Principle of nongravitating vacuum energy and some of its consequences

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For Einstein's general relativity (GR) or the alternatives suggested up to date, the vacuum energy gravitates. We present a model where a new measure is introduced for integration of the total action in D-dimensional spacetime. This measure is built from D scalar fields φ_a. As a consequence of such a choice of the measure, the matter Lagrangian L_m can be changed by adding a constant while no gravitational effects, such as a cosmological term, are induced. Such a *nongravitating vacuum energy theory* has an infinite dimensional symmetry group which contains volume-preserving diffeomorphisms in the internal space of scalar fields φ_a. Other symmetries contained in this symmetry group suggest a deep connection of this theory with theories of extended objects. In general *the theory is different from GR* although for certain choices of L_m, which are related to the existence of an additional symmetry, solutions of GR are solutions of the model. This is achieved in four dimensions if L_m is due to fundamental bosonic and fermionic strings. Other types of matter where this feature of the theory is realized, are, for example, scalars without potential or subjected to nonlinear constraints, massless fermions, and point particles. The point particle plays a special role, since it is a good phenomenological description of matter at large distances. de Sitter space is realized in an unconventional way, where the de Sitter metric holds, but such de Sitter space is supported by the existence of a variable scalar field which in practice destroys the maximal symmetry. The only spacetime where maximal symmetry is not broken, in a dynamical sense, is Minkowski space. The theory has nontrivial dynamics in 1+1 dimensions, unlike GR. [S0556-2821(96)04712-1]

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I. INTRODUCTION

As is known, in the general theory of relativity energy is a source of gravity, which is described by the metric tensor $g_{\mu\nu}$. This makes an important difference to ideas developed for flat space physics where the origin with respect to which we measure energy does not matter; that is, the energy is defined up to an additive constant. For general relativity in contrast, all the energy has a gravitational effect; therefore, the origin with respect to which we define the energy is important.

In quantum mechanics, there is the so-called zero-point energy associated with the zero-point fluctuations. In the case of quantum fields, such zero-point fluctuations turn out to have an associated energy density which is infinite. In fact there is a zero-point vacuum energy-momentum tensor of the form $T^{\rm vac}_{\mu\nu}=A\eta_{\mu\nu}$ in flat space ($\eta_{\mu\nu}$ is the Minkowski metric), or $T^{\rm vac}_{\mu\nu}=Ag_{\mu\nu}+$(terms $\propto R$) in curved space. Here A is infinite.

Notice that the appearance of an energy-momentum tensor proportional to $g_{\mu\nu}$ in Einstein's equations is equivalent [1] to what Einstein called the ''cosmological constant'' or ''Λ term.'' It was introduced by Einstein [2] in the form

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\Lambda g_{\mu\nu}=\frac{\kappa}{2}T_{\mu\nu}. \tag{1}$$

Such a Λ term does not violate any known symmetry. Therefore, normally we would not consider excluding it, if we were to apply the arguments usually made in quantum field theory. However, we get into trouble if we note that the natural scale of such a term, obtained on dimensional grounds, is of the order of magnitude of the Planck density. The problem is more severe once we realize that the zero-point vacuum energy appears as an infinite quantity.

Indeed, in order to get agreement with observations, different sources of energy density have to compensate with each other almost exactly to high accuracy, thus creating an acute ''fine-tuning problem.''

In order to explain this so-called ''cosmological constant problem,'' a variety of ideas have been developed; see, for example, the reviews in Ref. [3]. Among these attempts, possible changes in gravity theory were studied [3], where the result was that the cosmological constant appeared as an integration constant, for example, in ''nondynamical $\sqrt{-g}$'' models. The reason why such a constant should be picked zero is unclear, however.

In this paper we will also suggest a modification of gravity theory by imposing the principle that the vacuum energy density, to be identified with the constant part of the Lagrangian density, should not contribute to the equation of motion. The realization of this idea in the model considered here, apart from leading to a geometrically interesting new theory, also leads to the possibility of new gravitational effects.

