Spin precession of ultrarelativistic electrons in a circularly polarized electromagnetic wave

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A drastic growth in the frequency of the spin precession of ultrarelativistic electrons (positrons) in the field of a counterpropagating circularly polarized electromagnetic wave is predicted and its connection with the known effect of the rotation of γ-quantum polarization in a polarized electron target is demonstrated. It is shown that the effect considered corresponds to a correction of order $\alpha = 1/137$ to the amplitude of the coherent forward Compton scattering and cannot be described by the Bargmann-Michel-Telegdi equation which corresponds to a correction of order α^2 to this amplitude and predicts a three order of magnitude smaller precession frequency value as well as both its wrong sign and dependence on the electron energy and wave frequency. The discussed growth of the electron spin precession makes it really possible to observe this phenomenon using available high energy electron beams and superintense subpicosecond lasers. [S0556-2821(96)04710-8]

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I. INTRODUCTION

The present broad interest in the interaction of intense laser beams with ultrarelativistic electrons is connected with working out the electron-photon and photon-photon colliders [1]. Polarized laser beams make it possible to effectively polarize electrons (positrons) as well as to measure their polarization both at linear [1] and circular [2] accelerators. Modern powerful laser systems [3] allow one to obtain very intense electromagnetic fields in the focus of a laser beam. After a Lorentz magnification in the proper frame of superrelativistic electrons these fields can cause a lot of new mostly nonlinear phenomena [4,5], which soon will be possible to observe experimentally [6–9]. In this paper we describe a phenomenon of a drastic growth of the ultrarelativistic electron spin precession frequency in a counterpropagating circularly polarized electromagnetic wave (CPW) [10]. This phenomenon is intimately connected with both the nature of the anomalous magnetic moment (AMM) and the problem of the applicability and accuracy of the Bargmann-Michel-Telegdi (BMT) equation [11]. The predicted growth of the precession frequency makes it possible to observe the ultrarelativistic electron spin rotation in a counterpropagating CPW in experiments [6–9]. We will also show that though this phenomenon, in fact, is a linear one, it needs nearly the same laser power as the nonlinear ones [9] for its observation.

The absence of references concerning the ultrarelativistic electron spin precession in a CPW can, perhaps, be ascribed to the intuitive notion that this phenomenon is not large. Indeed, we will show that the BMT equation predicts a small precession frequency proportional to the AMM squared. There is, however, reason to believe that the last equation has a limited domain of applicability and a region of particle energies exists in which the spin precession frequency of ultrarelativistic electrons in a CPW, in fact, greatly exceeds that, following from the BMT equation.

This statement is grounded on the results of studies [12–15] of the γ-quantum polarization rotation in a spin-polarized electron target, the phenomenon described by the real part of the same helicity amplitude of the coherent forward scattering as the electron spin precession in a CPW. Recall that in the 1950s [16,17] a contribution to this amplitude proportional both to the γ-quantum energy and squared AMM of a scattering particle was found. We will show that it is this contribution that exactly corresponds to the precession frequency following from the BMT equation which is, indeed, too small to be observed experimentally. In the 1960s [12], however, it was found that another contribution to the spin-dependent part of the amplitude of the coherent forward Compton scattering grows much faster than the γ-quantum energy $\hbar\omega$ and at $\hbar\omega \geqslant mc^2 = 0.511$ MeV, where m is the electron mass, by nearly three orders of magnitude exceeds the contribution found in [16,17]. The corresponding growth of the rotation angle of γ-quantum polarization was soon observed experimentally [14,15].

An intimate connection of Compton and inverse Compton scattering allows us to predict a drastic growth of the spin-dependent part of the coherent forward electron scattering amplitude when the photon energy ω in the proper electron frame reaches the value comparable with mc^2. As far as the electron spin precession frequency in a CPW is proportional to this amplitude, its growth indicates that this precession frequency will also grow drastically when $\omega \geqslant mc^2$. In an arbitrary reference frame the last condition can be written in the form $x \geqslant 1$ where[1]

$$x = \frac{2kp}{m^2} = \frac{2\omega}{m} = \frac{2\omega_0 \varepsilon (1 - v\cos\theta)}{m^2} \tag{1}$$

is the parameter widely used in the theory of the inverse Compton scattering [1,2], ω_0 and ε are energies of a laser photon and electron, respectively, and $k = (\omega_0, \omega_0 \mathbf{n})$ and $p = (\varepsilon, \varepsilon \mathbf{v})$ are their four-momenta in this reference frame. Throughout the paper ω is the photon energy in the electron

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[1] We use the system of units in which $\hbar = c = 1$.

rest frame. The angle θ is constituted by the photon propagation direction $\mathbf{n}$ and electron velocity $\mathbf{v}$.

In the laboratory frame of reference in which an ultrarelativistis electron collides head on with a laser photon with the energy ω_0 Eq. (1) reads

$$x \simeq 4\varepsilon\omega_0/m^2 \simeq 0.0153\varepsilon(\mathrm{GeV})\omega_0(\mathrm{eV}) \simeq 0.019\varepsilon(\mathrm{GeV})/\lambda(\mu\mathrm{m}). \tag{2}$$

If an excimer laser with a typical wavelength $\lambda \sim (0.2\text{–}0.3)$ μm is used as is recommended in [9], the condition $x \geqslant 1$ satisfies at a readily available electron energy $\varepsilon \geqslant 10$ GeV which, thus, allows us to observe the predicted electron spin precession frequency growth in a circularly polarized laser wave.

The paper is organized as follows. The ultrarelativistic electron spin precession in a CPW is considered in terms of the BMT equation in Sec. II. The phenomenon of γ-quantum polarization rotation in a polarized electron target is outlined in Sec. III. Its connection with the electron spin precession in the regime of nonrelativistic transverse motion in a CPW is considered in Sec. IV in which this phenomenon is also described in conventional terms of Feynman diagrams. The spin precession frequency growth in the opposite regime of ultrarelativistic transverse electron motion in a CPW is described in Sec. V on the basis of a semiclassical electron self-energy amplitude which also opens up the third possibility to obtain the basic formula for the electron spin precession frequency in a CPW in the regime of nonrelativistic transverse motion. In Sec. VI we discuss some details of a possible experiment on observation of the ultrarelativistic electron spin precession in a focus of a circularly polarized laser beam and summarize the results.

II. DESCRIPTION OF THE SPIN PRECESSION IN A CPW ON THE BASIS OF THE BMT EQUATION

We will consider in this section the ultrarelativistic electron (positron) spin precession in a CPW starting from the Bargmann-Michel-Telegdi (BMT) equation [11]

$$d\vec{\zeta}/dt = 2\left(\mu' + \frac{\mu_0}{\gamma}\right)[\vec{\zeta}\times\mathbf{H}] + \frac{2\mu'\gamma}{\gamma+1}(\mathbf{v}\cdot\mathbf{H})[\mathbf{v}\times\vec{\zeta}] + 2\left(\mu' + \frac{\mu_0}{\gamma+1}\right)[\vec{\zeta}\times[\mathbf{E}\cdot\mathbf{v}]] \tag{3}$$

which describes the evolution of the spin vector $\vec{\zeta}$ of an electron (positron) moving with the energy $\varepsilon = \gamma m$ and velocity $\mathbf{v}$ in slowly varying electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields. Here m and $e = \pm|e|$ are the electron (positron) mass and charge; $\mu_0 = e/2m$ and μ' are the normal and anomalous parts of their magnetic momenta, respectively ($|\mu_0|$ is equal to the Bohr magnetron). Remember that $\mu' \simeq \mu_0'$ in the weak field limit, where $\mu_0' = (\alpha/2\pi)\mu_0$ is the celebrated AMM value calculated by Schwinger in 1948.

