Galactic periodicity and the oscillating *G* model

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We consider the model involving the oscillation of the effective gravitational constant that has been put forward in an attempt to reconcile the observed periodicity in the galaxy number distribution with the standard cosmological models. This model involves a highly nonlinear dynamics which we analyze numerically. We carry out a detailed study of the bound that nucleosynthesis imposes on this model. The analysis shows that for any assumed value for Ω (the total energy density) one can fix the value of $\Omega_{\rm bar}$ (the baryonic energy density) in such a way as to accommodate the observational constraints coming from the ^{4}He primordial abundance. In particular, if we impose the inflationary value $\Omega=1$ the resulting baryonic energy density turns out to be $\Omega_{\rm bar}\sim0.021$. This result lies in the very narrow range $0.016\leqslant\Omega_{\rm bar}\leqslant0.026$ allowed by the observed values of the primordial abundances of the other light elements. The remaining fraction of Ω corresponds to dark matter represented by a scalar field. [S0556-2821(96)06010-9]

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I. INTRODUCTION

The recent observations, in deep pencil beam surveys [1], showing that the galaxy number distribution exhibits a remarkable periodicity of $128h^{-1}$ Mpc comes as a shocking development, since, if taken at face value it would imply that we live in the middle of a pattern consisting of concentric two-spheres that mark the maxima of the galaxy number density. This, of course, would be catastrophic for our cosmological conceptions. While it is true that such periodicity has been observed only in the few directions that have been explored so far, it would be a striking coincidence if it turns out that it is absent in other directions and we just happen to have chosen to explore the only directions in which that phenomenon occurs. Therefore it seems reasonable to assume that the periodicity is also present in the deep pencil beam surveys in other directions, thus taking us to the concentric spheres scenario. The seriousness of the situation is such that this type of scenario has indeed been put forward in a model where the formation of these concentric shells is a result of a ''spontaneous breakdown of the cosmological principle'' via a mechanism that results in the appearance of patches filled with the concentric spheres pattern, with these patches covering the Universe [2]. Again it would still be difficult to explain how we did come out living in the center of such a patch (more precisely inside the innermost sphere of one such patch).

The only known way out of this type of scenario is to assume that there is only an apparent spatial periodicity that is the result of a true temporal periodicity which shows up in our observations of distant points in the Universe and that is mistakenly interpreted as a spatial periodicity [3]. The models that have been put forward in order to achieve this temporal periodicity involve the oscillation of an effective coupling constant due to the contribution to it coming from the spectation value of some scalar field, that actually oscillates coherently in cosmic time in the bottom of its effective potential.

The specific models that have been proposed involve the oscillation of the effective electric charge, electron mass, galaxy luminosity or gravitational constant [3–6]. From these the first two have been shown to conflict with bounds arising from the test of the *equivalence principle* [7]. As for the third scenario, it would seem to involve a large number of hypotheses since the galactic luminosity is fixed by the number and type of stars present in the galaxy and their respective luminosities, and the latter are themselves functions of the standard physics coupling constants that control nuclear reaction rates (on the variation of which there are severe limits), and of the transport mechanisms: convection, radiation, etc., which are also determined by the standard physics coupling constants. Thus it seems that the only way to produce such a model requires the introduction of ''exotic'' particles (like axions or massive neutrinos [3]) that would act as a new transport mechanism and besides that, the hypothesis of a second mechanism that would produce the oscillation, an assumption which presumably involves the coupling of these hypothetical particles to the hypothetical cosmological scalar field.

In light of the complicated nature of the alternative scenario, it seems worthwhile to carry out a careful analysis of the viability of the oscillating gravitational constant model, despite the difficulties that seem to appear when confronting the predictions of the model with other experimental data. We will address these difficulties below.

The oscillating *G* model is based on a cosmological, massive scalar free field, that is nonminimally coupled to curvature and whose oscillations in cosmic time result in the oscillation of the effective gravitational constant. The model is governed by a system of nonlinear, ordinary, differential equations that we shall integrate numerically without analytical simplifications. In previous related works [5,3,4], only approximate solutions have been explored as in the work of Morikawa [4]. In this work, the dynamical equation for the scalar field is linearized into a Bessel equation. From this,

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the Hubble parameter for a flat matterless Universe is evaluated. The approximations result in a precision of 2% in the Hamiltonian constraint. While such procedure is an extremely simple way to study the implementation of the model it is however not totally satisfying. In addition to the analytical simplifications Morikawa made, we emphasize that his ''matterless'' (no baryonic or radiation energy density) Universe assumption is not useful for the study of the constraints we discuss below, notably the one imposed by nucleosynthesis. As we shall analyze in Sec. IV, it is actually the presence of matter that makes it possible to satisfy such a constraint. Furthermore, it is obvious that Morikawa's approach is no longer valid in early times where the linear approximations break down. Hence, this method cannot be used to describe the behavior of cosmological variables during the nonoscillating phase.

The data arising from the Viking radar echo experiments in addition to those related with the limits on the Brans-Dicke parameter [5] impose certain bounds on the value of $\dot{G}/G$. There is an apparent conflict between these bounds and the value they should have for reproducing the galaxy-amplitude counting. However, this problem can be overcome by imposing the unnatural extra requirement that our Galaxy is at a ''fortunate phase'' [5] where the scalar field is swinging very close to zero at our particular place and time in the Universe. There are additional bounds on the oscillating G model arising from Big Bang nucleosynthesis, but we will argue that these cannot be adequately addressed in the fashion described by Accetta *et al.* [8]. This constraint is obtained by considering the Helium abundance limits which depend on the neutron-proton ratio at the temperature of freeze-out. This temperature is itself obtained from the condition that the Hubble parameter equals the nucleon to proton weak reaction rate.

Crittenden and Steinhardt [5], based on a previous analysis by Accetta and collaborators that resulted in the bound $\Delta G/G<0.4$ [8], argue that the nucleosynthesis constraint is so stringent that it practically rules out the oscillating G model unless we assume a ''fine-tuning'' within the oscillations of the scalar field.

The problem with employing the analysis of Accetta *et al.* [8] directly to this model is that they study the change occurring in the Hubble parameter as a function of the temperature when one changes the value of G, but fails to take into account the fact that the model implies an equation for the Hubble parameter that contains terms other than the simple ones used by the authors. These terms are associated with the contributions from the scalar field to the effective energy density.

In this paper we carry out an analysis of the nucleosynthesis bound taking into account these extra terms and demonstrate that indeed such a ''fine-tuning'' is needed and moreover that it is possible. However, we will argue that this ''fine-tuning'' is not of the kind that should result in the dismissal of the model, but rather a natural adjustment of the initial conditions that will lead to the observational data extracted from our Universe today. In other words, scientific models that require a very precise choice of the numerical value of the initial conditions in order to reproduce a given qualitative behavior of the observational data, are models that would be consider unnatural and the choice of the specific initial data is justifiably described as ''fine-tuning.'' However, models that require a very precise choice of the numerical value of the initial conditions in order to reproduce a specific numerical observational data cannot be considered as unnatural, especially if for every conceivable value of the observational data (at least in some range) there is a corresponding value of the initial data. In this type of models, the particular ''preferred'' value at the initial data is just the result of a 1 to 1 correspondence between initial conditions and final outcome. We will argue that the present model could be of the latter type.

The organization of the paper is as follows: In Sec. II the model is described and the basic equations of evolution for the fields and matter are derived, in Sec. III the numerical implementation is discussed together with the error estimation analysis, in Sec. IV the results of the numerical integrations are analyzed and finally in Sec. V we give a brief discussion of the main features exhibited by the model, their physical significance and the overall viability of the model.

