# Perturbative pion-photon transition form factors with transverse momentum corrections

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We perform a perturbative QCD analysis of the quark transverse momentum effect on the pion-photon transition form factors $F_{\pi\gamma}$ and $F_{\pi\gamma^*}$ in the standard light-cone formalism, with two phenomenological models of the wave function as the input of the nonperturbative aspect of the pion. We point out that the transverse momentum dependence in both the numerator and the denominator of the hard scattering amplitude is of the same importance and should be considered consistently. It is shown that after taking into account the quark transverse momentum corrections, the results obtained from different model wave functions are consistent with the available experimental data at finite $Q^2$. [S0556-2821(96)05511-7]

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## I. INTRODUCTION

The pion-photon transition form factor $F_{\pi\gamma}(Q^2)$ is a simple example for the perturbative analysis to exclusive processes and was first analyzed by Lepage and Brodsky [1]. They predicted $F_{\pi\gamma}(Q^2)$ by neglecting $k_\perp$ relative to $q_\perp$:

$$F_{\pi\gamma}(Q^2)=\frac{2}{\sqrt{3}Q^2}\int\frac{[dx]}{x_1x_2}\phi_\pi(x)\left[1+O\left(\alpha_s,\frac{m^2}{Q^2}\right)\right],\quad (1)$$

and $Q^2F_{\pi\gamma}(Q^2)$ would be essentially constant as $Q^2\to\infty$. This approximation would be valid if the wave function is peaked at low $k_\perp$ ($k_\perp$ is the transverse momentum of quark) so that $x_1x_2Q^2$ in the hard scattering amplitude dominates the denominator. However, at the end-point region $x_i\to 0,1$ and $Q^2\sim$ a few GeV$^2$ the wave function does not guarantee the $k_\perp$ negligible. One should take into account $k_\perp$ corrections from both the hard scattering amplitude and the wave function.

Recently, Refs. [2,3] calculated the $\pi$-$\gamma$ transition form factor within the covariant hard scattering approach including transverse momentum effects and Sudakov corrections [4] by neglecting the quark masses, the mass of the pion meson, and the $k_\perp$ dependence in the numerator of $T_H$. Their results show that Sudakov suppression in the form factor $F_{\pi\gamma}(Q^2)$ is less important than in other exclusive channels and the Chernyak-Zhitnitsky (CZ) wave function should be discarded by fitting the experimental data. However, as we know, the $k_\perp$ dependence of the wave function in Ref. [2] is the same in the different models and it may be difficult to draw a conclusion which excludes the CZ wave function. We will reexamine this problem in the present paper.

The light-cone formalism provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and for the application of perturbative QCD (PQCD) to exclusive processes [5,6]. In this formalism, the hadronic wave function which describes the hadronic composite state at a particular $\tau$ is expressed in terms of a series of light-cone wave functions in Fock-state basis:

$$|\pi\rangle=\sum|q\overline{q}\rangle\psi_{q\overline{q}}+\sum|q\overline{q}g\rangle\psi_{q\overline{q}g}+\cdots,\quad (2)$$

and the temporal evolution of the state is generated by the light-cone Hamiltonian $H_{\rm LC}=P^-=P^0-P^3$. Furthermore the vacuum state in the light-cone Fock basis is an exact eigenstate of the full Hamiltonian $H_{\rm LC}$. Thus all bare quanta in a hadronic Fock state are part of the hadron. (This point is very different from that in the equal-$t$ perturbative theory in which the quantization is performed at a given time $t$.) Light-cone PQCD is very convenient for light-cone-dominated processes. For the detail quantization rules we refer to literatures [1,5–7]. The more important point for practical calculation is that the contributions coming from higher Fock states are suppressed by $1/Q^n$, therefore we can employ only the valence state to the leading order for large $Q^2$. In this paper, we analyze the quark transverse momentum effects on the pion-photon transition form factors $F_{\pi\gamma}$ and $F_{\pi\gamma^*}$ at finite $Q^2$ in the standard light-cone formalism, with two phenomenological models of wave function as the input of the nonperturbative aspect of the pion. We demonstrate that the PQCD predictions with different models of wave function are consistent with the available experimental data by taking into account the quark transverse momentum.

