Model-independent analysis of B-$\bar{B}$ mixing and CP violation in B decays

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We present a framework to analyze the effects of new physics beyond the standard model on B-$\bar{B}$ mixing and CP violation in B decays in a model-independent manner. Assuming that the tree level decay amplitudes are dominated by the standard model ones, new physics contributions to B-$\bar{B}$ mixing can be extracted from several measurements at B factories. Using this framework, we show the present constraint on new physics contributions to B-$\bar{B}$ mixing and illustrate the constraints expected to be given by future experiments at B factories. We also point out the possibility that CP asymmetries in $B \to \psi K_S$, $B \to \pi\pi$, and $B \to DK$ modes look consistent with the standard model, even if a large new physics contribution is present in B-$\bar{B}$ mixing. [S0556-2821(96)00211-1]

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The physics of B mesons provides several tests of the standard model and could give insight into new physics beyond it. In the standard model, the test of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] is especially important. As shown in Fig. 1, the unitarity of the CKM matrix is graphically expressed by a triangle. In the standard model, the lengths of the sides are related to several decay rates and/or the magnitude of the B-$\bar{B}$ mixing, while the angles are related to several CP asymmetries. When these quantities are measured at future B factories, we will be able to test the unitarity of the CKM matrix by seeing whether or not the triangle is closed.

Quantitatively, this analysis will be done along the χ^2 test: All the observables such as the decay rates and CP asymmetries are represented by standard model parameters such as the CKM parameters and some hadronic parameters such as the B meson decay constant f_B. The χ^2 is calculated, with experimental results on the rates and asymmetries and knowledge of the hadronic parameters, as a function of the standard model parameters. Then, this χ^2 is minimized by varying the standard model parameters, and we can see whether or not the standard model is consistent depending on the obtained $\chi^2_{\rm min}$.

Although it is quite straightforward and powerful, this method has some defects. Even if a larger value of $\chi^2_{\rm min}$ is found, this method itself does not tell us anything other than that the standard model is doubtful. No quantitative information about new physics can be obtained. Moreover, even if the standard model seems to be consistent, there may be new physics that evades the above χ^2 test. From these points of view, it is desirable to introduce some new physics effects into the analysis and to see how they are restricted by experiments.

For this purpose, two approaches are possible. One is a model-dependent approach, in which we specify a model of new physics and analyze all data in it. Although each model needs a specific analysis, this approach has a predictive power if the model does not have too many parameters. Another approach is a model-independent approach, in which we do not introduce any model. The data are analyzed based on rather general principles or assumptions. If we can parametrize the effects of new physics by a few model-independent quantities, this approach is quite suitable to select appropriate new physics among many possibilities based on experimental information. We pursue the latter approach here.

Weak interactions of B mesons are described by $|\Delta B|=1$ and $|\Delta B|=2$ amplitudes, i.e., B decay and B-$\bar{B}$ mixing amplitudes, respectively. In the standard model, the $|\Delta B|=1$ processes occur through the tree and penguin diagrams at the quark level. The effects of new physics tend to appear in a penguin diagram because it is a loop diagram. In fact, in some classes of new physics such as supersymmetry, tree-level B decay amplitudes are hardly affected by new physics. On the other hand, it is difficult to exclude the effects of new physics in the penguin diagrams.

In the model-independent approach, we separate B decay processes into two classes. Although we are considering general cases, it is useful to introduce class I and class II based on the properties of the quark-level amplitudes in the standard model. The class-I processes do not have the penguin part in its quark-level amplitude and the class-II processes do. The processes of $b \to q_d q_u \overline{q'_u}$ type, where q_d represents a down-type quark and q_u and q'_u denote different up-type quarks, are class-I processes. The ordinary semileptonic decays are also regarded as class-I processes because their amplitudes have no penguin part. Figure 2(a) shows the above hadronic class-I process. The processes of $b \to q_d q \bar{q}$ type, where q denotes a generic type of quark, are class-II pro-

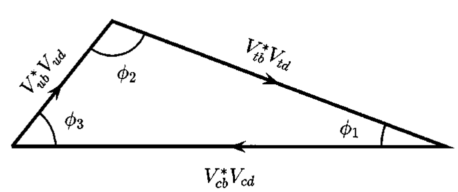

FIG. 1. Unitarity triangle.

