Effects of instantons on the excited baryons and two-nucleon systems

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The effects of the spin-orbit and the tensor parts of the instanton-induced interaction on the excited positive- and negative-parity nonstrange baryons and on the two-nucleon systems are investigated. The spin-orbit force from the instanton-induced interaction cancels most of that from the one-gluon exchange in the excited baryons while the spin-orbit force in the nucleon systems remains strong after the inclusion of the instanton-induced interaction. The model including the spin-orbit and the tensor terms of the instanton-induced interaction as well as the one-gluon exchange is found to reproduce successfully the excited baryon mass spectrum and the scattering phase shifts of two nucleons in the spin-triplet relative P-wave state. [S0556-2821(96)04811-4]

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I. INTRODUCTION

Valence quark models have been applied to low-energy light-quark systems and found to be successful in reproducing major properties of the hadrons and hadronic systems. The reason why such models can be so successful is not well understood. The empirical approach, however, suggests a few reasons. One of them is that the model space has an appropriate symmetry. Another reason is that a light quark has rather heavy effective mass. In such a low energy region, the effects of complicated configurations such as $q\bar{q}$ excitation and dynamical gluon effects are considered to be taken into account by employing constituent valence quarks and effective interactions among quarks with the required symmetry.

The empirical models mentioned above usually contain three terms: the kinetic term, the confinement term, and the effective one-gluon exchange (OGE) term. It is considered that OGE stands for the perturbative gluon effects and that the confinement force represents the long-range nonperturbative gluon effects.

It is well known that the color magnetic interaction (CMI) in OGE is responsible for producing many of the hadron properties. By adjusting the strength of OGE, CMI can reproduce the hyperfine splittings [(HFS), e.g., ground state N-Δ mass difference] [1–4] as well as the short range repulsion of the two-nucleon systems in the relative S wave [5–8]. It, however, is also known that the strength of OGE determined in this empirical way is much greater than 1, which makes it hard to treat it as the perturbative effect.

Moreover, the valence quark model including only OGE as an origin of HFS has a spin-orbit problem. The spin-orbit part of OGE is strong; it is just strong enough to explain the observed large spin-orbit force between two nucleons [5–7]. On the other hand, the experimental mass spectrum of the excited baryons, N^* and Δ^* resonances, indicates that such a strong spin-orbit force should not exist between quarks. A valence quark model in which the spin-orbit parts of the quark-quark interaction are removed by hands can well simulate the observed mass spectrum [1–4]. It was pointed out that the confinement force also produces the spin-orbit force, which may cancel the one from OGE in the excited baryons [2]. Suppose one takes the spin-orbit part of a two-body confinement force into account; however, it also cancels the spin-orbit part of OGE in the nuclear force [5,6]. To explain both of the features at the same time is highly nontrivial.

The QCD instantons were originally introduced in relation to the $U_A(1)$ problem. How these topological gluonic configurations behave in the actual QCD vacuum has not been derived directly from QCD. Under a few assumptions, such that the short-range repulsion among instantons and anti-instantons exists, and that the gluon condensate comes from the instantons, etc., however, the QCD vacuum can be regarded as liquid of small instantons and anti-instantons [9]. In this picture, the size of instantons is considered to be small enough compared to the low-energy hadronic scale, which enables us to treat them as a pointlike structure. Once the instanton configuration is fixed, the effects will be similar to the one by the dilute gas approximation, which was investigated in detail [10,11]. The existence of instantons in the vacuum changes the quark and gluon propagators [11] and produces couplings of instantons to the surrounding light-quark zero modes [10]. The latter leads a flavor-singlet interaction among quarks, which is believed to be an origin of the observed large mass difference of $\eta'-\eta$ mesons. It is also reported that the instantons essentially govern the behavior of the pions and the chiral symmetry of QCD [9,12]. This topological configuration is now known to relate many interesting features of QCD. Since it is hard to determine the quantitative feature of the instantons directly from QCD, empirical works on instanton-induced interaction will contribute also in understanding the structure of the QCD vacuum. A few phenomenological models with such interactions were proposed and found to reproduce the η' and η meson masses and their properties [13,14].

We focus our attention on the effect of the flavor-antisymmetric quark interaction induced from the instanton-light-quark couplings. How this instanton-induced interaction (III), which contributes by a few hundred MeV in the meson sector, affects other hadron systems is an interesting

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problem. Actually, several recent works indicate that the effect is also large in the baryon sector [14–20]. We argue that a valence quark model should include III as a short-range nonperturbative gluon effect in addition to the other aforementioned gluon effects.

In Ref. [16], we demonstrated that introducing III may solve the above difficulty in the P-wave systems due to the cancellation between OGE and III. In this paper, we discuss effects of the noncentral parts, especially the spin-orbit part of III on the excited nonstrange baryons and on the two-nucleon systems at the same time. We consider the excited baryons with the principal number=0 (the ground-state baryons), 1 (the negative-parity baryons), and 2 (the positive-parity excited baryons). We employ a quark potential model for that purpose because it can deal both with the single baryons and with the two-nucleon systems, and because the discussion based on the symmetry can be performed more clearly for the present subject. In Sec. II, we will show the model Hamiltonian. The discussion based on the symmetry is presented in Sec. III. The numerical results are shown in Sec. IV. The discussion and the summary are in Sec. V. Complicated calculation of the wave functions and the matrix elements are summarized in the Appendix.

II. QUARK MODEL WITH INSTANTON-INDUCED INTERACTION

We assume that both OGE and III are included in the quark model Hamiltonian:

$$H_{\rm quark}=K+(1-p_{\rm III})V_{\rm OGE}+p_{\rm III}V_{\rm III}+V_{\rm conf}, \tag{1}$$

where $p_{\rm III}$ is a parameter which represents the rate of the S-wave N-Δ mass difference explained by III. When one introduces the interaction strong enough to give the observed η-η' mass difference, $p_{\rm III}$ becomes 0.3–0.4 [14–16]. $V_{\rm OGE}$ and $V_{\rm III}$ are the Galilei invariant terms of the III and OGE potentials.

According to the instanton liquid model, the size of instantons is about 0.3 fm, which is a new scale of the low energy QCD phenomena [9]. The instantons and the anti-instantons couple to flavor-singlet light quarks. Assuming the instanton is small enough compared to the system we consider, one obtains the effective interaction between flavor-antisymmetric quarks arising from that coupling as [10,14–17]

$$H_{\rm III}=V_0^{(2)}\sum_{i<j}\overline{\psi_R}(i)\overline{\psi_R}(j)\,\frac{15}{8}\,\mathscr{A}^{\rm flavor}_{ij}\times\left(1-\frac{1}{5}\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)\psi_L(j)\psi_L(i)+(\mathrm{H.c.}), \tag{2}$$

where $V_0^{(2)}$ is the strength of the two-body part of III, and $\boldsymbol{\sigma}_i$ is the Pauli spin matrix for the ith quark. We obtain the following potential performing the nonrelativistic reduction to the lowest nonvanishing order in (p/m) for each operator of different spin structure [16]:

TABLE I. Parameters for the quark model and for the effective meson exchange potential (see text).

m_u [MeV]	α_s	$V_0^{(2)}$	b [fm]	
313	1.657	−483.8	0.62	
$p_{\rm III}$	$a_{\rm conf}$ [MeV/fm]	V_σ [MeV]	r_σ [fm]	r_a [fm]
0	43.84	−702.9	0.617	0.25
0.4	31.40	−528.7	0.579	0.25

$$V_{\rm III}=V_0^{(2)}\sum_{i<j}\mathscr{A}^{\rm flavor}_{ij}\left[\frac{15}{16}\left(1-\frac{1}{5}\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)-\frac{1}{m^2}\left\{\frac{3}{4}\,LS+\frac{q^2}{12}\left(1-\frac{3}{16}\,\lambda_i\cdot\lambda_j\right)S_{12}\right\}\right] \tag{3}$$

$$=\frac{1}{2}\,V_0^{(2)}\sum_{i<j}\left[\left(1+\frac{3}{32}\,\lambda_i\cdot\lambda_j+\frac{9}{32}\,\lambda_i\cdot\lambda_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)-\mathscr{A}^{\rm flavor}_{ij}\,\frac{1}{m^2}\left\{\frac{3}{4}\,LS+\frac{q^2}{12}\left(1-\frac{3}{16}\,\lambda_i\cdot\lambda_j\right)S_{12}\right\}\right], \tag{4}$$

where λ_i is the Gell-Mann matrix of the color SU(3) with $\lambda_i\cdot\lambda_j=\sum_{a=1}^{8}\lambda_i^a\lambda_j^a$. The same procedure for OGE leads [5,21] to

$$V_{\rm OGE}=4\pi\alpha_s\sum_{i<j}\frac{(\lambda_i\cdot\lambda_j)}{4}\left[\frac{1}{q^2}-\frac{\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{6m^2}+\frac{3}{2m^2q^2}\,LS+\frac{1}{12m^2}\,S_{12}\right] \tag{5}$$

with

$$LS=(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot i[\mathbf{q}\times(\mathbf{p}_i-\mathbf{p}_j)]/4, \tag{6}$$

$$S_{ij}=3(\boldsymbol{\sigma}_i\cdot\mathbf{q})(\boldsymbol{\sigma}_j\cdot\mathbf{q})/q^2-(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j). \tag{7}$$

Here $\mathscr{A}^{\rm flavor}_{ij}=(1-P^{\rm flavor}_{ij})/2$ is the antisymmetrizer in the flavor space, $m_u=m_d\equiv m$ is a constituent quark mass, and $\mathbf{q}$ is the three momentum transfer. The values of m, the strength of each interaction, α_s and $V_0^{(2)}$, with the size parameter b of the quark core of the baryon, are listed in Table I. Since we use a constituent quark model, the coupling of the interactions should also be determined empirically. The values are chosen as follows [5,7,14–16]: the quark mass is 1/3 of the nucleon mass; the size parameter b is taken to be a little smaller than the real nucleon size reflecting that the observed baryon size has contribution from the meson cloud; α_s and $V_0^{(2)}$ are determined to give the ground state N-Δ mass difference μ:

$$\frac{4}{3\sqrt{2\pi}}\,\frac{\alpha_s}{m^2b^3}=-\frac{9}{4\sqrt{2\pi}^3}\,\frac{V_0^{(2)}}{b^3}=293\ \ \mathrm{MeV}\equiv\mu; \tag{8}$$

$a_{\rm conf}$, the strength of the confinement potential, can be determined by $\delta m_N/\delta b=0$; and $p_{\rm III}$ is taken to give the η'-η mass difference.