II. FORMULATION OF THE MODEL

We will consider a model in which the effective gravitational constant becomes dependent on cosmic time due to a contribution to it coming from the spectation value of a scalar field. This can be achieved by considering a scalar field ϕ nonminimally coupled to gravity. One of the simplest models of this kind is obtained by taking a Lagrangian as follows:

$$\mathscr{L}=\left(\frac{1}{16\pi G_0}+\xi\phi^2\right)\sqrt{-g}R-\sqrt{-g}\left[\frac{1}{2}(\nabla\phi)^2+V(\phi)\right]+\mathscr{L}_{\rm mat}. \tag{1}$$

Here G_0 is the Newton's gravitational constant; ξ stands for the nonminimally coupling constant; and $V(\phi)$ is a scalar potential to be specified later (see Sec. III). In this model we are also including an schematic matter Lagrangian $\mathscr{L}_{\rm mat}$. Equation (1) shows that the introduction of the coupling term is equivalent to considering an effective gravitational constant which explicitly depends on the scalar field:

$$G_{\rm eff}=\frac{G_0}{1+16\pi G_0\xi\phi^2}. \tag{2}$$

The gravitational field equations following from the Lagrangian (1) can be written as

$$R^{\mu\nu}-\tfrac{1}{2}g^{\mu\nu}R=8\pi T^{\mu\nu}_{\rm eff}, \tag{3}$$

where

$$T^{\mu\nu}_{\rm eff}=G_{\rm eff}(4\xi T^{\mu\nu}_{\xi}+T^{\mu\nu}_{\rm sf}+T^{\mu\nu}_{\rm mat}), \tag{4}$$

$$T^{\mu\nu}_{\xi}=\nabla^{\mu}(\phi\nabla^{\nu}\phi)-g^{\mu\nu}\nabla_{\lambda}(\phi\nabla^{\lambda}\phi), \tag{5}$$

$$T^{\mu\nu}_{\rm sf}=\nabla^{\mu}\phi\nabla^{\nu}\phi-g^{\mu\nu}[\tfrac{1}{2}(\nabla\phi)^2+V(\phi)]. \tag{6}$$

The energy-momentum tensor of ''matter'' $T^{\mu\nu}_{\rm mat}$ will be composed of a combination of two noninteracting perfect fluids, one corresponding to pure baryonic matter $(i=1)$ and the other one representing a pure radiation field $(i=2)$:

$$T^{\mu\nu}_{\rm mat}=T^{\mu\nu}_{\rm bar}+T^{\mu\nu}_{\gamma}=\sum_{i=1,2}[(p_i+e_i)U^{\mu}U^{\nu}+p_i g^{\mu\nu}], \tag{7}$$

which possesses the symmetries of the spacetime. The scalar field will also be assumed to posses these symmetries.

Finally, the equation of motion for the scalar field becomes

$$\Box\phi+2\xi\phi R=\frac{\partial V(\phi)}{\partial\phi}. \tag{8}$$

We will focus on the Friedmann-Robertson-Walker (FRW) spacetimes which describe isotropic and homogeneous cosmological models

$$ds^2=-dt^2+a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\varphi^2\right], \tag{9}$$

where $a(t)$ is the scale factor and $k=1,0,-1$.

Our purpose is to study the behavior of the solutions of the gravitational, matter, and scalar field equations for the FRW line element (9). Since these equations are highly nonlinear, it is a difficult task to find analytic solutions; therefore, we will approach the problem via a numerical analysis.

One of the equations we find is the Hamiltonian constraint

$$\frac{\dot a^2}{a^2}+\frac{k}{a^2}=\frac{8}{3}\pi G_0E. \tag{10}$$

The dynamical equation for the single gravitational degree of freedom is

$$\frac{\ddot a}{a}+2\frac{\dot a^2}{a^2}+2\frac{k}{a^2}=4\pi G_0\left(E-\frac{1}{3}S\right), \tag{11}$$

where

$$E=\frac{G_{\rm eff}}{G_0}\left[e+\frac{1}{2}\dot\phi^2+V(\phi)-12\xi\phi\dot\phi\frac{\dot a}{a}\right], \tag{12}$$

is the total effective energy density and

$$S=\frac{3G_{\rm eff}}{G_0}\left[p+\tfrac{1}{2}\dot\phi^2-V(\phi)+4\xi\left(\dot\phi^2-\phi\dot\phi\frac{\dot a}{a}-\phi\Box\phi\right)\right]. \tag{13}$$

These source terms contain contributions from the three parts of the total energy-momentum tensor given in Eqs. (5)–(7).

Finally, the equation for the scalar field (8) can be written explicitly as

$$\ddot\phi+3\dot\phi\frac{\dot a}{a}+\frac{\partial V(\phi)}{\partial\phi}=16\pi G_0\xi\phi(E-S), \tag{14}$$

where we have replaced the scalar curvature in terms of the energy-momentum tensor quantities E and S.

It is worth mentioning that the contribution of the scalar field to the expression $T^{\mu\nu}_{\text{eff};\nu}=0$ [see Eq. (4)] vanishes identically and thus the energy momentum of the ordinary matter satisfies $T^{\mu\nu}_{\text{mat};\nu}=0$. We will moreover assume that the two perfect fluid components (baryons and photons) do not interact among themselves, thus each of their corresponding energy-momentum tensors is separately conserved leading to

$$\dot e_i+3(e_i+p_i)\frac{\dot a}{a}=0. \tag{15}$$

Equation (15) integrates immediately with respect the scale factor like in the standard cosmology case. We find then

$$e=e_{\rm bar}+e_\gamma=c_1\left(\frac{a_0}{a}\right)^3+c_2\left(\frac{a_0}{a}\right)^4, \tag{16}$$

$$p=p_{\rm bar}+p_\gamma=\frac{c_2}{3}\left(\frac{a_0}{a}\right)^4 \quad \text{with} \quad a_0:=a(t=t_0). \tag{17}$$

Here we have assumed an equation of state $p_\gamma=e_\gamma/3$ for the radiation part, whereas $p_{\rm bar}=0$ for the corresponding baryonic component. The first term of (16) represents thus the pure baryon energy density, while the second one the radiation contribution alone. The constants c_1 and c_2 are fixed by the ''initial'' conditions (i.e., the conditions today). In particular (at $t=t_0$) we will be assuming a baryonic energy density of one-tenth of the critical value (at least for the first numerical experiment, but later we will use c_1 as an adjustment parameter) and the radiation energy density corresponding to the 2.73 K cosmic background radiation (CBR) (see also Sec. III). Hereafter we shall refer as to *matter* the combination of both fluids.

With the aim of reducing the field equations into an initial value problem consisting of a system of first-order differential equations, we shall rearrange these conveniently. Here we present the final form of equations with source terms containing no second-order derivatives and introduce better-suited variables:

$$P_\alpha=-\dot\alpha(t)=-\frac{\dot a}{a}, \quad P_\phi=\dot\phi, \tag{18}$$

where

$$\alpha(t):=\ln\left[\frac{a(t)}{a_0}\right]. \tag{19}$$

The dynamic equation (11) then takes the form

$$\dot P_\alpha-P_\alpha^2=\frac{4}{3}\pi G_0(E+S), \tag{20}$$

where we have used the Hamiltonian constraint (10) in order to eliminate from Eq. (11) the term proportional to k. Notice that $P_\alpha\equiv-H(t)$.

Introducing Eqs. (18) into the scalar field equation we obtain

$$\dot P_\phi-3P_\alpha P_\phi+\frac{\partial V(\phi)}{\partial\phi}=16\pi G_0\xi\phi(E-S). \tag{21}$$

The source terms then take the following form:

$$E=\frac{G_{\rm eff}}{G_0}\left[e+\frac{1}{2}P_\phi^2+V(\phi)+12\xi\phi P_\phi P_\alpha\right], \tag{22}$$

and

$$S=\frac{3}{1+192\pi G_{\rm eff}\xi^2\phi^2}\frac{G_{\rm eff}}{G_0}\left[p+\frac{1}{2}P_\phi^2-V(\phi)+4\xi\left(\phi P_\phi P_\alpha+P_\phi^2-\phi\frac{\partial V(\phi)}{\partial\phi}\right)+64\pi G_0\xi^2\phi^2E\right]. \tag{23}$$

In obtaining the source (23) from (13) we have used the scalar field equation (21) in order to eliminate the term with $\Box\phi$.

Further analysis of this model requires the numerical integration of the field equations under appropriate initial conditions. This will be performed in the following sections. In the Appendix we provide the dimensionless form of the above equations.

III. INITIAL CONDITIONS AND FIXING OF PARAMETERS

The choice of our variables fixes in advance the values of α and P_α at present time. Then, the initial conditions are

$$\alpha|_{t=t_0}\equiv 0, \tag{24}$$

$$P_\alpha|_{t=t_0}\equiv -1. \tag{25}$$

It is worth emphasizing that by *initial conditions* we will mean throughout the paper the value of the field variables at present time and not their corresponding value near the Big Bang.

