Mass sum rules for heavy-flavored hadrons

D. B. Lichtenberg and R. Roncaglia
Physics Department, Indiana University, Bloomington, Indiana 47405

E. Predazzi
Dipartimento di Fisica Teorica, Università di Torino
and Istituto Nazionale di Fisica Nucleare, Sezione di Torino, I-10125, Torino, Italy

(Received 31 October 1995)

Regularities in hadron interaction energies are used to obtain formulas relating the masses of ground-state hadrons, most of which contain heavy quarks. Inputs are the constituent quark model, the Feynman-Hellmann theorem, and the structure of the color magnetic interaction of QCD. Some of the formulas can also be obtained from heavy quark effective theory or from hadron supersymmetry. Where data exist, the formulas agree quite well with experiment, but most of the sum rules provide predictions of heavy baryon masses that will be useful for future measurements. [S0556-2821(96)03111-6]

PACS number(s): 12.10.Kt, 12.40.Yx

Thus far, nobody has succeeded in solving continuum QCD in the nonperturbative regime necessary to evaluate the masses of hadrons. Consequently, hadron masses have been calculated in lattice approximations to QCD and in various models such as potential and bag models. We here adopt an alternative approach of exploiting observed regularities in the properties of ground-state hadrons to predict the masses of hadrons yet to be discovered.

It has been found [1,2] that if reasonable values of constituent quark masses are subtracted from the masses of the ground-state vector mesons, then the interaction energy is a smooth, monotonically decreasing function of the reduced mass of the quarks. An analogous result has been found [2,3] for the ground-state baryons of spin 3/2. Both these results can be understood [2,3] from application [4] of the Feynman-Hellmann theorem [5,6]. Here we exploit the regularities in the hadron interaction energies to obtain sum rules which relate the masses of different ground-state hadrons. The advantage in using sum rules is that they contain differences of hadron masses such that the quark masses cancel. Previous authors [7–9] have obtained sum rules relating hadron masses, but most of the ones we give here are new. Some of our formulas also follow from heavy quark effective theory [10,11] or from approximate hadron supersymmetry [12], sometimes called superflavor symmetry [13]. The effectiveness of our sum rules is, we believe, a sufficient motivation to propose them. They will prove extremely useful when the systematic search for heavy hadrons will have reached maturity: it will then be very convenient to have a set of predictions not linked to any specific theoretical model but based on general properties.

Although the Feynman-Hellmann theorem provides motivation for the systematic study of the vector mesons and spin-3/2 baryons, experimental regularities also appear in the masses of ground-state pseudoscalar mesons and spin-1/2 baryons. For this reason we also propose sum rules for the masses of these hadrons and for the masses of hadrons averaged over their spin states. For most mesons and for those baryons containing three different quark flavors, we can obtain unique expressions for the spin-averaged masses in terms of the physical hadron masses [14].

The meson and baryon interaction energies $E(ij)$ and $E(ijk)$ are defined by

$$E(ij)=M_M(ij)-\sum_i m_i, \quad E(ijk)=M_B(ijk)-\sum_i m_i, \tag{1}$$

where i, j, and k denote the quarks (or antiquarks) in a meson of mass M_M or baryon of mass M_B and m_i denote their constituent masses. It has been shown [2,3] that, for reasonable constituent quark masses, the $E(ij)$ of observed vector mesons and the $E(ijk)$ of observed spin-3/2 baryons are smooth, monotonically decreasing functions of a generalized reduced mass μ:

$$dE(ij)/d\mu \leqslant 0, \quad dE(ijk)/d\mu \leqslant 0, \tag{2}$$

where $1/\mu=\Sigma_i 1/m_i$. The motivation for examining the dependence of the interaction energies as a function of μ comes from the Feynman-Hellmann theorem [5,6], which enables us to obtain the inequalities (2) in a Hamiltonian formalism with certain restrictions on the flavor dependence of the interaction [2,3].