## II. THE PION-PHOTON TRANSITION FORM FACTORS $F_{\pi\gamma}$ AND $F_{\pi\gamma^*}$

The $\pi$-$\gamma$ transition form factor $F_{\pi\gamma}$ is defined from the $\pi^0\gamma\gamma^*$ vertex in the amplitude of $e\pi\to e\gamma$:

$$\Gamma_\mu=-ie^2F_{\pi\gamma}\epsilon_{\mu\nu\alpha\beta}p_\pi^\mu\epsilon^\alpha q^\beta,\quad (3)$$

where $p_\pi$ and $q$ are the momenta of the incident pion and the virtual photon, respectively, and $\epsilon$ is the polarization vector of the final (on-shell) photon. We adopt the standard momentum assignment at the ''infinite-momentum'' frame [1]:

$$p_\pi=(p^+,p^-,p_\perp)=(1,0,0_\perp),$$

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$$q=(0,q_\perp^2,q_\perp), \tag{4}$$

where $p^+$ is arbitrary. For simplicity we choose $p^+=1$, and we have $q^2=-q_\perp^2=-Q^2$. Then the $F_{\pi\gamma}$ is given by

$$F_{\pi\gamma}(Q^2)=\frac{\Gamma^+}{-ie(\epsilon_\perp\times q_\perp)}, \tag{5}$$

where $\epsilon=(0,0,\epsilon_\perp)$, $\epsilon_\perp\cdot q_\perp=0$ is chosen and $\epsilon_\perp\times q_\perp=\epsilon_{\perp1}q_{\perp2}+\epsilon_{\perp2}q_{\perp1}$. Since the contributions coming from higher Fock states are suppressed, we take into account only the conventional lowest Fock state of pion meson:

$$\psi_\pi=\frac{\delta_b^a}{\sqrt{n_c}}\frac{1}{\sqrt{2}}\left[\frac{u_\uparrow\overline{u_\downarrow}-u_\downarrow\overline{u_\uparrow}}{\sqrt{2}}-\frac{d_\uparrow\overline{d_\downarrow}-d_\downarrow\overline{d_\uparrow}}{\sqrt{2}}\right]\frac{\psi(x_i,k_\perp)}{\sqrt{x_1x_2}}. \tag{6}$$

The leading-order contribution to $F_{\pi\gamma}$ is calculated from Fig. 1 in light-cone PQCD [1]:

$$F_{\pi\gamma}(Q^2)=\frac{\sqrt{n_c}(e_u^2-e_d^2)}{i(\epsilon_\perp\times q_\perp)}\int_0^1[dx]\int_0^\infty\frac{d^2k_\perp}{16\pi^3}\psi(x_i,k_\perp)\times\left[\frac{\overline{v_\downarrow}(x_2,-k_\perp)}{\sqrt{x_2}}\not{\epsilon}\frac{u_\uparrow(x_1,k_\perp+q_\perp)}{\sqrt{x_1}}\frac{\overline{u_\uparrow}(x_1,k_\perp+q_\perp)}{\sqrt{x_1}}\gamma^+\frac{u_\uparrow(x_1,k_\perp)}{\sqrt{x_1}}\frac{1}{D}+(1\leftrightarrow2)\right], \tag{7}$$

where $[dx]=dx_1dx_2\delta(1-x_1-x_2)$, $e_{u,d}$ are the quark charges in units of $e$, and $D$ is the "energy denominator":

$$D=q_\perp^2-\frac{(k_\perp+q_\perp)^2+m^2}{x_1}-\frac{k_\perp^2+m^2}{x_2}. \tag{8}$$