As stated before, the initial condition for e corresponds to the value of the (baryon plus radiation) energy densities today and then

$$e_0:=e|_{t=t_0}=e_0^{\rm bar}+e_0^{\gamma}=c_1+c_2. \tag{26}$$

We fix c_2 by the value $T=2.73$ K from the CBR and choose for the moment a value of c_1 corresponding to $\Omega_{\rm bar}=0.1$. Thus, the values of the constants appearing in Eqs. (16) and (17) are

$$c_1=0.1e_c, \tag{27}$$

$$c_2\sim 4.2\times 10^{-5}e_c \quad \text{with} \quad e_c:=\frac{3c^2H_0^2}{8\pi G_0}. \tag{28}$$

Here e_c stands for the critical energy density in terms of the current value of the expansion rate H_0.

This choice will leave Ω (see below) as a free parameter which is adjusted in order to determine the initial conditions on ϕ which result in a model respecting the observed ^{4}He abundances. As we mentioned before, we will be able to follow (Sec. IV) a different strategy that consists in imposing $\Omega=1$ and using c_1 (i.e., $\Omega_{\rm bar}$) as an adjustment parameter.

The Viking data experiments constraint $\dot{G}/(GH)|_{\rm today}$ to be less than $0.3h^{-1}$ [5]. This imposes a bound on $\dot{\phi}$. The most conservative and the one we choose is

$$\dot{\phi}|_{t=t_0}=0. \tag{29}$$

However, it is straightforward to explore a less conservative initial condition.

The values of the initial scalar-field's amplitude and coupling constant are obtained from the observational data. The observed redshift-galaxy-count amplitude (see [5,3]) and the Hamiltonian constraint (10) at t_0 provide two algebraic equations which determine the values of ϕ_0 and ξ once we choose a value for Ω and $\Omega_{\rm bar}$. When assuming a harmonic scalar potential $V(\phi)=m^2\phi^2$, these equations are

$$\mathscr{A}_0=-\frac{G'_{\rm eff}\omega\phi_0}{2H_0G_{\rm eff}}, \tag{30}$$

$$\phi_0^2=\frac{e_0-\Omega}{16\pi\xi\Omega-m^2}, \tag{31}$$

where e_0 is the initial matter energy density in units of e_c; $\Omega:=E_0/e_c$ and $\omega^2\propto m^2$ is the oscillation frequency in units of H_0^{-1}.

Solving for ϕ_0 and ξ we obtain:

$$\xi=\frac{\mathscr{A}_0m^2}{16\pi[\mathscr{A}_0e_0+\omega(\Omega-e_0)]}, \tag{32}$$

$$\phi_0^2=\frac{\mathscr{A}_0}{16\pi\xi[\omega-\mathscr{A}_0]}. \tag{33}$$

The model will explain the observed galaxy distribution if $\mathscr{A}_0\geqslant 0.5$ [3]. The remaining parameter to be adjusted is ω which is obtained from the observed galaxy periodicity of 128 Mpc h^{-1}.

In order to test our numerical code we have first restricted ourselves to the standard cosmology where known analytical solutions exist. The field equations have been solved by means of a fourth-order scheme with an adaptive stepsize control. We have performed the integration of the equations with respect to time and also with respect to the variable α. The time integration produces relative errors on the dynamical variables, like the scale factor, energy density, and Hubble parameter which are of the order of 10^{-8}.

The choice of the variable α as integration parameter allows us to explore the evolution at very early stages ($\alpha<0$ region) while keeping the relative errors small. Indeed we stop the integration at $\alpha\sim -25$, a bit beyond the value at which the nucleosynthesis takes place, the calculations having started at $\alpha=0$.

While the integration of equations for flat ($k=0$) and hyperbolic ($k=-1$) Universes can be performed for an arbitrarily large value of α, for a closed Universe ($k=1$) the integration makes only sense for $\alpha\leqslant\alpha_{\rm max}$. The limiting value corresponds to the maximum size reached by the Universe and beyond which it starts recollapsing. We mention that the regions $\alpha>\alpha_{\rm max}$ are indeed not very interesting, first because the physics associated with them can be inferred from the $\alpha<\alpha_{\rm max}$ branch and then because no observational bounds arise from that region.

We use the Hamiltonian constraint as a test on the accuracy of the procedure. It is applied at every integration step and implemented by defining the *deviation* parameter (see the Appendix) as

$$\lambda:=\frac{(H/H_0)^2-(\Omega-1)e^{-2\alpha}}{E/e_c}. \tag{34}$$

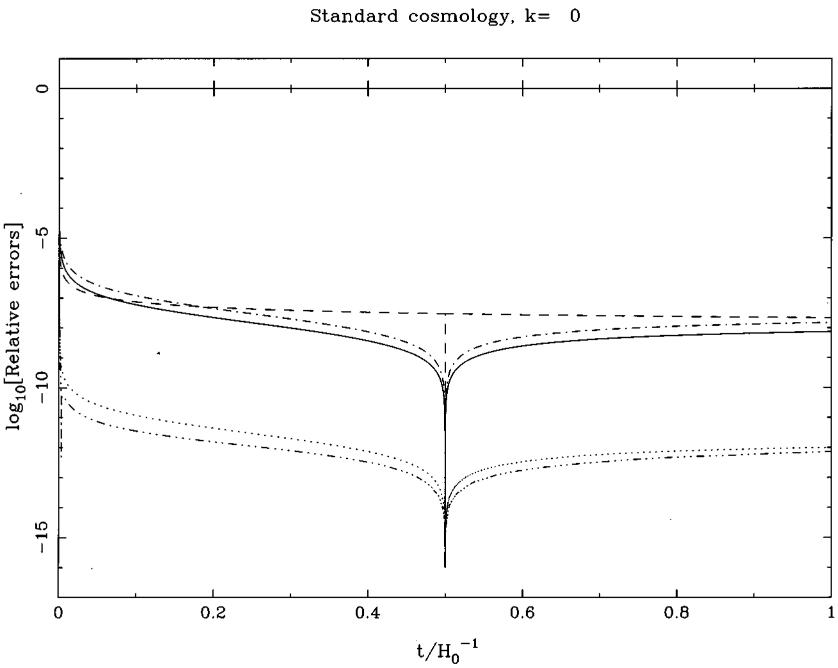

FIG. 1. Relative errors as a function of dimensionless time: solid lines (superposed): $\log_{10}(a(t)_{\rm an}-a(t)_{\rm num})/a(t)_{\rm an}|$, $\log_{10}|(\dot{a}(t)_{\rm an}-\dot{a}(t)_{\rm num})/\dot{a}(t)_{\rm an}|$; dashed line: $\log_{10}|(\alpha(t)_{\rm an}-\alpha(t)_{\rm num})/\alpha(t)_{\rm an}|$; dash-dotted line: $\log_{10}|(P_\alpha(t)^{\rm an}-P_\alpha(t)^{\rm num})/P_\alpha^{\rm an}|$; dotted line: $\log_{10}|((ea^3)_{\rm an}-(ea^3)_{\rm num})/(ea^3)_{\rm an}|$; dot-dashed line (bottom): $\log_{10}|1-\lambda|$. The initial conditions located at $t_0:=0.5H_0^{-1}$.

For an infinite accurate integration this parameter would equal one and the deviation from this value indicates the degree to which Eq. (10) fails to be satisfied.

Figure 1 shows the correlation between the relative errors found for the cosmological variables and that for the Hamiltonian constraint (i.e., in λ) as a function of time for the Einstein–de Sitter Universe. The deepest peaks (infinite precision) indicate the location of initial data. Only for convenience, in such points the precision has been arbitrarily set to be $\sim 10^{16}$.

We have also verified this type of correlation in closed and hyperbolic Universes. Because no analytical solutions are known for the oscillating models, we use Eq. (34) systematically (the ‘‘internal test’’) in order to verify the accuracy of the results.

IV. NUMERICAL RESULTS

As we will see below, for any given value of Ω it is possible to fix $\Omega_{\rm bar}$ such that the correct value of the ^{4}He abundance is recovered. Figure 2 shows the behavior of ϕ in some range of past and future epochs with $\Omega=1$ and the corresponding $\Omega_{\rm bar}\sim 0.021$. As the scale factor increases, the amplitude is damped due to redshifts while the frequency of oscillation (with respect to α), which is inversely proportional to H^2 [9], grows (see Fig. 3 for the behavior of the Hubble parameter). The amplitude and oscillation frequency will reach maximum and minimum values, respectively, as the scale factor decreases before entering into a stage (discussed below) in which the scalar field almost vanishes. This includes a late era in which matter is dominant over the scalar field energy density and an earlier one in which this is the opposite (see Fig. 4).

