Heavy quark fragmentation functions in the heavy quark effective theory

A. P. Martynenko and V. A. Saleev
Samara State University, Samara 443011, Russia
(Received 17 October 1995)

We calculate fragmentation functions for a b quark to fragment into color-singlet P-wave bound states $\overline{c}b$ in the heavy quark effective theory with the exact account of $O(1/m_b)$ corrections. We demonstrate an agreement of the obtained results with the corresponding calculations carried out in quantum chromodynamics.
[S0556-2821(96)02111-X]

PACS number(s): 12.39.Hg, 13.85.Ni, 14.40.Nd

The study of heavy quarkonia properties is a subject of much current interest for understanding quark-gluon interaction dynamics. B_c mesons, consisting of b and c quarks, hold a unique position in heavy quarkonia physics [1]. In the first place, B_c mesons consist of two heavy quarks, and so the predictions of the potential models refer to J/Ψ and Υ mesons as well as to B_c mesons [2]. In the second place, B_c mesons are constructed from quarks of different flavors and masses, which essentially determines their decay characteristics [3].

Production of mesons with heavy quarks in e^+e^-, $\gamma\gamma$, and $p\overline{p}$ interactions may be described by nonrelativistic perturbative quantum chromodynamics. At present, two mechanisms have been investigated for the production of B_c mesons: recombination and fragmentation. In the first case, B_c mesons are formed from heavy quarks, produced independently in hard subprocesses. The fragmentational mechanism demands pair production of b or c quarks in the hard subprocess with their subsequent fragmentation to B_c mesons ($\overline{b}\rightarrow B_c\overline{c}$, $c\rightarrow B_c b$). The relative contributions of these mechanisms in the production cross sections are different in various reactions. In e^+e^- annihilation only the quark fragmentation is essential [4]. In $p\overline{p}$ and $\gamma\gamma$ interactions the fragmentational mechanism prevails also for B_c-meson production with large transverse momenta [5,6]. In the range of small transverse momenta the recombination is dominant and determines the total cross section of B_c-meson production in $\gamma\gamma$ and $p\overline{p}$ interactions. But experimental conditions of B_c-meson discovery are more limited in the large transverse momenta domain. So the study of heavy quark fragmentation into B_c mesons has attracted considerable interest. An approach for the calculation of the fragmentation functions $D_{\overline{b}\rightarrow B_c\overline{c}}, D_{c\rightarrow B_c b}$ in nonrelativistic perturbative quantum chromodynamics was suggested in [7].

At the same time, in the last years there was suggested, based on QCD, the heavy quark effective theory (HQET) [8,9] for the description of heavy hadron properties. HQET makes it possible to obtain a finite analytical result with some accuracy even for the complicated processes of the quark-gluon interaction. In this approach the matrix elements of the different processes may be decomposed on degrees of two small parameters: the strong coupling constant $\alpha_s(m_Q)$ and $\Lambda_{\rm QCD}/m_Q$, where m_Q is the mass of the heavy quark. In the limit $m_Q\rightarrow\infty$ the effective Lagrangian, which describes the strong interactions of heavy quarks, has an exact spin-flavor symmetry [8]. HQET is successfully used for the investigation of exclusive and inclusive hadron decays [9]. Recently it was shown [10] that HQET may be used for the study of b-quark fragmentation into S-wave pseudoscalar and vector mesons and the corresponding nonpolarized fragmentation functions were calculated. The HQET calculation of b-quark fragmentation into transverse and longitudinal polarized S-wave B_c^* mesons was done in [11]. In this paper we have calculated the fragmentation functions of b-quark into P-wave color-singlet states $(\overline{c}b)$ at next to leading order in the heavy quark mass expansion using the methods of HQET.