III. SYMMETRY

One of the reasons that the nonrelativistic quark model can successfully predict the properties of the low energy system is that the model has an appropriate symmetry. We discuss here whether the observed properties in the spin-orbit force, small in the single baryons and large in the two-nucleon systems, can be explained by discussion based on the symmetry.

The interactions (4) and (5) consist of operators which conserve the flavor symmetry. Thus the spin-orbit or the tensor part of the two-body interactions for a quark pair, which requires the quark pair to be symmetric in the spin space, can be decomposed as

$$\mathcal{O}=\overline{\mathcal{O}}^{A}+\mathcal{O}^{S}+\overline{\mathcal{O}}^{S}+\mathcal{O}^{A}, \tag{9}$$

where

$$\overline{\mathcal{O}}^{A}\equiv\mathcal{O}\mathcal{A}^{\rm orb}\mathcal{S}^{\rm spin}\mathcal{A}^{\rm color}\mathcal{A}^{\rm flavor}, \tag{10}$$

$$\mathcal{O}^{S}\equiv\mathcal{O}\mathcal{A}^{\rm orb}\mathcal{S}^{\rm spin}\mathcal{S}^{\rm color}\mathcal{S}^{\rm flavor}, \tag{11}$$

$$\overline{\mathcal{O}}^{S}\equiv\mathcal{O}\mathcal{S}^{\rm orb}\mathcal{S}^{\rm spin}\mathcal{A}^{\rm color}\mathcal{S}^{\rm flavor}, \tag{12}$$

$$\mathcal{O}^{A}\equiv\mathcal{O}\mathcal{S}^{\rm orb}\mathcal{S}^{\rm spin}\mathcal{S}^{\rm color}\mathcal{A}^{\rm flavor}, \tag{13}$$

with antisymmetrizers $\mathcal{A}$'s and symmetrizers $\mathcal{S}$'s. For a quark pair with the relative-odd partial wave, the first two terms in the right-hand side of Eq. (9) are relevant, while the last two terms are for relative-even partial-wave pairs. The operator with a bar is for color anti-symmetric pairs, which is relevant to the single baryons. Those for the flavor-antisymmetric (-symmetric) quark pairs are marked by $A(S)$.

The noncentral term of III contains only flavor-antisymmetric components, $\overline{\mathcal{O}}^{A}$ and $\mathcal{O}^{A}$; OGE has all of the components in Eq. (9). Since OGE is vector-particle exchange and III is alike to scalar-particle exchange, their noncentral term has an opposite sign. Thus there is a spin-orbit cancellation where both OGE and III survive. Actually, it occurs only for the flavor-antisymmetric color-antisymmetric relative-p-wave quark pairs. Since the range of III is assumed to be δ function like, the noncentral part of III for the relative-l-partial wave pairs vanishes when $l\geqslant 2$.

To see the properties of the single baryons and of the short range part of the two-nucleon systems, we evaluate the energy of systems by the Gaussian wave functions where the center-of-mass motion is eliminated: $(0p)(0s)^2$ for the negative-parity single baryons, $(1s)(0s)^2$, $(0p)^2(0s)$, and $(0d)(0s)^2$ for the positive-parity single baryons, and and $(0p)(0s)^5$ for the six-quark systems. In Table II, the contribution of matrix element of each operator in Eq. (9) is listed for the negative-parity baryon $N^*(5/2^-)$, for the positive-parity excited baryon $N^*(7/2^+)$, and for the six-quark state with the relative P-wave two-nucleon quantum number. The first low corresponds to the contribution of the spin-orbit force from the flavor-antisymmetric color-antisymmetric relative-$0p$ quark pairs. The OGE and III contributions to $N^*(5/2^-)$ are evaluated by $1/2\mu C_{N=1}(1-p_{\rm III})$ and $-1/(3m^2b^2)\mu C_{N=1}p_{\rm III}$, respectively, with the parameter $p_{\rm III}$ with Eq. (1). Those to $N^*(7/2^+)$ are obtained by substituting $C_{N=1}$ by $C_{N=2}$, while C_{NN3P} should be used for the two-nucleon system. The entries in other rows should read in a similar way. Once the parameters are taken to satisfy Eq. (8), the contribution from OGE can be expressed only by μ, the ground state N-Δ mass difference; the contribution from III contains also the dimensionless parameter mb together with μ (see Appendix).

TABLE II. Matrix elements of the OGE and III spin-orbit and the tensor terms in units of their contribution to the ground state Δ-N mass difference. Their contribution to $N(5/2^-)$, $N(7/2^+)$ and the six quark state for the spin-triplet relative-P-wave two-nucleon system are listed as $C_{N=1}$, $C_{N=2}$, and C_{NN3P}. The columns under OGE and III correspond to the radial contribution of each quark pair, $\chi_{nl}/3$ and $\theta_{nl}/3$ [Eqs. (A78)–(A80)], from OGE and III, respectively. The states forbidden by the color or the flavor symmetry are left blank.

	OGE	III	$C_{N=1}$	$C_{N=2}$	C_{NN3P}
$\langle\overline{\mathcal{O}}^{A}_{LS}\rangle_{0p}$	$\frac{1}{2}$	$-\frac{1}{3}\frac{1}{m^2b^2}$	$\frac{3}{2}$	$\frac{3}{2}$	0.12
$\langle\mathcal{O}^{S}_{LS}\rangle_{0p}$	$-\frac{1}{4}$				1.52
$\langle\overline{\mathcal{O}}^{S}_{LS}\rangle_{0d}$	$\frac{1}{5}$		0	$\frac{3}{2}$	0
$\langle\mathcal{O}^{A}_{LS}\rangle_{0d}$	$-\frac{1}{10}$	0			0
$\langle\overline{\mathcal{O}}^{A}_{\rm tens}\rangle_{0p}$	$\frac{1}{12}$	$-\frac{5}{18}\frac{1}{m^2b^2}$	$-\frac{3}{5}$	$-\frac{3}{5}$	-0.16
$\langle\mathcal{O}^{S}_{\rm tens}\rangle_{0p}$	$-\frac{1}{24}$				0.84
$\langle\overline{\mathcal{O}}^{S}_{\rm tens}\rangle_{0d}$	$\frac{1}{30}$		0	$-\frac{3}{7}$	0
$\langle\overline{\mathcal{O}}^{S}_{\rm tens}\rangle_{1s-0d}$	$\frac{1}{12\sqrt{10}}$		0	0	0
$\langle\mathcal{O}^{A}_{\rm tens}\rangle_{0d}$	$-\frac{1}{60}$	0			0
$\langle\mathcal{O}^{A}_{\rm tens}\rangle_{1s-0d}$	$-\frac{1}{24\sqrt{10}}$	$-\frac{5}{6\sqrt{10}}\frac{1}{m^2b^2}$			0

As seen that C_{NN3P} for $\langle\mathcal{O}^{S}_{LS}\rangle_{0p}$ is by one order larger than C_{NN3P} for $\langle\mathcal{O}^{A}_{LS}\rangle_{0p}$, the contribution of the color symmetric spin-orbit operator is found to be dominant in the six-quark state. Since there is no OGE-III cancellation for that operator, the spin-orbit reduction of the relative P-wave two-nucleon systems is small. Within the single baryon, of course, only the operators with a bar are relevant. The OGE-III cancellation occurs in the spin-orbit part operating on the odd-wave quark pairs in the single baryons. It is the spin-orbit force between the odd-wave quark pairs that should disappear in the single baryon as we will show in the next section. Thus we expect that this cancellation will lead the observed properties in the spin-orbit force.

IV. RESULTS

A. Single baryons

We investigate the mass spectrum of the excited nonstrange baryons by a nonrelativistic quark model with the spin-spin, the spin-orbit, and the tensor parts of OGE and III. The central spin-independent part has been modified. The central part of the original model Hamiltonian Eq. (14) does not produce the correct zeroth-order splitting seen in the excited positive parity baryons. It is mainly because the spin-independent contact interaction from the one-gluon exchange is strong and has a wrong sign. As Refs. [1–3] pointed out, the deviation of the spin-independent force from harmonic will be very important and expressed by the following parameterization. We use the same method in Ref. [3] with the same values except for a little modified E_0:

$$H_{\mathrm{quark}}=H_c+(1-p_{\mathrm{III}})\widetilde{V_{\mathrm{OGE}}}+p_{\mathrm{III}}\widetilde{V_{\mathrm{III}}}, \tag{14}$$

$$H_c=E_0+N\Omega+\delta U, \tag{15}$$

$$E_0=1090\ \ \mathrm{MeV}+p_{\mathrm{III}}\mu/2, \tag{16}$$

$$\Omega=440\ \ \mathrm{MeV}, \tag{17}$$

$$\delta=400\ \ \mathrm{MeV}, \tag{18}$$

$$U=\begin{cases} -1 \\ -\frac{1}{2} \\ -\frac{2}{5} \\ -\frac{1}{5} \\ 0 \end{cases} \text{for } (D^{\mathrm{SF}},L^P)=\begin{cases} (56',0^+) \\ (70,0^+) \\ (56,2^+) \\ (70,2^+) \\ \text{otherwise} \end{cases}, \tag{19}$$

where $\widetilde{V_{\mathrm{OGE}}}(\widetilde{V_{\mathrm{III}}})$ is the spin-spin, the spin-orbit, and the tensor part of $V_{\mathrm{OGE}}(V_{\mathrm{III}})$, and N is the principal quantum number of the harmonic oscillator wave function.

