On the origin of the outgoing black hole modes

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The question of how to account for the outgoing black hole modes without drawing upon a trans-Planckian reservoir at the horizon is addressed. It is argued that the outgoing modes must arise via conversion from ingoing modes. It is further argued that the back reaction must be included to avoid the conclusion that particle creation cannot occur in a strictly stationary background. The process of ''mode conversion'' is known in plasma physics by this name and in condensed matter physics as ''Andreev reflection'' or ''branch conversion.'' It is illustrated here in a linear Lorentz noninvariant model introduced by Unruh. The role of interactions and a physical short distance cutoff is then examined in the sonic black hole formed with helium-II. [S0556-2821(96)04012-X]

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I. THE TRANS-PLANCKIAN RESERVOIR

A fundamental problem in black hole physics is to account for the origin of the outgoing modes. In ordinary field theory these modes arise from a reservoir of arbitrarily high frequency, short wavelength, degrees of freedom (in a free-fall frame) propagating just outside the event horizon and exponentially redshifting until they finally escape. There are good reasons to doubt the existence of such a trans-Planckian reservoir, however. Yet without this reservoir to draw upon, it seems that a short time after the formation of a black hole there would be a dearth of outgoing modes [1]. Perhaps this is just what happens. But it would produce a catastrophic breakdown of the usual vacuum structure outside a black hole horizon, would preclude the existence of Hawking radiation, and would invalidate any semiclassical analysis of black holes. While this possibility can perhaps not (yet) be ruled out conclusively by observations, it seems terribly unlikely. Underlying this paper is the assumption that the outgoing modes do indeed exist, and the problem is to account for them without drawing upon a reservoir of trans-Planckian degrees of freedom.

It is easiest to describe the problem in free field terms, where a ''mode'' of the field has an autonomous identity, but the problem also exists for interacting fields. One way to describe it is to think of the interacting theory perturbatively. Then the issue is that, in ordinary field theory, the value of low energy observables far from the hole at late times depends on trans-Planckian features of the propagator near the horizon [2]. Nonperturbatively, one can view the field theory as a set of coupled equations for the correlation functions. In this framework, the operator equations of motion presumably still imply that such observables depend on the trans-Planckian structure of the correlation functions just outside the horizon.

Two good reasons to doubt the existence of a trans-Planckian reservoir are (i) field theory divergences and (ii) divergence of black hole entropy in ordinary field theory. It is generally believed that the short distance divergences of quantum field theory are due to an unphysical assumption about the short distance physics, namely, that there are an infinite number of degrees of freedom localizable in any volume, no matter how small that volume may be. The existence of these degrees of freedom also leads in general to a divergent contribution to the entropy of a black hole. This can be viewed in terms of entanglement entropy [3–5], or thermal entropy of acceleration radiation [6], or simply in terms of the renormalization of Newton's constant (and the coefficients of curvature squared terms in the effective action) [7,8].[1] In this last guise it appears that the divergence of black hole entropy is merely one aspect of the divergence problem of quantum field theory. The cure for this problem is to reject the assumption that there exists an infinite density of localizable states.

II. NONLOCALITY AND LORENTZ NONINVARIANCE

Local Lorentz invariance requires that there be an infinite number of degrees of freedom in any volume, no matter how small that volume may be. To evade the conclusion that the density of states is infinite it seems that one must give up either locality or Lorentz invariance. Perturbative string theory is an example of a nonlocal theory in which Lorentz invariance is maintained but the the density of localizable states is finite (in fact zero) [10]. As currently formulated, string theory actually has many *more* states than ordinary field theory (see, e.g., [11]); however, each state is fundamentally completely nonlocal. (There exist conjectures that this is a condensed phase of a more fundamental theory that has a truly finite density of states [12].) The role of this nonlocality in accounting for the outgoing string states

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[1]In fact it seems that due to effects of ''curvature coupling,'' the entanglement and thermal entropies are not identical. It is the ''thermal entropy'' that appears more fundamental when it is recognized as just one contribution to the total entropy expression resulting from one piece of the low energy effective gravity action (see, e.g., [9]).

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around a black hole has not yet to my knowledge been clarified (see however [13,10,14]).

