinger terms; terms proportional to the propagator $\langle\varphi(x_3)\varphi(x_4)\rangle$, i.e., anomalous terms induced by C-number Schwinger terms. These terms are nonzero only in the spaces of even space-time dimensions.

In what follows we consider each group of these terms separately. For example, for the first group one should prove that[1]

[1]The same is true for the terms proportional to $\delta(x_1-x_4)$.

$$-\{\delta(x_1-x_3)\partial_\nu^{x_3}+b\partial_\nu^{x_3}\delta(x_1-x_3)\}\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle=(x_1^2)^{-(D+1)}(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\nu\tau}(x_1)g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_3)$$
$$\times\{-[\delta(Rx_1-Rx_3)\partial_\tau^{Rx_3}+b\partial_\tau^{Rx_1}\delta(Rx_1-Rx_3)]$$
$$\times\langle T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle\}. \qquad \text{(A8)}$$

Taking into account the transformation law for the Green function $\langle T_{\alpha\beta}(x_2)\varphi(x_3)\varphi(x_4)\rangle$

$$\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle=(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_3)\langle T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle$$

as well as the formulas

$$g_{\mu\tau}(x)g_{\tau\nu}(x)=\delta_{\mu\nu}, \quad \delta(Rx)=(x^2)^D\delta(x),$$

one can rewrite the RHS of (A8) as

$$\text{RHS of (A8)}=(x_1^2)^{-(D+1)}(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\nu\tau}(x_1)g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_3)$$
$$\times\{-[(x_1^2)^D\delta(x_1-x_3)x_3^2g_{\tau\lambda}(x_3)\partial_\lambda^{x_3}+bx_1^2g_{\tau\lambda}(x_1)\partial_\lambda^{x_1}((x_1^2)^D\delta(x_1-x_2))]\langle T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle\}$$
$$=(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_3)\left\{-\left[2bD\frac{(x_1)_\nu}{x_1^2}\delta(x_1-x_3)+\delta(x_1-x_3)\partial_\nu^{x_3}+b\partial_\nu^{x_1}\delta(x_1-x_3)\right]\right.$$
$$\left.\times\langle T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle\right\}$$
$$=-\left[(2bD+2d)\frac{(x_1)_\nu}{x_1^2}\delta(x_1-x_3)+\delta(x_1-x_3)\partial_\nu^{x_3}+b\partial_\nu^{x_1}\delta(x_1-x_3)\right]\{(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_2)$$
$$\times\langle T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_3)\rangle\}$$
$$=-\left\{2(bD+d)\frac{(x_1)_\nu}{x_1^2}\delta(x_1-x_3)+\delta(x_1-x_3)\partial_\nu^{x_3}+b\partial_\nu^{x_1}\delta(x_1-x_3)\right\}\langle T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle\}.$$

For the latter expression to coincide with the LHS of (A8), the first term in the braces should vanish, implying that

$$b=-\frac{d}{D}. \qquad \text{(A9)}$$

Now consider the second group in (A7). Analogous, but slightly more tedious calculations show that

$$(x_1^2)^{-(D+1)}(x_2^2)^{-D}(x_3^2)^{-d}(x_4^2)^{-d}g_{\nu\tau}(x_1)g_{\rho\alpha}(x_2)g_{\sigma\beta}(x_2)[\partial_\mu^{Rx_1}\langle T_{\mu\tau}(Rx_1)T_{\alpha\beta}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle]_{\text{second group}}$$
$$=[\partial_\mu^{x_1}\langle T_{\mu\tau}(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle]_{\text{second group}}+\left\{[-2D(1-2a_1)+8a_2+8a_3]\frac{(x_1)_\nu}{x_1^2}\delta_{\rho\alpha}\delta_{\sigma\beta}\right.$$
$$\left.+(2-4a_2+4a_3)\frac{(x_1)_\tau}{x_1^2}[(\delta_{\nu\rho}\delta_{\sigma\beta}+\delta_{\nu\sigma}\delta_{\rho\beta})\delta_{\tau\alpha}-(\delta_{\rho\tau}\delta_{\sigma\beta}+\delta_{\sigma\tau}\delta_{\rho\beta})\delta_{\nu\alpha}]\right\}\delta(x_1-x_2)\langle T_{\alpha\beta}(x_2)\varphi(x_3)\varphi(x_3)\rangle.$$
$$\text{(A10)}$$

Thus, the second group of terms will be R invariant only when

$$-2D(1-2a_1)+8a_2+8a_3=0, \quad 2-4a_2+4a_3=0.$$

The solution of the above system can be written as

$$a_1=\frac{D-2}{2D}-\frac{4}{D}a, \quad a_2=\frac{1}{2}+a, \quad a_3=a, \qquad \text{(A11)}$$

where a is a free parameter.