The quark masses relative to $Q^2$ can be neglected since they are the current quark masses in PQCD calculation. Thus Eq. (7) becomes

$$F_{\pi\gamma}(Q^2)=2\sqrt{n_c}(e_u^2-e_d^2)\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}\psi(x_i,k_\perp)\times T_H(x_1,x_2,k_\perp), \tag{9}$$

where

$$T_H(x_1,x_2,k_\perp)=\frac{q_\perp\cdot(x_2q_\perp+k_\perp)}{q_\perp^2(x_2q_\perp+k_\perp)^2}+(1\leftrightarrow2). \tag{10}$$

The leading behavior of $T_H$ (at large $Q^2$) is obtained by neglecting $k_\perp$ relative to $x_iq_\perp$ [6]:

$$T_H^{\rm LO}(x_1,x_2,k_\perp)=\frac{1}{x_1x_2Q^2}. \tag{11}$$

Thus Eq. (9) becomes Eq. (1) to the leading order. The higher twist corrections to $T_H^{\rm LO}$ take the forms of $(k_\perp/x_iQ)^n$. Equation (10) tells us that there are two factors to contribute for the $k_\perp$ dependence. One is from the PQCD hard scattering amplitude $T_H(x_i,Q,k_\perp)$, and another one is from the nonperturbative wave function $\psi(x_i,k_\perp)$. Although one hopes that the end-point behavior of the wave function can guarantee the reliability of neglecting these higher twist corrections and can suppress the end-point singularity, these corrections may substantially modify the predictions for $F_{\pi\gamma}$ at the momentum transfer $Q$ of a few GeV, especially for the wave function with a milder suppression factor in the end-point region. It should be emphasized that the $k_\perp$ dependence in the numerator and the denominator of $T_H$ is of the same importance. Thus one cannot simply ignore the $k_\perp$ term in the numerator of $T_H$ and Eq. (10) gives the complete expression in the leading order.

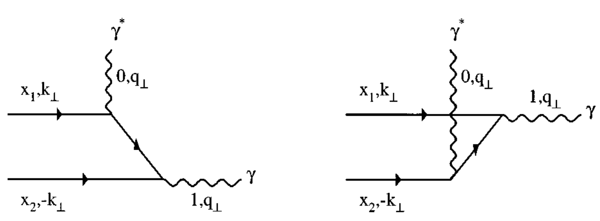


FIG. 1. The lowest order diagrams contributing to $F_{\pi\gamma}$ in light-cone PQCD.

The $\pi$-$\gamma^*$ transition form factor $F_{\pi\gamma^*}$ is extracted from the $\pi^0\gamma^*\gamma^*$ vertex in the two-photon physics. Once again, we employ the standard momentum assignment at the "infinite-momentum" frame:

$$p_\pi=(p^+,p^-,p_\perp)=(1,0,0_\perp),$$

$$q=(0,q_\perp^2-Q'^2,q_\perp),$$

$$q'=(1,q_\perp^2-Q'^2,q_\perp), \tag{12}$$

where $q$ and $q'$ are the momenta of the two photons, respectively, and $q^2=-q_\perp^2=-Q^2$, $q'^2=-Q'^2$. $F_{\pi\gamma^*}$ may be calculated from Fig. 1 by substituting a virtual photon $\gamma^*$ for the on-shell photon $\gamma$, which gives

$$F_{\pi\gamma}(Q^2,Q'^2)=2\sqrt{n_c}(e_u^2-e_d^2)\int_0^1[dx]\int\frac{d^2k_\perp}{16\pi^3}\psi(x_i,k_\perp)\times\left[\frac{q_\perp\cdot(x_2q_\perp+k_\perp)}{q_\perp^2[(x_2q_\perp+k_\perp)^2+x_1x_2q_\perp'^2}+(1\leftrightarrow2)\right]. \tag{13}$$