II. THE MODEL

A. A new measure and generally coordinate invariant action

All approaches to the cosmological constant problem have been made under the assumption that the invariant measure to be used for integrating the total Lagrangian density in

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the action is $\sqrt{-g}d^Dx$. In the present paper, this particular assumption will be modified, and the result will be that, by an appropriate generally coordinate invariant choice of the measure, the theory will not be sensitive to a change in the Lagrangian density by the addition of a constant, in contrast with the Einstein-Hilbert action, where such a change generates a cosmological term.

Let the measure of integration in a D-dimensional spacetime be chosen as $\Phi\ d^Dx$, where Φ is a yet unspecified scalar density of weight 1. In order to achieve the result that the vacuum energy does not gravitate, we will start from the demand that the addition of a constant in the Lagrangian density does not affect the dynamics of the theory. This means that $\int L\Phi\ d^Dx$ and $\int(L+\text{const})\Phi\ d^Dx$ must reproduce the same equations of motion. This is of course achieved if Φ is a total derivative.

The simplest choice for a scalar density of weight 1, which is as well a total derivative, may be realized by using D scalar fields $\varphi_a(x)$ and then defining

$$\Phi\equiv\varepsilon_{a_1a_2\cdots a_D}\varepsilon^{\alpha_1\alpha_2\cdots\alpha_D}(\partial_{\alpha_1}\varphi_{a_1})(\partial_{\alpha_2}\varphi_{a_2})\cdots(\partial_{\alpha_D}\varphi_{a_D}). \tag{2}$$

Here $\varepsilon^{\alpha_1\alpha_2\cdots\alpha_D}=1$ if $\alpha_1=0$, $\alpha_2=1,...,\alpha_D=D-1$ and ±1 according to whether $(\alpha_1,\alpha_2,...,\alpha_D)$ is an even or an odd permutation of $(0,1,...,D-1)$. Likewise for $\varepsilon_{a_1a_2\cdots a_D}$ $(a_i=1,2,\ldots,D)$. Therefore the total action we will consider is

$$S=\int\left(-\frac{1}{\kappa}R+L_m\right)\Phi\ d^Dx, \tag{3}$$

where $\kappa=16\pi G$, R is the scalar curvature, and we will take L_m not to depend on any of the scalars $\varphi_a(x)$. Notice that if we consider parity or time reversal transformations, then $S\to-S$, which does not affect the classical equations of motion. Quantum mechanically (considering for example the path integral approach), such transformation will transform *total* amplitudes into their complex conjugates, therefore leaving probabilities unchanged.

Notice that Φ is the Jacobian of the mapping $\varphi_a=\varphi_a(x^\alpha)$, $a=1,2,\ldots,D$. If this mapping is nonsingular ($\Phi\neq0$) then (at least locally) there is the inverse mapping $x^\alpha=x^\alpha(\varphi_a)$, $\alpha=0,1,...,D-1$. Since $\Phi\ d^Dx=D!\ d\varphi_1\wedge d\varphi_2\wedge\ldots\wedge d\varphi_D$ we can think of $\Phi\ d^Dx$ as integrating in the internal space variables φ_a. In addition, if $\Phi\neq0$ then there is a coordinate frame where the coordinates are the scalar fields themselves.

The field Φ is invariant under the volume preserving diffeomorphisms in internal space: $\varphi'_a=\varphi'_a(\varphi_b)$ where

$$\varepsilon_{a_1a_2\cdots a_D}\frac{\partial\varphi'_{b_1}}{\partial\varphi_{a_1}}\frac{\partial\varphi'_{b_2}}{\partial\varphi_{a_2}}\cdots\frac{\partial\varphi'_{b_D}}{\partial\varphi_{a_D}}=\varepsilon_{b_1b_2\cdots b_D}. \tag{4}$$