The other possibility is to abandon Lorentz invariance rather than locality. It might be considered foolish to question Lorentz invariance. After all, we certainly have no observations that challenge its validity, and indeed most physicists seem inclined to believe that it is an *exact* symmetry of nature. However, it should be remembered that the Lorentz group is noncompact, so it would take observations at arbitrarily large boosts to confirm Lorentz invariance. (At present the upper bound to the boost factors that have been probed is around $\gamma \sim 10^{12}$ in transforming to the center of mass frame in cosmic ray collisions.) Of course if Lorentz invariance is only an approximate symmetry it will be necessary to explain why it appears to hold so well where it has been tested. In this connection it is interesting to note that effective field theories governed by a Lorentzian metric are known to occur in many condensed matter systems where there is actually a preferred frame. If the spacetime metric is of this nature, and if the preferred frame is the cosmic rest frame, then it is clearly relevant that the Earth is essentially at rest in this frame. However, one would still have to explain why we do not see several different effective Lorentz metrics for different low energy fields. Of possible relevance is the observation [15,16] that there is a tendency for the effective metrics for different interacting fields to flow to a common metric in the infrared.

In this paper we will consider the consequences of Lorentz noninvariance for the issue of the origin of the outgoing modes. These considerations may also be relevant for a nonlocal theory such as string theory, since nonlocally realized Lorentz invariance might look somewhat like plain old Lorentz noninvariance in some domain. We shall see that there is a mechanism, operating within the Lorentz noninvariant model theories considered, whereby the outgoing modes originate from certain ingoing modes, by a counterintuitive but rather simple process.

III. CONVERSION OF INGOING TO OUTGOING MODES

In this section we consider in general terms the question of how the outgoing modes could possibly be restored in a theory with a finite density of states. This question was raised in Ref. [1], where it was proposed that a process of ''mode regeneration'' must take place, but the nature of this process remained obscure. It was conjectured that it necessarily involves *interactions*, and perhaps corresponds to the time reverse of a decay process in which low frequency outgoing modes fuse to form high frequency ones. There are (at least) two serious objections to this conjecture. First, one still needs to account for the low frequency outgoing modes, so not much has been explained. The other objection, pointed out by Ford [17], is that the outcome of such a process would undoubtedly depend on the details of the interactions, for instance on the coupling constants. The regeneration of the outgoing modes would only be partial, which would be inconsistent with the standard vacuum structure and would lead to violation of the generalized second law because the hole could not emit the full thermal spectrum of Hawking radiation. The only counter argument [1] was to further conjecture that the regeneration is like an equilibration process, which yields the usual spectrum of vacuum fluctuations provided enough time passes. However this does not evade the first objection.

How many outgoing modes must be accounted for? If the black hole background were static, the number would be infinite, since they could come out at any time. A black hole left alone would presumably evaporate in a time of order M^3 (in Planck units), however one can in principle easily maintain the static background by sending energy into the hole at a rate equal to the Hawking luminosity. Thus it seems that one must be able to account for a truly infinite number of independent outgoing modes. Supposing that the ultimate density of states is no greater than one per Planck volume, it follows that the modes must originate in an infinite volume.[2]

It seems that there are only two ways to account for these outgoing modes without drawing upon a trans-Planckian reservoir: (1) modes may emerge from the singularity and propagate superluminally out across the horizon; (2) ingoing modes may be changed into outgoing ones.

One can construct dispersive linear field theory models [18–21] where either possibility occurs. The first possibility would require the specification of boundary conditions at the singularity, and does not seem as natural as the second. (It is not appropriate to reject *a priori* the first possibility, since we are questioning Lorentz invariance.) The second possibility probably occurs for a sonic black hole [22] constructed with real atomic fluid, as will be discussed below.

Conversion from ingoing to outgoing modes is a phenomenon that is known in other areas of physics. In plasmas it is called ''mode conversion,'' which occurs when waves propagate in an inhomogenous plasma [23,24]. In condensed matter it is called ''Andreev relection,'' or ''branch conversion,'' which occurs in superfluid systems where the order parameter is position dependent, such as a normal-superconducting interface [25], or a superflow or other ''texture'' in superfluid ^{3}He [26]. In fact, a simple analog of the outgoing black hole modes is the quasiparticle modes with energy $\epsilon < \Delta$ propagating in a normal conductor away from an interface with a superconducting phase with energy gap Δ. The origin of these modes is to be found in ingoing quasihole modes which undergo Andreev reflection.

