Confinement-deconfinement transition in three-dimensional QED

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We argue that, at finite temperature, parity-invariant electrodynamics with massive electrons in 2+1 dimensions can exist in both confined and deconfined phases and has a confinement-deconfinement phase transition of Berezinskii-Kosterlitz-Thouless type. We show that an order parameter for the confinement-deconfinement phase transition is a version of the Polyakov loop operator whose average measures the free energy of an external charge that is not an integral multiple of the electron charge.

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The phenomenon of confinement of color charges and the possibility of a confinement-deconfinement phase transition in gauge field theories at finite temperature has been extensively investigated [1,2]. In either pure Yang-Mills theory, or any other gauge theory where all colored fields transform under the adjoint representation of the gauge group, the question of whether the theory exists in a confined or deconfined phase at a given temperature is known to be intimately related to the realization of a certain effective global symmetry of the Euclidean path integral related to the center of the gauge group [3–6]. An order parameter which probes the realization of this symmetry is the Polyakov loop operator

$$P(\vec{x}) \equiv \mathrm{tr}\left[\mathscr{P}\exp\left(i\int_0^{1/T} d\tau A_0(\tau,\vec{x})\right)\right], \tag{1}$$

whose correlators in finite temperature Yang-Mills theory are defined by the Euclidean path integral[1]

$$\langle P(\vec{x}_1)\cdots P(\vec{x}_m)P^\dagger(\vec{y}_1)\cdots P^\dagger(\vec{y}_n)\rangle = \frac{\int dA_\mu \exp(-\int_0^{1/T}\mathrm{tr}F^2/4)P(\vec{x}_1)\cdots P(\vec{x}_m)P^\dagger(\vec{y}_1)\cdots P^\dagger(\vec{y}_n)}{\int dA_\mu \exp(-\int_0^{1/T}\mathrm{tr}F^2/4)} \tag{2}$$

with periodic boundary conditions, $A_\mu(1/T,\vec{x})=A_\mu(0,\vec{x})$. The gauge field remains periodic under the gauge transformation

$$A'_\mu(\tau,\vec{x})=g^{-1}(\tau,\vec{x})A_\mu(\tau,\vec{x})g(\tau,\vec{x})+ig^{-1}(\tau,\vec{x})\nabla_\mu g(\tau,\vec{x})$$

when $g(\tau,\vec{x})$ is periodic up to an element of the center of the group, $g(1/T,\vec{x})=g(0,\vec{x})e^{2\pi in/N}$. The measure, action, and boundary conditions in (2) are invariant under such gauge transformations. However, the loop operator (1) is not invariant, but transforms under the element of the center, which is Z_N for SU(N): $P'(\vec{x})=P(\vec{x})e^{2\pi in/N}$. Therefore, if the Z_N symmetry is not spontaneously broken, the correlators

$$e^{-F(\vec{x}_1,\ldots,\vec{x}_m,\vec{y}_1,\ldots,\vec{y}_n)/T}=\langle P(\vec{x}_1)\cdots P(\vec{x}_m)\times P^\dagger(\vec{y}_1)\cdots P^\dagger(\vec{y}_n)\rangle \tag{3}$$

vanish unless $m=n$ modulo N. $F(\vec{x}_1,\ldots,\vec{y}_n)$ is the free energy in the presence of an array of classical fundamental representation quark and antiquark sources at positions $\vec{x}_i$ and $\vec{y}_i$, respectively. If the Z_N symmetry is unbroken, the expectation value of a single loop, $\langle P(\vec{x})\rangle$, vanishes and consequently the free energy, $F(\vec{x})$, of a single quark source is infinite—a signal of confinement. On the other hand, if the Z_N symmetry is spontaneously broken, $F(\vec{x})$ is finite and characterizes the deconfined phase. (This statement is strictly true only when the theory has a finite ultraviolet cutoff. In the limit where the cutoff is removed, the Polyakov loop operator will generally require a multiplicative renormalization.)