B. Equations of motion

The equations of motion obtained by variation of the action (3) with respect to the scalar fields φ_b are

$$A^\mu_b\partial_\mu\left(-\frac{1}{\kappa}R+L_m\right)=0, \tag{5}$$

where

$$A^\mu_b\equiv\varepsilon_{a_1a_2\cdots a_{D-1}b}\varepsilon^{\alpha_1\alpha_2\cdots\alpha_{D-1}\mu}(\partial_{\alpha_1}\varphi_{a_1})\times(\partial_{\alpha_2}\varphi_{a_2})\cdots(\partial_{\alpha_{D-1}}\varphi_{a_{D-1}}). \tag{6}$$

It follows from (2) that $A^\mu_b\partial_\mu\varphi_{b'}=D^{-1}\delta_{bb'}\Phi$ and taking the determinant of both sides, we get $\det(A^\mu_b)=(D^{-D}/D!)\Phi^{D-1}$. Therefore if $\Phi\neq0$, which we will assume in what follows, the only solution for (5) is

$$-\frac{1}{\kappa}R+L_m=\text{const}\equiv M. \tag{7}$$

Variation of $S_g\equiv-(1/\kappa)\int R\Phi\ d^Dx$ with respect to $g^{\mu\nu}$ leads to the result [4]

$$\delta S_g=-\frac{1}{\kappa}\int\Phi[R_{\mu\nu}+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)]\delta g^{\mu\nu}d^Dx. \tag{8}$$

In order to perform the correct integration by parts we have to make use of the scalar field $\chi\equiv\Phi/\sqrt{-g}$, which is invariant under continuous general coordinate transformations, instead of the scalar density Φ. Then integrating by parts and ignoring a total derivative term which has the form $\partial_\alpha(\sqrt{-g}P^\alpha)$, where P^α is a vector field, we get

$$\frac{\delta S_g}{\delta g^{\mu\nu}}=-\frac{1}{\kappa}\sqrt{-g}[\chi R_{\mu\nu}+g_{\mu\nu}\Box\chi-\chi_{,\mu;\nu}]. \tag{9}$$

In a similar way varying the matter part of the action (3) with respect to $g^{\mu\nu}$ and making use of the scalar field χ we can express a result in terms of the standard matter energy-momentum tensor $T_{\mu\nu}\equiv(2/\sqrt{-g})[\partial(\sqrt{-g}L_m)/\partial g^{\mu\nu}]$. Then after some algebraic manipulations we get instead of Einstein's equations

$$G_{\mu\nu}=\frac{\kappa}{2}\{T_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}[T^\alpha_\alpha+(D-2)L_m]\}+\frac{1}{\chi}\left(\frac{D-3}{2}g_{\mu\nu}\Box\chi+\chi_{,\mu;\nu}\right), \tag{10}$$

where $G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}$.

By contracting (10) and using (7), we get

$$\Box\chi-\frac{\kappa}{D-1}\{M+\tfrac{1}{2}[T^\alpha_\alpha+(D-2)L_m]\}\chi=0. \tag{11}$$

By using Eq. (11) we can now exclude $T^\alpha_\alpha+(D-2)L_m$ from Eq. (10):

$$G_{\mu\nu}=\frac{\kappa}{2}[T_{\mu\nu}+Mg_{\mu\nu}]+\frac{1}{\chi}[\chi_{,\mu;\nu}-g_{\mu\nu}\Box\chi]. \tag{12}$$

Notice that Eqs. (5) and (10) are invariant under the addition to L_m a constant piece, since the combination $T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}[T^\alpha_\alpha+(D-2)L_m]$ is invariant. However, by fixing the constant part of the difference L_m-M in the solution (7) of Eq. (5), we break this invariance. To give a definite physical meaning to the integration constant M, we conventionally take the constant part of L_m equal to zero.