Heavy b-quarks may fragment into bound states of two heavy quarks ($\overline{c}b$ states) with the orbital momentum $L=1$. There are four such states: ${}^1P_1, {}^3P_J$ $(J=0,1,2)$. Heavy quark fragmentation functions into P-wave B_c mesons were calculated by Chen [12] and Yuan [13] in QCD. Consider the calculation of the fragmentation functions in HQET. Let $q=m_b v+k$ be the four-momentum of virtual heavy quarks, $p_1=(1-r)Mv+\rho$ and $p_2=rMv-\rho$ the four-momenta of b and $\overline{c}$ quarks, correspondingly, and ρ the four-momentum of the relative motion. Let also $l=k-\rho$ be the four-momentum of the virtual gluon and k the residual momentum of the fragmenting heavy quark. The fragmentation functions for the process $b\rightarrow B_c+c$ are determined by the next expression [7]

$$D(z)=\frac{1}{16\pi^2}\int ds\,\theta\left(s-\frac{M^2}{z}-\frac{m_c^2}{1-z}\right)\lim_{q_0\rightarrow\infty}\frac{\Sigma|T|^2}{\Sigma|T_0|^2}, \quad (1)$$

where $M=m_b+m_c$ is the mass of B_c meson, T is the matrix element for production $B_c+\overline{c}$ from an off-shell b^* quark with virtuality $s=q^2$, and T_0 is the matrix element for producing an on-shell b quark with the same three-momentum $\vec{q}$. The calculation can be greatly simplified by using the axial gauge with gauge parameter $n_\mu=(1,0,0,-1)$ in the frame where $q_\mu=(q_0,0,0,\sqrt{q_0^2-s})$:

$$D_{\sigma\lambda}(k)=\frac{1}{k^2+i0}\left[g_{\sigma\lambda}-\frac{k_\sigma n_\lambda+k_\lambda n_\sigma}{k\cdot n}+\frac{n^2k_\sigma k_\lambda}{(k\cdot n)^2}\right]. \quad (2)$$

The part of amplitude T that involves production of the virtual b^* quark can be treated as an unknown Dirac spinor

 © 1996 The American Physical Society

Γ. The same spinor factor Γ appears in the matrix element $T_0=\overline{\Gamma v(q)}$, which leads to its cancellation in (1).

Let consider the fragmentation of a b quark into a color-singlet bound state $(\overline{c}b)$ $^1P_1$. The amplitude of such process involves the spinor factor $v(p_1)\overline{u(p_2)}$. To project the pair of quarks on the $^1P_1$ bound state we have used the next substitution [14]:

$$v(p_2)\overline{u(p_1)}\to\sqrt{M}\frac{\not p_2-m_c}{2m_c}\gamma_5\frac{\not p_1+m_b}{2m_b}. \tag{3}$$

The HQET Lagrangian, including the leading and the $1/m_b$ terms, is given by [8,9]

$$L=\overline{h_v}\left\{iv\cdot D+\frac{1}{2m_b}\left[C_1(iD)^2-C_2(v\cdot iD)^2-\frac{C_3}{2}g_s\sigma_{\mu\nu}G^{\mu\nu}\right]\right\}h_v, \tag{4}$$

where

$$C_1=1,\quad C_2=3\left(\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right)^{-8/(33-2n_f)}-2,$$

$$C_3=\left(\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right)^{-9/(33-2n_f)}. \tag{5}$$

All of these coefficients are equal to 1 at the heavy quark mass scale $\mu=m_b$. When we concern the fragmentation into P-wave mesons, it is necessary to decompose the projecting operator (3) and the gluon propagator (2) on the relative motion momentum ρ. Using the Feynman rules, derived from the HQET Lagrangian (4), we may write the full fragmentation amplitude into a $^1P_1$ state [14]:

$$iM(n^1P_1)=\frac{\sqrt{4\pi M}\alpha_s}{3m_cm_b}R_1'(0)\epsilon_\alpha(L_z)\frac{\partial}{\partial\rho_\alpha}\left\{\frac{1}{l^2}\left(-g_{\mu\nu}+\frac{n_\nu l_\mu}{n\cdot l}\right)\overline{u(p')}\gamma^\nu(m_c\not v-\not\rho-m_c)\gamma_5(m_b\not v+\not\rho+m_b)\times\left[v^\mu+\frac{C_1}{2m_b}(\rho+k)^\mu-\frac{C_2}{2m_b}v\cdot(\rho+k)v^\mu+i\frac{C_3}{2m_b}\sigma^{\mu\lambda}(\rho-k)_\lambda\right]\frac{1+\not v}{2}\Gamma\frac{i}{v\cdot k+\frac{C_1}{2m_b}k^2-\frac{C_2}{2m_b}(v\cdot k)^2}\right\}\Bigg|_{\rho=0}, \tag{6}$$