When dynamical quarks in the fundamental representation of the gauge group are present, the Polyakov loop operator cannot be used to characterize confinement, since the antiperiodic boundary condition which a quark field would have, $\psi(1/T,\vec{x})=-\psi(0,\vec{x})$, is left unchanged only by strictly periodic gauge transformations. This is interpreted as fundamental quarks explicitly breaking the Z_N symmetry and, physically, as the possibility of pair production of fundamental quarks screening the color of an external source, so that the free energy of the source is always finite.

In this paper, we shall argue that, on the other hand, the

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[1]We use units where Planck's constant, the speed of light, and Boltzmann's constant are one. In the present discussion, we shall consider gauge group SU(N). Generalization to other compact gauge groups is straightforward. For a discussion of the path integral formulation of finite-temperature gauge theory, see [7].

Abelian analog of the Polyakov loop operator can be used to study confinement in noncompact quantum electrodynamics even when dynamical electrons are present. It is only necessary that the charges of all dynamical fields be integer multiples of some basic charge. A further technical requirement is that the matter fields are massive. When there is one species of electron, the free energy of a distribution of external charges is given by

$$e^{-F(\vec{x}_1,\ldots,\vec{x}_n)/T}=\frac{\int dA_\mu d\psi d\overline{\psi}\exp\left(-\int_0^{1/T}\left[\frac{1}{4}F_{\mu\nu}^2+\overline{\psi}(\gamma\cdot(\nabla-ieA)+m)\psi\right]\right)\exp\left(i\sum e_i\int_0^{1/T}d\tau A_0(\tau,\vec{x}_i)\right)}{\int dA_\mu d\psi d\overline{\psi}\exp\left(-\int_0^{1/T}\left[\frac{1}{4}F_{\mu\nu}^2+\overline{\psi}(\gamma\cdot(\nabla-ieA)+m)\psi\right]\right)} \tag{4}$$

with (anti)periodic boundary conditions $A_\mu(1/T,\vec{x})=A_\mu(0,\vec{x})$, $\psi(1/T,\vec{x})=-\psi(0,\vec{x})$, $\overline{\psi}(1/T,\vec{x})=-\overline{\psi}(0,\vec{x})$. The gauge transformation $A'_\mu(\tau,\vec{x})=A_\mu(\tau,\vec{x})+\nabla_\mu\chi(\tau,\vec{x})$, $\psi'(\tau,\vec{x})=e^{ie\chi(\tau,\vec{x})}\psi(\tau,\vec{x})$ $\overline{\psi}'(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})e^{-ie\chi(\tau,\vec{x})}$ is a symmetry of the action, measure and boundary conditions when $\vec{\nabla}_\mu\chi(1/T,\vec{x})=\nabla_\mu\chi(0,\vec{x})$ and $\chi(1/T,\vec{x})=\chi(0,\vec{x})+2\pi n/e$. The cosets of the group of all time-dependent gauge transformations modulo those which are periodic form the additive group of the integers, Z, which is the analog of the Z_N symmetry of Yang-Mills theories. Under the action of an element of a coset, the Abelian Polyakov loop operator transforms as $\exp[ie_j\int_0^{1/T}d\tau A'_0(\tau,\vec{x}_j)]=\exp[ie_j\int_0^{1/T}d\tau A_0(\tau,\vec{x}_j)]e^{(2\pi i n e_j/e)}$. If Z is not spontaneously broken, $F(\vec{x}_1,\ldots,\vec{x}_n)$ defined by (4) is infinite when the total charge of the external distribution is not an integral multiple of the electron charge, $\Sigma_i e_i\neq\text{integer}\times e$. When the symmetry is broken, $F(\vec{x}_1,\ldots,\vec{x}_n)$ can be finite. The presence or absence of symmetry breaking can be related to cluster decomposition of neutral correlators. Thus, the nature of the realization of Z tests the ability of the electrodynamic system to screen charges which are not integral multiples of the electron charge.