It is very important to note that the terms depending on the matter fields in Eq. (12) as well as in Eq. (10) do not contain the χ field, in contrast with the usual scalar-tensor theories, such as the Brans-Dicke theory. As a result of this feature of the nongravitating vacuum energy (NGVE) theory, the gravitational constant does not suffer space-time variations. However, the matter energy-momentum tensor $T_{\mu\nu}$ is not conserved. Actually, taking the covariant divergence of both sides of Eq. (12) and using the identity $\chi^{\alpha}_{;\nu;\alpha}=(\Box\chi)_{,\nu}+\chi^{\alpha}R_{\alpha\nu}$, Eqs. (12) and (11), we get the equation of matter nonconservation

$$T_{\mu\nu}{}^{;\mu}=-2\,\frac{\partial L_m}{\partial g^{\mu\nu}}\,g^{\mu\alpha}\partial_\alpha \ln\chi. \tag{13}$$

C. Volume preserving symmetries and associated conserved quantities

From the volume preserving symmetries $\varphi'_a=\varphi'_a(\varphi_b)$ defined by Eq. (4) which for the infinitesimal case implies

$$\varphi'_a=\varphi_a+\lambda\varepsilon^{aa_1\cdots a_D}\frac{\partial F_{a_1a_2\cdots a_{D-1}}(\varphi_b)}{\partial\varphi_{a_D}} \tag{14}$$

($\lambda\ll 1$), we obtain, through Noether's theorem, the conserved quantities

$$j^{\mu}_{V}=A^{\mu}_{a}\left(-\frac{1}{\kappa}R+L_m\right)\varepsilon_{aa_1\cdots a_D}\frac{\partial F_{a_1a_2\cdots a_{D-1}}(\varphi_b)}{\partial\varphi_{a_D}}. \tag{15}$$

D. Symmetry transformations with the total Lagrangian density as a parameter and associated conserved quantities

Let us consider the following infinitesimal shift of the fields φ_a by an arbitrary infinitesimal function of the total Lagrangian density $L\equiv-(1/\kappa)R+L_m$, that is

$$\varphi'_a=\varphi_a+\epsilon g_a(L),\qquad \epsilon\ll 1. \tag{16}$$

In this case the action is transformed according to

$$\delta S=\epsilon D\int A^{\mu}_{a}L\,\partial_\mu g_a(L)\,d^Dx=\epsilon\int\partial_\mu\Omega^{\mu}d^Dx, \tag{17}$$

where $\Omega^{\mu}\equiv DA^{\mu}_{a}f_a(L)$ and $f_a(L)$ being defined from $g_a(L)$ through the equation $L(dg_a/dL)=df_a/dL$. To obtain the last expression in the Eq. (17) it is necessary to note that $\partial_\mu A^{\mu}_{a}\equiv 0$. By means of the Noether's theorem, this symmetry leads to the conserved current

$$j^{\mu}_{L}=A^{\mu}_{a}(Lg_a-f_a)\equiv A^{\mu}_{a}\int_{L_0}^{L}g_a(L')\,dL'. \tag{18}$$

For certain matter models, the symmetry structure is even richer (see discussion of the ''Einstein symmetry'' in the next section). The complete understanding of the group structure and the consequences of these symmetries is not known to us at present, but we expect to report on this in future publications.

III. EINSTEIN SYMMETRY AND EINSTEIN SECTOR OF SOLUTIONS

A. Einstein condition

We are interested now in studying the question of whether there is an Einstein sector of solutions; that is, are there solutions that satisfy Einstein's equations? First of all, we see that Eqs. (12) coincide with Einstein's equations with cosmological constant κM only if the χ field is a constant. From Eq. (11) we conclude that this is possible only if an essential restriction on the matter model is imposed: $2M+T^{\alpha}_{\alpha}+(D-2)L_m\equiv 2[M+g^{\mu\nu}(\partial L_m/\partial g^{\mu\nu})-L_m]=0$. However, we have not found any reasonable matter model where this condition is satisfied for $M\neq 0$ (recall that the constant part of L_m is taken to be zero). In the case $M=0$ this condition becomes

$$g^{\mu\nu}\frac{\partial L_m}{\partial g^{\mu\nu}}-L_m=0, \tag{19}$$

which means that L_m is a homogeneous function of $g^{\mu\nu}$ of degree one, in any dimension. If condition (19) is satisfied then the equations of motion allow solutions of general relativity (GR) to be solutions of the model, that is χ=const and $G_{\mu\nu}=(\kappa/2)T_{\mu\nu}$.