where $\epsilon_\alpha(L_z)$ is the polarization vector of the $^1P_1$ state. To calculate the amplitude (6) it is convenient to divide it into two parts on the degrees of the small parameter $1/m_b$. When $m_b\to\infty$ in the vertex function and in the propagator of heavy quarks, we obtain the main contribution to the fragmentation amplitude of b quarks in the form

$$iM_1(n^1P_1)=\frac{\sqrt{4\pi M}\alpha_s 2R_1'(0)}{3r^2(s-m_b^2)^3}\epsilon^*_\alpha(L_z)\overline{u(p')}W^\alpha\gamma_5\Gamma,$$

$$W_\alpha=(s-m_b^2)\left[(\not v+1)\gamma_\alpha-\frac{v\cdot k}{n\cdot k}\not n(\not v+1)\gamma_\alpha\right]+4mk_\alpha\left[1+\frac{k\cdot v}{k\cdot n}\not n\right](\not v-1)-2Mr(s-m_b^2)v_\alpha\frac{1}{n\cdot k}\times\not n(\not v-1)+2Mr(s-m_b^2)n_\alpha\frac{v\cdot k}{(n\cdot k)^2}\not n(\not v-1). \tag{7}$$

All calculations of the fragmentation functions, which are rather complicated, were done by means of the system REDUCE. Substituting (7) into (1), we obtain

$$D_1(n^1P_1)(y)=N_1\frac{(1-y)^2}{ry^8}(9y^4-4y^3+40y^2+96), \tag{8}$$

$$N_1=\frac{\alpha_s^2|R_{nP}'(0)|^2}{54\pi r^5M^5},$$

where $y=(1-z+rz)/rz$ is the so-called Yaffe-Randall variable [15], and $r=m_c/M$.

The amplitude of bound $\overline{c}b$ state n^3P_J production may be derived from (6), changing $\gamma_5\to\not\epsilon(S_z)$, where $\epsilon^\mu(S_z)$ is the spin wave function:

$$iM(n^3P_J)=\frac{\sqrt{4\pi M}\alpha_s}{3m_cm_b}R_1'(0)\sum_{S_z,L_z}\langle 1,L_z;1,S_z|J,J_z\rangle \times\epsilon^*_\beta(S_z)\epsilon^*_\alpha(L_z),$$

$$\frac{\partial}{\partial\rho_\alpha}\Bigg\{\frac{1}{l^2}\left(-g_{\mu\nu}+\frac{n_\nu l_\mu}{n\cdot l}\right)\overline{u(p')}\gamma^\nu(m_c\not v-\not\rho-m_c)\times\gamma_\beta(m_b\not v+\not\rho+m_b)\Bigg[v^\mu+\frac{C_1}{2m_b}(\rho+k)^\mu-\frac{C_2}{2m_b}v\cdot(\rho+k)v^\mu+i\frac{C_3}{2m_b}\sigma^{\mu\lambda}(\rho-k)_\lambda\Bigg]\times\frac{1+\not v}{2}\Gamma\frac{i}{v\cdot k+\frac{C_1}{2m_b}k^2-\frac{C_2}{2m_b}(v\cdot k)^2}\Bigg\}\Bigg|_{\rho=0}, \tag{9}$$

where we have expressed the Clebsch-Gordon coefficients and $\epsilon^*_\beta(S_z),\epsilon^*_\alpha(L_z)$ by the bound state polarizations [14]

$$\sum_{S_z,L_z}\langle 1,L_z;1,S_z|J,J_z\rangle\epsilon^*_\beta(S_z)\epsilon^*_\alpha(L_z)=\begin{cases}\frac{1}{\sqrt{3}}(g_{\alpha\beta}-v_\alpha v_\beta), & J=0,\\ \frac{i}{\sqrt{2}}\epsilon_{\alpha\beta\lambda\rho}v_\lambda\epsilon^*_\rho(J_z), & J=1,\\ \epsilon_{\alpha\beta}(J_z), & J=2.\end{cases} \tag{10}$$