At $T=0$, and for the physical value of the electromagnetic coupling constant, (3+1)-dimensional electrodynamics is in the deconfined Coulomb phase at zero temperature and is thought to form a Debye plasma at all finite temperatures. The Z symmetry is therefore always spontaneously broken. On the other hand, in 1+1 dimensions, the Coulomb interaction is confining at the tree level and it is known that, when the electron has mass, it remains confining after quantum corrections and the Z symmetry is unbroken at any temperature [8]. One might expect that, in the intermediate case of (2+1)-dimensional electrodynamics both confined and deconfined phases are possible. In that case, even at the classical level, the Coulomb potential is a marginally confining logarithm. Its entire spectrum is bound states, but the bound states can have arbitrarily large size. The free energy of a gas of classical charged particles is $F_{\text{cl}}=-\frac{1}{2}\Sigma_{i,j}e_ie_j(1/2\pi)\ln|\vec{x}_i-\vec{x}_j|$ and the partition function has the scaling form

$$e^{-F_{\text{cl}}/T}=\text{const}\times\prod_{i<j}|\vec{x}_i-\vec{x}_j|^{e_ie_j/2\pi T} \tag{5}$$

with temperature-dependent exponent, reminiscent of the spin-wave correlators in Gaussian spin wave theory in two dimensions [9].

It is interesting to ask how this result would be changed by radiative corrections and by thermal fluctuations. This can be answered in the context of quantum electrodynamics by computing the effective action for the Polyakov loop operator. This is done by first fixing a gauge where A_0 is independent of the Euclidean time. Then, the other degrees of freedom, $\vec{A},\psi$, are integrated from the path integral. What remains is an effective action for a static field $A_0(\vec{x})$ which can be used to evaluate correlators of Polyakov loops, $e^{ieA_0(\vec{x})/T}$. The Z symmetry is a periodicity of the effective action under $A_0(\vec{x})\rightarrow A_0(\vec{x})+2\pi T/e$ and the central question is whether or not this symmetry is spontaneously broken, and if both phases exist, what is the nature of the phase transition between them.

The effective action is nonlocal and nonpolynomial in $A_0(\vec{x})$. When the matter fields have a mass, it has a local expansion in powers of derivatives of $A_0(\vec{x})$ divided by masses. This expansion is accurate when the momentum scales of interest are much smaller than the masses and the effective field theory for $A_0(\vec{x})$ can be approximated by a local field theory where the ultraviolet cutoff is taken to be the masses of the charged matter fields.

In 2+1 dimensions, the fermion mass operator constructed from the minimal two-component Dirac fermions is a pseudoscalar and therefore violates parity [10,11]. If included in the action, they can generate a parity violating topological mass for the photon by radiative corrections [12]. In this paper, we wish to study the case where the electron has mass but the photon is massless. For this purpose, we shall use parity-invariant four-component fermions. Consider the Euclidean action

$$S=\int d^3x\left[\frac{1}{4}F_{\mu\nu}^2+\overline{\psi}(\gamma\cdot(\nabla-ieA)+m)\psi\right]. \tag{6}$$

If $m=0$ in (6) there is a "chiral" symmetry under the transformation $\psi'(x)=\gamma_5\psi(x)$. The latter symmetry can be broken at $T=0$ [13] by a chiral transition which generates parity-invariant fermion masses. However, the mass generation also breaks a continuous chiral symmetry which, by the Coleman-Mermin-Wagner theorem, must be unbroken at any finite temperature in 2+1 dimensions. It has been argued that, at finite temperature, even though the condensate vanishes, $\langle\bar\psi\psi\rangle=0$, the chiral transition can be replaced by a Berezinskii-Kosterlitz-Thouless (BKT) transition [14]. We shall consider the opposite limit of large parity-invariant mass, $m>>e^2$, where there is no chiral symmetry, and show that there is a BKT transition corresponding to a confinement-deconfinement transition at some value of temperature T. For $m>>T,e^2$ we shall see that the critical line has the equation $T_{\rm crit}=e^2/8\pi(1+e^2/12\pi m+\cdots)$. When $T<T_{\rm crit}$ the theory is confining, whereas when $T>T_{\rm crit}$ it is in a deconfined phase. It is interesting to speculate that this transition is in some way related to the chiral transition, particularly that it is in fact on the same critical line of BKT transitions as [14] which would be encountered if one lowers the mass m to zero.