B. Einstein symmetry

It is interesting to observe that when condition (19) is satisfied, a new symmetry of the action (3) appears. We will call this symmetry ''Einstein symmetry'' [because (19) leads to the existence of an Einstein sector of solutions]. Such symmetry consists of the scalings

$$g^{\mu\nu}\to\lambda g^{\mu\nu}, \tag{20}$$

$$\varphi_a\to\lambda^{-(1/D)}\varphi_a, \tag{21}$$

where λ=constant.

To see that this is indeed a symmetry, note that from definition of scalar curvature it follows that $R\to\lambda R$ when the transformation (20) is performed. Since condition (19) means that L_m is a homogeneous function of $g^{\mu\nu}$ of degree 1, we see that under the transformation (20) the matter Lagrangian $L_m\to\lambda L_m$. From this we conclude that (20) and (21) are indeed a symmetry of the action (3) when (19) is satisfied.

C. Examples

The situation described in the two previous subsections can be realized for special kinds of bosonic matter models.

(1) Scalar fields without potentials, including fields subjected to nonlinear constraints, like the σ model. The general coordinate invariant action for these cases has the form $S_m=\int L_m\Phi\; d^Dx$ where $L_m=\frac{1}{2}\sigma_{,\mu}\sigma_{,\nu}g^{\mu\nu}$.

(2) Matter consisting of fundamental bosonic strings. The condition (19) can be verified by representing the string action in the D-dimensional form where $g_{\mu\nu}$ plays the role of a background metric. For example, bosonic strings, according to our formulation, where the measure of integration in a D-dimensional spacetime is chosen to be $\Phi\; d^Dx$, will be governed by an action of the form

$$S_m = \int L_{\rm string} \Phi \; d^D x,$$

$$L_{\rm string} = -T \int d\sigma \, d\tau \, \frac{\delta^D(x - X(\sigma,\tau))}{\sqrt{-g}} \times \sqrt{\det(g_{\mu\nu} X^\mu_{,a} X^\mu_{,b})}, \qquad (22)$$

where $\int L_{\rm string}\sqrt{-g}\; d^D x$ would be the action of a string embedded in a D-dimensional spacetime in the standard theory; a, b label coordinates in the string world sheet and T is the string tension. Notice that under a scaling (20) (which means that $g_{\mu\nu} \to \lambda^{-1} g_{\mu\nu}$), $L_{\rm string} \to \lambda(D-2/2) L_{\rm string}$, therefore concluding that $L_{\rm string}$ is a homogeneous function of $g_{\mu\nu}$ of degree one, that is Eq. (19) is satisfied, if $D=4$.

(3) It is possible to formulate *the point particle model* of matter in a way such that Eq. (19) is satisfied. This is because for the free falling point particle a variety of actions are possible (and are equivalent in the context of general relativity). The usual actions are taken to be $S = -m\int F(y) ds$, where $y = g_{\alpha\beta}(dX^\alpha/ds)(dX^\beta/ds)$ and s is determined to be an affine parameter except if $F = \sqrt{y}$, which is the case of reparametrization invariance. In our model we must take $S_m = -m\int L_{\rm part} \Phi \; d^4 x$ with $L_{\rm part} = -m\int ds[\delta^4(x - X(s))/\sqrt{-g}] F(y[X(s)])$ where $\int L_{\rm part}\sqrt{-g}\; d^4 x$ would be the action of a point particle in four dimensions in the usual theory. For the choice $F = y$, condition (19) is satisfied. Unlike the case of general relativity, different choices of F lead to unequivalent theories.