Since the phenomenon of the electron spin precession in a CPW can be most readily observed in a multi-GeV region $x \geqslant 1$, we will use the ultrarelativistic approximation expanding the right-hand side of Eq. (3) into series in respect with the small parameter $1/\gamma < 10^{-4} \ll \mu'/\mu_0$. One can also assume that the longitudinal electron velocity shift in a CPW has an order of $1/\gamma^2$ in a real experimental situation (see below).

In order to obtain a value to compare with, let us first touch on the spin precession in a static uniform electric and transverse uniform magnetic field (spin precession in a longitudinal one is much slower). Introducing a transverse vector $\mathbf{F}_1 = \mathbf{H} - [\mathbf{E}\times\mathbf{v}]$ the frequency of the spin precession about this vector can be written in a form

$$\Omega_{uf} \simeq -2\left(\mu' + \frac{\mu_0}{\gamma}\right)F_1 \simeq -2\mu' F_1. \tag{4}$$

In fact, two types of contributions to the ultrarelativistic electron spin precession in a counterpropagating CPW can be distinguished. The first one is not connected with the electron velocity perturbation by the wave and can be described by the equation $d\vec{\zeta}/dt = [\vec{\zeta}\times\mathbf{h}]$, where $\mathbf{h} \simeq 4\mathbf{H}(\mu' + \mu_0/\gamma)$ in the ultrarelativistic approximation. This contribution to the spin precession frequency reminds one of the Bloch-Siegert shift of the magnetic resonance frequency (see, for example, [18]) and can be easily found if one considers the reference frame which rotates about the wave propagation direction $\mathbf{n}$ ($|\mathbf{n}| = 1$) with the frequency $-2\lambda_2\omega_0$ of the counterpropagating CPW field rotation ''seen'' by the electron (the positive spin rotation direction is chosen to appear as a counterclockwise from the end of the velocity vector; $\lambda_2 = \pm 1$ is the CPW polarization and ω_0 is its frequency). The vector $\mathbf{h}$ stops to rotate in this reference frame and acquires a large longitudinal component $2\lambda_2\omega_0\mathbf{n}$. As far as $h \ll \omega_0$, the electron spin rotates in this reference frame about the vector $\mathbf{n} + \mathbf{h}/2\lambda_2\omega_0 \simeq \mathbf{n}$ with the frequency $\lambda_2\sqrt{4\omega_0^2 + h^2} \simeq \lambda_2(2\omega_0 + h^2/4\omega_0)$, giving rise to the equal to $4\lambda_2(\mu' + \mu_0/\gamma)^2 H^2/\omega_0$ contribution to the spin rotation frequency in the laboratory frame of reference.

The second contribution to this frequency is connected with the electron velocity perturbation by the CPW [19] $\mathbf{v}_\perp = -e\lambda_2\mathbf{H}/\epsilon\omega_0$, which is usually characterized by the parameter [4,5] $\xi_0 = |e|H/m\omega_0$ (Sec. IV). As far as the last does not exceed one considerably even in the most intense available subpicosecond laser pulses [3,6–9], a transverse velocity component $v_\perp$ will not exceed $1/\gamma \ll 1$ much allowing one to neglect the longitudinal velocity shift in the CPW $\delta v_\parallel \simeq -(v_\perp^2 + 1/\gamma^2)/2 \sim 1/\gamma^2$ in the ultrarelativistic approximation. Substituting $\mathbf{v}_\perp$ into Eq. (3) one finally obtains

$$\Omega_{\mathrm{CPW}}^{\mathrm{BMT}} \simeq 4\lambda_2\frac{H^2}{\omega_0}\left[(\mu' + \mu_0/\gamma)^2 - 2\mu'\frac{\mu_0}{\gamma} - \frac{\mu_0^2}{\gamma^2}\right] = 4\lambda_2\mu'^2\frac{H^2}{\omega_0} = \left(\frac{\alpha}{2\pi}\right)^2\lambda_2\omega_0\xi_0^2 \tag{5}$$

for the sum of both considered contributions to the electron (positron) spin precession frequency about the average particle velocity. The detailed form of Eq. (5) shows that the terms proportional both to $\mu'\mu_0/\gamma$ and $(\mu_0/\gamma)^2$ cancel out. The last equality in Eq. (5) assumes that $\mu' = \mu_0'$ and shows that the BMT equation predicts a decrease of the spin precession frequency in a CPW with the growth of the wave frequency as well as its independence on the electron energy.

Let us compare the spin precession frequencies in a transverse uniform field (4) and a CPW (5). Assuming $\xi_0=1$ and $F_1=H$ one obtains an estimate $|\Omega^{\mathrm{BMT}}_{\mathrm{CPW}}/\Omega_{uf}| \simeq(\alpha/2\pi)\xi_0\sim10^{-3}$ showing that the BMT equation predicts a three order of magnitude smaller precession frequency in a CPW than in a transverse uniform field.

Such a small frequency makes it really difficult to observe the effect considered. Indeed, as far as in the case of $\xi_0\leqslant1$, the energy of a scattered photon can reach $\omega_{\max}\simeq x\varepsilon/(1+x)$; an electron will lose a substantial part of its energy in a single collision with a photon in the most interesting region of $x\geqslant1$. As a consequence, the interaction length with a wave cannot considerably exceed the characteristic length of the inverse Compton scattering $l_C=1/[2\sigma_C\rho_\gamma]\simeq\lambda/(\alpha\pi\xi_0^2)$, where $\rho_\gamma=H^2/(4\pi\omega_0)$ is the laser photon density (a number of photons in a cubic centimeter) and σ_C is the cross section of the inverse Compton scattering. Using Eq. (5) one can easily find that the spin precession angle on this length $\vartheta=l_C\Omega^{\mathrm{BMT}}_{\mathrm{CPW}}\simeq\alpha/\pi^2\sim1$ mrad is indeed too small for the experimental observation. Our further consideration will, however, show that Eq. (5) becomes inadequate already when $x\geqslant0.01$, and at $x\geqslant1$ the interaction with the field of radiation leads up to a nearly ($2\pi/\alpha\simeq10^3$) fold increase of the precession frequency and typical spin rotation angle, making this phenomenon really observable.