We introduce effective energy densities by

$$E_\xi := 12\xi\phi P_\phi P_\alpha \frac{G_{\rm eff}}{G_0}, \tag{35}$$

$$E_\phi := \frac{G_{\rm eff}}{G_0}\left[\frac{1}{2}P_\phi{}^2+m^2\phi^2\right], \tag{36}$$

$$E_{\rm mat} := \frac{G_{\rm eff}}{G_0}\, e. \tag{37}$$

Figure 4 shows how these fractions of the total effective energy density (depicted in Fig. 5) vary with α.

We emphasize that E_ξ (a ‘‘coupling energy density’’) is not positive definite and since it contributes to the total effective energy density E [see Eq. (22)], there are some regions where the fraction E_ξ/E is negative (dash-dotted line) while E_ϕ/E and $E_{\rm mat}/E$ exceed one (dashed and solid lines, respectively). In particular, when comparing Figs. 2 and 4

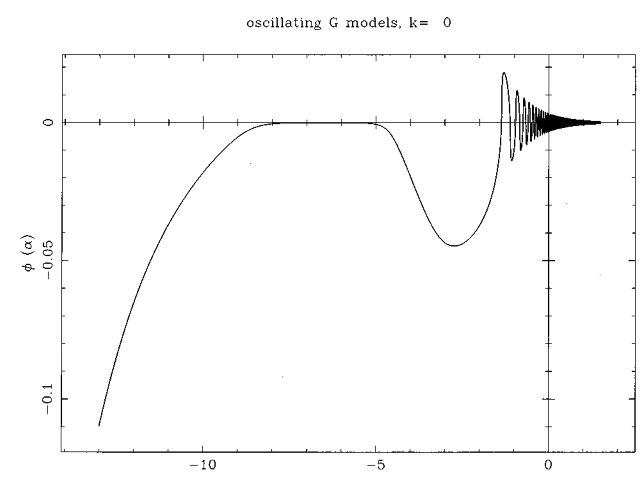

FIG. 2. Fine-tuned scalar field amplitude as a function of $\ln[a/a_0]$ for a flat Universe ($\Omega=1$) with $\Omega_{\rm bar}=0.021\,012\,641\,182\,345$ and $\mathscr{A}_0=0.5$ at the onset of oscillations. At present time ($\alpha=0$) the initial amplitude is $\phi_0\sim 3.288\times 10^{-3}$ and $\xi\sim 6.267$. Computations were stopped at $\alpha\sim 1.5$.

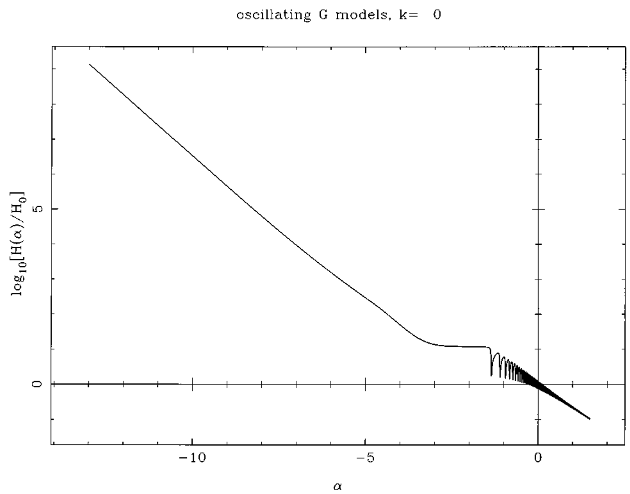

FIG. 3. The Hubble parameter obtained by using the same initial values as in Fig. 2 and $H(\alpha=0)=H_0$.

we note that when the amplitude $|\phi|$ is large (in the region $\alpha\in[-4,-3]$), the net scalar-field's contribution to E represented by $E_\xi+E_\phi$ is small because the negative ''energy density'' E_ξ compensates the contribution from E_ϕ. Indeed, it is the matter contribution $E_{\rm mat}$ which becomes dominant in this era.

When the scale factor is still small ($\alpha\in[-8,-5]$), $|\phi|$ falls down dramatically by entering what we called the ''fine-tuning era'' (see Sec. IV A) where $E_{\rm mat}$ dominates completely. The effective gravitational constant which was reduced in ~38% of its current value G_0 during the maxima of $|\phi|$, recovers its normal value again (Fig. 6). Finally, for still smaller α, the scalar field becomes dominant again, resulting in $G_{\rm eff}\rightarrow 0$ as we approach the Big Bang singularity. The Hubble parameter decreases monotonically in the early era ($\alpha\in[-\infty,3]$) (Fig. 3).

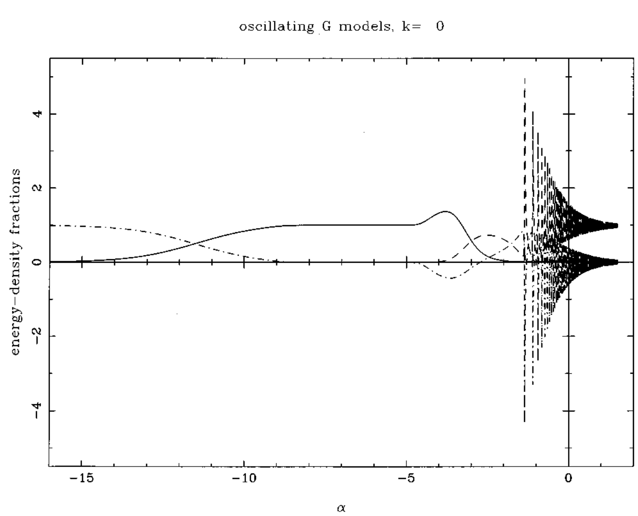

FIG. 4. Starting from the same initial values as in Fig. 2, this figure depicts the fractions of effective energy densities: $E_{\rm mat}$ (solid line), E_ϕ (dashed line), and E_ξ (dash-dotted line) (see text for definitions).

A. The nucleosynthesis bound and ''fine-tuning''

In previous investigations, Crittenden and Steinhardt [5] have brought attention to a couple of much more severe constraints on the oscillating models that those considered before by Morikawa [4] and Hill *et al.* [3]. The first of these constraints regarded the bound imposed by the Brans-Dicke parameter tests which are even more stringent that the one coming from the Viking radar echo experiments. However, the authors argue that it can be eluded by assuming a ''fortunate phase'' for the oscillation of the scalar field when the nonminimally coupling function varies quadratically with the changes of ϕ. The second new bound they mentioned arises from the fact that prior to the onset of its oscillatory behavior, the amplitude of ϕ had to be small enough in order to prevent $|\Delta G|/G>0.4$ at nucleosynthesis as described by Accetta *et al.* [8], but at the same time, the evolution of ϕ has to be such that its present amplitude is large enough to accommodate the value of $\mathscr{A}_0$ needed to generate the observed peaks in the deep pencil survey [1]. Crittenden and Steinhardt suggested that both conditions might be possible by ''fine-tuning'' the amplitude of ϕ at nucleosynthesis. This amplitude might then be amplified by the effect of the curvature during the matter-dominated era and afterwards damped by the redshift's effects.

Although the nucleosynthesis bound can be thought as imposing a limit on the variation of G, this is not entirely precise. The true bound that comes from nucleosynthesis is a limit on the deviation of the expansion rate of the Universe from its value given by the standard cosmological model. There exists a very narrow region for which the expansion rate of the Universe and the transition rate for the weak interaction, which convert neutrons to protons, traduce themselves into a freezeout temperature that reproduces the observed ^{4}He abundance (see [10] for a review). In the standard cosmology framework, for a radiation dominated era, the ratio between those rates, is

$$\frac{\Gamma}{H}\sim\left(\frac{\kappa T}{0.7\ \ \mathrm{MeV}}\right)^3. \tag{38}$$

Thus the temperature at which those weak interactions freezeout the n/p ratio is $\kappa T_F\sim 0.7$ MeV. At this temperature, the neutron-proton ratio is given by its equilibrium value

$$\left(\frac{n}{p}\right)=\exp(-Q/T_F)\approx 1/6, \tag{39}$$

where Q is the mass difference of neutrons and protons. This ratio can decrease to 1/7 if we take into account the ''natural'' neutron decay due to weak interactions. This value predicts approximately one ^{4}He nuclei for each 16H nuclei. Therefore the primordial abundance of ^{4}He-H ratio gives $Y_P\approx 25\%$, i.e., very close to the observed value, varying in some small amount depending on the number of neutrino species considered.

