Some remarks on the BRST quantization of massive Abelian two-form gauge fields

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In this paper we quantize massive Abelian two-form gauge fields in six dimensions following the antifield BRST formalism. The quantization procedure is based on the quantization of a first-class system associated with the original theory. This first-class system is obtained by converting the original second-class constraints into some first-class ones in a way to preserve the reducibility of free Abelian three-form gauge fields. The path integral of the first-class system is identical to the original one in an appropriate gauge-fixing fermion. Finally, it is shown that this first-class system leads to the one derived by Bizdadea and Saliu. [S0556-2821(96)05410-0]

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I. INTRODUCTION

The quantization of purely second-class constrained systems has attracted much attention lately. The most efficient tool in achieving the quantization of such theories, as well as of the gauge ones, was proved to be the Becchi-Rouet-Stora-Tyutin (BRST) formalism [1–5]. In order to incorporate the purely second-class systems within the BRST formalism, we have to turn the original system into a first-class one: (i) in the initial phase space [6], or (ii) in a larger phase space [7,8]. Many authors [9–18] have employed the methods from [7,8] to solve various models. At the same time, the methods based on enlarging the phase space have been extended recently to cover the systems preserving the reducibility relic of a first-class theory [19,20], as massive Abelian p-form gauge fields. There are many reasons for studying Abelian p-form gauge fields. For example, the significance of free Abelian two-form gauge fields consists in their natural appearance in the study of dual resonance models, relativistic strings, black holes, vortices, and extended supergravity [21–29]. As was shown in [19], the quantization of massive Abelian two-form gauge fields in four dimensions implies in fact the BRST quantization of the system

$$S^L_{0_{1,0}}[A,B]=\int d^4x\left(-\frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho}-\frac{1}{4}(MA_{\mu\nu}-F_{\mu\nu})\times(MA^{\mu\nu}-F^{\mu\nu})\right), \quad (1)$$

where $F_{\mu\nu\rho}=\partial_{[\mu}A_{\nu\rho]}$ and $F_{\mu\nu}=\partial_{[\mu}B_{\nu]}$. Action (1) comes from the gauging of the rigid symmetries $\delta_\epsilon B_\mu=M\epsilon_\mu$ for the Abelian gauge field action. At the Hamiltonian level, the conservation of the currents corresponding to the last symmetries, but for action (1), results in the constraint $-\partial^i\pi_i=0$, with π_μ the canonical momenta of the B^μ 's. There can be shown also in general that the quantization of massive Abelian p-form gauge fields induces the introduction of Abelian $(p-1)$-form gauge fields, the interacting term being of the type current-current [19,20]. However, it has never been realized a conversion method for the second-class constraints of the massive Abelian p -form gauge fields such that their quantization implies the BRST quantization of a reducible theory describing Abelian p-form gauge fields interacting with Abelian $(p+1)$-form gauge fields. This is the aim of our paper for $p=2$. Namely, in this work we quantize massive Abelian two-form gauge fields in D dimensions (1) transforming the original system into a first-class one in the original phase space, and (2) associating to the above system a one-parameter family of first-class systems, and subsequently applying the BRST formalism to the first-class family. As it will be seen, the conversion procedure (2) will lead to a system describing Abelian two-form interacting with three-form gauge fields in six dimensions. Related to the BRST quantization, we follow the general lines from [5].

II. THE RIGID SYMMETRIES OF FREE ABELIAN THREE-FORM GAUGE FIELDS

In this section we investigate some rigid symmetries of free Abelian three-form gauge fields in D dimensions. These symmetries lead to some conserved currents and to some quantities derived with their help to be further employed in the next section in order to accomplish the conversion procedure. The starting point of this section is the Lagrangian action

$$S^L_{0_3}[B]=-\int d^Dx\frac{1}{2\times 4!}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}, \quad (2)$$

with $F_{\mu\nu\rho\lambda}=\partial_{[\mu}B_{\nu\rho\lambda]}$ and the $B_{\mu\nu\rho}$'s antisymmetric in all indices. Action (2) possesses the rigid (Noether) symmetries

$$\delta_{\bar\epsilon}B_{\mu\nu\rho}=\epsilon_{\mu\nu\rho\lambda_1\cdots\lambda_{D-3}}\overline{\epsilon^{\lambda_1\cdots\lambda_{D-3}}}, \quad (3)$$

where $\epsilon_{\lambda_1\cdots\lambda_D}$ denotes the completely antisymmetric symbol in D dimensions, while $\overline{\epsilon^{\lambda_1\cdots\lambda_{D-3}}}$ are constant parameters. The conserved currents corresponding to (3) are of the form

$$j^\mu_{\lambda_1\cdots\lambda_{D-3}}=\frac{(-)^{3(D-3)}}{3!}\epsilon_{\lambda_1\cdots\lambda_{D-3}\alpha\beta\gamma}F^{\mu\alpha\beta\gamma}. \quad (4)$$