Our results in this subsection are affected by only a few parameters: E_0 to give the ground state energy, Ω to give the difference among the ground states, the $N=1$ negative-parity baryons and the $N=2$ positive-parity baryons, δ to split the $N=2$ baryons, μ in Eq. (8) for the hyperfine splittings, $(mb)^2$, and p_{III} to give the relative strength of III. This estimate by the harmonic oscillator wave function is affected only by the above combinations of the parameters listed in Table I. The values of the parameters here are reasonable. E_0 is close to $3m_u$, and the strength of OGE, $\alpha_s(1-p_{\mathrm{III}})$, becomes smaller when $p_{\mathrm{III}}=0.4$.

In Figs. 1(a) and 1(b), the mass spectra of the negative-parity and the positive-parity baryons are shown. The ground state mass is kept 940 MeV for the nucleon and 1240 MeV for Δ in each parameter set. The observed mass spectrum is shown by stars (the weighted average of the observed values) and boxes (possible error) [22]. The number of the stars corresponds to reliability of existence of the states: the four-star state means that its existence is certain while the one-star state means that evidence of its existence is poor.

The next right to the experiment is the mass spectrum given by H_c (denoted by B). This three-parameter model gives an excellent prediction for the excited baryons. The third spectrum (C) is derived from the Hamiltonian which contains H_c and the tensor term of OGE ($p_{\mathrm{III}}=0$): it corresponds to the one in Refs. [2, 3], where the spin-orbit term is omitted by hand. The introduction of the tensor term gives little change in the spectrum. Actually, one cannot conclude that the tensor term is necessary only from the mass spectrum; it was included so as to give correct decay modes [23].

The fourth column (D) contains the central, the spin-spin, and the spin-orbit term with $p_{\mathrm{III}}=0$. The spectrum, especially for the nucleons, is destroyed completely; which is the reason why the spin-orbit term had to be removed in Refs. [2, 3]. The question is, however, whether all the spin-orbit terms should be removed or not. The excited Δ^* mass spectrum is better than the nucleons'. There, all the quark pairs are in the flavor symmetric; the spin-orbit term exists only for the relative $0d$-wave pairs.

The Hamiltonian of the fifth spectrum (E) is the same as the fourth one except that we remove all the spin-orbit force between the relative $0p$-wave quark pairs, namely, flavor-antisymmetric pairs. The remaining spin-orbit term affects only the relative $0d$-quark pairs. The excited nucleon spectrum changes drastically; most of the spin-orbit effects, which destroy the spectrum, are found to come from the relative $0p$-wave quark pairs. The remaining effects of the spin-orbit force are still somewhat stronger than the best fit, but the spectrum becomes much closer to the realistic one.

As we showed in the previous section, the spin-orbit force between the relative odd partial wave pairs reduces by introducing III. The sixth column (F) corresponds to the Hamiltonian (14) with $p_{\mathrm{III}}=0.4$; it includes the central (parameterized), the spin-spin, the spin-orbit, and the tensor terms of both OGE and III. In this choice, the strength of the spin-orbit force between $0p$ pairs reduces by 0.32 from the $p_{\mathrm{III}}=0$ case while that between $0d$ pairs reduces by $(1-p_{\mathrm{III}})=0.6$. Note that this simple model does only have six parameters [E_0, D_0, δ, μ, $(mb)^2$, and p_{III}] for the whole nonstrange baryons up to the $N=2$ including both of the positive parity and the negative parity states. We do not adjust the relative strength of the spin-spin, the spin-orbit, and the tensor term. The result is reasonably consistent with the experiments. Thus we can conclude that the flavor-singlet interaction plays an essential role in reducing the strong spin-orbit force in the excited baryons to the observed strength.

In Refs. [2, 3, 23], the tensor part of OGE was introduced so as to give an appropriate decay rates. One of their examples is the relative strength of the πN decay from two $\Delta(5/2^+)$: $\Delta(1905)$ and $\Delta(2000)$. We estimate the ratio of the decay matrix elements for those states using the transition operator defined in [23], where they assumed the pointlike pion is emitted from single quark. The calculated matrix element for the higher $\Delta(5/2^+)$ is found by about 30% smaller than that of the lower state. The experimental partial decay width of the lower energy state to the πN channel is 32 to 39 MeV; two experiments are reported for the decay width from the higher energy state: 5 and 28 MeV [22]. It seems that the higher state decays more weakly to πN than the lower state. This decay-rate ratio for these states is consistent with the experiments, though it should be considered as a very rough estimate.

A possible flaw of our model as well as that in Refs. [2, 3, 23] is $\Delta(3/2^+)$. Two $\Delta(3/2^+)$ are seen experimentally: $\Delta(1600)$ and $\Delta(1920)$; both of them decay to the πN channel rather strongly. The lowest energy level of the predicted states is 1734 MeV. Inclusion of the spin-orbit term and III has made the state lower by about 150 MeV, but the level is still higher than it should be by about 100 MeV. The esti-

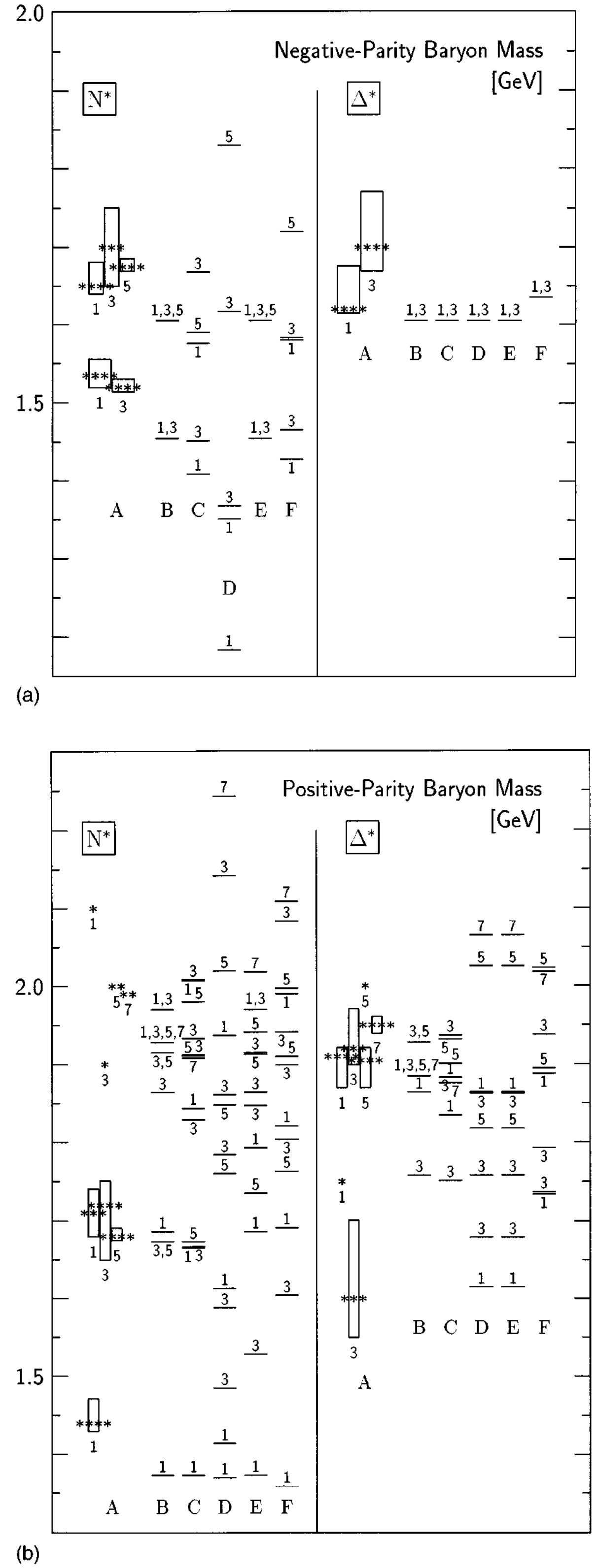

FIG. 1. Mass spectra for (a) the negative-parity nonstrange baryons and (b) the positive-parity ($N=2$) nonstrange baryons. A: the observed mass spectrum is shown by stars. The columns correspond (from left to right) to B: H_c; C: H_c+tensor term of OGE; D: H_c+LS and the tensor term of OGE; E: H_c+LS of OGE for $0d$-quark pairs only; F: H_c+LS and the tensor terms of OGE and III with $p_{\rm III}=0.4$. Each number corresponds to the spin, $2J$, of the level.

mated decay matrix elements from both of the two lower states to the πN channel are large, but that from the highest level is small. The couplings to the baryon-meson channel, such as πN, may be important [24]. The experiments have a large error also for these states. Further investigation both in the experiments and in the theories is necessary to clarify this problem.

B. Two nucleons

The main purpose of this paper is to show that the inclusion of the instanton-induced interaction gives the channel specific cancellation of the spin-orbit force between quarks. In the symmetry consideration we show that the spin-orbit force in the two-nucleon scattering does not reduce much by introducing III. In this section we show that a realistic quark cluster model including III can actually reproduce the two-nucleon scattering phase shift for the triplet P-wave states.

The wave function is the same as those in Refs. [5–7, 15]:

$$\overline{\Psi}=\mathscr{A}_q\{\phi_N^2\chi(R)\}. \tag{20}$$

The notation is the same as Eq. (A102) in the Appendix, except for $\chi(R)$, the relative wave function, which is now to be solved.