At finite temperature, (2+1)-dimensional QED contains three parameters with the dimension of mass, the electron mass m, the gauge coupling e^2, and temperature T. The loop expansion is super-renormalizable [10] and is an expansion in the smaller of the dimensionless ratios e^2/m and e^2/T. We can compute the effective action for $A_0(\vec{x})\equiv a(\vec{x})\sqrt{T}$ in a double expansion in the number of loops and in powers of derivatives of $a(\vec{x})$. To order one-loop and up to quadratic order in derivatives the effective action is

$$S_{\rm eff}[A_0]=\int d\vec{x}\left(Z(m,ea/\sqrt{T})\frac{1}{2}\vec{\nabla}a\cdot\vec{\nabla}a-V(m,ea/\sqrt{T})\right). \tag{7}$$

Here V is the effective potential for A_0 arising from the fermion determinant and Z is obtained from expansion of the temporal components of the vacuum polarization function to linear order in $-\vec{\nabla}^2$.

$$V(m,eA_0/T)=\frac{1}{(\rm Vol)}\ln\det[(-i\partial_0-eA_0)^2-\nabla^2+m^2], \tag{8}$$

where the fermions have antiperiodic boundary conditions in the 0 direction. The determinant can be computed by considering the ratio [15]

$$\Delta(m,eA_0/T)=\det[(-i\partial_0-eA_0)^2-\nabla^2+m^2]/\det(-\partial_0^2-\nabla^2+m^2)=\prod_{\vec{k}}\left[1-\frac{\sin^2(eA_0/2T)}{\cosh^2(\lambda_k/2T)}\right]\equiv\prod_{\vec{k}}\Delta_{\vec{k}}(m,eA_0/T), \tag{9}$$

where $\lambda_k^2=\vec{k}^2+m^2$ are the eigenvalues of $-\nabla^2+m^2$. Equation (9) holds in any dimensions. In 2+1 dimensions one can perform the integral on $\vec{k}$ arising in $\ln\Delta(m,eA_0/T)$, after taking the infinite volume limit:

$$V(m,eA_0/T)=\int_{-\infty}^{+\infty}\frac{d^2\vec{k}}{(2\pi)^2}\ln\Delta_{\vec{k}}(m,eA_0/T)=-\frac{T^2}{\pi}\left[\frac{m}{T}\mathrm{Li}_2(e^{-m/T},eA_0/T+\pi)+\mathrm{Li}_3(e^{-m/T},eA_0/T+\pi)\right], \tag{10}$$

where $\mathrm{Li}_2(r,\theta)=-\int_0^r dx\ln(1-2x\cos\theta+x^2)/2x$ and $\mathrm{Li}_3(r,\theta)=\int_0^r dx\,\mathrm{Li}_2(x,\theta)/x$ are the real parts of the dilogarithm and trilogarithm according to the convention of Ref. [16]. As expected, Eq. (10) shows the periodicity of the effective potential for $eA_0/T\to eA_0/T+2\pi$. (In 1 and 3 spatial dimensions the integral in $\vec{k}$ can only be performed analytically for $m=0$ in which case it gives simple polynomial expressions. In the limit $m=0$, the effective potential for A_0 has been discussed in [17].)

It is also straightforward to compute the term which contributes the leading order in derivatives to the effective action:

$$Z(m,ea/\sqrt{T})=1+\frac{e^2}{12\pi m}\left(1-m\frac{\partial}{\partial m}\right)\frac{\sinh m/T}{\cosh m/T+\cos ea/\sqrt{T}}. \tag{11}$$

The critical behavior of the two-dimensional model defined by Eqs. (7), (10), and (11) can be understood by comparing it with the sine-Gordon model in two dimensions. That this comparison can be reliably performed can be seen by the study of the harmonic content of (10):

$$V(m,ea/\sqrt{T})=-\frac{T^2}{\pi}\sum_{n=1}^{\infty}\frac{e^{-nm/T}}{n^3}\left(1+\frac{nm}{T}\right)\cos[n(ea/\sqrt{T}+\pi)]. \tag{12}$$