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Because $j^0_{i_1\cdots i_{D-3}}=0$, the conservation of the spatial part of the last currents reads $\partial_i j^i_{i_1\cdots i_{D-3}}=0$. In terms of the canonical momenta $\Pi_{\mu\nu\rho}$ conjugated to the fields $B_{\mu\nu\rho}$, the above conservation laws read

$$\partial_i \Pi^{ijk}=0. \tag{5}$$

Anticipating a bit, we shall impose relations (5) as constraints for the extravariables to be introduced in Sec. III. They are analogous to the constraint $\partial_i \pi^i=0$ implemented in the context of action (1) for Abelian one-form gauge fields [19].

With the aid of the currents (4) we build the ''currents''

$$j_{\lambda_1\cdots\lambda_{D-4}}=j^{\mu}_{\lambda_1\cdots\lambda_{D-4}\mu}. \tag{6}$$

The above ''currents'' satisfy the relations

$$\partial^{\lambda_1} j_{\lambda_1\cdots\lambda_{D-4}}=0, \tag{7}$$

$$\partial^{\lambda_2}\partial^{\lambda_1} j_{\lambda_1\cdots\lambda_{D-4}}=0, \tag{8}$$

$$\vdots$$

$$\partial^{\lambda_{D-4}}\cdots\partial^{\lambda_2}\partial^{\lambda_1} j_{\lambda_1\cdots\lambda_{D-4}}=0. \tag{9}$$

We remark that actually, relations (7) are not conservation laws, but merely identities. Indeed, they hold independently of the equations of motion ($\partial_\mu F^{\mu\nu\rho\lambda}=0$), and, moreover, there are no rigid symmetries of action (2) revealing such conservations. This is the reason of putting the word ''current'' between inverted commas. Relations (8) and (9) also represent some identities. They are clearly reducibility relations associated to (7). For this reason it is natural to choose all these relations to contribute to the reducibility relations of a certain first-class system to be constructed in the next section starting with massive Abelian two-form gauge fields.

III. THE FIRST-CLASS FAMILY ASSOCIATED WITH MASSIVE ABELIAN TWO-FORM GAUGE FIELDS

Here, we convert the second-class constraints of massive Abelian two-form gauge fields into some first-class constraints by extending the original phase space. We notice that the conversion will be achieved following the general lines exposed in [30], which will be extended now to reducible theories in a way different from the one presented there. To begin with, we take the action

$$S^L_{0_2}[A]=\int d^Dx\left(-\frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho}-\frac{M^2}{4}A_{\mu\nu}A^{\mu\nu}\right), \tag{10}$$

where $F_{\mu\nu\rho}=\partial_{[\mu}A_{\nu\rho]}$. The canonical analysis of action (10) furnishes the second-class constraints

$$G^j\equiv\pi^{0j}=0, \quad \text{primary}, \tag{11}$$

$$C_j\equiv 2\partial^i\pi_{ij}-M^2A_{0j}=0, \quad \text{secondary}, \tag{12}$$

together with the canonical Hamiltonian

$$H=\int d^{D-1}x\left(-\pi_{ij}\pi^{ij}+\frac{1}{12}F_{ijk}F^{ijk}+\frac{M^2}{4}A_{\mu\nu}A^{\mu\nu}-2A^{0j}\partial^i\pi_{ij}\right). \tag{13}$$

For later convenience we denote $C^{(0)}_j\equiv 2\partial^i\pi_{ij}$ and $C^{(1)}_j\equiv -M^2A_{0j}$.

Starting with (11)–(13), our conversion method consists of two steps. First, we construct in the original phase space a first-class system described by the first-class Hamiltonian [6]

$$\bar{H}=H-\frac{1}{M^2}\int d^{D-1}x\left(C^{(0)}_jC^{(1)j}+\frac{1}{2}C^{(1)}_jC^{(1)j}\right), \tag{14}$$

as well as the first-class constraints (11). It is simple to see that $[\bar{H},G^j]=0$ strongly, where the symbol $[,]$ denotes the Poisson bracket. Second, we associate to the prior system a one-parameter family of first-class systems in a larger phase-space built as follows.