The Hamiltonian for the valence quarks is Eq. (1), except that we omit the tensor terms of OGE and III because the tensor force between the quarks is not dominant in the two-nucleon system [7]. We use the linear confinement potential for $V_{\rm conf}$ in Eq. (1):

$$V_{\rm conf}=\sum_{i<j} a_{\rm conf} r_{ij}\,. \tag{21}$$

After integrating out the internal coordinates of the nucleons, we have the resonating group method equation

$$\{H_q+N^{1/2}V_{\rm EMEP}N^{1/2}-EN\}\chi=0, \tag{22}$$

where H_q is the Hamiltonian kernel for $H_{\rm quark}$ and N is the normalization kernel. The effective meson exchange potential $V_{\rm EMEP}$ is multiplied by $N^{1/2}$ because the potential $V_{\rm EMEP}$ should be added to the equation in the Schrödinger form. Here we take $V_{\rm EMEP}$ to have the central and the tensor parts of the one-pion exchange with the form factor corresponding to the size parameter b, and the Gaussian-type central attraction [7]:

$$V_{\rm EMEP}(R)=(\tau\cdot\tau)(\sigma\cdot\sigma)V_\pi^C(R)+(\tau\cdot\tau)V_\pi^T(R)S_{12}+V_g(R), \tag{23}$$

$$V_\pi^C(R)=\frac{g_\pi^2}{4\pi}\,\frac{1}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{\exp[-m_\pi R]}{R}\,\frac{1}{2}\{\text{erfc}(\alpha_-) - \exp[2m_\pi R]\text{erfc}(\alpha_+)\}, \tag{24}$$

$$V_\pi^T(R)=\frac{g_\pi^2}{4\pi}\frac{1}{3}\left(\frac{m_\pi}{2m_N}\right)^2\frac{\exp[-m_\pi R]}{R}$$
$$\times\frac{1}{2}\left[\left\{1+\frac{3}{m_\pi R}+\frac{3}{(m_\pi R)^2}\right\}\mathrm{erfc}(\alpha_-)\right.$$
$$-\left\{1-\frac{3}{m_\pi R}+\frac{3}{(m_\pi R)^2}\right\}\exp[2m_\pi R]\mathrm{erfc}(\alpha_+)$$
$$\left.-\left\{1+\frac{6\beta}{(m_\pi R)^2}\right\}\frac{m_\pi R}{\sqrt{\pi\beta^3}}\exp[-\alpha_-^2]\right], \tag{25}$$

$$\alpha_\pm=\frac{m_\pi b}{\sqrt{3}}\pm\frac{\sqrt{3}R}{2b}, \tag{26}$$

$$\beta=(m_\pi b)^3/3, \tag{27}$$

$$V_g(R)=V_\sigma\exp\left[-\left(\frac{R}{r_\sigma+r_a}\right)^2\right]$$
$$-\{V_\sigma+V_\pi^C(0)\}\exp\left[-\left(\frac{R}{r_\sigma-r_a}\right)^2\right]. \tag{28}$$

The parameters V_σ and r_σ in V_{EMEP} are determined by fitting the experimental phase shifts of the triplet partial-odd wave states with fixed r_a (Table I). The coupling constant $g_\pi^2/(4\pi)=13.7$ is taken from Ref. [25]. The other parameters in H_{quark} are the same as in the previous section, which are also listed in Table I.

To see the contribution from the spin-orbit term more clearly, we recompile the phase shift as [5–7]

$$\delta(^3P_C)=\frac{1}{9}\,\delta(^3P_0)+\frac{1}{3}\,\delta(^3P_1)+\frac{5}{9}\,\delta(^3P_2), \tag{29}$$

$$\delta(^3P_T)=-\frac{5}{36}\,\delta(^3P_0)+\frac{5}{24}\,\delta(^3P_1)-\frac{5}{72}\,\delta(^3P_2), \tag{30}$$

$$\delta(^3P_{LS})=-\frac{1}{6}\,\delta(^3P_0)-\frac{1}{4}\,\delta(^3P_1)+\frac{5}{12}\,\delta(^3P_2). \tag{31}$$

The calculated phase shifts are shown in Fig. 2 together with those of the energy-dependent phase shift analysis for the low-energy region, VZ40, taken from SAID database [25]. The central part seems to require more sophisticated effective meson exchange than the two-ranged Gaussian potential. The one-pion exchange can give enough strength to the tensor part of the two-nucleon system. The spin-orbit force is reproduced by the quark model well, even when III is included by $p_{\mathrm{III}}=0.4$. As seen in Table II, the cancellation occurs only for the color-antisymmetric quark pairs, which play minor roles in the P-wave two-nucleon systems. The III spin-orbit part for the color-antisymmetric pairs has the same sign as that of OGE. Thus the reduction of the spin-orbit effect by including III is less than $(1-p_{\mathrm{III}})$.

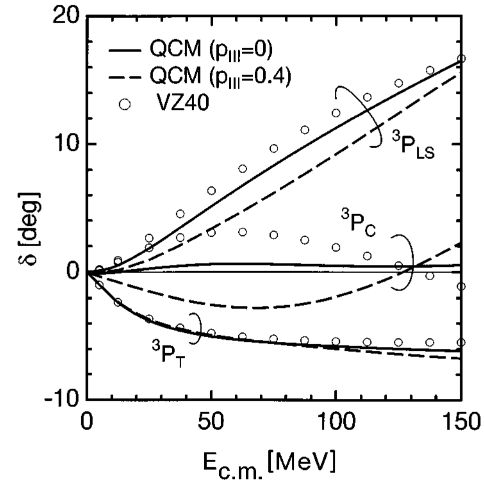

FIG. 2. Two-nucleon scattering phase shifts for the spin-triplet relative-P-wave states. The phase shifts are recompiled to present the strength of the central (^3P_C), the spin-orbit ($^3P_{LS}$), and the tensor part (^3P_T) (see text). The circles correspond to the experiments VZ40 [25], the solid lines are for $p_{\mathrm{III}}=0$, and the dashed lines are for $p_{\mathrm{III}}=0.4$.

V. DISCUSSION AND SUMMARY

We investigate the effects of the noncentral part of the interaction induced from the instanton-light-quark coupling on the excited nonstrange baryons and two-nucleon systems. It is found that the III spin-orbit force cancels the OGE spin-orbit force in the excited baryons while the spin-orbit force in the two-nucleon systems remains strong after the inclusion of III, which is favored by the experiments.

The spin-orbit and the tensor part of III survives only for the color-antisymmetric quark pairs in a relative-odd partial wave state. There, the cancellation of the spin-orbit and tensor parts occurs between OGE and III, which leads to the particular cancellation required. The relative strength of each term may change if we take other effects into account. The channel-specific cancellation is, however, explained based on the symmetry; the overall nature will still be valid with the change of the model parameters.

The other possible source of the noncentral part is the confinement force. To produce such channel dependence in the confinement force, however, one has to introduce a three-body confinement, such as a flip-flop model [6]. There, the confinement force in the hidden color configuration and in the baryonic configuration can be different from each other. It allows us to have an extra free parameter, which cannot be measured by the experiments. The two-body confinement force, which has the factor $(\lambda\cdot\lambda)$, shows similar channel dependence to OGE and cannot produce the cancellation required here. We neglect the spin-orbit part of the confinement here, because its mechanism is not known and the spin-orbit force cannot be determined. But the effect itself may not be small and will have to be considered in future.

The meson exchange also produces the noncentral parts. Usually, the σ, ρ, and ω mesons are considered as the main source of the spin-orbit force between the nucleons. This picture, however, cannot be applied to the quark systems in a straightforward way. These mesons are not pointlike; one cannot safely assume that they interact directly to quarks.

Moreover, their couplings to the nucleons are determined mainly from the two-nucleon scattering data empirically. In a model which includes only meson and baryon degrees of freedom, the ωN coupling is usually taken to be strong so as to produce the short-range repulsion between two nucleons. In a quark model the repulsion is explained by the quark Pauli-blocking effect, by OGE, and by III. The σ-meson exchange, which produces the intermediate attraction, also stands for complicated modes such as two-pion exchange, coupling to the $\pi\Delta$ channel, and even the attraction from III. It is hard to determine the genuine coupling to the mesons. Here we take a quark-meson hybrid picture; the short-ranged properties are explained by the nature of a quark model and the long-ranged properties are explained by the meson-exchange model. The modes which can be presented by the mesons will be taken into account as the meson clouds. The ρ-meson cloud has similar dependence to OGE because the isospin factor $(\tau\cdot\tau)$ shows similar channel dependence to $(\lambda\cdot\lambda)$; it will not produce the channel-specific cancellation to OGE. The spin-orbit force of the ω or σ cloud may produce such a cancellation. Their strength, however, becomes smaller in such a picture. The contribution from meson clouds is unlikely to produce a major part of the large effects required here.

On the contrary, there is clear evidence for the existence of the interaction between the flavor-singlet quarks in the meson mass spectrum, i.e., η'-η mass difference; this effect is considered to come not from the meson exchange but, at least, mainly from the instanton-light-quark coupling, whose role we investigate here. It is natural to think that there are large effects from III also on the properties of the baryons or baryon systems; one of which, we argue, is the cancellation in the spin-orbit and the tensor force.

The other effects from the instantons, e.g., the deformation of the quark and gluon propagators should be considered. Also, since the instanton has a small but finite size, the correction to the interaction from the pointlike instantons employed here has to be examined. Moreover, because their effective masses themselves are considered to come from the instanton-light-quark coupling in the instanton-liquid picture [9], the massless zero-mode quarks around instantons are assumed to couple to the constituent quarks. Though we assume the coupling of the constituent quarks and instantons is simple, it may actually have some momentum dependence. Let us, however, again mention that our discussion is based on the symmetry, which will survive reasonable change of the model parameters.

The symmetry consideration has to be reexamined when one considers the relativistic systems or the systems including strangeness. The estimate by the MIT bag model indicates that the major effect of the cancellation still exists in the negative-parity baryons [16]. It is interesting to investigate the role of the instanton-induced interaction in the excited baryons with the strangeness, especially in the flavor-singlet states, which will be presented elsewhere.

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APPENDIX: MATRIX ELEMENTS BY THE HARMONIC OSCILLATOR WAVE FUNCTIONS

Here we evaluate each operator defined by Eq. (9).