In the black hole model to be discussed below, certain ingoing short wavelength modes are converted to outgoing short wavelength modes just outside the horizon, which then redshift down to long wavelengths as they climb away from the hole. These ingoing modes are actually *outgoing* as viewed in the free-fall frame of the black hole. Thus, from

[2]Allowing the black hole to evaporate, the number is instead finite. Consider just the s waves, and divide the frequency range into intervals of a small size ϵ. Below the Planck frequency there are ϵ^{-1} frequency intervals, and each interval defines a wave packet with time spread $\sim \epsilon^{-1}$. In the evaporation time M^3 there are, therefore, $\epsilon^{-1}(M^3/\epsilon^{-1}) = M^3$ independent s-wave modes that emerge. Since the high angular momentum modes have little cross section for absorption by the hole, the total number of modes should also be of order M^3. Again assuming one Planck volume per mode, this requires a total volume M^3. That is, the minimum distance from the horizon at which the modes can originate is of order M.

the viewpoint of a free-fall observer, the conversion does not lead to a deficit of ingoing modes.

Mode conversion from ingoing to outgoing modes may provide a satisfactory mechanism for the mode regeneration outside black hole horizons. It can happen even in *free* field theory, so it does not suffer form Ford's objection, it presumably survives in the presence of interaction, and the mechanism is sufficiently universal to account for the outgoing modes in a wide range of theories.

IV. THE STATIONARITY PUZZLE

If all outgoing wave packets can be traced backwards in time to ingoing ones that have never encountered the collapse process which formed the black hole, then for them the black hole spacetime appears stationary. Since Killing frequency is conserved, particle creation for them appears impossible. How can particle creation occur in the presence of a conserved Killing frequency?

The mere existence of a conserved Killing frequency is not the problem. For instance in de Sitter space, a space of maximal symmetry, ordinary quantum fields are excited even in the Killing vacuum [27]. This can happen because the Killing frequency does not define the relevant notion of particle states. A black hole spacetime is asymptotically flat, however, and the Killing time agrees at infinity with the relevant time for defining the asymptotic particle states (assuming any preferred frame at infinity is the one in which the black hole is at rest). It would therefore appear there can be no particle production by an eternal stationary black hole. The annihilation operator $a_{\rm out}$ for an outgoing positive Killing frequency wave packet would be expressible in terms of the annihilation operator $a_{\rm in}$ for an ingoing positive Killing frequency wave packet. With the standard in-vacuum boundary condition, $a_{\rm in}|\Psi\rangle=0$, the number $N_{\rm out}|\Psi\rangle$ would therefore vanish.

Three possible escape routes from this no-creation conclusion might be imagined: (1) the ingoing modes that give rise to the outgoing modes do *not* originate at infinity, (2) quantum field theory breaks down at short distances, (3) the back reaction destroys the Killing symmetry and decoheres positive Killing frequency superpositions.

In the presence of a cutoff on the density of states, as argued above, the modes must originate in an infinite volume, so escape route (1) is precluded due to an ''overcrowding'' problem (assuming the modes do not emerge from the singularity rather than coming in from infinity). Escape route (2) is that, as a wave packet is followed backwards in time out to infinity, it may blueshift so far that the field theory description breaks down. After that, the connection between ''Killing frequency at infinity'' and norm in Hilbert space might dissolve. This escape route does not seem too promising. While it is plausible that the field theory description breaks down, one would still expect to have a conserved Killing energy, and this may be sufficient for a no-creation argument.

Escape route (3) is by far the most promising. In the real problem the gravitational field is dynamical and couples to all other fields. The quantum evolution of the coupled system does not preserve the Killing symmetry of the classical background. The high wave vector wave packets that form the incoming mode are presumably correlated to different states of the gravitational field at short distances, and these states lack the Killing symmetry of the background. Furthermore, there is the effect of decoherence. Neglecting back reaction and self-interaction, an outgoing positive Killing frequency wave packet evolves backwards in time to a sum of ingoing wave packets $\phi_{+}+\phi_{-}$, where ϕ_{+} and ϕ_{-} have positive and negative norms, respectively (although each has positive Killing frequency). If the back reaction is taken into account, the states corresponding to these two wave packets are presumably correlated to different, perhaps orthogonal, states of the gravitational field. In this case it would be incorrect simply to sum them together in forming the annihilation operator $a(\phi_{+}+\phi_{-})=(\phi_{+}+\phi_{-},\hat{\Phi})$ [where (,) is the conserved norm and $\hat{\Phi}$ is the field operator], even if the Killing symmetry were preserved.