To derive (A10) we have used the formula

$$g_{\rho\lambda_1}(x)g_{\sigma\lambda_2}(x)\partial_\nu^x[g_{\lambda_1\alpha}(x)g_{\lambda_2\beta}(x)]=-2\left[\delta_{\alpha\beta}\left(\delta_{\nu\rho}\frac{x_\alpha}{x^2}-\delta_{\nu\alpha}\frac{x_\rho}{x^2}\right)+\delta_{\rho\alpha}\left(\delta_{\nu\sigma}\frac{x_\beta}{x^2}-\delta_{\nu\beta}\frac{x_\sigma}{x^2}\right)\right]. \qquad \text{(A12)}$$

Now let us consider the third group in (A7). Much more cumbersome algebraic calculations lead to the relations between the coefficients $c_1, c_2, \ldots, f_2$. Here we only mention the basic formulas used in this derivation:

$$\langle P_T^{D-2}(x_2)\varphi(x_3)\varphi(x_4)\rangle=(x_2^2)^{-(D-2)}(x_3^2)^{-d}(x_4^2)^{-d}\langle P_T^{D-2}(Rx_2)\varphi(Rx_3)\varphi(Rx_4)\rangle, \tag{A13}$$

$$\frac{1}{(x_1^2)^{(D+1)}}g_{\nu\omega}(x_1)\partial_{\lambda_1}^{Rx_1}\delta(Rx_1-Rx_2)=g_{\nu\omega}(x_2)g_{\lambda_1\beta_1}(x_2)\partial_{\beta_1}^{x_1}\delta(x_1-x_2)+2\left(\delta_{\nu\lambda_1}\frac{(x_2)_\omega}{x_2^2}+\delta_{\omega\lambda_1}\frac{(x_2)_\nu}{x_2^2}-\delta_{\nu\omega}\frac{(x_2)_{\lambda_1}}{x_2^2}\right)\delta(x_1-x_2), \tag{A14}$$

$$\begin{aligned}\frac{1}{(x_1^2)^{(D+1)}}g_{\nu\omega}(x_1)\partial_{\lambda_1}^{Rx_1}\partial_{\lambda_2}^{Rx_2}\delta(Rx_1-Rx_2)={}&x_2^2g_{\nu\omega}(x_2)g_{\lambda_1\beta_1}(x_2)g_{\lambda_2\beta_2(x_2)}\partial_{\beta_1}^{x_1}\partial_{\beta_2}^{x_1}\delta(x_1-x_2)\\&+2\left\{\left(\delta_{\nu\lambda_1}(x_2)_\omega+\delta_{\omega\lambda_1}(x_2)_\nu-2\frac{(x_2)_\omega(x_2)_\nu(x_2)_{\lambda_1}}{x_2^2}\right)g_{\lambda_2\tau}(x_2)+\left(\delta_{\nu\lambda_2}(x_2)_\omega\right.\right.\\&\left.\left.+\delta_{\omega\lambda_2}(x_2)_\nu-2\frac{(x_2)_\omega(x_2)_\nu(x_2)\lambda_2}{x_2^2}\right)g_{\lambda_1\tau}(x_2)+\delta_{\lambda_1\lambda_2}(x_2)_\tau g_{\nu\omega}(x_2)\right\}\partial_\tau^{x_1}\delta(x_1-x_2)\\&+2(\delta_{\nu\omega}\delta_{\lambda_1\lambda_2}-\delta_{\nu\lambda_1}\delta_{\omega\lambda_2}-\delta_{\omega\lambda_1}\delta_{\nu\lambda_2})\delta(x_1-x_2),\end{aligned} \tag{A15}$$