Let us emphasize that one of the greatest achievements of the standard cosmology is the predicted light element abundance that we observe today. Therefore, the oscillating model would be considered as viable if it preserves also these nice features.

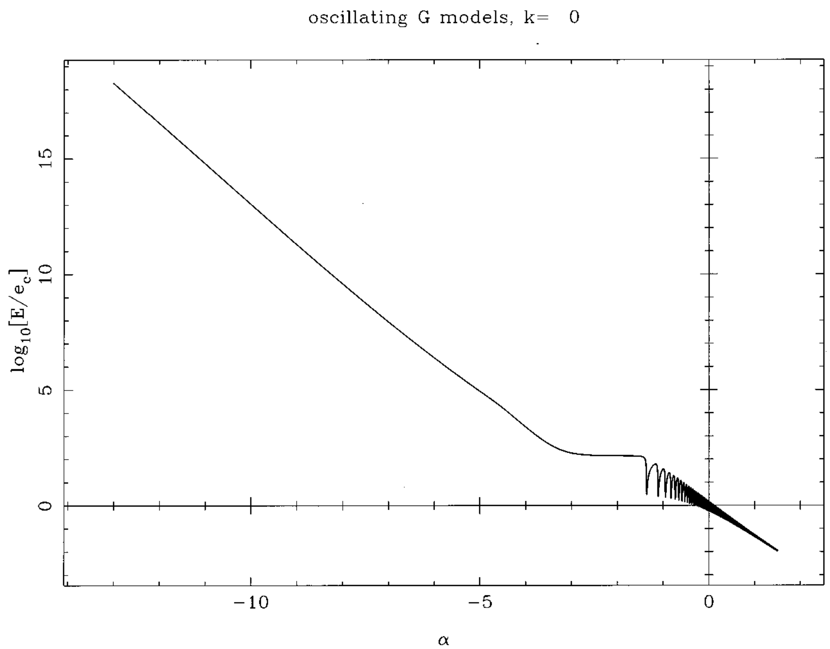

FIG. 5. The evolution of the total effective energy density resulting from the numerical integration of the field equations of Sec. II (see also the Appendix), with initial values as in Fig. 2.

Because of the exponential relation (39), a small variation of the freezeout temperature will produce a large deviation of the observed neutron-proton ratio. Deviations of the freezeout temperature will arise as a result of a small change of the expansion rate. The greater the expansion rate, the greater the freezeout temperature and so larger the ^{4}He abundance. This is the main reason why alternative cosmological models can fail.

We showed in the previous section that the initial condition ϕ_0 and the value of ξ depends upon $\mathscr{A}_0$, ω, Ω, and $\Omega_{\rm bar}$. The parameters ω and $\mathscr{A}_0$ are fixed in order to satisfy the observed galaxy periodicity and the galaxy peak amplitude whereas $\phi_0=0$ prevents conflicts with the Viking experiment's constraint. So we can try to adjust Ω or $\Omega_{\rm bar}$ in order to ensure the satisfaction of the nucleosynthesis bound. Initially we considered fixing $\Omega_{\rm bar}=0.1$ and then adjusting Ω. Another perhaps more natural possibility (see discussion) is to take a value of Ω (in particular $\Omega=1$ as suggested by inflation) and then adjusting $\Omega_{\rm bar}$. As we will see below, we have done this obtaining $\Omega_{\rm bar}\sim 0.021$.

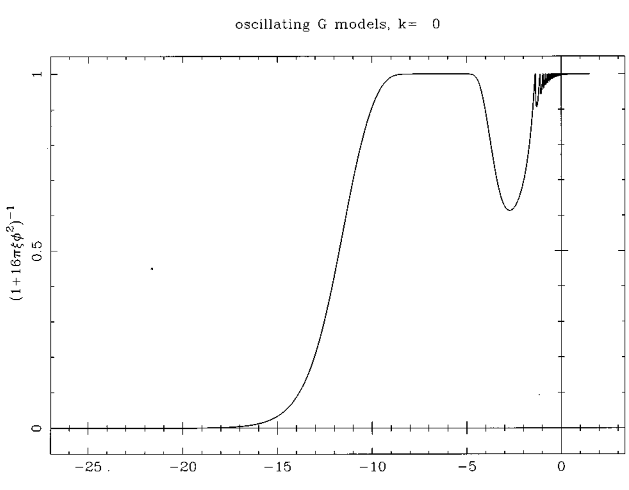

FIG. 6. Behavior of the effective gravitational ''constant'' in units of Newton's constant G_0. The initial values have been chosen as in Fig. 2.

In the process of integrating towards small values of α we find initially the following behavior: for some choices of $\Omega_{\rm bar}$, the value of ϕ goes monotonically to $+\infty$, or to $-\infty$ for some others. This suggested to us that there exists some value of $\Omega_{\rm bar}$ at which the transition from one behavior to the other takes place. We found that for $\Omega=1$ this ''fine-tuned'' value turns out to be $\Omega_{\rm bar}\sim 0.021$. Figure 7 shows the behavior of ϕ for three values of $\Omega_{\rm bar}$ about this point.

This transition point, traduces in a very special ''initial'' conditions for which the ϕ amplitude is ''squeezed'' to zero, before the onset of the oscillatory behavior. The search of that transition value is what we call ''*fine-tuning*.'' Our numerical experience indicates that it is possible to generate a finite region during which $\phi\sim 0$ before growing or decreasing monotonically (in the direction of decreasing α) and that the extent of that region depends on the improvement of the ''fine-tuning.''

Different ''experiments'' in ''fine-tunings'' are depicted in Fig. 8. The larger the plateau for which $G_{\rm eff}\to G_0$ (i.e., for which $\phi\to 0$) the closer the freezeout temperature is to the standard cosmology prediction 0.7 MeV. We have found that $\Omega_{\rm bar}=0.021+\epsilon$ results in a freezeout temperature which agrees with 0.7 MeV and therefore giving a ^{4}He abundance that approximates best the observed value (see Fig. 9).

In light of the extreme sensitivity of T_F to the value of $\Omega_{\rm bar}$ we are not able to improve the ''fine-tuning'' due to the limitations in the numerical precision. However, even with a noninfinitely precise ''fine-tuning'' the model is able to recover the ^{4}He abundance from nucleosynthesis. Figure 9 shows the freezeout temperature (~ 0.7 MeV) predicted by the best ''fine-tuning'' we explored.

The $\Omega=1$ scenario (with $\Omega_{\rm bar}\sim 0.021$) corresponds to an age of the Universe of $\sim 0.8H_0^{-1}$ (see Fig. 10). Figure 11 shows a typical curve of the Hubble parameter as a function

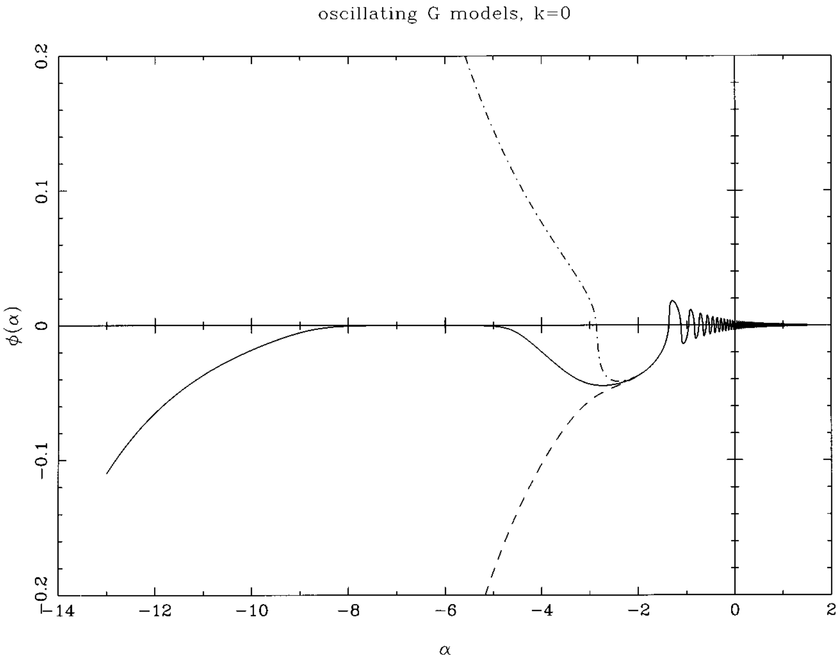

FIG. 7. Behavior of the scalar field for three different values of $\Omega_{\rm bar}$ within the $\Omega=1$ scenario. The solid line represents the best ''fine-tuning'' obtained with $\Omega_{\rm bar}=0.021\,012\,641\,182\,345$ (see Fig. 2); the dashed line for $\Omega_{\rm bar}=0.022$, and the dash-dotted line with $\Omega_{\rm bar}=0.020$.

of redshifts. The transition between the matter and radiation epochs is clearly appreciated in Fig. 12 [see also Eq. (16)].