Taking in (9) only the terms of leading order on $1/m_b$ and doing necessary differentiation on ρ_α, we have obtained

$$iM_1(n^3P_J)=\frac{\sqrt{4\pi M}\alpha_s 2R_1'(0)}{3r^2(s-m_b^2)^3}\times\sum_{S_z,L_z}\langle 1,L_z;1,S_z|J,J_z\rangle\epsilon^*_\beta(S_z)\epsilon^*_\alpha(L_z),$$

$$\overline{u(p')}\Bigg\{(s-m_b^2)\left(1-\frac{k\cdot v}{k\cdot n}\not n\right)\gamma_\alpha\gamma_\beta-4k_\alpha M\left(1+\frac{k\cdot v}{k\cdot n}\not n\right)\gamma_\beta-4rM^2\left(\frac{k\cdot v}{k\cdot n}\right)^2 n_\alpha\not n\gamma_\beta\Bigg\}(1+\not v)\Gamma. \tag{11}$$

This amplitude determines the basic contribution to fragmentation functions of b quarks into the ^3P_J- state:

$$D_1(n^3P_0)(y)=N_1\frac{(y-1)^2}{ry^8}(y^4-4y^3+8y^2+32), \tag{12}$$

$$D_1(n^3P_1)(y)=N_1\frac{2(y-1)^2}{ry^8}(3y^4-4y^3+16y^2+48), \tag{13}$$

$$D_1(n^3P_2)(y)=N_1\frac{20(y-1)^2}{ry^8}(y^4+4y^2+8). \tag{14}$$

TABLE I. The fragmentation probabilities of b-quark into P-wave color-singlet states $(\overline{c}b)$.

$P_{b\to\overline{c}b}(2^1P_1)$	$P_{b\to\overline{c}b}(2^3P_0)$	$P_{b\to\overline{c}b}(2^3P_1)$	$P_{b\to\overline{c}b}(2^3P_2)$
6.4×10^{-5}	2.5×10^{-5}	7.3×10^{-5}	10.5×10^{-5}

Our results (8), (12)–(14) coincide with the leading $1/r$ terms of the fragmentation functions, calculated in [12,13]. The calculation of the next-to-leading order contributions $O(1/m_b)$ to the P-wave fragmentation functions was performed on the basis of the amplitudes (6) and (9). Omitting the details of our calculations we can represent the obtained fragmentation functions with $C_1=C_3=1$ in the following manner:

$$D(n^1P_1)(y)=\frac{\alpha_s^2|R'_{nP}(0)|^2}{54\pi r^5M^5}\frac{(y-1)^2}{ry^8}\times[(9y^4-4y^3+40y^2+96)-r(3y^5-31y^4+32y^3+8y^2-192y+96)]. \tag{15}$$

$$D(^3P_0)(y)=\frac{\alpha_s^2|R'_{nP}(0)|^2}{54\pi r^5M^5}\frac{(y-1)^2}{ry^8}\left[(y^4-4y^3+8y^2+32)+\frac{r}{3}(3y^5-11y^4+392y^2+192y-96)\right], \tag{16}$$

$$D(^3P_1)(y)=\frac{\alpha_s^2|R'_{nP}(0)|^2}{27\pi r^5M^5}\frac{(y-1)^2}{ry^8}[(3y^4-4y^3+16y^2+48)+r(3y^5+5y^4+8y^3+32y^2+96y-48)], \tag{17}$$

$$D(^3P_2)(y)=\frac{10\alpha_s^2|R'_{nP}(0)|^2}{27\pi r^5M^5}\frac{(y-1)^2}{ry^8}\times\left[(y^4+4y^2+8)+\frac{r}{15}(-3y^5-y^4+36y^3-164y^2+240y-120)\right]. \tag{18}$$

As it follows immediately from (15)–(18) that our results coincide with the QCD calculations of Yuan [13] and Chen [12] with the accuracy $O(r)$, if we take into account that the nonperturbative factor of (15)–(18) in [13] contains the reduced mass μ contrary to our factor rM. Using the obtained fragmentation functions (15)–(18), we may calculate the fragmentation probabilities of the corresponding $(\overline{c}b)$ mesons [7]:

$$P_{b\to\overline{c}b}(nP)=\int_0^1 dz D_{b\to\overline{c}b}(nP)(z,r). \tag{19}$$

Omitting the analytical expressions for the P-wave fragmentation probabilities (19) and putting $|R'_{2P}(0)|^2=0.201$ GeV^5, $m_c=1.5$ GeV, $m_b=4.9$ GeV, and $\alpha_s(2m_c)=0.38$ ($2m_c$ is a minimal energy of exchanged gluon), we have obtained the numerical value of the fragmentation probabilities, which are presented in Table I. We see that our integral probabilities of P-wave $\overline{cb}$ meson production, founded by means of the b-quark fragmentation functions in HQET, are in good agreement with the results of QCD calculations of [13].