$$\begin{aligned}&\frac{1}{(x_1^2)^{(D+1)}}g_{\nu\omega}(x_1)\partial_{\lambda_1}^{Rx_1}\partial_{\lambda_2}^{Rx_2}\partial_{\lambda_3}^{Rx_2}\delta(Rx_1-Rx_2)\\
&=(x_2^2)^2g_{\nu\omega}(x_2)g_{\lambda_1\beta_1}(x_2)g_{\lambda_2\beta_2}(x_2)g_{\lambda_3\beta_3}(x_2)\partial_{\beta_1}^{x_1}\partial_{\beta_2}^{x_1}\partial_{\beta_3}^{x_1}\delta(x_1-x_2)+2x_2^2\{2g_{\nu\omega}(x_2)[(x_2)_{\lambda_3}g_{\lambda_1\beta_1}(x_2)g_{\lambda_2\beta_2}(x_2)\\
&\quad+(x_2)_{\lambda_2}g_{\lambda_1\beta_1}(x_2)g_{\lambda_3}\beta_2(x_2)+(x_2)_{\lambda_1}g_{\lambda_3\beta_1}(x_2)g_{\lambda_2\beta_2}(x_2)]+[\delta_{\nu\lambda_3}(x_2)_\omega+\delta_{\omega\lambda_3}(x_2)_\nu-\delta_{\nu\omega}(x_2)_{\lambda_3}]g_{\lambda_1\beta_1}(x_2)g_{\lambda_2\beta_2}(x_2)\\
&\quad+[\delta_{\nu\lambda_1}(x_2)_\omega+\delta_{\omega\lambda_1}(x_2)_\nu-\delta_{\nu\omega}(x_2)_{\lambda_1}]g_{\lambda_3\beta_1}(x_2)g_{\lambda_2\beta_2}(x_2)+[\delta_{\nu\lambda_2}(x_2)_\omega+\delta_{\omega\lambda_2})(x_2)_\nu-\delta_{\nu\omega}(x_2)_{\lambda_2}]g_{\lambda_1\beta_1}(x_2)g_{\lambda_3\beta_2}(x_2)\\
&\quad+(x_2)_{\beta_1}g_{\nu\omega}(x_2)[\delta_{\lambda_1\lambda_3}g_{\lambda_2\beta_2}(x_2)+\delta_{\lambda_2\lambda_3}g_{\lambda_1\beta_2}(x_2)+\delta_{\lambda_1\lambda_2}g_{\lambda_3\beta_2}(x_2)]\}\delta_{\beta_1}^{x_1}\delta_{\beta_2}^{x_1}\delta(x_1-x_2)\\
&\quad-2x_2^2\left\{\left[g_{\nu\lambda_1}(x_2)g_{\omega\lambda_3}(x_2)+g_{\nu\lambda_3}(x_2)g_{\omega\lambda_1}(x_2)-2\delta_{\lambda_1\lambda_3}\frac{(x_2)_\omega(x_2)_\nu}{x_2^2}\right]g_{\lambda_2\tau}(x_2)\right.\\
&\quad+\left[g_{\nu\lambda_2}(x_2)g_{\omega\lambda_3}(x_2)+g_{\nu\lambda_3}(x_2)g_{\omega\lambda_2}(x_2)-2\delta_{\lambda_2\lambda_3}\frac{(x_2)_\omega(x_2)_\nu}{x_2^2}\right]g_{\lambda_1\tau}(x_2)\\
&\quad+\left[g_{\nu\lambda_1}(x_2)g_{\omega\lambda_2}(x_2)+g_{\nu\lambda_2}(x_2)g_{\omega\lambda_1}(x_2)-2\delta_{\lambda_1\lambda_2}\frac{(x_2)_\omega(x_2)_\nu}{x_2^2}\right]g_{\lambda_3\tau}(x_2)\\
&\quad-2\frac{(x_2)_\tau}{x_2^2}\left[(x_2)_\omega(\delta_{\nu\lambda_1}\delta_{\lambda_2\lambda_3}+\delta_{\nu\lambda_2}\delta_{\lambda_1\lambda_3}+\delta_{\nu\lambda_3}\delta_{\lambda_1\lambda_2})+(x_2)_\nu(\delta_{\omega\lambda_1}\delta_{\lambda_2\lambda_3}+\delta_{\omega\lambda_2}\delta_{\lambda_1\lambda_3}+\delta_{\omega\lambda_3}\delta_{\lambda_1\lambda_2})\right.\\
&\quad\left.\left.-2\frac{(x_2)_\omega(x_2)_\nu}{x_2^2}[(x_2)_{\lambda_1}\delta_{\lambda_2\lambda_3}+(x_2)_{\lambda_2}\delta_{\lambda_1\lambda_3}+(x_2)_{\lambda_3}\delta_{\lambda_1\lambda_2}]\right]\right\}\partial_\tau^{x_1}\delta(x_1-x_2).\end{aligned} \tag{A16}$$

Again, resultantly we get the system of algebraic equations, having the solution which can be written in the form

$$c_1=\frac{2}{D-1},\quad c_2=-\frac{D}{2(D-1)},\quad c_3=-\frac{D}{2(D-1)},$$

$$c_4=-\frac{1}{D-2}\left(bD+\frac{D^2+2D-2}{D-1}\right),\quad e_1=-(bD+D+2),$$

$$e_2=b+2,\quad e_3=-\frac{D}{2},\quad e_4=1,$$

$$f_1=(D-2)(1+b),\quad f_2=\frac{D-2}{2}, \tag{A17}$$

where b is a free parameter.