It seems quite plausible that some combination of these two effects of the back reaction can evade the stationarity paradox. Indeed there has already been much interesting work on the consequences of the back reaction on the phase of the Bogoliubov coefficients in the Hawking effect, and the existence of the kind of decoherence effect being invoked here [28]. Particularly encouraging is Parentani's work which establishes that decoherence due to the back reaction gives rise to an energy flux from a ''uniformly'' accelerated particle detector [29] or mirror [30], even though such systems radiate no energy in the background field approximation [31,32]. (It should be admitted however that in these systems there is a number expectation value for the outgoing modes even in the background field approximation, whereas in our black hole scenario even the number vanishes.) Our conclusion is that it seems possible in principle to produce the Hawking radiation even if outgoing modes arise from modes that are ingoing after the collapse that formed the black hole.

V. DISPERSIVE MODELS

In this section we discuss the process of mode conversion in a free field model that violates local Lorentz invariance on account of the presence in the action of higher spatial derivative terms in the free-fall frame of a black hole. For simplicity the model is restricted to two spacetime dimensions. The model arose from considerations of Unruh's sonic black hole analog [22], which we now briefly describe.

In Unruh's analogy, the perturbations of a stationary background fluid flow are quantized. If the background flow goes supersonic there is a ''sonic horizon,'' from beyond which sound cannot escape. Equating the sound field to a massless free field, Unruh argued that the sonic horizon will emit thermal Hawking phonons at a temperature $v'/2\pi$, where v' is the gradient of the background velocity field at the horizon. One can begin to take into account the atomic nature of a real fluid via the departure from linearity of the dispersion relation $\omega(k)$ for phonons [1]. The key point is that the slope, which gives the group velocity of wave packets, is not constant, and in fact initially decreases as k increases. This dispersion relation holds in the comoving frame of the fluid, and leads to the phenomenon of mode conversion from ingoing to outgoing modes as demonstrated by Unruh [18]. The model considered in [18] is not a real fluid but rather free field theory with higher spatial derivative terms designed to produce a nonlinear dispersion relation.

Unruh's model can be reinterpreted without the fluid flow interpretation as a theory of a free quantum field in a black hole spacetime. Let us describe the model in a slightly generalized form[3] [20,21]. The model consists of a free, Hermitian scalar field propagating in a two-dimensional black hole spacetime. The dispersion relation for the field lacks Lorentz invariance, and is specified in the free-fall frame of the black hole, that is, the frame carried in from the rest frame at infinity by freely falling trajectories.[4] Let u^α denote the unit vector field tangent to the infalling worldlines, and let s^α denote the orthogonal, outward pointing, unit vector, so that $g^{\alpha\beta}=u^\alpha u^\beta - s^\alpha s^\beta$. The action is assumed to have the form

$$S=\frac{1}{2}\int d^2x\sqrt{-g}g^{\alpha\beta}\mathcal{D}_\alpha\phi^*\mathcal{D}_\beta\phi, \tag{1}$$

where the modified differential operator $\mathcal{D}_\alpha$ is defined by

$$u^\alpha\mathcal{D}_\alpha = u^\alpha\partial_\alpha, \tag{2}$$

$$s^\alpha\mathcal{D}_\alpha = \hat{F}(s^\alpha\partial_\alpha). \tag{3}$$

The time derivatives in the local free-fall frame are thus left unchanged, but the orthogonal spatial derivatives are replaced by $\hat{F}(s^\alpha\partial_\alpha)$. The function $\hat{F}$ determines the dispersion relation. Invariance of the action (1) under constant phase transformations of ϕ guarantees that there is a conserved current for solutions and a conserved ''inner product'' for pairs of solutions to the equations of motion.