We also use a result of Bertlmann and Martin [15] that in a nonrelativistic approximation the spin-averaged meson masses satisfy the inequalities ($m_i<m_j<m_k$)

$$2M(ij)-M(ii)-M(jj)\geqslant 0, \tag{3}$$

$$M(ii)+M(jk)-M(ij)-M(ik)\leqslant 0. \tag{4}$$

Furthermore, we use a generalization of (3) and (4) to baryons [16]:

$$2M(ijk)-M(ijj)-M(ikk)\geqslant 0, \tag{5}$$

$$M(iii)+M(ijk)-M(iij)-M(iik)\leqslant 0. \tag{6}$$

Where data are available, the inequalities (3)–(6) hold also for hadrons of definite spin configuration. Using (1), we see that (3)–(6) should hold for the corresponding meson and baryon interaction energies because the quark masses cancel. We use these ideas to motivate our proposed sum rules for hadron masses. In particular, we use quantitative estimates of the behavior of $E(ij)$, $E(ijk)$, and their derivatives taken from the known hadrons and extrapolate to unknown hadrons.

We begin with the mesons. From the Feynman-Hellmann theorem we deduce that inequality (3) is most likely to be an equality in the case in which the value of μ in the different terms varies least. This is the case in which the i quark is either u or d and the j quark is s. (We neglect the mass difference between u and d, and denote them by q.) Using (3) as an equality for vector mesons, we obtain

$$2K^* - \phi - \rho = 0, \tag{7}$$

where the symbol for a hadron denotes its mass. This sum rule agrees quite well with the data [17], but it does not involve mesons containing heavy quarks and may not be new. Because of the larger changes in μ for the heavy c or b quarks, sum rules analogous to Eq. (7) do not hold for mesons containing these quarks, but only the inequality (3). In obtaining (7), we assume that the ϕ is an $s\bar{s}$ state. We cannot obtain a corresponding sum rule for the pseudoscalars because neither the η nor the η' is a pure $s\bar{s}$ state. (They are mixtures of $q\bar{q}$ and $s\bar{s}$ and perhaps glueball as well.)

Sum rules from (4) for vector and pseudoscalar mesons containing one heavy quark are

$$D_s^* - D^* + B^* - B_s^* = 0, \quad D_s - D + B - B_s = 0. \tag{8}$$

According to heavy quark effective theory [10,11,13], the interaction energy in a hadron should not change appreciably when a c quark is replaced by a b quark. Because the quark masses cancel in (8), these sum rules follow from heavy quark theory. All the mesons appearing in (8) have been seen experimentally so these equations can be tested. The result of the comparison with the data [17,18] is shown in Table I (where all the comparisons between the data, where available, and the sum rules to be discussed in what follows, are also given).

We have many more possibilities to obtain sum rules for baryons than for mesons because for a fairly large number of baryons, the variation of the generalized μ is small. Examining the systematics, we find that (5) is approximately an equality if none of the quarks is heavy. We obtain the two sum rules for spin-3/2 baryons

$$2\Sigma^* - \Delta - \Xi^* = 0, \quad 2\Xi^* - \Sigma^* - \Omega = 0. \tag{9}$$

These two sum rules are well known, and together are just the Gell-Mann–Okubo baryon decuplet mass formula.

We next turn to sum rules involving at least two baryons containing a heavy quark. We obtain the sum rules from the systematics

$$\begin{gathered}\Omega + \Sigma_b^* - \Xi^* - \Xi_b^* = 0, \;\; \Sigma_c^* + \Omega_b^* - \Xi_c^* - \Xi_b^* = 0, \\ \Xi_c^* + \Xi_b^* - \Omega_c^* - \Sigma_b^* = 0, \;\; \Omega + \Xi_c^* - \Xi^* - \Omega_c^* = 0, \\ \Xi^* + \Sigma_b^* - \Sigma^* - \Xi_b^* = 0, \;\; \Xi^* + \Xi_c^* - \Sigma^* - \Omega_c^* = 0.\end{gathered} \tag{10}$$

TABLE I. Test of sum rules when all particles' masses are known from experiment. Column one gives the sum rule; column two lists the actual value in MeV obtained by using experimental mass values from the Particle Data Group [17] and from a recent conference report by Jarry [18].