Only for completeness we mention that, for example, if we choose $\Omega_{\rm bar}=0.1$ and fine-tune Ω this results in $\Omega\sim1.67$ which corresponds to a closed Universe. Figure 13 shows that the age of the Universe based in this scenario is about $0.7H_0^{-1}$. Somewhat younger than the age predicted by a Universe that includes only baryonic matter and radiation energy densities (a hyperbolic-standard-cosmology model) which corresponds to $\sim0.9H_0^{-1}$. Figure 14 shows the freezeout temperature ($\sim$0.8 MeV) predicted by the best ''fine-tuning'' we explored in the $\Omega_{\rm bar}=0.1$, $\Omega\sim1.67$ scenario. This temperature is very close to the one predicted by the standard cosmology ($\sim$0.7 MeV). This examplifies the fact that for any choice of Ω (or $\Omega_{\rm bar}$) the freezeout temperature of standard cosmology can be recovered by adjusting the other parameter $\Omega_{\rm bar}$ (or Ω).

V. CONCLUSION AND DISCUSSION

At first sight the model could not be less appealing: a ''fortunate phase'' and an incredibly precise ''fine-tuning.'' However, we must judge it in the proper context by contemplating the alternatives.

First the observed periodicity might be a statistical fluke, then of course that would be the end of the story. However, as we argued in the Introduction it would be quite a coincidence that we just happened to look at the couple of directions that exhibit such patterns. In any event this is a matter that only further observations will answer.

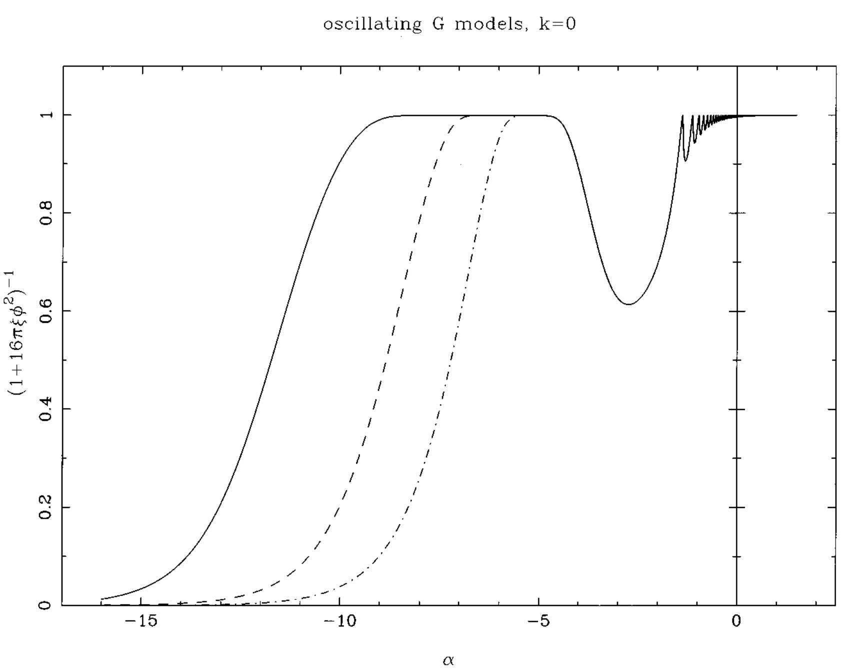

FIG. 8. Effective gravitational ''constant'' in units of Newton's constant G_0 for different fine-tunings: solid line obtained with $\Omega_{\rm bar}=0.021\,012\,641\,182\,345$, dashed line for $\Omega_{\rm bar}=0.021\,012\,641\,18$, dash-dotted line with $\Omega_{\rm bar}=0.021\,012\,6$.

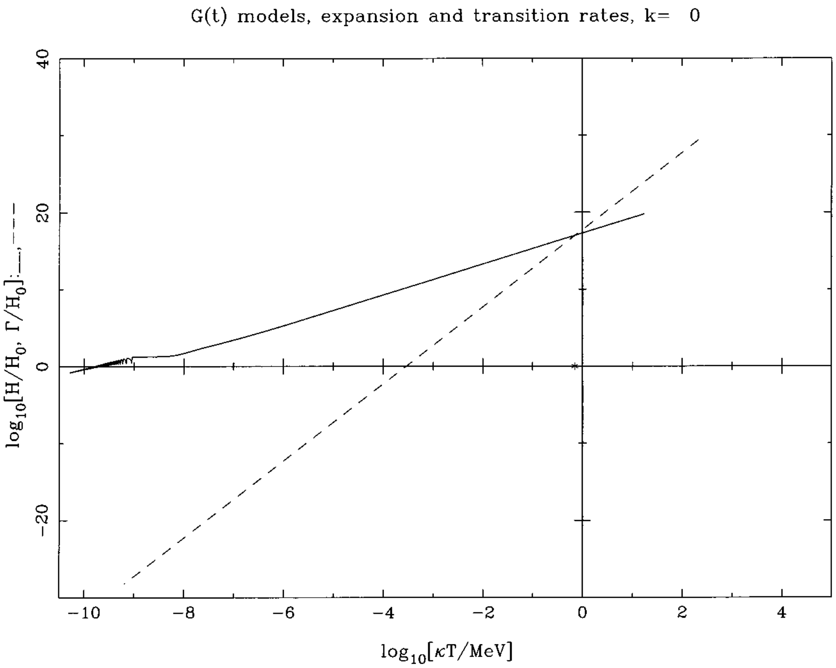

FIG. 9. Expansion (solid line) and transition (dashed line) rates in terms of black-body temperature. The asterisk depicts the freezeout temperature ~0.7 MeV at which nucleosynthesis takes place as predicted by the standard cosmological models. The cross-point of both curves indicates the corresponding freezeout temperature ~0.7 MeV for the oscillating model of previous figures.

Next we have the ''spontaneous breaking of the cosmological principle'' [2], clearly a major departure from our cosmological conceptions which besides that, requires something even more fortunate than the ''fortunate phase'' of the oscillating *G* models: We happened to be ''fortunate'' enough to be born in a galaxy which happens to lay in the middle of a concentric collection of shells of maximal galaxy density.

And finally the *galactic luminosity oscillation* model, which as we said at the beginning, more than a model is a vaguely specified scenario which nevertheless seems to require at least two new hypotheses: a new type of star cooling mechanism, and also (as the other alternatives do) a driving oscillating cosmological scalar field to turn on and off that mechanism periodically in cosmic time. Needless to say that once the scenario is implemented with a specific model, unforsen new bounds might also have to be overcome.

In this light, the oscillating *G* model does not look as clearly dismissable. Furthermore, while it is true that the fortunate phase will have to remain such, for the other problem, ''the fine-tuning,'' we will argue below that there are scenarios in which this problem is not present.