1. Single baryons

A. Wave functions

a. S-wave baryons. There are flavor-decaplet states with $J^P=3/2^+$ and flavor-octet states with $1/2^+$. By writing the orbital angular momentum L and intrinsic spin S explicitly with the dimension in the flavor space $\mathbf{D}^F$ and the dimension in the spin-flavor space D^{SF} as $|\mathbf{D}^F;D^{\mathrm{SF}}(LS)J^P\rangle$, they are represented as

$$|1\rangle\equiv\left|\mathbf{10};56\left(0\,\frac{3}{2}\right)\frac{3}{2}^+\right\rangle=|[1^3]^C\rangle|[3]^O\rangle|[3]^F\rangle|[3]^S\rangle, \tag{A1}$$

$$|2\rangle\equiv\left|\mathbf{8};56\left(0\,\frac{1}{2}\right)\frac{1}{2}^+\right\rangle=|[1^3]^C\rangle|[3]^O\rangle\{|[21]^F\rangle|[21]^S\rangle\}_{[3]}. \tag{A2}$$

b. P-wave baryons. There are flavor-decaplet states with $J^P=1/2^-$, $3/2^-$ and flavor-octet states with $J^P=(1/2^-)^2$, $(3/2^-)^2$, $5/2^-$. For future use, we listed flavor-singlet states with $1/2^-$, $3/2^-$ for the strangeness -1 systems. In the same representation above, they are

$$|3\rangle\equiv\left|\mathbf{10};70\left(1\,\frac{1}{2}\right)J^-\right\rangle=|[1^3]^C\rangle\{|[21]^O\rangle|[3]^F\rangle|[21]^S\rangle\}_{[3]}, \tag{A3}$$

$$|4\rangle\equiv\left|\mathbf{8};70\left(1\,\frac{3}{2}\right)J^-\right\rangle=|[1^3]^C\rangle\{|[21]^O\rangle|[21]^F\rangle\}_{[3]}|[3]^S\rangle, \tag{A4}$$

$$|5\rangle\equiv\left|\mathbf{8};70\left(1\,\frac{1}{2}\right)J^-\right\rangle=|[1^3]^C\rangle\{|[21]^O\rangle|[21]^F\rangle|[21]^S\rangle\}_{[3]}, \tag{A5}$$

$$|6\rangle\equiv\left|\mathbf{1};70\left(1\,\frac{1}{2}\right)J^-\right\rangle=|[1^3]^C\rangle\{|[21]^O\rangle|[1^3]^F\rangle|[21]^S\rangle\}_{[1^3]}, \tag{A6}$$

c. N=2 positive parity baryons. There are flavor decaplet states with $J^P=(1/2^+)^2$, $(3/2^+)^3$, $(5/2^+)^2$, and $7/2^+$, flavor octet states with $(1/2^+)^4$, $(3/2^+)^5$, $(5/2^+)^3$, and $7/2^+$, and flavor singlet states with $(1/2^+)^2$, $(3/2^+)^2$, and $(5/2^+)^2$:

$$|7_L\rangle\equiv\left|\mathbf{10};56\left(L\,\frac{3}{2}\right)J^+\right\rangle=|[1^3]^C\rangle|[3]^O\rangle|[3]^F\rangle|[3]^S\rangle, \tag{A7}$$

$$|8_L\rangle\equiv\left|\mathbf{10};70\left(L\,\frac{1}{2}\right)J^+\right\rangle=|[1^3]^C\rangle\{|[21]^O\rangle|[3]^F\rangle|[21]^S\rangle\}_{[3]}, \tag{A8}$$

$$|9_L\rangle \equiv \left|\mathbf{8};70\left(L\ \frac{3}{2}\right)J^+\right\rangle = |[1^3]^C\rangle\{|[21]^O\rangle|[21]^F\rangle\}_{[3]}|[3]^S\rangle, \tag{A9}$$

$$|10_L\rangle \equiv \left|\mathbf{8};56\left(L\ \frac{1}{2}\right)J^+\right\rangle = |[1^3]^C\rangle|[3]^O\rangle\{|[21]^F\rangle|[21]^S\rangle\}_{[3]}, \tag{A10}$$

$$|11_L\rangle \equiv \left|\mathbf{8};70\left(L\ \frac{1}{2}\right)J^+\right\rangle = |[1^3]^C\rangle\{|[21]^O\rangle|[21]^F\rangle|[21]^S\rangle\}_{[3]}, \tag{A11}$$

$$|13_L\rangle \equiv \left|\mathbf{1};70\left(L\ \frac{1}{2}\right)J^+\right\rangle = |[1^3]^C\rangle\{|[21]^O\rangle|[1^3]^F\rangle|[21]^S\rangle\}_{[1^3]}, \tag{A12}$$

with $L=0$ and 2, and

$$|12\rangle \equiv \left|\mathbf{8};20\left(1\ \frac{1}{2}\right)J^+\right\rangle = |[1^3]^C\rangle|[1^3]^O\rangle\{|[21]^F\rangle|[21]^S\rangle\}_{[1^3]}, \tag{A13}$$

$$|14\rangle \equiv \left|\mathbf{1};20\left(1\ \frac{3}{2}\right)J^+\right\rangle = |[1^3]^C\rangle|[1^3]^O\rangle|[1^3]^F\rangle|[3]^S\rangle, \tag{A14}$$

for $L=1$.

Here

$$\{|[21]^\alpha\rangle|[21]^\beta\rangle\}_{[3]} = \frac{1}{\sqrt{2}}\{|[21]^\alpha MS\rangle|[21]^\beta MS\rangle + |[21]^\alpha MA\rangle|[21]^\beta MA\rangle\}, \tag{A15}$$

$$\{|[21]^\alpha\rangle|[21]^\beta\rangle\}_{[1^3]} = \frac{1}{\sqrt{2}}\{|[21]^\alpha MS\rangle|[21]^\beta MA\rangle - |[21]^\alpha MA\rangle|[21]^\beta MS\rangle\}, \tag{A16}$$

$$\begin{aligned}\{|[21]^\alpha\rangle|[21]^\beta\rangle|[21]^\gamma\rangle\}_{[3]} = \frac{1}{2}\{&-|[21]^\alpha MS\rangle|[21]^\beta MS\rangle|[21]^\gamma MS\rangle\\ &+|[21]^\alpha MS\rangle|[21]^\beta MA\rangle|[21]^\gamma MA\rangle\\ &+|[21]^\alpha MA\rangle|[21]^\beta MS\rangle|[21]^\gamma MA\rangle\\ &+|[21]^\alpha MA\rangle|[21]^\beta MA\rangle|[21]^\gamma MS\rangle\}.\end{aligned} \tag{A17}$$

The orbital wave function in the coordinate space can be written by Jacobi's coordinates, $\boldsymbol{\xi}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$, $\boldsymbol{\eta}=(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)/\sqrt{6}$, and $\mathbf{R}_G=(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)/\sqrt{3}$. When we write $|NL[f]\rangle$, they are

$$|00[3]\rangle = \psi_{0s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}), \tag{A18}$$

$$|01[21]MS\rangle = \psi_{0s}(\boldsymbol{\xi})\psi_{0p}(\boldsymbol{\eta}), \tag{A19}$$

$$|01[21]MA\rangle = \psi_{0p}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}), \tag{A20}$$

$$|01[1^3]\rangle = [\psi_{0p}(\boldsymbol{\xi})\times\psi_{0p}(\boldsymbol{\eta})]^1, \tag{A21}$$

$$|02[3]\rangle = \frac{1}{\sqrt{2}}\{\psi_{0d}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}) + \psi_{0s}(\boldsymbol{\xi})\psi_{0d}(\boldsymbol{\eta})\}, \tag{A22}$$

$$|02[21]MS\rangle = \frac{1}{\sqrt{2}}\{\psi_{0d}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}) - \psi_{0s}(\boldsymbol{\xi})\psi_{0d}(\boldsymbol{\eta})\}, \tag{A23}$$

$$|02[21]MA\rangle = [\psi_{0p}(\boldsymbol{\xi})\times\psi_{0p}(\boldsymbol{\eta})]^2, \tag{A24}$$

$$|10[3]\rangle = \frac{1}{\sqrt{2}}\{\psi_{1s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}) + \psi_{0s}(\boldsymbol{\xi})\psi_{1s}(\boldsymbol{\eta})\}, \tag{A25}$$

$$|10[21]MS\rangle = \frac{1}{\sqrt{2}}\{\psi_{1s}(\boldsymbol{\xi})\psi_{0s}(\boldsymbol{\eta}) - \psi_{0s}(\boldsymbol{\xi})\psi_{1s}(\boldsymbol{\eta})\}, \tag{A26}$$

$$|10[21]MA\rangle = [\psi_{0p}(\boldsymbol{\xi})\times\psi_{0p}(\boldsymbol{\eta})]^0, \tag{A27}$$

where $[a_l\times b_{l'}]^L_M = \Sigma(lml'm'|LM)a_{lm}b_{l'm'}$.