III. THE BARYSHEVSKY-LYUBOSHITS EFFECT

We will consider in this section the effect of rotation of γ-quantum polarization in a polarized electron target [12–15] which is intimately connected with the electron spin precession in a CPW. It is well known that polarization rotation stems from the difference of indices of refraction $n_\pm(\omega)$ of photons with right $(+)$ and left $(-)$ circular polarizations. In its turn this difference is connected with that of the real parts of the amplitudes $f_\pm(\omega,0)$ of the coherent forward scattering of circularly polarized photons by the relation

$$n_+(\omega)-n_-(\omega)=\frac{2\pi\rho}{\omega^2}\mathrm{Re}[f_+(\omega,0)-f_-(\omega,0)], \quad (6)$$

where ρ is the density of scattering centers the role of which is played by electrons in the case of γ-quantum polarization rotation in a polarized electron target. As far as we consider only the coherent forward scattering, a zero argument for the scattering angle will be omitted everywhere below (it will be used, instead, for the photon energy). Since polarized electrons can be considered as nearly not bound to nuclei in the most interesting photon energy region $\omega\geqslant m$, the spin dependence of the photon absorption in a polarized electron target is correctly described by the difference of the total cross sections of scattering of photons polarized parallel and antiparallel to the polarization vector of free electrons

$$\frac{1}{2}[\sigma_{\uparrow\downarrow}(x)-\sigma_{\uparrow\uparrow}(x)] = \frac{\sigma_0}{x}\left[2\left(1+\frac{2}{x}\right)\ln(1+x)-5+\frac{2}{1+x}-\frac{1}{(1+x)^2}\right], \quad (7)$$

where $\sigma_0=\pi(\alpha/m)^2\simeq2.5\times10^{-25}\ \mathrm{cm}^2$ and x is the invariant parameter introduced by Eq. (1) which reads $x=2\omega/m$ in the electron rest frame considered in this section. The difference (7) determines the imaginary part of the spin-dependent contribution

$$\mathrm{Im}f_2(\omega)=\frac{\omega}{4\pi}\frac{1}{2}[\sigma_{\uparrow\downarrow}(x)-\sigma_{\uparrow\uparrow}(x)] \quad (8)$$

to the coherent forward scattering amplitude

$$f_\pm(\omega)=f_1(\omega)\mp f_2(\omega)(\vec{\zeta}\cdot\mathbf{n}), \quad (9)$$

where $\vec{\zeta}$ and $\mathbf{n}$ are the electron polarization and the unit vector of the photon propagation direction, respectively. As far as we use Gell-Mann's *et al.* notation [20] (see also [12,13]), attention should be drawn to the fact that in Ref. [20] the signs are not consistent in Eqs. (5.9) and (5.15) playing the same role as our Eqs. (8) and (9). The angle of the γ-quantum polarization rotation on the length l,

$$\vartheta_\gamma=-\omega(n_+-n_-)l=\frac{4\pi}{\omega}(\vec{\zeta}\cdot\mathbf{n})\mathrm{Re}f_2(\omega)l, \quad (10)$$

is proportional to the real part of $f_2(\omega)$ which can be found from the dispersion relation [12,20]

$$\mathrm{Re}f_2(\omega)=f_2'(0)\omega+\frac{2\omega^3}{\pi}P\int_0^\infty\frac{\mathrm{Im}f_2(\omega')}{\omega'^2(\omega'^2-\omega^2)}d\omega', \quad (11)$$

where $f_2'(0)=df_2(\omega)/d\omega$ at $\omega=0$. Substituting Eqs. (7) and (8) into Eq. (11) one easily finds [12,13] that the leading contribution to the amplitude (11) has the second order in the fine structure constant $\alpha=1/137$ and reads

$$\begin{aligned}\mathrm{Re}f_2^{(2)}(\omega)&=\mathrm{Re}f_2^{(2)}(x)\\&=-\frac{\alpha^2}{4\pi m}\left\{\left(\frac{2}{x}-1\right)\left[F(x-1)+\frac{\pi^2}{6}\right]\right.\\&\quad-\left(\frac{2}{x}+1\right)[F(x)-\ln x\ln(1+x)]\\&\quad\left.+\frac{x}{1-x^2}+\frac{2x^3\ln x}{(1-x^2)^2}\right\},\end{aligned} \quad (12)$$

where $x=2\omega/m$ and

$$F(x)=\int_0^x\ln(1+t)\frac{dt}{t}. \quad (13)$$

The same result can be also obtained in a more customary way. Indeed, the scattering amplitude (9) which determines the refraction indexes (6) is connected by the relation $f(\omega)=M_{fi}/8\pi m$ (see Sec. IV) with the amplitude M_{fi} following from the QED Feynman rules. The second order in α of the amplitude (12) indicates that it corresponds to the lowest order radiative correction to the coherent forward Compton scattering, described by the diagrams shown in Fig. 1. The method of calculating this correction at arbitrary scattering angle was given in [21,22] (see also [23]) and leads exactly to Eq. (12). This approach will be considered in more

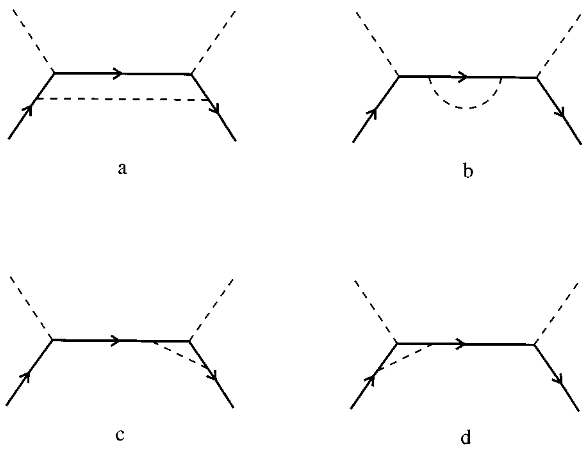

FIG. 1. Feynman diagrams contributing to the lowest order radiative correction to Compton scattering and describing the radical growth with the parameter x [see Eqs. (1) and (2)] of both the angle of the photon polarization rotation in a polarized electron target and the frequency of the ultrarelativistic electron spin precession in a CPW.

detail in application to the inverse coherent forward Compton scattering in the next section.

The spin-dependent part of the coherent forward scattering amplitude (12) describes an impressive growth of the rotation angle (10) of γ-quantum polarization in the energy region $\omega\sim m$. This growth was observed experimentally [14,15] and can be illustrated by both the low and high energy limits of Eq. (12). Indeed, when $x\ll 1$ or $\omega\ll m$ one has

$$\mathrm{Re}f_2^{(2)}(x\ll 1)=\frac{\alpha^2}{144\pi}\frac{x^3}{m}\left(60\ln\frac{1}{x}-37\right)\propto\omega^3\ln\omega \tag{14}$$

and the angle (10) grows proportionally to $\omega^2\ln\omega$ with the increase of the photon energy.

In the opposite limit of $x\gg 1$ or $\omega\gg m$ one has

$$\mathrm{Re}f_2^{(2)}(x\gg 1)=\frac{\pi\alpha^2}{8m} \tag{15}$$

and the angle (10) decreases inversely proportionally to ω showing that the phenomenon of the γ-quantum polarization rotation is indeed most pronounced in the region of $\omega\sim m$.

Since $df_2^{(2)}/d\omega=0$ at $\omega=0$ [see Eq. (14)], the second order amplitude (12) gives no contribution to the first term in the right-hand side of Eq. (11). As a consequence, this amplitude does not contribute to the interaction of the electron spin with the low energy circularly polarized photons to describe which next order in α contribution to the amplitude (11) should be considered.

In fact, the third order in α contribution to this amplitude was found in [16,17] in the low energy limit. Generally speaking, the scattering amplitude was evaluated there without use of the perturbation theory for the case of an arbitrary spin-1/2 particle not specifying the nature of fields and interactions in detail. As a consequence, when one considers Compton scattering, this amplitude contains the terms of different order in α. However, in the case of the forward Compton scattering, all the terms proportional both to $\mu_0^2\propto\alpha$ and $\mu_0\mu_0'\propto\alpha^2$ cancel out leaving only the third order in α contribution

$$f_2^{(3)}=f_2'(0)\omega=-2\mu_0'^2\omega=-\frac{\alpha^3 x}{16\pi^2 m} \tag{16}$$

[recall that terms of different order also cancel out in a similar manner in Eq. (5)]. As would be expected, the ratio $-f_2^{(2)}(x\ll 1)/f_2^{(3)}=(\pi x^2/\alpha)\,[60\ln(1/x)-37]/9$ reaches π/α and the angle (10) by several hundred times exceeds the value $\vartheta_\gamma^{(3)}=4\pi/\omega(\vec{\zeta}\mathbf{n})\mathrm{Re}f_2^{(3)}(\omega)l$ in the region of $x\sim 1$.