To start, we must stress that while the ''fine-tuning'' is completely unnatural when approached, as we have, from the present to the past, when looked from the opposite and more natural direction, the situation is quite different. In fact all that seems to be required is for some mechanism to drive the scalar field to an extremely low value before the era of nu-

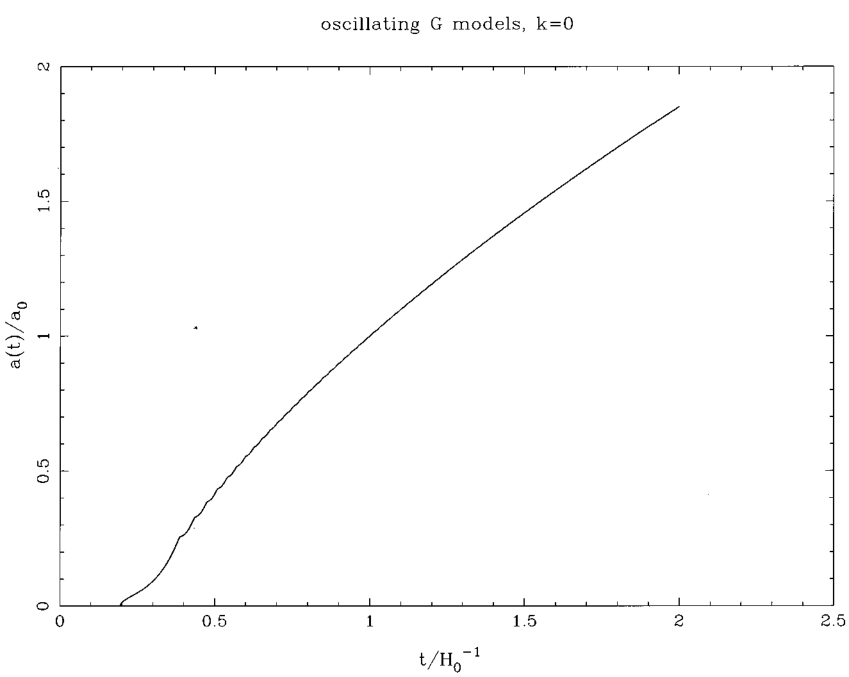

FIG. 10. Scale factor of the oscillating model of previous figures in units of its value today as a function of cosmic time (in units of H_0^{-1}). The present time t_0 has been placed at $t=1H_0^{-1}$.

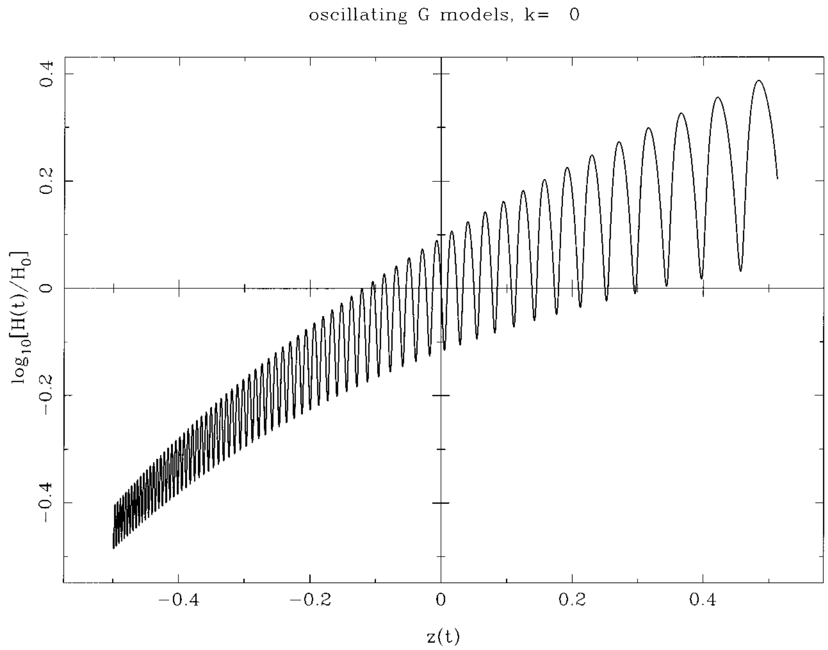

FIG. 11. The Hubble parameter as a function of redshifts. The initial conditions of Figs. 3 and 10 determine the initial value $H/H_0=1$ at $z(t_0)=0$.

cleosynthesis, for then our calculations show the field will remain at that value up to and beyond that era so will have $G_{\rm eff}\approx G_0$ and then the success of ''Big Bang nucleosynthesis'' will be recovered naturally. The amplitude $|\phi|$ will later be amplified by the curvature coupling precisely before the onset of the oscillatory behavior.

Thus it is possible that starting from an arbitrary value of ϕ near the Big Bang, a mechanism (no different from that required to solve the other problems of the standard Big Bang model, including inflation itself) would drive the scalar field to a value near zero, at which it will remain until just before $H\approx m$ when the amplification and then oscillations would occur. This type of scenario might be combined with the inflationary prediction $\Omega=1$ and a corresponding ''fine-tuning'' on $\Omega_{\rm bar}$. This has been done and the ''fine-tuning'' yielded a value $\Omega_{\rm bar}\sim 0.021$, surprisingly in the very narrow range $0.016\leqslant\Omega_{\rm bar}\leqslant 0.026$ that results in a successful nucleosynthesis of the light elements other than ^{4}He. But the point is that this would really be no ''fine-tuning'' at all (if looked in the right perspective), because all that it will mean is that, given the physical constant *precise values*, the inflationary mechanism will ensure that the energy densities of the various components, scalar field and ordinary matter, add up to $\Omega=1$, which will correspond then to a Universe at our time with *precise* values of the densities, expansion rate, etc. In particular the precise current value of the scalar field amplitude and phase arises from a particular precise value of the parameters at early times, among them the baryon content of the Universe. The fact that the corresponding baryonic density today turns out to lay in a very narrow range consistent

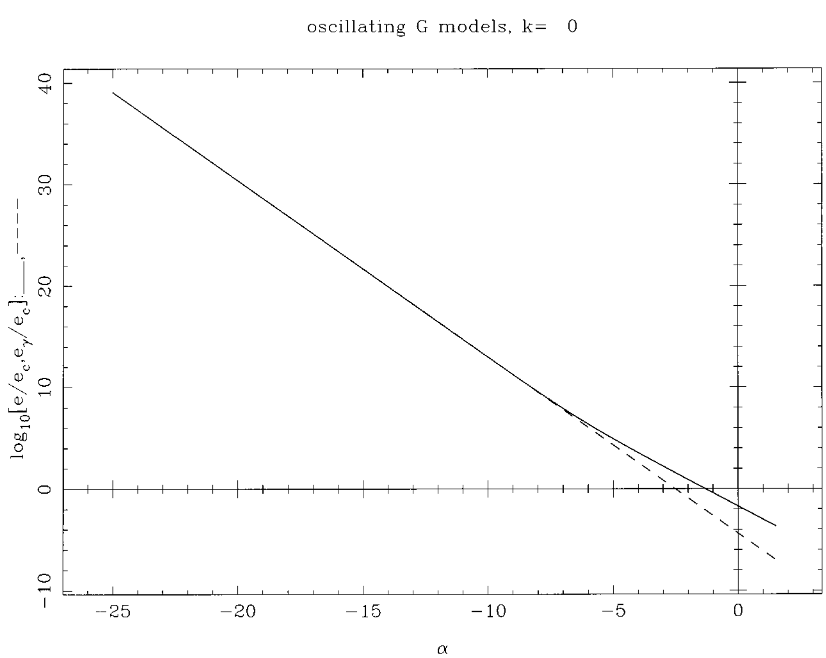

FIG. 12. The combined matter-radiation (solid line) and pure radiation (dashed line) energy densities with initial values as in Fig. 2.

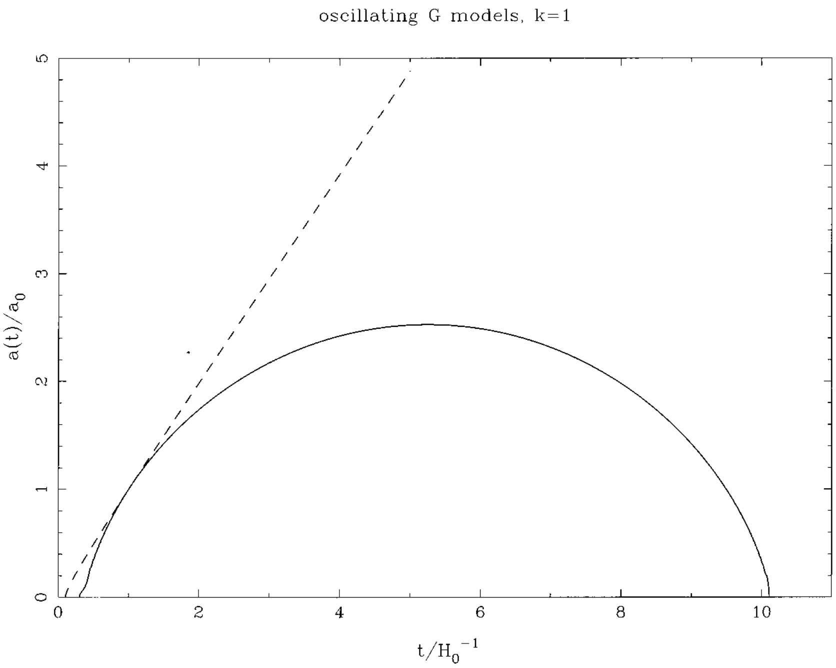

FIG. 13. Scale factor for the closed oscillating Universe with $\Omega \sim 1.67$ and $\Omega_{bar}=0.1$ (solid line). The dashed line corresponds to the case with no scalar field (standard cosmology). As in Fig. 10, the present time t_0 has been placed at $t=1H_0^{-1}$.

with the light element's nucleosynthesis suggests that the model should be taken seriously. In this respect we would like to point out that the *a priori* probability for this happening just by chance is about 1 in 100 (the range [0.016, 0.026] which is observationally allowed for Ω_{bar} represents approximately 1 part in 100 inside the numerical range allowed in principle [0,1]).