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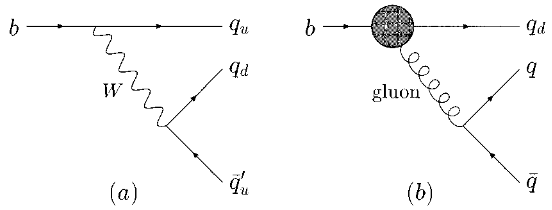

FIG. 2. Diagrams of hadronic b decays: (a) the class-I process and (b) the (gluonic) penguin contribution in the class-II process, where the blob may contain exotic quanta in addition to the standard ones.

cesses. Figure 2(b) shows the (gluonic) penguin contribution to the class-II process. If q is an up-type quark, $b\to q_d q\bar{q}$ consists of the penguin part and the tree part. If q is a down-type quark, $b\to q_d q\bar{q}$ contains only the penguin part.[1]

In the following, we assume that the class-I processes are described by the standard W-exchange diagrams as shown in Fig. 2(a). On the other hand, we do not exclude the possibility that the class-II processes are affected by new physics that may be present in the blob shown in Fig. 2(b). In addition, we assume that the unitarity of the CKM matrix is saturated by the first three generations. This assumption is necessary to determine the quark-mixing parameters from class-I processes and to apply them to evaluate the standard model contribution to the B-$\bar{B}$ mixing, as described below. All possible extensions of the standard model obviously satisfy this assumption unless they contain one or more extra quarks whose mixing with the ordinary quarks is not negligible. Thus, taking the known hierarchical structure of the CKM matrix into account, the three-generation unitarity seems to be a plausible assumption.

New physics can also contribute to the B-$\bar{B}$ mixing amplitude M_{12}, which can be written as

$$M_{12}=|M_{12}|e^{i\phi_M}=M_{12}^{\mathrm{SM}}+M_{12}^{\mathrm{new}}, \tag{1}$$

where M_{12}^{SM} is the standard model contribution and M_{12}^{new} represents contributions from new physics.[2] M_{12}^{new} is the model-independent parameter that describes effects of new physics in the B-$\bar{B}$ mixing. Note that Eq. (1) is enough to determine a phase convention for M_{12}^{new} once the phase convention in evaluating M_{12}^{SM} is fixed, because the relative phase between M_{12}^{SM} and M_{12}^{new} is a physically meaningful quantity. We use the following expression of M_{12}^{SM}:

$$M_{12}^{\mathrm{SM}}=\frac{G_F^2}{12\pi^2}m_W^2 m_B f_B^2 B_B \eta_B (V_{td}^* V_{tb})^2 S(m_t^2/m_W^2), \tag{2}$$

where

$$(V_{td}^* V_{tb})^2=\lambda^6 A^2(1-\rho+i\eta)^2 \tag{3}$$

in the Wolfenstein parametrization [3] and

$$S(x)=x\left[\frac{1}{4}+\frac{9}{4(1-x)}-\frac{3}{2(1-x)^2}\right]-\frac{3}{2}\frac{x^3}{(1-x)^3}\ln x \tag{4}$$

is the Inami-Lim function [4].

In the following, we see how M_{12}^{new} is constrained by the present experiments and how the constraint will be improved by future experiments. For this purpose, we have to determine the Wolfenstein parameters A, ρ, and η simultaneously with M_{12}^{new}. The ordinary analysis in which the standard model is assumed cannot be applied if we consider the effects of new physics. However, according to the above two assumptions, i.e., the dominance of the standard model contributions in the class-I processes and the three-generation unitarity, we can extract information about the Wolfenstein parameters without being bothered by the unknown new physics in the class-II processes, which is not parametrized in our analysis.

First, let us consider the semileptonic decay of the B meson $\bar{B}\to X_c \ell \bar{\nu}$, which is one of the class-I processes and free from new physics. We can determine the CKM matrix element $|V_{cb}|=\lambda^2 A$ from its width. Second, the charmless semileptonic decay of the B meson $\bar{B}\to X_u \ell \bar{\nu}$ is also considered to be free from new physics and we can obtain a constraint on $|V_{ub}/V_{cb}|=\lambda\sqrt{\rho^2+\eta^2}$ from its rate. These exhaust the presently available constraints that are not affected by new physics in our framework.