It is remarkable that the third order in α contribution (16) exceeds the second order one (12), (14) at $x<0.01$ (more precisely, at $x<0.00926$) or $\omega<2.44$ keV. As a consequence, polarization rotation changes its direction at $\omega\simeq 2.44$ keV. Note that though the third order contribution also becomes comparable with the second order one at $x\simeq 2\pi^3/\alpha\simeq 8500$, Eq. (16) can hardly be used in this energy region since it was obtained [16,17] in the limit of low γ-quantum energies and a more thorough consideration of the third order in α contribution, based on evaluation of the next correction to the diagrams given in Fig. 1, is necessary in the high energy region.

A proportionality of the third order in α contribution (16) to the AMM squared indicates its intimate connection with the frequency (5) of the electron spin rotation, following from the BMT equation. This connection will be substantiated in the next section along with the interrelation of the described behavior of the coherent forward scattering amplitude (11) with the specific effects accompanying the electron spin rotation in a CPW.

IV. ELECTRON SPIN PRECESSION IN THE LIMIT OF NONRELATIVISTIC TRANSVERSE MOTION IN A CPW

We will show in this section that the amplitude (12) correctly describes the electron spin rotation in a CPW in the limit of nonrelativistic transverse motion in a CPW $\xi_0^2\ll 1$. Here

$$\xi_0^2=\left(\frac{eH}{m\omega_0}\right)^2=3.65\times 10^{-19} I\left(\frac{W}{\mathrm{cm}^2}\right)\lambda^2(\mu\mathrm{m}) \tag{17}$$

is the dimensionless parameter [3–9] characterizing the intensity I of a CPW. Subpicosecond laser systems allow one to generate the pulses with $\xi_0^2\simeq 1$ at present and soon will be able to reach the values $\xi_0^2\gg 1$ in a sufficiently extended laser focus region [9]. Recall that the transverse electron motion in a wave is nonrelativistic when $\xi_0^2\ll 1$ and ultrarelativistic when $\xi_0^2\gg 1$. The character of both the spin evolution and radiation process in a CPW differs essentially in these two limits which are analyzed in this and the next sections, respectively.

It is known [4,5] (see also the next section) that the electron interaction with an electromagnetic wave can be treated as a scattering on independent photons if $\xi_0^2\ll 1$. By analogy with the effect of rotation of γ-quantum polarization in a polarized electron target we will consider the electron interaction with a laser wave in this limit as a coherent forward electron scattering by a polarized ''photon target.'' A concentration of the scattering centers (photons) in the last can be comparable with that in a condensed medium in the case of a superintense laser pulse. As far as the photon energy ω_0 of the most powerful laser systems does not far exceed 5 eV [9], parameter (2) will reach one at the ultrarelativistic electron energies $\varepsilon \geqslant 10$ GeV.

Also by analogy with [24] and Sec. III, the coherent interaction of polarized electrons with the ''photon target'' can be described by the indexes of refraction $n_{\pm}(\varepsilon)$ of electrons polarized parallel (+) and antiparallel (–) to their momentum **p**. The difference of these refraction indexes describes the rotation of the transverse electron spin component and is connected with the difference of the real parts of the amplitudes $f_{\pm}(\varepsilon)$ of the coherent forward scattering of a polarized electron by the polarized ''photon target'' by the relation

$$n_{+}(\varepsilon)-n_{-}(\varepsilon)=\frac{4\pi\rho_{\gamma}}{p^2}\mathrm{Re}[f_{+}(\varepsilon)-f_{-}(\varepsilon)], \qquad (18)$$

where the doubling of the photon density $\rho_{\gamma}=H^2/4\pi\omega_0$ in a CPW [compare with Eq. (6)] arises due to the head-on motion of the electron and ''photon target.'' Note that both the refraction indices and scattering amplitudes of electrons and photons are distinguished here only by their arguments (ε for electrons and ω for photons).

It is convenient to introduce a spin-dependent part $f_2(\varepsilon)$ of the electron scattering amplitude $f_{\pm}(\varepsilon)$ according to the equation [compare with Eq. (9)]

$$f_{\pm}(\varepsilon)=f_1(\varepsilon)\pm\lambda_2 f_2(\varepsilon). \qquad (19)$$

Note that the difference of signs in Eqs. (9) and (19) is connected with the opposite directions of the electron and photon propagation. The knowledge of the difference (18) of the electron refraction indices allows one to find their spin precession frequency

$$\begin{aligned}\Omega&=-pv[n_{+}(\varepsilon)-n_{-}(\varepsilon)]\\&=-\frac{8\pi}{\varepsilon}\rho_{\gamma}\lambda_2\mathrm{Re}f_2(\varepsilon)=-\frac{2H^2}{\varepsilon\omega_0}\lambda_2\mathrm{Re}f_2(\varepsilon).\end{aligned} \qquad (20)$$

We will use two ways to evaluate the spin-dependent part of the amplitude (19). The first one will demonstrate a strong connection of the electron spin precession in a CPW with the Baryshevsky-Lyuboshits effect. This way consists of transformation of the amplitude of coherent forward Compton scattering $f_{\pm}(\omega)$ to that of the inverse coherent forward Compton scattering $f_{\pm}(\varepsilon)$. The second way is a straight evaluation of the last using the Feynman rules.

Following the first way one has to obtain the ratio of forward scattering amplitudes in different reference frames. We will proceed from the written in an arbitrary reference frame differential cross section of the forward scattering of a ''first'' particle by a ''second'' one:

$$\frac{d\sigma(0)}{do_1}\equiv|f|^2=\frac{|M_{fi}|^2v_1^2\varepsilon_1}{64\pi^2\varepsilon_2[\varepsilon_1^2\varepsilon_2^2(1-v_1v_2\cos\theta)^2-m_1^2m_2^2]^{1/2}|v_1-v_2\cos\theta|}. \qquad (21)$$

Here $m_{1,2}$, $\varepsilon_{1,2}$, and $v_{1,2}$ are, respectively, particle masses, energies, and velocities, constituting the angle θ. The forward scattering amplitude $M_{fi}=M_{fi}(s)$ is a function of the invariant parameter $s=(p+k_0)^2=(1+x)m^2$, where x is the parameter (1) used in the theory of the inverse Compton scattering [1]. The sought-for amplitude

$$f(\varepsilon)=\frac{M_{fi}(s)v}{8\pi\omega_0(1-v\cos\theta)^{1/2}|v-\cos\theta|^{1/2}}\simeq\frac{M_{fi}(s)}{16\pi\omega_0} \qquad (22)$$

of scattering of an electron of energy ε and velocity v by a laser photon of frequency ω_0 follows from Eq. (21) after the substitutions $v_1=v$, $v_2=1$, $\varepsilon_1=\varepsilon$, $\varepsilon_2=\omega_0$, $m_1=m$, and $m_2=0$. The same amplitude $M_{fi}(s)$ is connected with the forward Compton scattering amplitude $f(\omega)=M_{fi}/8\pi m$ of a photon ($v_1=1$, $m_1=0$) of the energy $\omega=\omega_0(\varepsilon/m)(1-v\cos\theta)$, corresponding to the same value of the parameter (1) in the electron rest frame ($v_2=0$, $\varepsilon_2=m_2=m$). Excluding $M_{fi}(s)$ one obtains the relation

$$f(\varepsilon)=\frac{mvf(\omega)}{\omega_0(1-v\cos\theta)^{1/2}|v-\cos\theta|^{1/2}}\simeq\frac{mf(\omega)}{2\omega_0}, \qquad (23)$$

allowing one to transform the spin-dependent part $f_2(\omega)$ of the amplitude (9) of the coherent forward photon scattering by an electron in rest to the spin-dependent part $f_2(\varepsilon)$ of the amplitude (19) of the coherent forward electron scattering by a laser photon. Note that the last equalities in Eqs. (22) and (23) correspond to the case of a head-on collision ($\cos\theta=-1$) of an ultrarelativistic electron with a photon.