In view of the previous arguments we may conclude by saying that the ''fine-tuning'' should be seen as simply recovering the ''initial'' conditions corresponding to the observational data and that, moreover, this has resulted in the specific prediction $\Omega_{bar} \sim 0.021$. Therefore, this is certainly the most attractive of all the models considered in order to explain the observed periodicity in the galactic distribution, and should also be considered as a missing mass model with the scalar field playing the role of dark matter which is, however, indirectly observable in the oscillation of the galactic distribution.

APPENDIX: DIMENSIONLESS FORM OF FIELD EQUATIONS

1. Einstein's dynamical equation

It is easy to check that when restoring factors of c and introducing characteristic lengths, Eq. (20) becomes

$$\dot{\tilde{P}}_\alpha = \tilde{P}_\alpha^2 + \tfrac{1}{2}[\tilde{E}+\tilde{S}], \tag{A1}$$

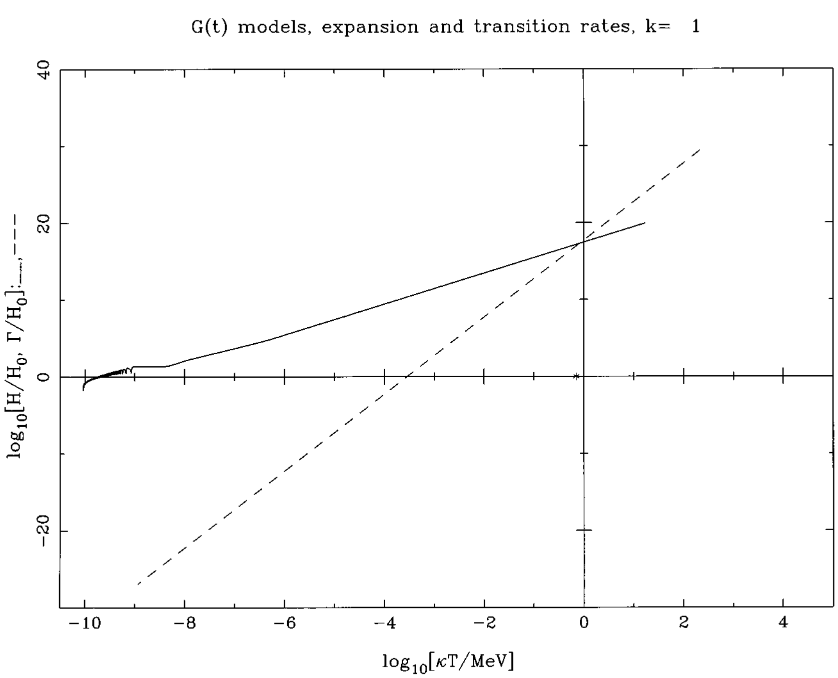

FIG. 14. Similar to Fig. 9 for the closed Universe with $\Omega_{bar}=0.1$ and $\Omega \sim 1.67$. Here the predicted freezeout temperature closely agrees with ~ 0.7 MeV.

with

$$\tilde{t}:=tH_0, \tag{A2}$$

$$\tilde{P}_\alpha:=-\dot{\tilde{\alpha}}, \tag{A3}$$

$$\tilde{E}:=\frac{E}{e_c}, \tag{A4}$$

$$\tilde{S}:=\frac{S}{e_c}, \tag{A5}$$

$$e_c:=\frac{3c^2H_0^2}{8\pi G_0}. \tag{A6}$$

Here a dot over a tilde means derivation with respect the dimensionless time $\tilde{t}$.

2. Scalar field's equation of motion

Using (A2)–(A6) and introducing

$$\tilde{P}_\phi:=\dot{\tilde{\phi}}, \tag{A7}$$

Eq. (21) can be written

$$\dot{\tilde{P}}_\phi=3\tilde{P}_\alpha\tilde{P}_\phi-\frac{3}{8\pi}\tilde{V}'(\phi)-6\xi\phi[\tilde{S}-\tilde{E}]. \tag{A8}$$

The scalar field ϕ now turns to be a dimensionless quantity and

$$\tilde{V}(\phi):=\frac{V(\phi)}{e_c}. \tag{A9}$$

Thus, in the case of a scalar potential $V(\phi)=\Lambda\phi^n$, Λ has units of energy density. In particular if $\Lambda:=m^2$ and $n=2$, the dimensionless harmonic frequency of oscillation is given by

$$\tilde{\omega}:=\frac{\omega}{H_0}=\sqrt{\frac{3}{4\pi}}\,\tilde{m}. \tag{A10}$$

Moreover, with such a choice of units, the coupling constant ξ is also dimensionless. Finally, the dimensionless source terms read explicitly

$$\tilde{S}=\frac{3\tilde{G}}{1+192\pi\tilde{G}\xi^2\phi^2}\Bigg[\tilde{p}+\frac{4\pi}{3}\tilde{P}_\phi^2-\tilde{V}(\phi)+\frac{32\pi}{3}\xi\left(\phi\tilde{P}_\phi\tilde{P}_\alpha+\tilde{P}_\phi^2-\frac{3}{8\pi}\phi\tilde{V}'(\phi)\right)\Bigg]+\frac{192\pi\tilde{G}\xi^2\phi^2\tilde{E}}{1+192\pi\tilde{G}\xi^2\phi^2}, \tag{A11}$$

$$\tilde{E}=\tilde{G}\left[\tilde{e}+\frac{4\pi}{3}\tilde{P}_\phi^2+\tilde{V}(\phi)+32\pi\xi\phi\tilde{P}_\phi\tilde{P}_\alpha\right], \tag{A12}$$

$$\tilde{G}=[1+16\pi\xi\phi^2]^{-1}. \tag{A13}$$

Equations (A1), (A3), (A7), (A8) with source terms (A11)–(A13) and the equation of conservation of energy (A20) are then the complete set of equations best suited to be solved numerically. With a trivial manipulation we write these equations in terms of derivatives with respect to α instead of $\tilde{t}$.

3. The Hamiltonian constraint and the deviation parameter

With these notations, the dimensionless form of the Hamiltonian constraint (10) reads

$$\tilde{P}_\alpha^2+\frac{k}{a^2H_0^2}=\tilde{E}. \tag{A14}$$

Now, at $t=t_0$ this becomes

$$1+\frac{k}{a_0^2H_0^2}=\tilde{E}_0. \tag{A15}$$

We can now replace k in Eq. (A14) in terms of hereabove initial conditions to get

$$\tilde{P}_\alpha^2+(\tilde{E}_0-1)e^{-2\alpha}-\tilde{E}=0, \tag{A16}$$

where we have employed our variable α instead of a.

We can introduce also the dimensionless deceleration parameter $\tilde{q}(t)$ in terms of sources

$$\tilde{q}(t)=\frac{1}{2\tilde{P}_\alpha^2}[\tilde{E}+\tilde{S}]. \tag{A17}$$

At $t=t_0$, we can also rewrite (A17) in the form

$$\tilde{E}_0=2\tilde{q}_0-\tilde{S}_0. \tag{A18}$$

Moreover, the deviation parameter introduced in (34) take the form

$$\lambda:=\frac{\tilde{P}_\alpha^2-(\Omega-1)e^{-2\alpha}}{\tilde{E}}, \tag{A19}$$

where we stress $\tilde{P}_\alpha\equiv-H(t)/H_0$ and $\Omega\equiv\tilde{E}_0$.

4. Conservation equations of matter and radiating fields

Equation (15) becomes

$$\dot{\tilde{e}}_i=3(\tilde{e}_i+\tilde{p}_i)\tilde{P}_\alpha. \tag{A20}$$

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