The remaining constraint relevant to our analysis is the one given by the observation of B-$\bar{B}$ mixing. From the experiments, we obtain a constraint on the absolute value of the B-$\bar{B}$ mixing amplitude $|M_{12}|=|M_{12}^{\mathrm{SM}}+M_{12}^{\mathrm{new}}|$. Since the possible range of M_{12}^{SM} is limited by the above-mentioned constraints on the Wolfenstein parameters and the knowledge of the top quark mass and the hadronic matrix element, we can

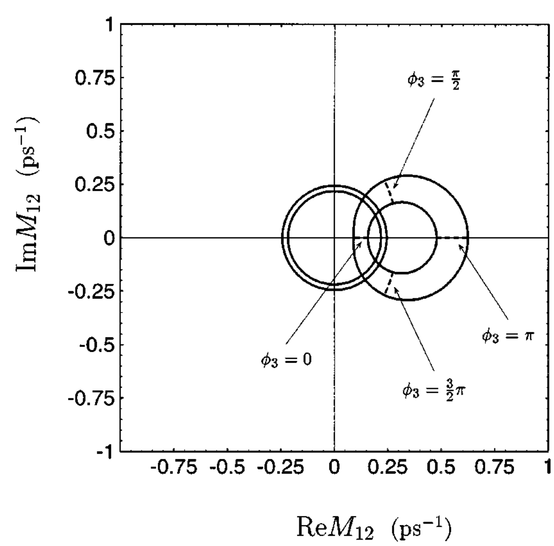

FIG. 3. Experimentally allowed region and standard model prediction for M_{12}. The circles whose centers are at the origin show the direct experimental constraint and the distorted circles show the standard model prediction. The dashed lines show the standard model predictions for several fixed ϕ_3 values. (The angle ϕ_3 is defined in Fig. 1.)

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[1]The processes $b\to dd\bar{s}$ and $b\to ss\bar{d}$ do not appear in the standard model within the lowest order of the weak interaction. Even if these processes are caused by new physics, they do not affect the following discussions.

[2]A similar expression has been introduced in Ref. [2] in a different context.

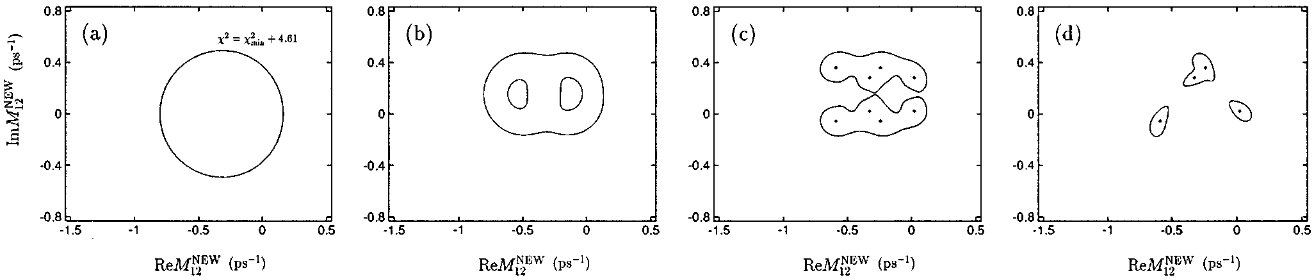

FIG. 4. Allowed regions of $M_{12}^{\rm new}$. The lines that $\chi^2=\chi^2_{\rm min}+4.61$ are shown. (a) The presently allowed region corresponding to the inputs in Table I. (b) The expected constraint from $B\to\psi K_S$ in addition to the inputs in Table I. (c) The expected constraint from $B\to\psi K_S$ and $B^\pm\to DX_s^\pm$ in addition to the inputs in Table I. The dots represent the solutions obtained from the central values of the inputs. (d) The expected constraint from all the inputs listed in Tables I and III. The dots have the same meaning as in (c).

obtain an allowed region of $M_{12}^{\rm new}$. We do not include the information from the K-$\bar{K}$ mixing into our analysis because we need to introduce another model-independent parameter $M_{12}^{\rm new}$ for the K-$\bar{K}$ mixing.

Here we present the result of an analysis that follows the above strategy. We summarize the inputs in Table I. λ is determined by the semileptonic kaon and hyperon decays [5], which are free from new physics in our framework. A and $\sqrt{\rho^2+\eta^2}$ are constrained by the semileptonic B decays as mentioned above [6]. $|M_{12}|$ is obtained from the measurement of the B^0-$\bar{B}^0$ mass difference Δm [7]. The top quark mass m_t is given by the direct observations [8]. $f_B\sqrt{B_B\eta_B}$ is estimated by several theoretical methods. We take the value used in Ref. [9]. We omitted experimental errors and/or theoretical uncertainties of λ, A, m_t, and $f_B\sqrt{B_B\eta_B}$ for simplicity.