Sum rule	Violation (MeV)
$2K^* - \phi - \rho = 0$	1 ± 3
$D_s^* - D^* + B^* - B_s^* = 0$	5 ± 5 or 12 ± 4 [a]
$D_s - D + B - B_s = 0$	4 ± 5 or 11 ± 2 [a]
$2\Sigma^* - \Delta - \Xi^* = 0$	5 ± 3
$2\Xi^* - \Sigma^* - \Omega = 0$	9 ± 3
$\Sigma_c^* - \Xi^* + \phi - D^* = 0$	8 ± 2
$\Sigma_c^* - \Sigma_b^* + B^* - D^* = 0$	4 ± 18 [a]
$\Sigma_b^* - \Sigma_c^* + D_s^* - B_s^* = 0$	8 ± 18 [a]
$\Sigma_b - N + K - B_s = 0$	6 ± 18 [a]
Eq. (14)	1 ± 3
Eq. (15)	2 ± 3
Eq. (16)	8 ± 18 [a]
Eq. (17)	3 ± 18 [a]

[a]Results which use any data from Ref. [18].

Because each of these sum rules contains the mass of a baryon not yet discovered, they cannot be tested at this time. On the other hand, these relations (like most of those that follow) provide approximate values for the masses of the unknown baryons and await experimental verification. Our predictions are summarized in Table II.

The sum rules for spin-1/2 baryons are different from those in Eq. (10) because the color magnetic energies, which depend on quark masses and spin configuration, are different. If all three quarks in a baryon have different flavors, then two distinct baryons exist with a given quark content. For any pair of spin-1/2 baryons containing three different flavors, the lighter baryon is the one in which the two lightest quarks have spin 0, and the heavier baryon is the one in which the two lightest quarks have spin 1, as with the Λ and Σ.

We have not been able to find any sum rules involving four baryons if the two lightest quarks have spin 0 (Λ-type symmetry). Sum rules for the case in which the two lightest quarks have spin 1 (Σ-type symmetry) are

$$\begin{gathered}\Sigma + \Omega_c - \Xi - \Xi_c' = 0, \;\; \Xi + \Sigma_c - \Sigma - \Xi_c' = 0, \\ \Xi + \Xi_b' - \Sigma - \Omega_b = 0, \;\; \Xi_c' + \Xi_b' - \Sigma_b - \Omega_c = 0, \\ \Omega_c + \Sigma_b - \Sigma_c - \Omega_b = 0.\end{gathered} \tag{11}$$

(When two spin-1/2 baryons exist with the same quark content and the same Greek symbol [17], a prime denotes the configuration in which the two lightest quarks have spin 1.) Only the last of the sum rules in (11) follows from heavy quark theory. Data do not yet exist to test these sum rules. Again, our predictions are listed in Table II.

We have obtained a large number of mass formulas involving two baryons and two mesons. We give here those in which no baryon contains more than one heavy quark. We begin with formulas for spin-3/2 baryons and spin-1 mesons

TABLE II. Predicted baryon masses in MeV. Here, M_A and M_S denote the two spin-1/2 baryons, M^* denotes spin-3/2 baryons, and M_B denotes spin-averaged baryons. The predicted masses are determined, where possible, using mass sum rules in which the values of all masses but one are known from experiment. If more than one formula exist for a given hadron, an average is taken. The results are exploited to obtain new mass values from sum rules containing more than one hadron with masses not yet measured. The errors in the predicted masses are estimated to be up to 20 MeV for a baryon containing one heavy quark, 30 MeV if it contains two c quarks, 40 MeV if it contains one c and one b quark, and 50 MeV if it contains two b quarks.

Quark content and symbol				M_B	M_A	M_S	M^*
qqq		N	Δ	1086		939[a]	1232[a]
qqs	Λ	Σ	Σ^*	1270	1116[a]	1193[a]	1385[a]
ssq		Ξ	Ξ^*			1318[a]	1533[a]
sss			Ω				1672[a]
qqc	Λ_c	Σ_c	Σ_c^*	2450	2285[a]	2453[a]	2530[a]
qsc	Ξ_c	Ξ_c'	Ξ_c^*	2588	2468[a]	2582[b]	2651[b]
ssc		Ω_c	Ω_c^*			2710[a]	2775
qqb	Λ_b	Σ_b	Σ_b^*	5783	5627[a]	5818[a]	5843[a]
qsb	Ξ_b	Ξ_b'	Ξ_b^*			5955	5984
ssb		Ω_b	Ω_b^*			6075	6098
ccq		Ξ_{cc}	Ξ_{cc}^*			3676	3746
ccs		Ω_{cc}	Ω_{cc}^*			3787	3851
qcb	Ξ_{cb}	Ξ_{cb}'	Ξ_{cb}^*	7062	7029	7053	7083
scb	Ω_{cb}	Ω_{cb}'	Ω_{cb}^*	7151	7126	7148	7165
bbq		Ξ_{bb}	Ξ_{bb}^*				10398
bbs		Ω_{bb}	Ω_{bb}^*				10483

[a]Input mass from experiment [17,18].