(i) For every pair $(G^j,C^{(0)}_j)$ we introduce the canonical bosonic pairs (B^{ijk},Π_{ijk}) antisymmetric in their indices such that the new secondary constraints become

$$\gamma_j\equiv\lambda K_j(B,\Pi)-C^{(0)}_j, \tag{15}$$

with $K_j(B,\Pi)$ some functions to be further identified, and λ the nonvanishing parameter of the first-class family. As the functions $C^{(0)}_j$ are first-order reducible

$$\partial^j C^{(0)}_j=0, \tag{16}$$

and we wish to obtain a reducible first-class family, it appears natural that the functions $K_j(B,\Pi)$ be taken also first-order reducible, with the same reducibility functions as in (16). In order to find some reducible functions $K_j(B,\Pi)$ we make use of relations (7)–(9). Taking $D=6$, we observe that the ''currents'' $j_{\lambda_1 0}$ derived from (6) satisfy precisely the identities

$$\partial^k j_{k0}=0, \tag{17}$$

the identities (8) and (9) being trivial in this case. Thus we choose

$$K_j(B,\Pi)\equiv-\frac{1}{4}j_{j0}, \tag{18}$$

such that the functions γ_j become first-order reducible, namely

$$\partial^j\gamma_j=0. \tag{19}$$

(ii) For every relation (5) we introduce a new bosonic canonical pair (B^{0ij},Π_{0ij}) together with the supplementary constraints

$$\bar{G}_{ij}\equiv\Pi_{0ij}=0, \tag{20}$$

such that their consistencies imply, as new secondary constraints, the relations (5) (up to a factor)

$$\bar{\gamma}_{ij}\equiv-3\partial^k\Pi_{kij}=0. \tag{21}$$

In this way, we managed to associate with the original system a one-parameter family of first-class systems, with the first-class constraints (11), (15), (20), and (21), with $K_j(B,\Pi)$ from (15) given by (18). The above constraints are second-order reducible. Indeed, apart from (19) there still exist the reducibility relations

$$\partial^i \overline{\gamma_{ij}} = 0, \quad \partial^j \partial^i \overline{\gamma_{ij}} = 0. \tag{22}$$

At this point we build the first-class Hamiltonian of the first-class family in a way to obtain (15) and (21) as secondary constraints. Then, the first-class Hamiltonian is expressed by

$$H^* = \overline{H} + \int d^5x (A^{0j}\gamma_j + B^{0ij}\overline{\gamma_{ij}}) + g \equiv \int d^5x h^*, \tag{23}$$

with g a function verifying the equations

$$[G^j, g] = 0, \quad [\overline{G^{ij}}, g] = 0, \tag{24}$$

$$[\gamma_j, H^*] = 0, \tag{25}$$

$$[\overline{\gamma_{ij}}, g] = 0. \tag{26}$$

For solving the last system, we represent g as a series of powers in Π_{ijk}'s with coefficients functions of A^{ij}'s only plus a function depending strictly on B^{ijk}'s

$$g = \int d^5x \left(\overset{(1)}{g}{}^{ijk} \Pi_{ijk} + \overset{(2)}{g}{}^{ijk}_{i_1 j_1 k_1} \Pi_{ijk} \Pi^{i_1 j_1 k_1} + \cdots + f(B^{ijk}) \right). \tag{27}$$

It is clear from (27) that (24) are automatically fulfilled. Introducing (27) in (25), we get

$$g = \int d^5x \left(-\frac{3M^2}{\lambda^2} \Pi_{ijk} \Pi^{ijk} + \frac{M^2}{2\lambda} \epsilon^{0ijklm} A_{lm} \Pi_{ijk} + f(B^{ijk}) \right). \tag{28}$$

Requiring now for (28) to check (26), we derive $f(B^{ijk})$ of the form

$$f(B^{ijk}) = \frac{\lambda^2}{2 \times 4! M^2} F_{ijkl} F^{ijkl}, \tag{29}$$

where $F_{ijkl} = \partial_{[i} B_{jkl]}$. In principle, in (29) there can appear a polynomial in $F_{ijkl}F^{ijkl}$. We took into consideration only the first term due to the first term in (28), which signalizes that only (29) is in accordance with this term. This determines completely the function g, and thus H^*.