The flavor part is the same as in Ref. [3]. For the proton, it is

$$|[3]\rangle = (duu+udu+uud), \tag{A28}$$

$$|[21]MS\rangle = \frac{1}{\sqrt{6}}(-udu-duu+2uud), \tag{A29}$$

$$|[21]MA\rangle = \frac{1}{\sqrt{2}}(udu-duu). \tag{A30}$$

B. Matrix elements

The operators we consider can be written as

$$\begin{aligned}\mathcal{O} = \sum_{(i<j)}\{&\mathcal{O}^C_c(\mathcal{S}^F\mathcal{O}^O_c + \mathcal{A}^F\overline{\mathcal{O}^O_c})1\\ &+\mathcal{O}^C_{ss}(\mathcal{S}^F\mathcal{O}^O_{ss} + \mathcal{A}^F\overline{\mathcal{O}^O_{ss}})\sigma\cdot\sigma\\ &+\mathcal{O}^C_{ls}(\mathcal{S}^F\mathcal{O}^O_{ls} + \mathcal{A}^F\overline{\mathcal{O}^O_{ls}})L\cdot S\\ &+\mathcal{O}^C_{t}(\mathcal{S}^F\mathcal{O}^O_{t} + \mathcal{A}^F\overline{\mathcal{O}^O_{t}})S_{12}\}.\end{aligned} \tag{A31}$$

Then, the matrix element

$$\mathcal{O}(n'_L,n_L) \equiv \frac{1}{\sqrt{2J+1}}\langle n'(LS')J\|\mathcal{O}\|n(LS)J\rangle \tag{A32}$$

is reduced to the sum of the two-body matrix elements. Here n is the number expressing the states defined by Eqs. (A1–A14).

a. Flavor-decaplet positive parity baryons.

$$\mathscr{O}(1,1)=A_{0s}+\Delta_{0s}, \tag{A33}$$

$$\mathscr{O}(7_0,7_0)=\frac{1}{2}\{A_{0s}+A_{1s}+\Delta_{0s}+\Delta_{1s}\}, \tag{A34}$$

$$\mathscr{O}(7_0,7_2)=\frac{1}{2}C_t\left(0\,\frac{3}{2}\,2\frac{3}{2}\,J\right)\sqrt{5}\,\theta_{1s-0d}, \tag{A35}$$

$$\mathscr{O}(7_0,8_2)=\frac{1}{2\sqrt{2}}C_t\left(0\,\frac{3}{2}\,2\frac{1}{2}\,J\right)\sqrt{10}\theta_{1s-0d}, \tag{A36}$$

$$\mathscr{O}(8_0,8_0)=\frac{1}{4}\{A_{0s}+A_{1s}+2A_{0p}+\Delta_{0s}+\Delta_{1s}-6\Delta_{0p}\}, \tag{A37}$$

$$\mathscr{O}(8_0,7_2)=\frac{1}{2\sqrt{2}}C_t\left(0\,\frac{1}{2}\,2\,\frac{3}{2}\,J\right)(-\sqrt{10})\,\theta_{1s-0d}, \tag{A38}$$

$$\mathscr{O}(7_2,7_2)=\frac{1}{2}\Bigg\{A_{0s}+A_{0d}+\Delta_{0s}+\Delta_{0d}+C_{ls}\left(2\,\frac{3}{2}\,2\,\frac{3}{2}\,J\right)\left(-\sqrt{\frac{3}{2}}\right)\chi_{0d}+C_t\left(2\,\frac{3}{2}\,2\,\frac{3}{2}\,J\right)\left(-\sqrt{\frac{10}{7}}\right)\theta_{0d}\Bigg\}, \tag{A39}$$

$$\mathscr{O}(7_2,8_2)=\frac{1}{2\sqrt{2}}\Bigg\{C_{ls}\left(2\,\frac{3}{2}\,2\frac{1}{2}\,J\right)\left(-\sqrt{\frac{3}{5}}\right)\chi_{0d}+C_t\left(2\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{20}{7}}\right)\theta_{0d}\Bigg\}, \tag{A40}$$

$$\mathscr{O}(8_2,8_2)=\frac{1}{4}\Bigg\{A_{0s}+A_{0d}+2A_{0p}+\Delta_{0s}+\Delta_{0d}-6\Delta_{0p}+C_{ls}\left(2\,\frac{1}{2}\,2\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{12}{5}}\right)\chi_{0d}\Bigg\}. \tag{A41}$$

b. Flavor-octet positive parity baryons.

$$\mathscr{O}(2,2)=\frac{1}{2}\{A_{0s}+\Delta_{0s}+\overline{A_{0s}}-3\overline{\Delta_{0s}}\}, \tag{A42}$$

$$\mathscr{O}(9_0,9_0)=\frac{1}{4}\{A_{0s}+A_{1s}+2\overline{A_{0p}}+\Delta_{0s}+\Delta_{1s}+2\overline{\Delta_{0p}}\}, \tag{A43}$$

$$\mathscr{O}(9_0,12)=-\frac{1}{2}C_{ls}\left(0\,\frac{3}{2}\,1\,\frac{1}{2}\,J\right)\sqrt{\frac{1}{3}}\,\overline{\chi_{0p}}, \tag{A44}$$

$$\mathscr{O}(9_0,9_2)=\frac{1}{4}C_t\left(0\,\frac{3}{2}\,2\,\frac{3}{2}\,J\right)\{(-\sqrt{2})2\,\overline{\theta_{0p}}+\sqrt{5}\,\theta_{1s-0d}\}, \tag{A45}$$

$$\mathscr{O}(9_0,10_2)=\frac{1}{4}C_t\left(0\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\sqrt{10}\theta_{1s-0d}, \tag{A46}$$

$$\mathscr{O}(9_0,11_2)=\frac{1}{4\sqrt{2}}C_t\left(0\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\{(-\sqrt{4})2\,\overline{\theta_{0p}}-\sqrt{10}\theta_{1s-0d}\}, \tag{A47}$$

$$\mathscr{O}(10_0,10_0)=\frac{1}{4}\{A_{0s}+A_{1s}+\overline{A_{0s}}+\overline{A_{1s}}+\Delta_{0s}+\Delta_{1s}-3\overline{\Delta_{0s}}-3\overline{\Delta_{1s}}\}, \tag{A48}$$

$$\mathscr{O}(10_0,11_0)=\frac{1}{4\sqrt{2}}\{A_{0s}-A_{1s}-\overline{A_{0s}}+\overline{A_{1s}}+\Delta_{0s}-\Delta_{1s}+3\overline{\Delta_{0s}}-3\overline{\Delta_{1s}}\}, \tag{A49}$$

$$\mathscr{O}(10_0,9_2)=\frac{1}{4}C_t\left(0\,\frac{1}{2}\,2\,\frac{3}{2}\,J\right)(-\sqrt{10})\,\theta_{1s-0d}, \tag{A50}$$

$$\mathscr{O}(11_0,11_0)=\frac{1}{8}\{A_{0s}+A_{1s}+2A_{0p}+\overline{A_{0s}}+\overline{A_{1s}}+2\overline{A_{0p}}+\Delta_{0s}+\Delta_{1s}-6\Delta_{0p}-3\overline{\Delta_{0s}}-3\overline{\Delta_{1s}}+2\overline{\Delta_{0p}}\}, \tag{A51}$$

$$\mathscr{O}(11_0,12)=-\frac{1}{2\sqrt{2}}C_{ls}\left(0\,\frac{1}{2}\,1\,\frac{1}{2}\,J\right)\sqrt{\frac{4}{3}}\,\overline{\chi_{0p}}, \tag{A52}$$

$$\mathscr{O}(11_0,9_2)=\frac{1}{4\sqrt{2}}C_t\left(0\,\frac{1}{2}\,2\,\frac{3}{2}\,J\right)\{\sqrt{4}\quad 2\,\overline{\theta_{0p}}-(-\sqrt{10})\,\overline{\theta_{1s-0d}}\}, \tag{A53}$$

$$\mathscr{O}(9_2,9_2)=\frac{1}{4}\Bigg\{A_{0s}+A_{0d}+2\overline{A_{0p}}+\Delta_{0s}+\Delta_{0d}+2\overline{\Delta_{0p}}+C_{ls}\left(2\,\frac{3}{2}\,2\,\frac{3}{2}\,J\right)\left[\left(-\sqrt{\frac{3}{8}}\right)2\,\overline{\chi_{0p}}+\left(-\sqrt{\frac{3}{2}}\right)\chi_{0d}\right]+C_t\left(2\,\frac{3}{2}\,2\,\frac{3}{2}\,J\right)\times\left[\left(-\sqrt{\frac{7}{10}}\right)2\,\overline{\theta_{0p}}+\left(-\sqrt{\frac{10}{7}}\right)\theta_{0d}\right]\Bigg\}, \tag{A54}$$

$$\mathscr{O}(9_2,10_2)=\frac{1}{4}\Bigg\{C_{ls}\left(2\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{3}{5}}\right)\chi_{0d}+C_t\left(2\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{20}{7}}\right)\theta_{0d}\Bigg\}, \tag{A55}$$

$$\mathcal{O}(9_2,11_2)=\frac{1}{4\sqrt{2}}\left\{C_{ls}\left(2\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\left[\left(-\sqrt{\frac{3}{20}}\right)2\overline{\chi_{0p}}-\left(-\sqrt{\frac{3}{5}}\right)\chi_{0d}\right]+C_t\left(2\,\frac{3}{2}\,2\,\frac{1}{2}\,J\right)\times\left[\left(-\sqrt{\frac{7}{5}}\right)2\overline{\theta_{0p}}-\left(-\sqrt{\frac{20}{7}}\right)\theta_{0d}\right]\right\}, \tag{A56}$$

$$\mathcal{O}(9_2,12)=-\frac{1}{2}\left\{C_{ls}\left(2\,\frac{3}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{1}{12}}\right)\overline{\chi_{0p}}+C_t\left(2\,\frac{3}{2}\,1\,\frac{1}{2}\,J\right)(-\sqrt{3})\overline{\theta_{0p}}\right\}, \tag{A57}$$

$$\mathcal{O}(10_2,10_2)=\frac{1}{4}\left\{A_{0s}+A_{0d}+\overline{A_{0s}}+\overline{A_{0d}}+\Delta_{0s}+\Delta_{0d}-3\overline{\Delta_{0s}}-3\overline{\Delta_{0d}}+C_{ls}\left(2\,\frac{1}{2}\,2\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{12}{5}}\right)\chi_{0d}\right\}, \tag{A58}$$

$$\mathcal{O}(10_2,11_2)=\frac{1}{4\sqrt{2}}\left\{A_{0s}-A_{0d}-\overline{A_{0s}}+\overline{A_{0d}}+\Delta_{0s}-\Delta_{0d}+3\overline{\Delta_{0s}}-3\overline{\Delta_{0d}}-C_{ls}\left(2\,\frac{1}{2}\,2\,\frac{1}{2}\,J\right)\times\left(-\sqrt{\frac{12}{5}}\right)\chi_{0d}\right\}, \tag{A59}$$

$$\mathcal{O}(11_2,11_2)=\frac{1}{8}\left\{A_{0s}+A_{0d}+2A_{0p}+\overline{A_{0s}}+\overline{A_{0d}}+2\overline{A_{0p}}+\Delta_{0s}+\Delta_{0d}-6\Delta_{0p}-3\overline{\Delta_{0s}}-3\overline{\Delta_{0d}}+2\overline{\Delta_{0p}}+C_{ls}\left(2\,\frac{1}{2}\,2\,\frac{1}{2}\,J\right)\left[\left(-\sqrt{\frac{3}{5}}\right)2\overline{\chi_{0p}}\right]+\left(-\sqrt{\frac{12}{5}}\right)\chi_{0d}\right\}, \tag{A60}$$

$$\mathcal{O}(11_2,12)=-\frac{1}{2\sqrt{2}}\,C_{ls}\left(2\,\frac{1}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{1}{3}}\right)\overline{\chi_{0p}}, \tag{A61}$$

$$\mathcal{O}(12,12)=\frac{1}{2}\left\{A_{0p}+\overline{A_{0p}}-3\Delta_{0p}+\overline{\Delta_{0p}}+C_{ls}\left(1\,\frac{1}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{1}{3}}\right)\overline{\chi_{0p}}\right\}. \tag{A62}$$

c. Flavor-singlet positive parity baryons.