The black hole line element we shall consider is static and has the form

$$ds^2=dt^2-[dx-v(x)dt]^2, \tag{4}$$

where $v(x)$ is negative and increasing to the right, going to a constant $v_0<0$ at infinity. The black hole is at rest in these coordinates if v_0 vanishes. This is a generalization of the Lemaître line element for the Schwarzschild spacetime, which is given by $v(x)=-\sqrt{2M/x}$ (together with the usual angular part). ∂_t is a Killing vector, of squared norm $1-v^2$, and the event horizon is located at $v=-1$. The curves given by $dx/dt=v$ are timelike free-fall worldlines which are at rest (tangent to the Killing vector) where $v=0$. Since we assume $v<0$ these are *ingoing* trajectories. v is their coordinate velocity, t measures proper time along them, and they are everywhere orthogonal to the constant t surfaces. We shall refer to the function $v(x)$ as the *free-fall velocity*. The asymptotically flat region corresponds to $x\to\infty$.

In terms of the notation above, the orthonormal basis vectors adapted to the free-fall frame are given by $u=\partial_t+v\partial_x$ and $s=\partial_x$, and and in these coordinates $g=-1$. Thus the action (1) becomes

$$S=\frac{1}{2}\int dtdx[|(\partial_t+v\partial_x)\phi|^2-|\hat{F}(\partial_x)\phi|^2]. \tag{5}$$

If we further specify that $\hat{F}(\partial_x)$ is an odd function of ∂_x, then integration by parts yields the field equation

$$(\partial_t+\partial_x v)(\partial_t+v\partial_x)\phi=\hat{F}^2(\partial_x)\phi. \tag{6}$$

The behavior of wave packets in this model can be understood qualitatively as follows. Assume a solution to the field equation (6) of the form $\phi=e^{-i\omega t}f(x)$ and solve the resulting ordinary differential equation (ODE) for $f(x)$ by the WKB approximation. That is, write $f(x)=\exp[i\int k(x)dx]$ and assume the quantities $\partial_x v$ and $\partial_x k/k$ are negligible compared to k. The resulting equation is the position-dependent dispersion relation

$$[\omega-v(x)k]^2=F^2(k), \tag{7}$$

where $F(k)\equiv -i\hat{F}(ik)$. This is just the dispersion relation in the local free-fall frame, since the free-fall frequency ω' is related to the Killing frequency ω by

$$\omega'=\omega-v(x)k. \tag{8}$$

The choice of the function $F(k)$ completes the definition of the model. The ordinary wave equation corresponds to $F(k)=k$. Expanding in k, one has

$$F(k)=k-k^3/k_0^2+\cdots \tag{9}$$

(assuming reflection invariance). The wave vector k_0 characterizes the scale of ''new physics.'' The only qualitative choice being made here is that the cubic term is negative. In many condensed matter systems the dispersion relation behaves in this way, and it is necessary in order that (for an interacting field) the excitations be stable against decay into longer wavelength ones [33]. The group velocity in the free-fall frame is $dF/dk=1-3k^2/k_0^2+\cdots$, which decreases initially (at least) as the wave vector grows. This decrease in the group velocity is the essential feature for us.

Unruh's choice [18] for the function $F(k)$ was

$$F_{\text{Unruh}}(k)=k_0\{\tanh[(k/k_0)^n]\}^{1/n} \tag{10}$$

for various integers n. For every n this has the feature that the group velocity vanishes for large wave vectors and the frequency approaches a maximum. This dispersion relation is in some ways like the dispersion relation for superfluid helium-4, with the roton minimum taken out. [Other choices for $F(k)$ are studied in Ref. [21], but these will not be discussed here.] This dispersion relation is plotted in Fig. 1 along with the dispersion relations for the ordinary wave equation and for quasiparticle excitations of superfluid helium-4.

The dispersion relation is useful for understanding the motion of wave packets that are somewhat peaked in both position and wave vector. The change in position can be found by integrating the group velocity $d\omega/dk=v(x)+dF/dk$ while satisfying the dispersion relation (7). A graphical method we have employed is described in Refs. [20,21]. The same method was used by Brout, Massur,

[3]All of my understanding of this model has been developed in collaboration with Corley.

[4]A related model was invented in Ref. [19], which imposes the altered dispersion relation in Eddington-Finkelstein coordinates rather than free-fall coordinates.