[b]After this work was completed, we learned that the Ξ_c' and Ξ_c^* have been observed [19]. Preliminary values of their masses are 2573 ± 4 and 2643 ± 4, respectively, in agreement with our predictions within our stated errors.

$$\Xi^*-\Xi_c^*+D^*-K^*=0,\quad \Omega-\Omega_b^*+B^*-K^*=0,$$
$$\Sigma_c^*-\Xi^*+\phi-D^*=0,\quad \Sigma_c^*-\Xi_c^*+K^*-\rho=0,$$
$$\Sigma_c^*-\Sigma_b^*+B^*-D^*=0,\quad \Omega_c^*-\Xi^*+\rho-D^*=0,$$
$$\Omega_c^*-\Xi_c^*+K^*-\phi=0,\quad \Omega_c^*-\Xi_b^*+B^*-D_s^*=0,$$
$$\Sigma_b^*-\Sigma_c^*+D_s^*-B_s^*=0,\quad \Xi_b^*-\Omega+\phi-B^*=0,$$
$$\Xi_b^*-\Omega_b^*+K^*-\rho=0,\quad \Omega_b^*-\Omega_c^*+D^*-B^*=0,$$
$$\Omega_b^*-\Sigma_b^*+\rho-\phi=0. \tag{12}$$

Several of these formulas can be justified from hadron supersymmetry or from heavy quark effective theory.

The formulas involving two spin-1/2 baryons (Σ-type symmetry) and two pseudoscalar mesons are

$$\Xi_c'-\Omega_b+B_s-D=0,\quad \Sigma_b-N+K-B_s=0,$$
$$\Xi_b'-\Sigma_c+D-B_s=0. \tag{13}$$

Of these, the first and the third descend from hadron supersymmetry. We were unable to find formulas involving two spin-1/2 baryons (Λ-type symmetry) containing only up to one heavy quark, and two pseudoscalar mesons.

Finally, the sum rules involving spin-averaged baryons and mesons are

$$(2\Sigma^*+\Sigma+\Lambda)/4-(N+\Delta)/2+(3\rho+\pi)/4 -(3K^*+K)/4=0, \tag{14}$$

$$(2\Sigma_c^*+\Sigma_c+\Lambda_c)/4-(2\Sigma^*+\Sigma+\Lambda)/4+(3K^*+K)/4 -(3D^*+D)/4=0, \tag{15}$$

$$(2\Sigma_c^*+\Sigma_c+\Lambda_c)/4-(2\Sigma_b^*+\Sigma_b+\Lambda_b)/4+(3B^*+B)/4 -(3D^*+D)/4=0, \tag{16}$$

$$(2\Sigma_b^*+\Sigma_b+\Lambda_b)/4-(2\Sigma_c^*+\Sigma_c+\Lambda_c)/4+(3D_s^*+D_s)/4 -(3B_s^*+B_s)/4=0. \tag{17}$$

The first three of these equations have been obtained previously [9]. The last two can also be derived from heavy quark theory. Furthermore, Eq. (17) follows from (16) with the help of (8).

Some of our formulas contain only masses of known hadrons. We test these formulas using data from the Particle Data Group [17] and more recent preliminary data from conference talks by Jarry [18] and Alam [19]. We give our results in Table I. As can be seen from this table, those of our sum rules which can be tested agree with experiment within about 10 MeV or less.

Although several theoretical papers have been written about baryons containing two heavy quarks [7,8,13,16,20], none has yet been observed. We have been able to find a rather large number of sum rules involving some baryons which contain two heavy quarks [21], but do not reproduce them here.

If all but one of the hadrons entering a formula have been observed, we use the formula to predict the mass of the unknown hadron. We put these predicted masses in other sum rules to obtain still further predictions, using the sum rules of Ref. [21] as well as those appearing here. The predicted masses arising from this procedure are given in Table II. Our estimated errors are 20 MeV or less for sum rules not involving any baryons containing two heavy quarks. The caption to Table II gives our estimated errors each case.