Notice that in the case of point particles (taking $F = y$), a geodesic equation (and therefore the equivalence principle) is satisfied in terms of the metric $g^{\rm eff}_{\alpha\beta} \equiv \chi g_{\alpha\beta}$ even if χ is not constant. It is interesting also that in the four-dimensional case $g^{\rm eff}_{\alpha\beta}$ is invariant under the Einstein symmetry described by Eqs. (20) and (21). Furthermore, since for point particles the theory allows an Einstein sector, we seem to have a formulation where at macroscopic distances there will be no difference with the standard GR, but where a substantial difference could appear in microphysics. The theory could be useful in suggesting new effects that may be absent in GR and that could constitute nontrivial tests of the nongravitating vacuum energy (NGVE) theory and of GR. Deviations from GR may have also interesting cosmological consequences for the early Universe [5].

In all the above cases 1, 2, and 3, solutions where the field $\chi \equiv \Phi/\sqrt{-g}$ is constant exist if $M=0$ and then solutions of Eq. (12) coincide with those of the Einstein theory with zero cosmological constant. Theories of fermions (including fermionic string theories) appear also can serve as candidate matter models where there will be an Einstein-Cartan sector of solutions, as will be studied elsewhere.

IV. THE COSMOLOGICAL CONSTANT PROBLEM

A. The de Sitter solution in the context of the NGVE theory

If we allow nonconstant χ, we expect to obtain *effects not present in Einstein gravity and cosmology*. For example, as we will see, in the NGVE theory de Sitter space is realized in an unconventional way, where the de Sitter metric holds, but such de Sitter space is supported by the existence of a variable scalar field χ which in practice destroys the maximal symmetry.

Effects of a nonconstant χ can be studied first in the case where there is no matter ($L_m = T_{\mu\nu} = 0$). If we require maximal symmetry of the space-time metric in such a case, the field χ must satisfy the condition $\chi_{,\mu;\nu} = c\chi g_{\mu\nu}$ where c is a constant. Geometry allows [6] such χ only for the value of $c = -(R/12)$ and it is then possible to have maximally symmetric metric with any value of $R = -\kappa M$. For example a de Sitter solution of Eqs. (11) and (12) (taking $D=4$) is $ds^2 = dt^2 - a^2 d\vec{x}^2$, where $a = a_0 \exp(\lambda t)$, $\chi = \chi_0 \exp(\lambda t)$, and $\lambda^2 = \kappa M/12$.

Let us consider the point particle model (considered at the end of Sec. III) which satisfies the Einstein condition. In this case the physics of the de Sitter space is described by geodesics with respect to the effective metric $g^{\rm eff}_{\alpha\beta}$. Notice that such a metric corresponds to a power law inflation: $ds^2_{\rm eff} = d\tau^2 - \tau^6 d\vec{x}^2$, where $\tau = \tau_0 \exp(\lambda t)$ and $\lambda^2 = \kappa M/12$. By examining the physical metric $g^{\rm eff}_{\alpha\beta}$ we notice that there are not the 10 Killing vectors of the de Sitter space. We see that although the metric $g_{\mu\nu}$ is maximally symmetric, the physical geometry is not maximally symmetric.

B. Particle creation, possible instability of the de Sitter space, and the Parker condition

Some years ago, Parker [7] suggested a possible mechanism, based on particle production in the early Universe, for selecting a zero cosmological constant. The basic assumption Parker made was that of the existence of an underlying theory where the cosmological constant can be dynamically adjusted by a process based on something like the familiar ''Lenz's law,'' which requires that the equilibrium condition be achieved after the cosmological constant relaxes to a certain value.

The idea is based on the fact that for a four-dimensional homogeneous, isotropic cosmological background, there is not real particle creation for massless conformally coupled scalar fields (MCCSF's) satisfying

$$\Box \phi + \frac{R}{6} \phi = 0. \qquad (23)$$

If there are massless minimally coupled scalar fields (MMCSF's) or gravitons, in general particle creation takes place. However, when considering particle creation effect in a given background metric with zero scalar curvature, the MCCSF and MMCSF theories behave in the same way [7]. Also for such a background $R=0$, one expects no graviton production [7]. The existence of radiation with $T^\mu_\mu = 0$ does not affect these conclusions [7].