Figure 3 shows the experimentally allowed region and the standard model prediction for M_{12} in the complex M_{12} plane. The circles whose centers are at the origin show the direct experimental constraint on $|M_{12}|$ in Table I and the distorted circles show the standard model ''prediction,'' which is calculated by Eq. (2) with the inputs in Table I. A vector from a point in the region of the standard model prediction to a point in the experimentally allowed region corresponds to a possible complex value of $M_{12}^{\rm new}$. From this figure we can see that the value of $M_{12}^{\rm new}$ is certainly limited. By integrating out ρ and η in the maximum likelihood method, we can obtain an allowed region of $M_{12}^{\rm new}$. The result is shown in Fig. 4(a), where we show the line $\chi^2=\chi^2_{\rm min}+4.61$, which is usually regarded as being of the 90% confidence level. This figure tells us that the contribution of $M_{12}^{\rm new}$ to the total B-$\bar{B}$ mixing amplitude M_{12} can be comparable with or larger than that of $M_{12}^{\rm SM}$; besides that, the standard model is completely consistent. Also note that a positive value of Re$M_{12}^{\rm new}$ tends to be disfavored.

TABLE I. Inputs corresponding to the present experimental and theoretical knowledge.

λ	0.220 [5]
$\|V_{cb}\|=\lambda^2 A$	0.038 [6]
m_t	174 GeV [8]
$f_B\sqrt{B_B\eta_B}$	165 MeV [9]
$\|V_{ub}/V_{cb}\|=\lambda\sqrt{\rho^2+\eta^2}$	0.08 ± 0.02 [6]
$\Delta m=2\|M_{12}\|$	0.462 ± 0.026 ps^{-1} [7]

Now, let us discuss effects of new physics on CP asymmetries in B decays, which are planned to be measured in future experiments at B factories. We start with the class-I process $B\to DX_s$, for which we can use the standard model decay amplitudes. For the charged B meson mode $B^\pm\to DX_s^\pm$, since this decay mode is free from new physics provided that no significant CP violation in D^0-$\bar{D}^0$ mixing is caused by new physics, the method by Gronau and Wyler [10] can be applied to extract information of the angle ϕ_3 defined in Fig. 1. We can uniquely determine $\sin^2\phi_3$ by using two strange states that make phase shifts that are different from each other, e.g., K and K^*, for X_s. For the neutral B meson mode $B^0(\bar{B}^0)\to DK_S$ [11], the situation is a little different owing to the new physics contribution to the B-$\bar{B}$ mixing, $M_{12}^{\rm new}$. From the time-dependent decay rate, however, we can uniquely determine $\sin\phi_M$, $\sin(\phi_M+2\phi_3)$, and $\sin^2\phi_3$.[3]

For the $B\to\pi\pi$ process, the decay amplitude can be affected by new physics because it is a class-II process. However, we can extract the $\Delta I=3/2$ part of the amplitude by the isospin analysis [12]. Since the $\Delta I=3/2$ part of the amplitude does not contain the penguin contribution, it can be regarded as a class-I process. Although this analysis depends on the assumption that the penguin part of the amplitude is $\Delta I=1/2$, it seems to be plausible even in the presence of new physics. In other words, as in the standard model [13], we do not expect that the electromagnetic penguin, the electroweak penguin, and the box diagram play important roles in this process even in the presence of new physics. In such a case, we can extract $\sin(\phi_M+2\phi_3)$ with fourfold ambiguity. If the penguin contribution turns out to be small, this fourfold ambiguity disappears. We can also resolve it by comparing it with the $\sin(\phi_M+2\phi_3)$ obtained from $B^0(\bar{B}^0)\to DK_S$.

The last process discussed here is $B^0(\bar{B}^0)\to\psi K_S$, which is classified as a class-II process. In the standard model, the

[3]In the standard model, which is considered in Ref. [11], ϕ_M, defined in Eq. (1), corresponds to $2\phi_1$ in our phase convention.

TABLE II. Quantities obtainable from several processes of CP violation search.