Using Eqs. (20) and (23) one can easily see that the contribution (16) of order α^3 to the amplitude (11) leads to the electron spin precession frequency

$$\Omega^{(3)}=\left(\frac{\alpha}{2\pi}\right)^2\lambda_2\omega_0\xi_0^2 \qquad (24)$$

which exactly coincides with that following from the BMT equation [see Eq. (5)]. In its turn, the contribution (12) of order α^2 to the amplitude (11) leads to the electron spin precession frequency

$$\Omega^{(2)} = -\frac{2H^2}{\varepsilon\omega_0}\lambda_2 f_2^{(2)}(\varepsilon) = -\frac{mH^2}{\varepsilon\omega_0^2}\lambda_2 f_2^{(2)}(\omega)$$
$$= \frac{\alpha}{\pi}\lambda_2\omega_0\xi_0^2\frac{1}{x}\left\{\left(\frac{2}{x}-1\right)\left[F(x-1)+\frac{\pi^2}{6}\right]-\left(\frac{2}{x}+1\right)[F(x)-\ln x\ln(1+x)]+\frac{x}{1-x^2}+\frac{2x^3\ln x}{(1-x^2)^2}\right\}, \quad (25)$$

where the parameter x should be taken from Eq. (2) and the function $F(x)$ is given by Eq. (13).

A second way to obtain this frequency is based on Eq. (22) and the well-known Feynman rules allowing one to evaluate the amplitude M_{fi}, corresponding to the lowest order radiative correction to the inverse Compton scattering. This correction is described by four Feynman diagrams (a)–(d) from Fig. 1 in which an electron first absorbs the incoming photon and then emits the outgoing one. In addition, the same four diagrams in which an electron first emits a photon and subsequently absorbs the incident one also have to be taken into account. As far as the coherent forward inverse Compton scattering is considered, the momenta of initial and final photons (and electrons) are equal (it is interesting that the diagrams given in [21,22] describe just this case).

The amplitudes corresponding to the diagrams given in Fig. 1 were first considered in [21,22] (see also [23]). In fact, to describe the electron spin precession in a CPW we need only the spin-dependent parts of these amplitudes which do not contain divergent integrals. The methods of analytical calculations allow one to further simplify their evaluation which together with Eq. (22) leads to the following contribution of the diagram (a) to the amplitude $f_2^{(2)}(\varepsilon)$

$$\mathrm{Re} f_2^{(2a)}(\varepsilon) = -\frac{\alpha^2}{8\pi\omega_0}\left\{\left(\frac{4}{x}-1\right)\left[F(x-1)+\frac{\pi^2}{6}\right] -\left(\frac{4}{x}+1\right)[F(x)-\ln x\ln(1+x)]\right\}. \quad (26)$$

As far as the amplitudes of the electron self-energy [25] and vertex part with one electron out of the mass shell [23] are well known, only the spurs of γ-matrices have to be evaluated in order to obtain the contribution of the diagram (b),

$$\mathrm{Re} f_2^{(2b)}(\varepsilon) = -\frac{\alpha^2 x}{8\pi\omega_0}\left[\frac{1}{1-x^2}+\frac{2\ln x}{(1-x^2)^2}\right], \quad (27)$$

as well as the equal contributions of the diagrams (c) and (d)

$$\mathrm{Re} f_2^{(2c)}(\varepsilon) = \mathrm{Re} f_2^{(2d)}(\varepsilon) = \frac{\alpha^2}{8\pi\omega_0}\frac{1}{x}\left[F(x-1)+\frac{\pi^2}{6}-F(x) +\ln x\ln(1+x)+\frac{x^2\ln x}{1-x^2}\right]. \quad (28)$$

One can easily see from Eq. (20) that the sum of the contributions (26)–(28) leads to (25).

Leaving aside the diagrams (a) and (b) note that the diagrams (c) and (d) alone can explain the reason why the BMT equation as well as the similar classical equations taking into account the field inhomogeneity cannot, in principle, describe the electron spin precession in a CPW in the most interesting quantum region of $x \geqslant 1$. Recall that all the classical equations mentioned contain the electron AMM which is closely connected with the low photon energy limit of the vertex part with both initial and final electrons on the mass shell. However, the diagrams (c) and (d) show that the electron spin precession in a CPW is connected with the vertex part with only one electron on the mass shell. What is more, the low photon energy limit $x \ll 1$ of this vertex part cannot describe the real experimental situation which corresponds to the region $x \geqslant 1$ indicating the quantum nature of the considered phenomenon.

When the two ways to obtain the basic Eq. (25) have been outlined, one can proceed to the analysis of the main features of the electron spin precession in a CPW. From the asymptote

$$\Omega^{(2)}(x \ll 1) = -\frac{\alpha}{\pi}\lambda_2\omega_0\xi_0^2 x^2\left(\frac{5}{3}\ln\frac{1}{x}-\frac{37}{36}\right) \quad (29)$$

one can see that the third order contribution (24) exceeds the second order one (25) in the region of $x \ll 0.01$ [or ε (GeV) $\ll \lambda(\mu m)/2$] in which the BMT equation correctly describes the spin precession in a CPW with an accuracy equal to the ratio $R = -\Omega^{(2)}(x \ll 1)/\Omega^{(3)}$. The opposite signs of the contributions (24) and (25) result in the change of sign of the combined frequency $\Omega = \Omega^{(3)} + \Omega^{(2)}$ as well as of the direction of the spin precession at $x \simeq 0.01$. It is interesting to note that the last condition precisely corresponds to the technical possibilities which were available in the 1970s. Namely, to the wavelength $\lambda = 1.06$ μm of the most powerful in the 1970s Nd:glass lasers and the energy $\varepsilon \simeq 0.5$ GeV of the storage rings at which the first polarized beams of ultrarelativistic electrons had been obtained in 1970.

The most interesting features of the electron spin evolution in a CPW manifest themselves in the region $x \gg 0.01$ in which the second order in α contribution (25) to the spin precession frequency dominates strongly over (24). According to Eq. (25) the electron spin precession frequency $\Omega \simeq \Omega^{(2)}$ grows as fast as $\omega_0\varepsilon^2\ln 1/x$ with ω_0 and ε, while the frequency (5) does not depend on ε at all and decreases in inverse proportion to ω_0 if one assumes $\mu' = \mu_0'$. The dependence of the ratio of these frequencies on the parameter x is given in Fig. 2. After nearly a 700-fold increase in the region $x \sim 1$ up to the value $\sim(\alpha/\pi)\omega_0\xi_0^2$, the spin precession frequency in a CPW with $\xi_0 \sim 1$ becomes comparable with that in the transverse uniform field of the strength $F \sim (\alpha/\pi)\omega_0/\mu_0' = xF_0/\gamma \sim F_0/\gamma$ in which the field in the proper electron frame reaches the typical value [4,5] $F_0 = 4.41\times 10^{13}$ G. Only recently such intense fields were reached in a focus of subpicosecond laser pulses [3,6–9] and the investigation of particle interaction with the fields of comparable strength began in crystals [26].