C. Realization of the Parker cosmological condition in the Einstein sector of the NGVE theory

In the context of the NGVE theory, we have states with $R=0$ that exist among many other possible states, provided the integration constant M is chosen zero and the matter Lagrangian on shell equals zero [see Eq. (7)]. The last condition is satisfied, for example, for the case of massless fermions and for electromagnetic radiation with

$T^{\mu}_{(\mathrm{em})\mu}=L_{\mathrm{em}}=E^2-B^2=0$. Such a state may be realized apparently for the Universe filled by ultrarelativistic matter.

As Parker [7] points out, a de Sitter space, although not satisfying the condition $R=0$, has a chance however of being stable due to its maximal symmetry. This seems in accordance with the calculations of Candelas and Raine [8]. In our case however even this possibility seems to be excluded. Indeed, for the de Sitter space in the NGVE theory we have $\chi=\chi(t)\neq$const which means explicit breaking of maximal symmetry since for a maximally symmetric space a scalar field must obey [9] the condition $\partial_\mu\chi=0$. Therefore we expect that in the NGVE theory the de Sitter space suffers from the above mentioned instability towards particle production. Only flat spacetime allows the possibility of maximal symmetry since in this case χ may be constant, which is clear from Eq. (11).

It is interesting to observe that the Parker condition in the framework of the NGVE theory, when applied to the above mentioned case of an early Universe dominated by ultrarelativistic matter (massless fermions and electromagnetic radiation with $T^{\mu}_{(\mathrm{em})\mu}=L_{\mathrm{em}}=E^2-B^2=0$), is a particular realization of the Einstein condition (19). Notice that once the Einstein condition is satisfied, the direct coupling of the χ field with matter disappears [see Eq. (11)]. When this decoupling does not exist, as for example is apparent from Eq. (13), for the small matter perturbations around a de Sitter space, we expect a tendency for any homogeneous χ state to lose energy to inhomogeneous degrees of freedom (from the point of view of thermodynamics a transfer of energy from homogeneous degree of freedom of the χ field into inhomogeneous degrees of freedom is more preferable than the reverse since the inhomogeneous modes are more numerous). An effective way to describe this would be the introduction of a friction term in the equation of motion for χ. We expect this would lead to the decay of the de Sitter space towards a Friedmann epoch with $M=0$ and $\chi=$const. The nonequilibrium dynamics explaining the details of how the relaxation of the cosmological constant is achieved, or in our case how the constant M is changed to zero, is a subject for future studies.

V. DISCUSSION

There are many directions in which research concerning the NGVE theory could be expanded. A subject of particular interest consists of the study of the NGVE theory in $1+1$ dimensions. In this case, the model gives equations that coincide with those of Jackiw and Teitelboim [10] when the constant of integration κM in the NGVE theory is identified with the constant scalar curvature which is imposed on the vacuum solutions of that model [10] and where our field χ plays the same role, in the equations, as the Lagrange multiplier field in the Jackiw-Teitelboim model [10].

Another model that resembles the NGVE theory studied here is the nondynamical $\sqrt{-g}$ theory (NDSQR) (for reviews, see articles of Weinberg and Ng [3]). For the NDSQR theory, also any de Sitter space is a solution of the theory and the constant curvature of the vacuum solutions also appears as an integration constant. The differences between the NDSQR theory and the NGVE theory are however very obvious. To mention just some: (a) The NDSQR theory does not exist as a nontrivial theory in $1+1$ dimensions, while the NGVE theory does; (b) The four-dimensional de Sitter solutions do not possess the maximal symmetry for the NGVE theory case (due to the fact that the scalar field χ is not constant) while maximal symmetry is respected in the NDSQR theory where flat space and de Sitter space have the same symmetry; (c) In the NGVE theory, the matter energy-momentum tensor is not covariantly conserved, while in the NDSQR theory it is; (d) In addition, the NDSQR theory is for all practical purposes indistinguishable from GR, while the NGVE theory is really a new physical theory, which could contain an Einstein sector of solutions, but in addition there is the potential of finding out new gravitational effects.