Consider then the large m limit, T/m and e^2/m small with finite e^2/T. In this limit, the higher harmonics are small perturbations to the potential

$$V(m,ea/\sqrt{T})=\frac{Tm}{\pi}e^{-m/T}\cos(ea/\sqrt{T}), \tag{13}$$

which is the sine-Gordon potential. Amit *et al.* [18]. showed that in the sine-Gordon model any perturbations of the type $\cos(n\beta\phi)$ to a sine-Gordon potential $\alpha\cos(\beta\phi)/\beta^2$ are irrelevant for the critical behavior of the model. By analogy with the spin wave plus Coulomb gas model, it was also proven in Refs. [18] that a critical line for a BKT [19] phase transition in the sine-Gordon model with a logarithmic potential starts at the point $(\alpha,\beta^2)=(0,8\pi)$. We can then conclude that also in 2+1 QED at finite temperature there is a BKT phase transition, with a critical line in the $(m/T, e^2/T)$ plane starting at $(m/T, e^2/T) = (\infty, 8\pi)$. The critical temperature for this transition (up to one-loop order) can be computed from Eqs. (11) and (13) as $T_{\rm crit} = e^2/8\pi(1 + e^2/12\pi m + \cdots)$. This is the critical value of the coupling constant originally found by Coleman in his discussion of bosonization of the massive Thirring model [20]. It is interesting that the phase transition is accessible to perturbative analysis. (To our knowledge it is the only confinement-deconfinemt transition which is so accessible.) This is a consequence of the fact that the coefficient of the cosine term in the effective action in the large mass, weak coupling limit is in the range of critical parameters of the sine-Gordon theory.

The vacuum expectation value of A_0 in the deconfined phase, where the Z symmetry is spontaneously broken, is $\langle A_0 \rangle = 2\pi n T/e$. In a semiclassical analysis, this expectation value contributes an imaginary chemical potential for the electron. However, this chemical potential can be absorbed by shifting the Matsubara frequency of the electron by $2\pi n T$. Thus, in the case of electrodynamics, the semiclassical thermodynamics of the deconfined phase do not suffer from the difficulties of negative entropy and imaginary thermodynamic potential that plague the metastable Z_N phases of QCD [21].

We wish to point out the distinction between our present results and the well-known fact that, in the absence of dynamical matter fields, compact three-dimensional QED has a finite temperature phase transition between a confining and deconfined phase [5,1]. That transition is also known to be of BKT type. In our case, it is essential that the U(1) gauge symmetry be noncompact so that the order parameter with incommensurate charges is gauge invariant. It is also essential that the charges of dynamical matter field are all quantized in integer multiples of some basic unit. Although this means that in a realistic system, one could not actually do the experiment of introducing an incommensurate charge and measuring the required energy, the phase transition should be observable by other means. For example, the voltage A_0 in the deconfined phase fluctuates near some minimum of the potential and the electric screening is characterized by a Debye mass which can be obtained by expanding the effective potential. The BKT transition has a universal jump [18] in the parameters which characterize the Debye screening in the deconfined phase. This jump is a definite prediction of our analysis. In the confined phase, A_0 is a random field which suppresses screening of the long-ranged Coulomb interactions. Our results should be of interest, and are in principle testable in spin systems commonly studied in lower-dimensional condensed matter physics where U(1) gauge symmetries arise naturally [22].

An experimentally testable consequence of the spontaneous breaking of Z symmetry in the deconfined phase of $(2+1)$- and $(3+1)$-dimensional QED is the existence of domain walls. An important and outstanding problem, which has already been addressed for SU(N) gluodynamics [23] and massless QED [17], is the understanding of their physical properties in the present case of QED with massive matter.

It is interesting to ask what happens when the three-dimensional electrodynamics has a small topological mass which cuts of the long-ranged fluctuations of the gauge fields. It has been suggested that there is a phase transition at zero temperature as the topological mass is varied [24]. Whether such a phase transition could be observed at finite temperature is a subject of ongoing investigation.

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