$$\mathcal{O}(13_0,13_0)=\frac{1}{4}\{2\overline{A_{0p}}+\overline{A_{0s}}+\overline{A_{1s}}+2\overline{\Delta_{0p}}-3\overline{\Delta_{0s}}-3\overline{\Delta_{1s}}\}, \tag{A63}$$

$$\mathcal{O}(13_0,14)=-\frac{1}{\sqrt{2}}\,C_{ls}\left(0\,\frac{1}{2}\,1\,\frac{3}{2}\,J\right)\left(-\sqrt{\frac{1}{3}}\right)\overline{\chi_{0p}}, \tag{A64}$$

$$\mathcal{O}(13_2,13_2)=\frac{1}{4}\left\{2\overline{A_{0p}}+\overline{A_{0s}}+\overline{A_{0d}}+2\overline{\Delta_{0p}}-3\overline{\Delta_{0s}}-3\overline{\Delta_{0d}}+C_{ls}\left(2\,\frac{1}{2}\,2\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{3}{5}}\right)2\overline{\chi_{0p}}\right\}, \tag{A65}$$

$$\mathcal{O}(13_2,14)=-\frac{1}{\sqrt{2}}\left\{C_{ls}\left(2\,\frac{1}{2}\,1\,\frac{3}{2}\,J\right)\sqrt{\frac{1}{12}}\,\overline{\chi_{0p}}+C_t\left(2\,\frac{1}{2}\,1\,\frac{3}{2}\,J\right)\sqrt{3}\,\overline{\theta_{0p}}\right\}, \tag{A66}$$

$$\mathcal{O}(14,14)=\overline{A_{0p}}+\overline{\Delta_{0p}}+C_{ls}\left(1\,\frac{3}{2}\,1\,\frac{3}{2}\,J\right)\left(-\sqrt{\frac{5}{24}}\right)\overline{\chi_{0p}}+C_t\left(1\,\frac{3}{2}\,1\,\frac{3}{2}\,J\right)\sqrt{\frac{5}{6}}\,\overline{\theta_{0p}}. \tag{A67}$$

d. Negative parity baryons.

$$\mathcal{O}(3,3)=\frac{1}{2}\{A_{0s}+A_{0p}+\Delta_{0s}-3\Delta_{0p}\}, \tag{A68}$$

$$\mathcal{O}(4,4)=\frac{1}{2}\left\{A_{0s}+\Delta_{0s}+\overline{A_{0p}}+\overline{\Delta_{0p}}+C_{ls}\left(1\,\frac{3}{2}\,1\,\frac{3}{2}\,J\right)\times\left(-\sqrt{\frac{5}{6}}\right)\overline{\chi_{0p}}+C_t\left(1\,\frac{3}{2}\,1\,\frac{3}{2}\,J\right)\left(-\sqrt{\frac{10}{3}}\right)\overline{\theta_{0p}}\right\}, \tag{A69}$$

$$\mathcal{O}(4,5)=\frac{1}{2\sqrt{2}}\left\{C_{ls}\left(1\,\frac{3}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{1}{3}}\right)\overline{\chi_{0p}}+C_t\left(1\,\frac{3}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{20}{3}}\right)\overline{\theta_{0p}}\right\}, \tag{A70}$$

$$\mathcal{O}(5,5)=\frac{1}{4}\left\{A_{0s}+A_{0p}+\Delta_{0s}-3\Delta_{0p}+\overline{A_{0s}}+\overline{A_{0p}}-3\overline{\Delta_{0s}}+\overline{\Delta_{0p}}+C_{ls}\left(1\,\frac{1}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{4}{3}}\right)\overline{\chi_{0p}}\right\}, \tag{A71}$$

$$\mathcal{O}(6,6)=\frac{1}{2}\left\{\overline{A_{0s}}+\overline{A_{0p}}-3\overline{\Delta_{0s}}+\overline{\Delta_{0p}}+C_{ls}\left(1\,\frac{1}{2}\,1\,\frac{1}{2}\,J\right)\left(-\sqrt{\frac{4}{3}}\right)\overline{\chi_{0p}}\right\}. \tag{A72}$$

$C_{ls}(L'S'LSJ)$ and $C_t(L'S'LSJ)$ are defined as

$$C_\alpha(L'S'LSJ)=\sqrt{(2J+1)(2L'+1)(2L+1)(2S'+1)(2S+1)}\begin{Bmatrix} L' & S' & J \\ \lambda & \lambda & 0 \\ L & S & J \end{Bmatrix}, \qquad \text{(A73)}$$

with $\lambda=1$ and 2 for $\alpha=ls$ and t, respectively. The radial integrations are

$$A_{nl}=3\langle nl|\mathcal{O}^O_{c\,(ij)=(12)}|nl\rangle\langle\mathcal{O}^C_{c\,(ij)=(12)}\rangle, \qquad \text{(A74)}$$

$$\Delta_{nl}=3\langle nl|\mathcal{O}^O_{ss12}|nl\rangle\langle\mathcal{O}^C_{ss}\rangle, \qquad \text{(A75)}$$

$$\chi_{nl}=3\langle nl|\mathcal{O}^O_{ls12}|nl\rangle\langle\mathcal{O}^C_{ls}\rangle, \qquad \text{(A76)}$$

$$\theta_{nl}=3\langle nl|\mathcal{O}^O_{t12}|nl\rangle\langle\mathcal{O}^C_{t}\rangle, \qquad \text{(A77)}$$

$$\theta_{n'l'-nl}=3\langle n'l'|\mathcal{O}^O_{t12}|nl\rangle\langle\mathcal{O}^C_{t}\rangle. \qquad \text{(A78)}$$

Those with bars are similarly defined. From Eqs. (A33)–(A72), one can actually see that only the flavor-antisymmetric $0p$ pairs and the flavor-symmetric $0d$ pairs are relevant for the noncentral part in the single baryons.

For OGE, the terms in Eq. (A31) are

$$\mathcal{O}^O_{ss}=\overline{\mathcal{O}}^O_{ss}=-4\pi\alpha_s\,\frac{1}{6m^2}\,\delta(\mathbf{r}), \qquad \text{(A79)}$$

$$\mathcal{O}^O_{ls}=\overline{\mathcal{O}}^O_{ls}=-4\pi\alpha_s\,\frac{3}{2m^2}\,\frac{1}{4\pi r^3}, \qquad \text{(A80)}$$

$$\mathcal{O}^O_{t}=\overline{\mathcal{O}}^O_{t}=-4\pi\alpha_s\,\frac{1}{4m^2}\,\frac{1}{4\pi r^3}, \qquad \text{(A81)}$$

with

$$\mathcal{O}^C_\alpha=\frac{\lambda\cdot\lambda}{4}.$$

Therefore, the two-body matrix elements become

$$\Delta_{0s}=\overline{\Delta}_{0s}=\frac{1}{2}\,\mu_{\rm OGE}, \qquad \text{(A82)}$$

$$\Delta_{1s}=\overline{\Delta}_{1s}=\frac{3}{4}\,\mu_{\rm OGE}, \qquad \text{(A83)}$$

$$\Delta_{nl}=\overline{\Delta}_{nl}=0\quad(l>0), \qquad \text{(A84)}$$

$$\overline{\chi}_{0p}=\frac{3}{2}\,\mu_{\rm OGE}, \qquad \text{(A85)}$$

$$\chi_{0d}=\frac{3}{5}\,\mu_{\rm OGE}, \qquad \text{(A86)}$$

$$\overline{\theta}_{0p}=\frac{1}{4}\,\mu_{\rm OGE}, \qquad \text{(A87)}$$

$$\theta_{0d}=\frac{1}{10}\,\mu_{\rm OGE}, \qquad \text{(A88)}$$

$$\theta_{0d-1s}=\sqrt{\frac{1}{160}}\,\mu_{\rm OGE}, \qquad \text{(A89)}$$

with

$$\mu_{\rm OGE}=\frac{1}{2}\,(A_{0s}+\Delta_{0s}-\overline{A}_{0s}+3\overline{\Delta}_{0s})=\alpha_s\,\frac{4}{3}\,\frac{1}{\sqrt{2\pi}}\,\frac{1}{m^2b^3},$$

which corresponds to the contribution of OGE to the S-wave N-Δ mass difference (=293 MeV).