The extended action [5] of the second-order reducible first-class family associated to massive Abelian two-form gauge fields reads

$$S_0^E[A,B,\pi,\Pi,u,v] = \int d^6x (\dot{A}^{0i}\pi_{0i} + \dot{A}^{ij}\pi_{ij} + \dot{B}^{0ij}\Pi_{0ij} + \dot{B}^{ijk}\Pi_{ijk} - h^* - u_j G^j - u_{ij}\overline{G^{ij}} - v^j \gamma_j - v^{ij}\overline{\gamma_{ij}}). \tag{30}$$

Passing to the total action [5] corresponding to (30) (taking $v^j = v^{ij} = 0$) and eliminating the momenta and the remaining multipliers on their equations of motion [31], we infer the Lagrangian action of the first-class family as

$$S_{0_{2,3}}^L[A,B] = \int d^6x \left(-\frac{1}{12} F_{\mu\nu\rho} F^{\mu\nu\rho} - \frac{\lambda^2}{2 \cdot 4! M^2} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} + \frac{\lambda}{2 \times 4!} \epsilon_{\mu\nu\rho\lambda\alpha\beta} A^{\alpha\beta} F^{\mu\nu\rho\lambda} \right). \tag{31}$$

Action (31) describes a gauge theory of Abelian two and three-form gauge fields with topological coupling in six dimensions. We notice that if we set $\lambda = M$ and subsequently take the limit $M \to 0$, we decouple the above theories. It is precisely the fact that $M \neq 0$ which allowed us to couple the two systems mentioned before through our procedure. We fixed in this way the correct value of the parameter.

IV. THE BRST QUANTIZATION OF THE FIRST-CLASS FAMILY

Within this section we quantize the system described by action (31) in the light of the antifield BRST formalism [5]. This action is invariant under the gauge transformations

$$\delta_\epsilon A_{\mu\nu} = \partial_{[\mu} \epsilon_{\nu]}, \quad \delta_\epsilon B_{\mu\nu\rho} = \partial_{[\mu} \epsilon_{\nu\rho]}, \tag{32}$$

where ϵ_ν's and $\epsilon_{\nu\rho}$'s are arbitrary functions, the last ones being antisymmetric. The gauge transformations (32) are second-order reducible, the gauge generators and first and second-order reducibility matrices taking the form

$$Z^{\alpha_0}_{\alpha_1} = \begin{pmatrix} Z^{\alpha\beta\gamma}_{\mu\nu} & 0 \\ 0 & Z^{\mu\nu}_{\lambda} \end{pmatrix}, \quad Z^{\alpha_1}_{\alpha_2} = \begin{pmatrix} Z^{\mu\nu}_{\lambda} & 0 \\ 0 & Z^{\lambda} \end{pmatrix}, \quad Z^{\alpha_2}_{\alpha_3} = \begin{pmatrix} Z^{\lambda} \\ 0 \end{pmatrix}. \tag{33}$$

In the De Witt condensed notations, the functions $Z^{\alpha\beta\gamma}_{\mu\nu}$, $Z^{\mu\nu}_{\lambda}$, Z^{λ} read

$$Z^{\alpha\beta\gamma}_{\mu\nu} = \frac{1}{2}[(\delta^\beta_\mu \delta^\gamma_\nu - \delta^\beta_\nu \delta^\gamma_\mu)\partial^\alpha + (\delta^\gamma_\mu \delta^\alpha_\nu - \delta^\gamma_\nu \delta^\alpha_\mu)\partial^\beta + (\delta^\alpha_\mu \delta^\beta_\nu - \delta^\alpha_\nu \delta^\beta_\mu)\partial^\gamma], \tag{34}$$

$$Z^{\mu\nu}_{\lambda} = \delta^\nu_\lambda \partial^\mu - \delta^\mu_\lambda \partial^\nu, \quad Z^\lambda = \partial^\lambda. \tag{35}$$

Corresponding to the matrices (33), the minimal ghost and antifield spectra are given by

ghost	$\eta^{\mu\nu}$	$\mathscr{C}^\lambda$	η^λ	$\mathscr{C}$	η
gh	1	1	2	2	3
ϵ	1	1	0	0	1,