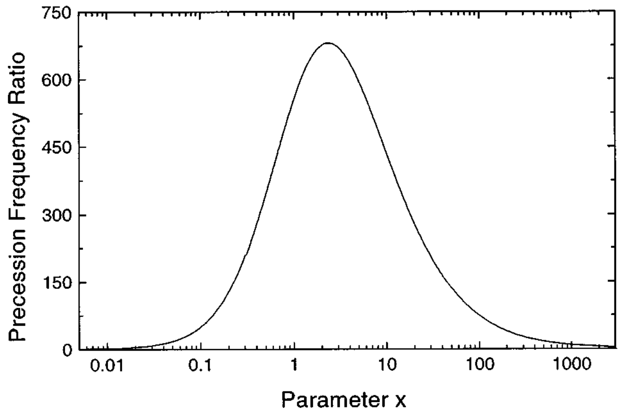

FIG. 2. The dependence on the parameter x of the ratio $R=-\Omega^{(2)}/\Omega^{(3)}$ of the contribution (25) of order α^2 to the electron spin precession frequency in a CPW in the limit of $\xi_0^2\ll 1$, to the contribution (24) of order α^3 equal to the spin precession frequency (5), following from the BMT equation. This curve also illustrates the growth of the proportional to $-f_2^{(2)}(\omega)/f_2^{(3)}(\omega)$ ratio of the contributions of order α^2 and α^3 to the angle (10) of the γ-quantum polarization rotation in a polarized electron target.

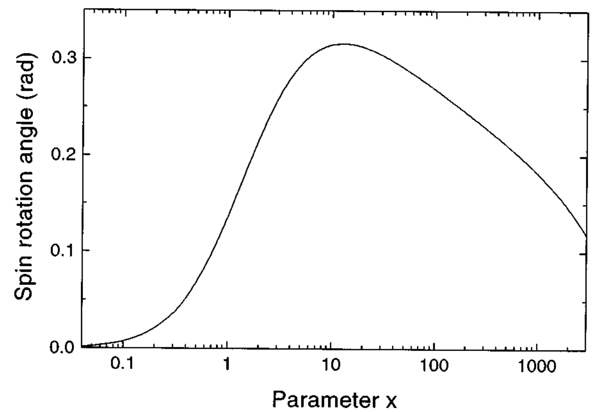

FIG. 3. The dependence on the parameter x of the rotation angle of transverse electron spin component in a CPW on a typical length of the inverse Compton scattering in the limit $\xi_0^2\ll 1$ of nonrelativistic transverse electron motion.

Thus the precession frequency in the region $x\sim 1$ becomes nearly a thousand times higher than (5) and reaches a value typical for the most intense available uniform fields. Note that if the assumption $\mu'=\mu_0'$ leading to the last equality in Eq. (5) is not used, the BMT equation predicts the frequency $\Omega_{\mathrm{CPW}}^{\mathrm{BMT}}=4\lambda_2\mu'^2H^2/\omega_0$ which allows to argue that the predicted spin precession frequency growth can, in some sense, be interpreted as a nearly 25-fold increase of the electron AMM. However, the opposite signs of frequencies (5) and (25) [see also Eq. (29)] make such an interpretation too artificial from our point of view.

As noted above, the optimal spin rotation angle is determined by the scattering length $l_C=1/[2\rho_\gamma\sigma_C(x)]$, where $\sigma_C(x)$ is the cross section of the inverse Compton scattering, corresponding to the given value of the parameter (2). The dependence of the typical spin rotation angle $\vartheta=\Omega l_C$ on the last is illustrated by Fig. 3. As far as the asymptotic cases are considered, since $\sigma_C(x\ll 1)\simeq 8\sigma_0/3$ the rotation angle $\vartheta=\Omega l_C\simeq 5x^2\ln(1/x)/4\pi$ grows like $(\omega_0\varepsilon)^2\ln(1/x)$ at $x\ll 1$. On the contrary, since $\sigma_C(x\gg 1)\simeq\sigma_0(2\ln x+1)/x$ and $\Omega^{(2)}(x\gg 1)\simeq-\alpha\pi\lambda_2\omega_0\xi_0^2/2x$, the angle $\vartheta\simeq\pi/(2\ln x+1)$ only slowly decreases with ω_0 and ε at $x\gg 1$ (but $x\ll 2\pi^3/\alpha\simeq 8500$; see below). Thus the energy region $x\sim 1$ is best suited for the experimental observation of the electron spin precession in a CPW, the possibility of which will be briefly analyzed in Sec. VI.

It should be also mentioned that $\Omega^{(2)}(x\gg 1)$ equates $-\Omega^{(3)}$ at $x\simeq 2\pi^3/\alpha\simeq 8500$. However, since the contribution (16) of order α^3 to the Compton scattering amplitude was found in the low energy limit [16,17], the only conclusion one can draw out from this circumstance concerns the importance of evaluation of this contribution at arbitrary values of the parameter x for the analysis of the considered effect in the high energy region.

V. ELECTRON SPIN PRECESSION IN THE LIMIT OF ULTRARELATIVISTIC TRANSVERSE MOTION IN A CPW

To describe a spin evolution in the case of arbitrary CPW intensity we will proceed from the spin-dependent part of the lowest order in α electron self-energy amplitude [4,5,27]. Owing to the classical character of the electron motion in the wave field we will use the expression, obtained for this amplitude by the semiclassical operator method [28,29]. Translational invariance of the problem allows one to use the spin-dependent part of the self-energy amplitude per unit of length

$$\mathrm{T}_{\vec\zeta}^{(2)}(\vec\zeta)=\frac{i\alpha}{4\pi}\int_0^{\varepsilon}d\omega\int_0^{\infty}\frac{d\tau}{\tau}\,\frac{\omega}{\varepsilon}\vec\zeta\left\{\frac{1}{\gamma}[\mathbf{v}(t+\tau)\times\mathbf{v}(t)]-\left(1+\frac{\varepsilon}{\varepsilon'}\right)[\mathbf{v}(t+\tau)-\langle\mathbf{v}\rangle_\tau]\times[\mathbf{v}(t)-\langle\mathbf{v}\rangle_\tau]\right\}$$
$$\times\exp\left\{-i\frac{\omega\varepsilon}{2\varepsilon'}\left[\frac{\tau}{\gamma^2}+\int_0^{\tau}(\mathbf{v}(t+\tau')-\langle\mathbf{v}\rangle_\tau)^2d\tau'\right]\right\}. \tag{30}$$

Here

$$\langle \mathbf{v}\rangle_\tau=\int_0^\tau \mathbf{v}(t+\tau')\frac{d\tau'}{\tau},$$

where $\mathbf{v}(t)$ is the electron velocity at a moment of time t, ω is the energy of a virtual photon, emitted by the electron in a CPW, $\varepsilon'=\varepsilon-\omega$, and $\gamma=\varepsilon/m$. Note that the amplitude (30) corresponds to the second order self-energy diagram in the Furry picture with the ''solid'' electron line which takes account of all the orders of the electron interaction with the external field. The perturbation theory is, thus, used in Eq. (30) only for the interaction with the field of radiation. According to [28] the self-energy amplitude is normalized in such a way that the doubled imaginary part of Eq. (30) is equal to the spin-dependent contribution to the total probability of photon emission by an electron in the external field. In its turn, the real part of Eq. (30) which describes the spin dependence of the coherent forward electron scattering by the external field is connected by a simple relation with the contribution to the electron spin precession frequency arising due to the interaction with the field of radiation (see below).