We are unable to use our mass formulas to get predictions for the baryons Ξ_{bb} and Ω_{bb}. However, we can estimate the difference $\Omega_{bb}-\Xi_{bb}$. From heavy quark symmetry, it should be $\Omega'_{cb}-\Xi'_{cb}=95$ MeV, while hadron supersymmetry suggests that $\Omega_{bb}-\Xi_{bb}=B_s-B=90$ MeV.

If any of our sum rules should turn out to be badly in error, would we learn anything? First, it is highly unlikely that such an event will happen because enough hadrons are already known to make us believe that the regularities in the interaction energies are much more than coincidences. Therefore, these regularities should persist in ground-state hadrons not yet discovered. Second, in the unlikely event that an exception is found, it will cast doubt on the flavor independence of the fundamental interaction between quarks.

In conclusion, relying on observed systematics of the interaction energies of known hadrons, we have obtained a large number of sum rules for the masses of known and unknown hadrons. In those cases in which the masses of all hadrons entering our sum rules are known from experiment, the sum rules agree with experiment within about 10 MeV or less. This fact gives us confidence that the predictions of unknown hadron masses which follow from the sum rules are likely to be correct within quite small errors compared to the masses themselves. We believe our predictions should be a useful guide to experimentalists searching for new hadrons, as our results depend on the regularities in observed hadrons persisting to hadrons not yet seen rather than on any specific model of quark interactions.

We thank Rick Van Kooten and Saj Alam for information about the latest experimental data. Part of this work was done while one of us (E.P.) visited Indiana University. This work was supported in part by the U.S. Department of Energy, in part by the U.S. National Science, Foundation, and in part by the Italian National Institute for Nuclear Physics and by MURST (Ministry of Universities, Research, Science and Technology) of Italy.

[1] W. Kwong and J. L. Rosner, Phys. Rev. D **44**, 212 (1991).
[2] R. Roncaglia, A. Dzierba, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **51**, 1248 (1995).
[3] R. Roncaglia, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **52**, 1722 (1995).
[4] C. Quigg and J. L. Rosner, Phys. Rep. **56**, 167 (1979).
[5] R. P. Feynman, Phys. Rev. **56**, 340 (1939).
[6] H. Hellmann, Acta Physicochim. URSS I, 6, 913 (1935); IV, 2, 225 (1936); Einführung in die Quantenchemie (Deuticke, Leipzig, 1937), p. 286.
[7] J. Franklin, Phys. Rev. D **12**, 2077 (1975), and references therein.
[8] T. Ito, T. Morii, and M. Tanimoto, Z. Phys. C **59**, 57 (1993).
[9] D. B. Lichtenberg and R. Roncaglia, Phys. Lett. B **358**, 106 (1995).
[10] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[11] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); B. Grinstein, Nucl. Phys. **B339**, 253 (1990); H. Georgi, Phys. Lett. B **240**, 447 (1990).
[12] H. Miyazawa, Prog. Theor. Phys. **36**, 1266 (1966); D. B. Lichtenberg, J. Phys. G **16**, 1599 (1990); **19**, 1257 (1993).
[13] H. Georgi and M. B. Wise, Phys. Lett. B **243**, 279 (1990); M. J. Savage and M. B. Wise, *ibid.* **248**, 177 (1990).
[14] M. Anselmino, D. B. Lichtenberg, and E. Predazzi, Z. Phys. C **48**, 605 (1990).
[15] R. A. Bertlmann and A. Martin, Nucl. Phys. **B168**, 111 (1980).
[16] E. Bagan, H. G. Dosch, P. Gosdzinsky, S. Narison, and J.-M. Richard, Z. Phys. C **64**, 57 (1994).
[17] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[18] P. Jarry, in *Physics in Collision 15*, edited by M. Rozanska and K. Rybicki (World Scientific, Singapore, 1996), p. 431.
[19] M. S. Alam, in *Baryons '95*, Proceedings of the Workshop, Santa Fe, New Mexico, 1995 (unpublished); and (private communication).
[20] J.-M. Richard, in *The Future of High Sensitivity Charm Experiments*, Proceedings of the Workshop, Batavia, Illinois, 1994, edited by D. M. Kaplan and S. Kwan (Fermilab, Batavia, 1994), p. 95.
[21] D. B. Lichtenberg, R. Roncaglia, and E. Predazzi, Report No. IUHET–321, 1995 (unpublished). Available on the Los Alamos phenomenology bulletin board as paper hep-ph/9511461.