We should also point out that a more thorough study of the infinite dimensional symmetry found here should be made. In particular the restrictions on the possible induced terms in the quantum effective action seem to be strong if the symmetries (14) and (16) remain unbroken after quantum corrections are also taken into account. In particular, symmetry under the transformations (16) seems to prevent the appearance of terms of the form $f(\chi)\Phi$ [except of $f(\chi)\propto 1/\chi$] in the effective action which although is invariant under volume preserving transformations (14), breaks symmetry (16). The case $f(\chi)\propto 1/\chi$ is not forbidden by symmetry (16) and appearance of such a term would mean inducing a ''real'' cosmological term, i.e., a term of the form $\sqrt{-g}\Lambda$ in the effective action. However, appearance of such a term seems to be ruled out because of having opposite parity properties to that of the action given in (3). Of course, in the absence of a consistently quantized theory, such arguments are only preliminary. Nevertheless it is interesting to note that if all these symmetry arguments are indeed applicable, this would imply that the scalar fields φ_a can appear in the effective action only in the integration measure, that is they preserve their geometrical role.

The physical meaning of the symmetry (16) is puzzling. To get a feeling of the meaning of this symmetry, it is interesting to notice that such transformation becomes nontrivial if $L\equiv-(1/\kappa)R+L_m$ is *not* a constant, in contrast to what we have studied so far in this paper. For the derivation of Eq. (7), which implies $L=$const, we have assumed that $\Phi\neq0$. Allowing $\Phi=0$ in some regions of space-time should be equivalent to allowing for ''defects'' which could be string-like or take the shape of other extended objects. In such event, the symmetry (16) would become nontrivial precisely at the location of such defects, a situation that reminds us of the reparametrization invariance of theories of extended objects.

In this context, it becomes then natural to explore the possibility that all the matter could arise as regions of space-time where $\Phi=0$, i.e., as defects described above. This could be a way to realize an idea of Einstein and Infeld [11] that matter should arise as singular points from a pure gravitational theory. In their words [11]: ''All attempts to represent matter by an energy-momentum tensor are unsatisfactory and we wish to free our theory from any particular choice of such a tensor. Therefore we shall deal here only with gravitational equations in empty space, and matter will be represented by singularities of the gravitational field.'' In GR, however, such a mechanism could lead only to singularities of the metric, i.e., black holes (if the cosmic censorship hypothesis [12] is correct). Although black holes are interesting objects, they are of course not satisfactory candidates for the description of the matter we see around. Defects that appear in the NGVE theory are singularities of the measure which are not necessarily singularities of the metric. Furthermore the existence of the symmetry (16) suggests to us a close connection between these singularities and reparametrization invariance of extended objects as mentioned above.

Finally, the crucial question concerning the large distance behavior of the model should be analyzed in detail. In particular, the study of general conditions under which the theory contains an Einstein sector of solutions or a set of solutions which deviate very little from those of the Einstein theory must be studied. In this respect, let us recall that we have already reviewed a number of cases where the Einstein condition and Einstein symmetry hold. In these cases the theory is guaranteed to contain an Einstein sector of solutions.

However, it would be interesting to study situations where the answer is not so clear. We have here in mind cases where neither the Einstein condition nor the Einstein symmetry are exactly satisfied, but that they could appear only in the long distance behavior of the theory, without being satisfied by the underlying microscopic theory. This possibility is suggested by the fact that the point particle limit allows a formulation consistent with the Einstein symmetry. In the point particle limit, however, the underlying microscopic (field theoretic) structure is ''integrated out'' and in this way disappears. This of course suggests that the integration of microscopic degrees of freedom could, under very general circumstances, lead to a macroscopic theory satisfying the Einstein symmetry. The answer to this last question will of course demand further research.

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