Finally, consider the fourth group in (A7), which is proportional to the propagator of the field φ. It is evident that the factor before $\langle\varphi(x_3)\varphi(x_4)\rangle$ in (A7) should transform in the same manner as $\partial_\mu^{x_1}\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle$ under the R transformation. So our aim is to derive the expression for the divergence of the propagator of stress-energy tensor. The latter reads

$$\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle = N\Big\{ g_{\mu\rho}(x_{12})g_{\nu\sigma}(x_{12}) + g_{\mu\sigma}(x_{12})g_{\nu\rho}(x_{12}) - \frac{2}{D}\delta_{\mu\nu}\delta_{\rho\sigma}\Big\} \times (x_{12})^{-D}. \quad \text{(A18)}$$

This function is ill defined in the spaces of even dimensions. However, the matter of our concern is its divergence. So let us consider the regularized expression for the propagator

$$\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle_{\text{reg}} = N\{g_{\mu\rho}(x_{12})g_{\nu\sigma}(x_{12}) + (\rho\leftrightarrow\sigma) - \text{trace in } \rho,\sigma\}(x_{12})^{-d_T}. \quad \text{(A19)}$$

The divergence of this function equals to

$$\partial^{x_1}_{\mu}\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle_{\text{reg}} = 2N(d_T-D)\left(\frac{(x_{12})_\rho}{x_{12}^2}g_{\nu\sigma}(x_{12}) + (\rho\leftrightarrow\sigma) - \text{trace in } \rho,\sigma\right) \times (x_{12})^{-d_T}. \quad \text{(A20)}$$

Here we used the formulas

$$\partial^x_\mu\left[\frac{1}{(x^2)^l}g_{\mu\nu}(x)\right] = 2(l-D+1)\frac{x_\nu}{(x^2)^{l+1}},$$

$$g_{\rho\mu}(x)\partial^x_\mu g_{\nu\sigma}(x) = -2\left[\frac{x_\sigma}{x^2}g_{\nu\rho}(x) + \delta_{\rho\sigma}\frac{x_\nu}{x^2}\right].$$

The RHS of (A20) may be put into the form

$$\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle_{\text{reg}} = N\frac{(d_T-D)}{(d_T-1)d_T(d_T+1)}\Big\{\partial^{x_1}_\nu\partial^{x_1}_\rho\partial^{x_1}_\sigma - \frac{(d_T-1)}{4(d_T-D/2)}(\delta_{\nu\rho}\partial^{x_1}_\sigma + \delta_{\nu\sigma}\partial^{x_1}_\rho) - \text{trace in } \rho,\sigma\Big\}(x_{12}^2)^{-(d_T-1)}. \quad \text{(A21)}$$

This expression is already well defined for all dimensions D. Taking the limit $d_T\leftrightarrow D$, we get

$$\lim_{d_T\to D}\langle T_{\mu\nu}(x_1)T_{\rho\sigma}(x_2)\rangle_{\text{reg}} = \begin{cases} 0 \quad \text{for odd } D, \\ -\dfrac{\pi^{D/2}}{2^{D-2}\Gamma(D/2)\Gamma(D+2)}N\Big\{\partial^{x_1}_\nu\partial^{x_1}_\rho\partial^{x_1}_\sigma - \dfrac{D-1}{2D}(\delta_{\nu\rho}\partial^{x_1}_\sigma + \delta_{\nu\sigma}\partial^{x_1}_\rho)\Box^{x_1} - \text{trace in } \rho,\sigma\Big\}\Box^{(D-2)/2}_{x^1}\delta(x_{12}) \quad \text{for even } D. \end{cases} \quad \text{(A22)}$$

Here we used

$$\Box^k(x^2)^{-l} = 4^k\frac{\Gamma(l+k)\Gamma\left(l-\frac{D}{2}+1+k\right)}{\Gamma(l)\Gamma\left(l-\frac{D}{2}+1\right)}(x^2)^{-(l+k)}, \quad \lim_{\epsilon\to 0}\epsilon(x^2)^{-(D/2+\epsilon)} = -\frac{\pi^{D/2}}{\Gamma\left(\frac{D}{2}\right)}\delta(x).$$

Comparing this result with the RHS of (A7) one finds

$$c_T = -\frac{\pi^{D/2}}{2^{D-2}\Gamma\left(\frac{D}{2}\right)\Gamma(D+2)}N, \quad q = -\frac{D-1}{2D}. \quad \text{(A23)}$$

Substituting now expressions (A9), (A11), (A17), and (A23) into (A7) we finally get the result, which coincides with (15).

Proceeding analogously one can derive the Ward identities for all the other functions. No additional relations of the type (A14)–(A17) are necessary. The only fact which remains to be stressed is that the evident relation

$$\partial^{x_1}_\mu\partial^{x_2}_\rho\langle j_\mu(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle = \partial^{x_2}_\rho\partial^{x_1}_\mu\langle j_\mu(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$$

should be satisfied in the derivation of the Ward identities for the Green function $\langle j_\mu(x_1)T_{\rho\sigma}(x_2)\varphi(x_3)\varphi(x_4)\rangle$.

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