In the presence of uniform electric and transverse uniform magnetic fields a transverse component of the ultrarelativistic electron velocity can be written in the form [29]

$$\mathbf{v}_\perp(t+\tau)=\frac{e\mathbf{F}\tau}{\epsilon}+\frac{1}{\gamma}\{\vec{\xi}_1\cos[2\omega_0(t+\tau)+\varphi_0] + \vec{\xi}_2\sin[2\omega_0(t+\tau)+\varphi_0]\}, \tag{31}$$

where $\mathbf{F}=\mathbf{E}-\mathbf{v}(\mathbf{v}\cdot\mathbf{E})/v^2+[\mathbf{H}\times\mathbf{v}]$ and φ_0 is the initial wave phase. Orthogonal vectors $\vec{\xi}_{1,2}$ constitute a right-hand triad with the unit vector $\mathbf{n}$ of the wave propagation direction and characterize the wave intensity $\xi_0^2=(\xi_1^2+\xi_2^2)/2$ and polarization, described by the Stokes parameters λ_3 and $\lambda_2=[\vec{\xi}_1\times\vec{\xi}_2]\cdot\mathbf{n}/\xi_0^2$. In the case of pure circular polarization $\lambda_3=0$, $\lambda_2=\pm1$, $\xi_1=\xi_2=\xi_0=|e|H/m\omega_0$ [see Eq. (17)] after the averaging over the phase φ_0 the real part of the spin-dependent contribution (30) to the self-energy amplitude can be written in the form [29]

$$\begin{aligned}\mathrm{Re}T_{\vec{\zeta}}^{(2)}&=-\frac{1}{2}\vec{\zeta}\cdot\vec{\Omega}^{(2)}\\ &=\mu_0'\vec{\zeta}[\mathbf{F}\times\mathbf{v}]+\frac{\alpha m^2}{2\pi\varepsilon^3}\lambda_2(\vec{\zeta}\cdot\mathbf{v})\xi_0^2\int_0^\varepsilon \omega d\omega\int_0^\infty\frac{dz}{z^2}(2\\ &\quad+u)\left(1-\cos z-\frac{z\sin z}{2}\right)\\ &\quad\times\sin\left\{\frac{uz}{s}\left[1+\xi_0^2+\frac{2\xi_0^2}{z^2}(\cos z-1)\right]\right\},\end{aligned} \tag{32}$$

where $u=\omega/(\varepsilon-\omega)$ and $z=s\tau$. We have assumed for simplicity that the uniform field is not very strong and neglected accordingly the AMM variation in it. According to [28] the pseudovector $\vec{\Omega}^{(2)}$ of frequency of the spin precession which arises due to the electron interaction with the field of radiation was introduced in Eq. (32). The corresponding equation of the spin evolution

$$d\vec{\zeta}/dt=[\vec{\Omega}^{(2)}\times\vec{\zeta}] \tag{33}$$

has a form similar to that of the BMT equation (3) but, in fact, is much more general and can be used to examine its applicability.

The uniform field has been introduced here merely to demonstrate that Eqs. (32) and (33) correctly describe a contribution $\vec{\Omega}^{(2)}=-2\mu_0'[\mathbf{F}\times\mathbf{v}]=-2\mu_0'\mathbf{F}_1$ of the field of radiation to the ultrarelativistic electron spin precession frequency (4) in this well-known case. Note that since Eq. (33) takes into account only the spin evolution under the influence of the field of radiation, it does not describe a contribution of the ''normal'' part $\mu_0=e/2m$ of the electron magnetic moment to the frequency (4).

In the case of zero uniform field the remaining term in the right-hand side of Eq. (32) is proportional to $\vec{\zeta}\cdot\mathbf{v}$. According to Eq. (33), it describes the electron spin precession about the average velocity vector with the frequency $\Omega^{(2)}$ which generalizes the expression (25) to the case of arbitrary CPW intensity. Indeed, if $\xi_0^2\ll1$ the electron interaction with the wave field can be treated as a perturbation [4,5] allowing one to expand Eq. (32) in powers of the parameter ξ_0^2. The proportional to ξ_0^2 main term of this expansion leads to the frequency (25) thoroughly discussed in Sec. IV. This term corresponds to the diagrams given in Fig. 1, two vertexes of which arise due to the electron interaction with the field of radiation and another two describe its interaction with the electromagnetic wave. The proportional to $\xi_0^4,\xi_0^6,\ldots$ higher order terms of the expansion of Eq. (32) correspond to the diagrams with also two vertexes of electron interaction with the field of radiation and four, six, or more vertexes of interaction with the electromagnetic wave.

The aforesaid allows one to interpret the index of the frequency $\Omega^{(2)}$ not only as that corresponding to the second order in α of the contribution of the diagrams given in Fig. 1, but also as that corresponding to the second order of the electron interaction with the field of radiation described by the self-energy amplitude (30). The latter interpretation is more general since it remains adequate for all the terms of Eq. (32) expansion in powers of the small parameter $\xi_0\ll1$ as well as in the case of $\xi_0\sim1$ when the electron interaction with the wave cannot be treated as a perturbation.

As mentioned above, the existing laser systems allow one to obtain the pulses with $\xi_0\sim1$ and their fast progress [3] will soon make it possible to reach the region $\xi_0\gg1$ in which Eq. (32) as well as the formulas [4,5,28] describing the radiation process in the wave field can also be substantially simplified. Indeed, as far as the angle of electron deflection by the wave exceeds the typical angle of radiation in the limit of $\xi_0\gg1$, the uniform field approximation can be applied. This means that if $\xi_0\gg1$ the integral (32) is forming in the region $z\ll1$ and all the trigonometric functions of the argument z can be expanded in powers of z leading to the simple expression

$$\Omega^{(2)} = -\frac{2\alpha}{3\pi}\lambda_2\omega_0\int_0^{\varepsilon}\left[1 - y\int_0^{\infty}\sin\left(yt + \frac{t^3}{3}\right)dt\right]\left(2 - \frac{\omega}{\varepsilon}\right)\frac{d\omega}{\varepsilon} \tag{34}$$

for the spin precession frequency in the second order in electron interaction with the field of radiation. Here

$$\chi = \frac{2\gamma F}{H_0}, \quad y = \left[\frac{\omega}{\chi(\varepsilon - \omega)}\right]^{2/3} \tag{35}$$

are the parameters widely used in the theory of radiation in the intense uniform field [4,5,28], the role of which is played here by the field of CPW $F = E = H$. The doubling of the field strength in Eq. (35) is connected with the summation of actions of electric and magnetic fields of a counterpropagating wave on the electron moment and spin (see also Sec. II). Note that three parameters (2), (17), and (35) characterizing the electron interaction with the wave are related by the equality

$$\chi = \frac{x\xi_0}{2}. \tag{36}$$

The energy dependence of the frequency (34) differs considerably in the regions $\chi < 1$ and $\chi > 1$. Indeed, the integral (34) asymptote in the former can be found by the method [4] (see also [28]) used to evaluate the AMM asymptote in the limit of weak uniform field. As a result one easily obtains for the ratio of the frequencies (34) and (5) in the same limit $R = -\Omega^{(2)}(\chi \ll 1)/\Omega_{\rm CPW}^{\rm BMT} \simeq (20\pi/3\alpha)x^2[\ln(1/\chi) + C + \ln 3/2 - 59/20]$ where $C = 0.5772\ldots$ is the Euler constant. This ratio differs from that obtained in the case of $\xi_0^2 \ll 1$ [see Eqs. (24) and (29)] only by a multiplier under the logarithm sign. As a consequence, most of the features of the effect considered coincide in the limits of $\xi_0^2 \ll 1$ and $\xi_0^2 \gg 1$ at small x.