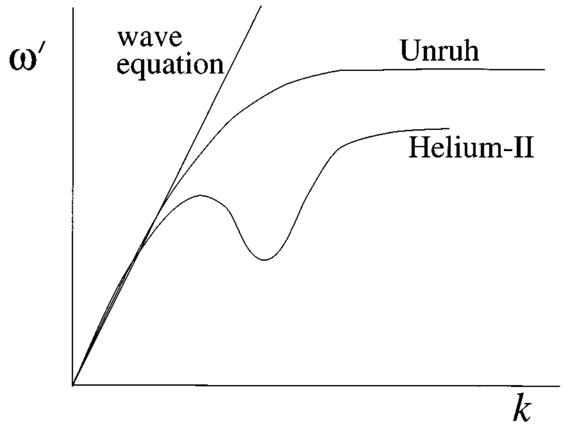

FIG. 1. Sketch of dispersion relations $\omega'(k)$ for the wave equation, the Unruh model, and helium-II at zero temperature and pressure. ω' is the frequency in the free-fall frame (8).

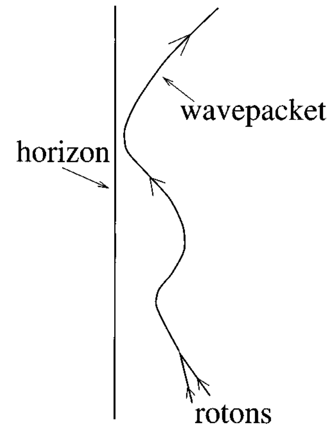

FIG. 2. Sketch of the history of an outgoing wave packet propagated with the dispersion relation for helium-II in Fig. 1.

Parentani, and Spindel (BMPS) [19], who also found a Hamiltonian formulation for the wave-packet propagation using Hamilton-Jacobi theory.

The model with the dispersion relation (10) was solved in Ref. [18] by numerical integration of the partial differential equation (PDE) (6), and the results can be reproduced qualitatively by the WKB methods. Propagating a low wave-vector outgoing wave packet backwards in time, the horizon is approached, the wave vector blueshifts to something of order k_0, and the group velocity drops to zero in the static frame. Mode conversion occurs near the stopping point [20,21], and the wave packet moves back away from the horizon with large positive and negative wave vector components. (Forwards in time these components are ingoing in the static frame but outgoing in the free-fall frame. This happens because their group velocity in the free-fall frame is smaller in magnitutde than $v(x)$ due to the flattening of the dispersion curve at large wave vectors.) These have positive and negative free-fall frequencies and (therefore) positive and negative norms, respectively. The magnitude of the component wave vectors grows without bound as the wave packet moves outward where $v(x)$ decreases, and the asymptotic group velocity is just $v(x)$. Were $v(x)$ to drop to zero, the wave vector would diverge. To avoid dealing with this one can impose the in-vacuum in the free-fall frame at nonzero $v(x)$. (This was done in [18] and [21].) No matter how small v gets, the same result is obtained for the (negative) norm of the negative wave vector piece, which is the Bogoliubov coefficient that determines the particle creation amplitude for the outgoing wave packet. Thus, even though the difference between the free-fall and Killing frames is going to zero as v goes to zero, the wave vector is diverging in such a way that the wave packet always maintains a negative free-fall frequency part of the same, negative, norm.

From this analysis we see that the Unruh model, while it entails a strict cutoff in free-fall frequency, involves in an essential way arbitrarily high wave vectors, i.e., arbitrarily short wavelengths. Insofar as we wish to explore the consequences of a fundamental short distance cutoff, this is an unsatisfactory feature of the model. The outgoing modes emerging from the black hole region still arise from arbitrarily short wavelength modes, albeit ingoing ones.

VI. HELIUM-II SONIC BLACK HOLE

For a physical model with a strict cutoff let us consider the behavior of liquid helium-4 at zero temperature. This is of course an interacting system, so is not nearly as simple as the Unruh model. Nevertheless, is is possible to make some reasonable conjectures based on the form of the quasiparticle spectrum in Fig. 1. (To avoid the need to consider interactions, one might study instead field theory on a lattice falling into a black hole.) In discussing the helium model, it is natural to go back to Unruh's original sonic analogy and think of the free-fall velocity $v(x)$ as the velocity of the background fluid flow. To begin with, let us ignore the existence of interactions and just follow modes as if they were free.