As for III all operators without a bar vanish. The flavor-antisymmetric operators are

$$\overline{\mathcal{O}}^O_{ss}=-V_0^{(2)}\,\frac{3}{16}\,\delta(\mathbf{r}), \qquad \text{(A90)}$$

$$\overline{\mathcal{O}}^O_{ls}=V_0^{(2)}\,\frac{9}{4m^2}\,\frac{\delta(r)}{4\pi r^4}, \qquad \text{(A91)}$$

$$\overline{\mathcal{O}}^O_{t}=V_0^{(2)}\,\frac{5}{4m^2}\,\frac{\delta(r)}{4\pi r^4}, \qquad \text{(A92)}$$

with

$$\mathcal{O}^C_{ss}=\mathcal{O}^C_{ls}=1, \qquad \text{(A93)}$$

$$\mathcal{O}^C_{t}=1-\frac{3}{4}\,\frac{\lambda\cdot\lambda}{4}. \qquad \text{(A94)}$$

Thus we obtain the two-body matrix elements for III as

$$\overline{\Delta}_{0s}=\frac{1}{4}\,\mu_{\rm III}, \qquad \text{(A95)}$$

$$\overline{\Delta}_{1s}=\frac{3}{8}\,\mu_{\rm III}, \qquad \text{(A96)}$$

$$\overline{\Delta}_{nl}=0\quad(l>0), \qquad \text{(A97)}$$

$$\overline{\chi}_{0p}=-\frac{1}{m^2b^2}\,\mu_{\rm III}, \qquad \text{(A98)}$$

$$\overline{\chi}_{nl}=0\quad(l>1), \qquad \text{(A99)}$$

$$\overline{\theta}_{0p}=-\frac{5}{6}\,\frac{1}{m^2b^2}\,\mu_{\rm III}, \qquad \text{(A100)}$$

$$\overline{\theta}_{nl}=0\quad(l>2), \qquad \text{(A101)}$$

with

$$\mu_{\rm III}=-\frac{9}{4}V_0^{(2)}\,(2\pi b^2)^{-3/2}.$$

The mass of baryons are obtained from Eqs. (A33)–(A72), by diagonalizing them if necessary, and adding the central part.

2. Two nucleons

Here we consider a six-quark system $(0s)^5(0p)$, with quantum number of two nucleons with relative partial P wave. The wave function is

$$\Psi=\mathcal{A}_q\Phi\equiv\mathcal{A}_q\{\phi_N^2\psi_{0p}(R)\}, \tag{A102}$$

where $\mathcal{A}_q$ is an antisymmetrizer with respect to all six quarks, ϕ_N is the nucleon wave function defined by the previous section, and the $\psi_{0p}(R)$ is a P-wave harmonic oscillator with size parameter b and $R=(r_1+r_2+r_3-r_4-r_5-r_6)/\sqrt{6}$.

Noncentral operators relevant to this state are orbitally antisymmetric two terms in Eq. (9). The expectation value by the above state can written as

$$\langle\Psi|\mathcal{O}|\Psi\rangle=\left\langle\Phi\left|\sum_{i<j}(\overline{\mathcal{O}}{}^A_{ij}+\mathcal{O}^S_{ij})\mathcal{A}_q\right|\Phi\right\rangle\Big/\langle\Phi|\mathcal{A}_q|\Phi\rangle \tag{A103}$$

$$=\left\langle\Phi\left|\sum_{i<j}(\overline{\mathcal{O}}{}^A_{ij}+\mathcal{O}^S_{ij})(1-9P_{36})\right|\Phi\right\rangle\Big/\langle\Phi|(1-9P_{36})|\Phi\rangle, \tag{A104}$$

where $P_{36}\equiv P^c_{36}P^f_{36}P^s_{36}P^O_{36}$ is the exchange operator for quark i, and j in the color, flavor, spin and orbital space. The numerator is

$$\begin{aligned}&9\langle\Phi|(\overline{\mathcal{O}}{}^A_{36}+\mathcal{O}^S_{36})|\Phi\rangle-9\{4\langle\Phi|(\overline{\mathcal{O}}{}^A_{14}+\mathcal{O}^S_{14})P_{36}|\Phi\rangle+\langle\Phi|(\overline{\mathcal{O}}{}^A_{36}+\mathcal{O}^S_{36})P_{36}|\Phi\rangle\}\\&=9\{2\langle\Phi|(\overline{\mathcal{O}}{}^A_{36}+\mathcal{O}^S_{36})|\Phi\rangle-4\langle\Phi|(\overline{\mathcal{O}}{}^A_{14}+\mathcal{O}^S_{14})P_{36}|\Phi\rangle\}\\&=18\{\langle\Phi|\overline{\mathcal{O}}{}^A_{36}|\Phi\rangle+\langle\Phi|\mathcal{O}^S_{36}|\Phi\rangle-2\langle\Phi|\mathcal{O}^S_{14}P_{36}|\Phi\rangle\}.\end{aligned} \tag{A105}$$

Here we use

$$\mathcal{O}_{16}P_{36}=P_{36}\mathcal{O}_{13}, \tag{A106}$$

$$\langle\Phi|\mathcal{O}_{13}|\Phi\rangle=\langle\Phi|\mathcal{O}_{13}P_{36}|\Phi\rangle=0, \tag{A107}$$

$$\langle\Phi|\mathcal{O}_{36}P_{36}|\Phi\rangle=-\langle\Phi|\mathcal{O}_{36}|\Phi\rangle. \tag{A108}$$

For the spin-orbit part, $\mathcal{O}_{ij}=f(r_{ij})L_{ij}\cdot S_{ij}$, each term in Eq. (A105) can be evaluated as

$$\langle\Phi|\mathcal{O}_{36}|\Phi\rangle=\frac{1}{16}\langle f(r_{36})L_{36}\cdot S_{36}(1\mp P^c_{36})(1\mp P^f_{36})(1+P^s_{36})\times(1-P^o_{36})\rangle \tag{A109}$$

$$=\frac{1}{2}(1\mp\langle P^c_{36}\rangle)\,\frac{1}{2}(\langle f(r_{36})L_{36}(1-P^o_{36})\rangle)\times\frac{1}{4}\langle S_{36}(1+P^s_{36}\mp P^f_{36}\mp P^{sf}_{36})\rangle, \tag{A110}$$

$$\langle\Phi|\mathcal{O}_{14}P_{36}|\Phi\rangle=\frac{1}{16}\langle f(r_{14})L_{14}\cdot S_{14}(1\mp P^c_{14})(1\mp P^f_{14})\times(1+P^s_{14})(1-P^o_{14})P_{36}\rangle \tag{A111}$$

$$=\frac{1}{2}(\langle P^c_{36}\rangle\mp\langle P^c_{14}P^c_{36}\rangle)\times\frac{1}{2}(\langle f(r_{14})L_{14}(1-P^o_{14})P^o_{36}\rangle)\times\frac{1}{4}\langle S_{14}(1+P^s_{14}\mp P^f_{14}\mp P^{sf}_{14})P^{sf}_{36}\rangle. \tag{A112}$$

The $\mp$ reads $-$ for $\overline{\mathcal{O}}{}^A$ and $+$ for $\mathcal{O}^S$. The color part is $\langle P^c_{36}\rangle=\langle P^c_{14}P^c_{36}\rangle=1/3$. The spin-flavor part can be calculated directly:

$$\langle p^\uparrow p^\uparrow|S_{36}(1+P^s_{36}\mp P^f_{36}\mp P^{sf}_{36})|p^\uparrow p^\uparrow\rangle=\frac{1}{4}\left\{\frac{1}{3}+\frac{1}{3}\mp\frac{7}{27}\mp\frac{7}{27}\right\}=\begin{cases}\dfrac{1}{27}\\[4pt]\dfrac{8}{27},\end{cases} \tag{A113}$$

$$\langle p^\uparrow p^\uparrow | S_{14}(1+P^s_{14}\mp P^f_{14}\mp P^{sf}_{14})P^{sf}_{36}|p^\uparrow p^\uparrow\rangle = \frac{1}{4}\left\{\frac{5}{81}+\frac{5}{81}\mp\frac{5}{81}\mp\frac{5}{81}\right\} = \begin{cases} 0 \\ \frac{5}{81}. \end{cases} \tag{A114}$$

Also,

$$\langle f(r_{36})L_{36}(1-P^o_{36})\rangle = \tfrac{2}{9}\overline{\chi^A_{0p}} \text{ (or } \tfrac{2}{9}\chi^S_{0p}) \text{ for } \overline{\mathcal{O}^A} \text{ (or } \mathcal{O}^S)$$

$$\text{and } \langle f(r_{14})L_{14}(1-P^o_{14})P^o_{36}\rangle = \tfrac{2}{9}\chi^S_{0p}.$$

The denominator of Eq. (A106) is 50/81 [5,7]. Thus the matrix element Eq. (A105) is

$$\langle\Psi|\mathcal{O}|\Psi\rangle = \frac{3}{75}\overline{\chi^A_{0p}} + \frac{38}{75}\chi^S_{0p}. \tag{A115}$$

The coefficients are listed as C_{NN3P} in Table II. The tensor part can similarly be obtained:

$$\langle\Psi|\mathcal{O}|\Psi\rangle = -\frac{4}{75}\overline{\theta^A_{0p}} + \frac{21}{75}\theta^S_{0p} \tag{A116}$$

by using

$$\langle p^\uparrow p^\uparrow|\sigma^+_3\sigma^+_6(1+P^s_{36}\mp P^f_{36}\mp P^{sf}_{36})|p^\downarrow p^\downarrow\rangle = \frac{1}{4}\left\{\frac{9}{81}+\frac{9}{81}+\mp\frac{17}{81}\mp\frac{17}{81}\right\} = \begin{cases} -\frac{4}{81} \\ \frac{13}{81}, \end{cases} \tag{A117}$$

$$\langle p^\uparrow p^\uparrow|\sigma^+_1\sigma^+_4(1+P^s_{14}\mp P^f_{14}\mp P^{sf}_{14})P^{sf}_{36}|p^\downarrow p^\downarrow\rangle = \frac{1}{4}\left\{\frac{5}{162}+\frac{5}{162}\mp\frac{5}{162}\mp\frac{5}{162}\right\} = \begin{cases} 0 \\ \frac{5}{162}. \end{cases} \tag{A118}$$

One can clearly see that the flavor symmetric part of the operator is dominant in this state both for the spin-orbit term and the-tensor term.

The actual value is obtained by substituting χ^A_{0p} by $\overline{\chi_{0p}}$ in Eqs. (A85) and (A98) and $\overline{\chi^S_{0p}}$ by $-3/4\mu_{\rm OGE}$, and by substituting θ^A_{0p} by θ_{0p} in Eqs. (A87) and (A100) and θ^S_{0p} by $-1/8\mu_{\rm OGE}$.

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