(36)

antifield	$B^*_{\alpha\beta\gamma}$	$A^*_{\mu\nu}$	$\eta^*_{\mu\nu}$	$\mathscr{C}^*_\lambda$	η^*_λ	$\mathscr{C}^*$	η^*
gh	-1	-1	-2	-2	-3	-3	-4
ϵ	1	1	0	0	1	1	0,

(37)

with the notations gh and ϵ denoting the ghost number and the Grassmann parity. The nonminimal solution of the master equation [5] takes the form

$$S_{\min}=S^{L}_{0_{2,3}}+\int d^6x(B^{*}_{\alpha\beta\gamma}\partial^{[\alpha}\eta^{\beta\gamma]}+A^{*}_{\mu\nu}\partial^{[\mu}\mathscr{C}^{\nu]}+\eta^{*}_{\mu\nu}\partial^{[\mu}\eta^{\nu]}+\mathscr{C}^{*}_{\lambda}\partial^{\lambda}\mathscr{C}+\eta^{*}_{\lambda}\partial^{\lambda}\eta. \tag{38}$$

At this stage we show that using a peculiar gauge-fixing fermion the path integral correspondent to action (38) leads to the path integral for massive Abelian two-form gauge fields. In this respect, we introduce a nonminimal sector slightly different from the general prescription in [5] such that the nonminimal solution of the master equation be expressed as

$$S_{\text{non min}}=S_{\min}+\int d^6x(\overline{\eta^{*}_{\alpha\beta\gamma}}b^{\alpha\beta\gamma}+\overline{\eta^{*}_{\alpha\beta}}b^{\alpha\beta}+\overline{\eta^{*}_{\alpha}}b^{\alpha}+\overline{\eta^{*}}b+\overline{\eta'}^{*}b'+\overline{\eta''}^{*}b''). \tag{39}$$

The appropriate gauge-fixing fermion is given by

$$\Psi=\int d^6x(\overline{\eta^{\alpha\beta}}(b_{\alpha\beta}-MA_{\alpha\beta})+\overline{\eta^{\alpha\beta\gamma}}B_{\alpha\beta\gamma}+\eta_{\mu\nu}\partial^{\mu}\overline{\eta^{\nu}}+\overline{\eta^{\nu}}\partial_{\nu}\overline{\eta''}+\mathscr{C}^{\nu}\partial_{\nu}\overline{\eta'}+\eta^{\nu}\partial_{\nu}\overline{\eta}). \tag{40}$$

Eliminating in the standard way the antifields from (39) with the help of (40) and integrating in the path integral associated to the gauge-fixed action, Z_{Ψ}, over all the fields except the $A_{\alpha\beta}$'s, we get

$$Z_{\Psi}=\int \mathscr{D}A_{\alpha\beta}\det(M)\exp(iS^{L}_{0_2}[A_{\alpha\beta}]). \tag{41}$$

Formula (41) represents the path integral of the first-class family and it coincides with that for massive Abelian two-form gauge fields. This result is identical with the one derived in [19]. There, the conversion mechanism is resulting in a first-class system described by action (1).

At this point we can clarify the meaning of the original system bearing the reducibility trace of a certain reducible first-class theory. From (41) it follows that the path integral of the original system comes from the BRST quantization of the first-class system (31) (with $\lambda=M$). We conclude then that the original system is obtained at the path integral level from the first-class one.

V. THE CLASSICAL ANALYSIS OF THE FIRST-CLASS SYSTEM

Here we point out some interesting aspects linked to the first-class system. Its action is invariant under the rigid transformations (3) in six dimensions. The corresponding conserved currents have the form

$$\overline{j^{\mu}_{\alpha\beta\gamma}}=-\frac{1}{3!}\epsilon_{\alpha\beta\gamma\nu\rho\lambda}F^{\mu\nu\rho\lambda}-M\delta^{\mu}_{[\alpha}A_{\beta\gamma]}. \tag{42}$$

Using (42), action (31) may be written under the form

$$S^{L}_{0_{2,3}}[A,B]=S^{L}_{0_2}[A]+\int d^6x\frac{1}{2\times 4!}\overline{j^{\mu}_{\alpha\beta\gamma}}\overline{j^{\alpha\beta\gamma}_{\mu}}. \tag{43}$$