In [1] it was argued that in the helium model a long wavelength outgoing wave packet, traced backward in time, would come to rest at an ''effective horizon'' where the comoving group velocity and fluid velocity are equal and opposite. For this part of the process, the difference between the helium dispersion relation and that of the Unruh model (10) is irrelevant, so Unruh's results [18], as well as those of [19,21], show that this expectation is incorrect. Rather, the blueshifting continues, the group velocity continues to drop, and the wave packet is swept back out away from the sonic horizon as a superposition of positive and negative wave-vector packets. Now let us continue to follow the progress of, say, the positive wave-vector part, backward in time, using the dispersion relation of liquid helium. The idea is sketched in Fig. 2. (The behavior of the negative wave-vector part is similar.) The packet will go over the first maximum of the dispersion curve, at which point its comoving group velocity changes sign, which only pushes it away from the horizon even faster. Eventually, however, it approaches another turn around point, near the roton minimum, where the free-fall frequency line becomes tangent once again to the dispersion curve. It seems reasonable to suppose that what happens here is another reversal of direction, with the wave packet continuing along the dispersion curve and falling back towards the horizon. As the wave vector rises and group velocity falls one more tangency point will be reached, where there is presumably one final reversal of direction. After that (still backward in time), the wave packet heads back away from the horizon with still blueshifting wave vector and finally runs off the end of the quasiparticle spectrum. The quasiparticle spectrum terminates when a decay channel into two rotons opens up, at twice the momentum and energy of the roton minimum [33,34]. Therefore, it appears that our outgoing mode originates as some two roton mode (with vanishing comoving group velocity) that is swept in by the flow. That is, the number operator for outgoing long wavelength phonons is dynamically related to the four-point function for ingoing rotons.

We were forced to incorporate the interactions when the end of the quasiparticle spectrum was reached, but of course the interactions play some role all along that we have ignored. One way to think about this is to ask about the stability of quasiparticles. If other decay channels are kinematically available, then presumably these are mixed in to the evolution of the vacuum correlation functions. There are indeed two regions of the quasiparticle spectrum that are unstable in addition to the end point. First, there is ''anomalous dispersion'' at low wave vectors, where $d^2\omega/dk^2$ is actually positive rather than negative [35], which leads to a finite phonon lifetime [33,34]. Second, past the roton minimum, there is a region where the group velocity just reaches the velocity of sound [36], leading to phonon emission. The existence of these processes presumably implies a mixing of the multipoint vacuum correlation functions. Thus an outgoing phonon mode really arises from a superposition of various numbers of phonon modes, each of which ultimately arises from ingoing multiroton modes.

In summary, we thus conjecture the following forward in time behavior. As particular superpositions of multiroton modes of the superfluid are swept in towards the horizon, the interactions and velocity gradient conspire to turn them (after some dancing around near the horizon) into outgoing phonon modes which are in (or very nearly in) their ground state. At this point the Hawking effect is responsible for populating them thermally as they climb away from the horizon. The negative energy flux across the horizon which is required by energy conservation must leave the superfluid in a state with lower energy density than the homogeneous superfluid ground state.

In fact, the true behavior of helium in the presence of a sonic horizon is complicated by the instability towards vortex and roton creation. Even if vortex creation can somehow be suppressed, a roton condensate will develop [37] when the flow velocity exceeds the Landau velocity $v\sim 60$ m/s which is much less than the long wavelength velocity of sound (238 m/s) in helium-II. This condensate depletes the superfluid component (and alters the dispersion relation), and the superfluid component vanishes entirely well before the speed of sound is reached. It thus appears that a long lived sonic horizon cannot actually be established in helium-II. Perhaps another condensed matter system can provide a physically realizable black hole analog.

VII. INFORMATION LOSS

Finally, we point out that in the models considered here, it appears that the presence of a cutoff and violation of Lorentz invariance do not change the picture with regard to information loss in black hole evaporation. The created particles still have a ''partner'' [19], which falls down into the singularity, to whom they are correlated.[5] We see no mechanism for recovering the information in those correlations based on these models.

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[5] It should be noted that the usual partner, obtained [38] by reflecting an out mode across the horizon, is another mode with trans-Planckian pedigree in ordinary field theory. In the dispersive models, this partner apparently [19] also arises from an ingoing (negative Killing frequency) mode.

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