$B^{\pm} \rightarrow DX_s^{\pm}$	$\sin^2\phi_3$
$B^0(\overline{B^0}) \rightarrow DK_S$	$\sin\phi_M$, $\sin(\phi_M+2\phi_3)$, $\sin^2\phi_3$
$B \rightarrow \pi\pi$	$\sin(\phi_M+2\phi_3)$ (fourfold in general)
$B^0(\overline{B^0}) \rightarrow \psi K_s$	$\sin\phi_M$ (assuming no penguin that has a nonstandard phase)

TABLE III. Inputs used for the illustration of the expected constraints from CP violation experiments at B factories.

$\sin\phi_M$	0.66 ± 0.08
$\sin^2\phi_3$	0.62 ± 0.25
$\sin(\phi_M+2\phi_3)$	0.57 ± 0.17

penguin contribution in this process does not cause any problem, because the weak phases of the tree and the penguin contributions are the same. However, they could differ from each other in the presence of new physics, so that we cannot extract information about ϕ_M from this process in general. Nevertheless, it is desirable to include this mode in our analysis because it is expected to be precisely measured. In the following, we assume that there is no significant penguin contribution that has a CP-violating phase different from that of the standard model in this process. With this assumption, we can uniquely determine $\sin\phi_M$ from the time-dependent decay rate. Note that the above assumption can be checked by comparing the $\sin\phi_M$ obtained in this process with that obtained in the $B^0(\overline{B^0}) \rightarrow DK_S$ process. Also, it can be tested by looking for the CP-violating rate differences (the direct CP violation) that may be seen in $B^{\pm} \rightarrow \psi K^{\pm}$ and the $\cos(\Delta mt)$ term in the time-dependent rate of $B^0(\overline{B^0}) \rightarrow \psi K_S$ decay depending on the relevant phase shifts. Table II summarizes the above arguments on the several modes of studying CP violation.

Now, let us illustrate how measurements of CP asymmetries in these modes at B factories constrain the model-independent parameter M_{12}^{new}. The inputs for this illustration in addition to those in Table I are given in Table III. The central values of these inputs are calculated by setting $M_{12}^{\text{new}}=0$ and $(\rho,\eta)=(0.23,0.29)$, which is a typical point allowed in the standard model analysis [9]. The errors in this table are taken from Ref. [14].

At an earlier stage of a B factory run, we will observe only the CP violation in the $B^0(\overline{B^0}) \rightarrow \psi K_S$ mode. In Fig. 4(b), we show the expected constraint on M_{12}^{new} from this mode, which gives the constraint on $\sin\phi_M$, and the information given in Table I. The expected constraint is not much stronger than that in Fig. 4(a), because ϕ_3 still remains free. In Fig. 4(c), we show the expected constraint by adding the information from $B^{\pm} \rightarrow DX_s^{\pm}$, i.e., the constraints on $\sin^2\phi_3$. In this case, we have eight solutions for M_{12}^{new} owing to the twofold ambiguity in the ϕ_M determination from $\sin\phi_M$ and the fourfold ambiguity in the ϕ_3 determination from $\sin^2\phi_3$. These solutions are indicated by the dots in Fig. 4(c). The rather larger allowed regions in Fig. 4(c) consist of those surrounding these eight solutions. Figure 4(d) shows the expected constraint from all the inputs given in Tables I and III. We have four solutions even in this case. If the allowed regions obtained with real data do not contain the origin ($M_{12}^{\text{new}}=0$), we can not only conclude that the standard model is excluded but also determine the new physics contribution quantitatively. An important observation from this illustration is that we cannot exclude the possibility of a significantly large M_{12}^{new} even if all the measurements considered here seem to be consistent with the standard model.

We have neglected the uncertainties of A, m_t, and $f_B\sqrt{B_B\eta_B}$ in the above analysis. The allowed regions in Figs. 4(a)–4(d) are changed if we vary these values. However, we expect that these uncertainties will be reduced enough by future experimental and theoretical developments.

In conclusion, we presented a framework of analyzing the B-$\overline{B}$ mixing and the CP violations in B decays in a model-independent manner. We introduced the model-independent parameter M_{12}^{new} and showed the constraint on it from the presently available experimental data. We also illustrated how M_{12}^{new} would be constrained by the future experiments of CP violation at B factories. We found that there remain some nontrivial solutions of M_{12}^{new} even for the inputs corresponding to the standard model case ($M_{12}^{\text{new}}=0$). This means that we cannot exclude the possibility of a significant contribution to the B-$\overline{B}$ mixing from new physics even if CP asymmetries in all the modes of $B \rightarrow \psi K_S$, $B \rightarrow DX_s$, and $B \rightarrow \pi\pi$ seem to be consistent with the standard model.

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