The distribution functions $f_{Q/H}$ of the heavy quark inside the heavy P-wave mesons, obtained by means of the fragmentation functions $D_{\underline{Q}\to H}$, Eqs. (15)–(18), have a pole of order 8 located at $x=\overline{r}$ [13]:

$$f_{Q/H}=xD_{Q\to H}\left(\frac{1}{x}\right)=\frac{a_8(r)}{(x-\overline{r})^8}+\text{less singular terms.} \tag{20}$$

Therefore, we can conclude from (15)–(18), (20) that the coefficients $a_8(r)$ and $a_7(r)$ of the Laurent series of $f_{Q/H}$ obey the Braaten-Levin spin-counting rules: 3:1:3:5.

So the performed calculations show that the heavy quark effective theory may be successfully used for the study of the heavy quark fragmentation. In this approach we may systematically take into account the $O(1/m_b)$ corrections in the amplitudes and the probabilities of the fragmentation, which increases the accuracy of HQET calculations. Moreover, it seems more important that HQET lead to the finite analytical answer, when we study complicated problems in heavy quark physics. The approach, based on HQET, may be used for the calculation of heavy quark fragmentation functions into L-wave mesons ($L=2,3,\ldots$), and for the investigation of B_c-meson hadroproduction.

We are grateful to R. N. Faustov, V. V. Kiselev, and A. K. Likhoded for useful discussions of B_c-meson physics and the heavy quark effective theory, and to E. Braaten, K. Cheung, and S. Fleming for the valuable information about obtained results. This work was done under the financial support of the Russian Fund of Fundamental Researches (Grant No. 93-02-3545) and by State Committee on High Education of Russian Federation (Grant No. 94-6.7-2015).

[1] S. S. Gershtein *et al.*, Usp. Fiz. Nauk **165**, 3 (1995).
[2] S. S. Gershtein *et al.*, Yad. Fiz. **48**, 515 (1988) [Sov. J. Nucl. Phys. **48**, 327 (1988)].
[3] V. V. Kiselev and A. V. Tkabladze, Yad. Fiz. **48**, 536 (1988) [Sov. J. Nucl. Phys. **48**, 341 (1988)].
[4] V. V. Kiselev, A. K. Likhoded, and M. V. Shevlyagin, Yad. Fiz. **57**, 733 (1994) [Sov. J. Nucl. Phys. **57**, 689 (1994)]; C.-H. Chang and Y.-Q. Chen, Phys. Lett. B **284**, 127 (1992).
[5] A. V. Berezhnoy, A. K. Likhoded, and M. V. Shevlyagin, Yad. Fiz. **58**, 730 (1995); K. Cheung, Phys. Rev. Lett. **71**, 3413 (1993).
[6] K. Kolodziej, A. Leike, and R. Ruckl, Phys. Lett. B **348**, 219 (1995).
[7] E. Braaten, K. Cheung, and T. C. Yuan, Phys. Rev. D **48**, R5049 (1993); K. Cheung and T. C. Yuan, Phys. Lett. B **325**, 481 (1994); E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993).
[8] M. Neubert, Phys. Rep. **245**, 261 (1994).
[9] B. Grinstein, Report No. UCSD/PTH 94-24 (unpublished).
[10] E. Braaten *et al.*, Phys. Rev. D **51**, 4819 (1995).
[11] A. P. Martynenko and V. A. Saleev, Izv. Vuz. Fiz. (to be published).
[12] Y.-Q. Chen, Phys. Rev. D **48**, 5158 (1993); **50**, 6013(E) (1994).
[13] T. C. Yuan, Phys. Rev. D **50**, 5664 (1994).
[14] J. H. Kuhn, J. Kaplan, and El G. O. Safiani, Nucl. Phys. **B157**, 125 (1979).
[15] R. Jaffe and L. Randall, Nucl. Phys. **B412**, 79 (1994).