We observe that at the classical level we reobtain from the first-class system the original one if $\overline{j^{\mu}_{\alpha\beta\gamma}}=0$. Redefining $\overline{j^{\mu}_{\alpha\beta\gamma}}$ as new fields we can emphasize the Wess-Zumino action associated with the original system under the form

$$S^{WZ}_{0}[A,B]=\int d^6x\frac{1}{2\times 4!}\overline{j^{\mu}_{\alpha\beta\gamma}}\overline{j^{\alpha\beta\gamma}_{\mu}}. \tag{44}$$

In the sequel we make the connection between the first-class system (43) and the first-class system (1). With the aid of (32) we obtain

$$\delta_{\epsilon}\left(\frac{1}{2\times 4!}\overline{j^{\mu}_{\alpha\beta\gamma}}\overline{j^{\alpha\beta\gamma}_{\mu}}\right)=\frac{M^2}{2}A^{\alpha\beta}\partial_{[\alpha}\epsilon_{\beta]}. \tag{45}$$

Because in (43) the dependence of $\overline{j^{\mu}_{\alpha\beta\gamma}}$'s is contained only in the term $[1/(2\times 4!)]\overline{j^{\mu}_{\alpha\beta\gamma}}\overline{j^{\alpha\beta\gamma}_{\mu}}$, we can replace the fields $\overline{j^{\mu}_{\alpha\beta\gamma}}$ with some other fields, V_{μ}, introduced implicitly through the relation

$$\frac{1}{2\times 4!}\overline{j^{\mu}_{\alpha\beta\gamma}}\overline{j^{\alpha\beta\gamma}_{\mu}}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{M}{2}F_{\mu\nu}A^{\mu\nu}, \tag{46}$$

with $F_{\mu\nu}=\partial_{[\mu}V_{\nu]}$ and $\delta_{\epsilon}V_{\nu}=\partial_{\nu}\epsilon+M\epsilon_{\nu}$. The legitimacy of the transformation (46) results from the equality between the gauge variations of both its sides

$$\delta_{\epsilon}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{M}{2}F_{\mu\nu}A^{\mu\nu}\right)=\delta_{\epsilon}\left(\frac{1}{2\times 4!}\overline{j^{\mu}_{\alpha\beta\gamma}}\overline{j^{\alpha\beta\gamma}_{\mu}}\right). \tag{47}$$

Introducing (46) in (43), we recover action (1) modulo the identifications $B_{\mu}\equiv V_{\mu}$, but in six dimensions. The Wess-Zumino action in the last case is

$$\overline{S^{W}_{0}}Z[A,V]=-\frac{1}{4}\int d^6x(\tilde{j}^{\mu}_{\nu}\tilde{j}^{\nu}_{\mu}-M^2A_{\mu\nu}A^{\mu\nu}), \tag{48}$$

where $\tilde{j}^{\mu}_{\nu}=F^{\mu}_{\nu}-MA^{\mu}_{\nu}$ is the gauge-invariant current corresponding to the rigid symmetries $\delta_{\epsilon}V_{\nu}=M\epsilon_{\nu}$ of action (1). From (47) and (48) we can conclude that we may regain the first-class system associated with massive Abelian two-form gauge fields obtained in [19] by making a field transformation such that the gauge variations of the Wess-Zumino actions obtained through the two above indicated conversion procedures are identical.

VI. CONCLUSION

Using the antifield BRST formalism, we proved that converting the second-class constraints of massive Abelian two-form gauge fields into some first-class ones we can quantize consistently our model in a way which preserves the reducibility relic of free Abelian three-form gauge fields. The conversion method exposed in Sec. III is directly connected with the manifest covariance of the action of the first-class system. The covariance is realized through the presence of $C^{(0)}_{j}$ within the new secondary constraints (15). If one does not introduce $C^{(0)}_{j}$ in (15), then the function g is vanishing, and thus is further leading to a noncovariant action for the first-class system. Another feature specific to our conversion method is revealed by implementing in the theory the reducibility relations of free Abelian three-form gauge fields with the aid of the ''currents'' (6). It is precisely the setting (18) which couples consistently Abelian two and three-form gauge fields through (23). The equivalence between the quantization procedure exposed in this paper with the one from [19] is accomplished by formula (41). At the same time, the equivalence at the classical level between the two first-class systems is achieved via the equality between the gauge variations of the corresponding Wess-Zumino actions. The procedure described in this work can also be applied starting with massive Abelian vector fields and establishing the equivalence with our mechanism for an irreducible second-class system [32]. More on Abelian p-form gauge fields can be found in [33].

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