First of all, the fast decrease of the frequency (34) at small x makes it necessary to take into consideration the next order in electron interaction with the field of radiation (compare with Secs. III and IV). We will assume that in the region $x \ll 1$ Eq. (5) correctly describes a contribution to the spin precession frequency of the next order radiative correction to the self-energy amplitude (32) in the limits of both $\xi_0^2 \ll 1$ (see Sec. IV) and $\xi_0^2 \gg 1$. One can easily see that the frequencies (34) and (5) equate, resulting in change of sign of the combined spin precession frequency $\Omega = \Omega_{\rm CPW}^{\rm BMT} + \Omega^{(2)}$ at $x^2 \simeq 3.5\times 10^{-4}/\ln(1/\chi)$.

At the higher values of x the ratio $R = -\Omega^{(2)}/\Omega_{\rm CPW}^{\rm BMT}$ grows proportionally to $(\omega_0\varepsilon)^2\ln(1/\chi)$ with the wave frequency and electron energy. However, on the contrary to the limit of $\xi_0^2 \ll 1$, the obtained asymptote remains valid only up to the region $x \sim 1/\xi_0 \ll 1$ [see Eq. (36)] in which the ratio R reaches the value of about $10^3/\xi_0^2$ and its further growth with the parameter x slows down at $x > 1/\xi_0$.

Indeed, as far as the integral in Eq. (34) approaches 3/2 at $\chi \gg 1$, one easily obtains an estimate $R = -\Omega^{(2)}(\chi \gg 1)/\Omega_{\rm CPW}^{\rm BMT} \simeq (8\pi/\alpha)/\xi_0^2 \simeq 3\times 10^3/\xi_0^2$ which shows that in the considered limit of ultrarelativistic transverse motion this ratio is nearly constant when $\chi > 1$ and $x > 1/\xi_0$ and does not reach the value $R \sim 2\pi/\alpha$ attainable in the opposite limit. The smaller value of the frequency ratio in the limit considered indicates that when the parameter ξ_0 exceeds one, the scale of the spin precession frequency growth becomes less than that in the case of nonrelativistic transverse electron motion. In addition, our numerical evaluation of the spin precession angle at a typical length of synchrotronlike radiation in the wave in the limit of $\xi_0^2 \gg 1$ also demonstrates a smaller value of this angle than in the case of $\xi_0^2 \ll 1$. Thus, one can conclude that the limit of relativistic transverse electron motion does not give considerable advantages to observe the effect of the electron spin precession in a CPW.

Note also that the ratio R independence on the electron energy at $x \gg 1$ allows one to expect that in contrast to the limit of $\xi_0^2 \ll 1$, the next order in α contributions to the self-energy amplitude and spin precession frequency will not be important at very high energies in the considered limit of relativistic transverse electron motion.

VI. EXPERIMENTAL POSSIBILITY TO OBSERVE THE SPIN PRECESSION OF ULTRARELATIVISTIC ELECTRONS IN A CPW

We will consider in this section some details of a possible experiment on observation of the electron spin precession in a counterpropagating circularly polarized laser wave. As far as the regime of relativistic transverse electron motion in the wave $\xi_0^2 \gg 1$ does not possess considerable advantages and, in addition, needs higher CPW power, we will restrict our consideration to the nonrelativistic limit of $\xi_0^2 \ll 1$.

A strong nonuniformity of the laser field in a focal region will not cause principal difficulties in this case. Indeed, since the precession frequency (25) is proportional to the local photon density ρ_γ, the spin precession angle $\vartheta = \int\Omega(t)dt \propto \int\rho_\gamma(t)dt$ will be proportional to the electron scattering probability $p = \sigma_C(x)\int\rho_\gamma(t)dt$. As a consequence, normalizing the measured spin rotation angle to the number of scattered photons one will bypass the problems [9] of stabilization and absolute measurement of the field strength of superintense subpicosecond laser pulses and extract a direct information on the spin precession frequency growth illustrated by Fig. 2, corresponding to the case of $p = 1$.

The value $x = 2.50$ at which the spin-dependent part of the cross section of the inverse Compton scattering vanishes [1] represents a reasonable choice corresponding to $\varepsilon \simeq 140$ GeV at $\lambda = 1.06$ μm (Nd:glass laser) and $\varepsilon \simeq 33$ GeV at $\lambda = 0.248$ μm (Kr-F excimer laser). Recall that the electron-positron pair production in the field of a laser pulse by the backward scattered photons will not complicate an experiment at $x < 4.8$ [1]. As far as at $x \gg 1$ an electron loses a considerable part of its energy in a single collision with a laser photon, the probability p of the electron scattering by a laser pulse cannot far exceed one as in the case of photon-photon colliders [1]. If one chooses $p(x) = 1$, the angle ϑ of the electron spin rotation in a circularly polarized laser pulse will be equal to 0.241 rad at $x = 2.50$ (see Fig. 3).

Assuming the Gaussian shape of the laser beam as well as the head-on collision which can be realized using mirrors with holes for the electron beam [1], the length of the laser focus region and the photon density in it can be estimated [1], respectively, as $l \sim 4\pi a_\gamma^2/\lambda$ and $\rho_\gamma \sim A/\pi a_\gamma^2 l_\gamma\omega_0$, where a_γ is the rms focal spot radius, A is the energy, and l_γ is the length of the laser pulse. On the contrary to the case of electron-photon conversion in the photon-photon colliders, l_γ has to considerably exceed the length l_e of the electron beam in order to provide the close conditions of interaction of all the electrons with the laser pulse. The condition $p \simeq 1$ for the probability of the electron scattering by laser photons leads to an estimate

$$A \simeq \frac{\pi l_\gamma}{2\sigma_C} \simeq 2.0 \frac{\sigma_0}{\sigma_C} l_\gamma [mm] J \tag{37}$$

for the laser flash energy. As far as $\sigma_C(2.5) = 1.04\sigma_0$ and $l_\gamma \gg l_e \sim 1$ mm, A has to be 10 J or more, l_γ / l_e times exceeding the energy necessary for an efficient electron-photon conversion [1]. The smaller pulse energies A' will, naturally, allow one to observe at $x = 2.5$ a smaller spin rotation angle $\vartheta = 0.241(A'/A)$ rad.

In contrast to the phenomenon of the AMM modification in an electromagnetic wave in the presence of a uniform field [5], the considered phenomenon of the electron spin precession in a CPW does not need an additional intensive field for its observation. It is remarkable that three order of magnitude spin precession frequency growth in a CPW is, in some sense, comparable in scale with a 25-fold growth of the AMM, whereas a modification of the last even in the sixth or seventh decimal place is considered in [5] as a phenomenon representing a considerable interest.

From the aforesaid it may be concluded that the described phenomenon of a drastic electron spin precession frequency growth in a CPW represents a fundamental interest and can be observed experimentally at present.

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