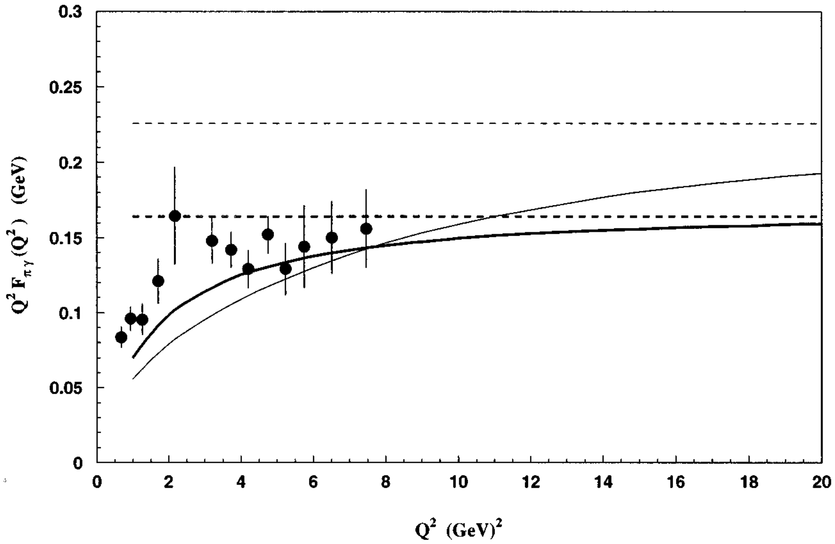


FIG. 2. The $\pi$-$\gamma$ transition form factor. The solid curves are obtained by taking into account the $k_\perp$ dependence, while the dashed curves are results without $k_\perp$ dependence. In both of the cases, the thick curves are calculated from the BHL wave function and the thin curves are for the CZ-like wave function. The data are taken from Refs. [10,11].

The leading order behavior of $F_{\pi\gamma^*}$ can be obtained from Eq. (13) by neglecting $k_\perp$ relative to $x_i q_\perp$ [1]:

$$F_{\pi\gamma^*}(Q^2,Q'^2)=2\sqrt{n_c}(e_u^2-e_d^2)\int_0^1[dx]\phi_\pi(x)\times\left[\frac{1}{x_2Q^2+x_1Q'^2}+(1\leftrightarrow 2)\right]. \quad (14)$$

Similar to the $F_{\pi\gamma}$, Eq. (13) may substantially modify the predictions obtained from Eq. (14).

## III. NUMERICAL CALCULATIONS

In order to see the transverse momentum corrections, we employ two models of wave function: (a) the Brodsky-Huang-Lepage (BHL) wave function [5]

$$\psi^{\rm BHL}(x,k_\perp)=A\exp\left[-\frac{k_\perp^2+m^2}{8\beta^2x(1-x)}\right], \quad (15)$$

where $A=32\ {\rm GeV}^{-1}$, $\beta=0.385$ GeV, and $m=289$ MeV [8]; (b) the CZ-like wave function [9]

$$\psi^{\rm CZ}(x,k_\perp)=A(1-2x)^2\exp\left[-\frac{k_\perp^2+m^2}{8\beta^2x(1-x)}\right], \quad (16)$$

where $A=136\ {\rm GeV}^{-1}$, $\beta=0.455$ GeV, and $m=342$ MeV [8]. These models express that the Fock state wave function $\psi(x_i,k_\perp)$ in the infinite momentum frame depends on the off-shell energy variable $\varepsilon=\Sigma_i^n(k_{\perp i}^2+m_i^2/x_i)$, which was pointed out in Ref. [5].

Substituting the models (15) and (16) into Eqs. (9), (10), and (13), one can get the transverse momentum corrections to the pion-photon transition form factor. The results of $F_{\pi\gamma}$ calculated with $\psi^{\rm BHL}$ and $\psi^{\rm CZ}$ are plotted in Fig. 2. The dashed curves are calculated from the hard scattering amplitude $T_H^{\rm LO}$ in the leading order without the transverse momentum corrections [see Eq. (1)], and the constant predictions with the different wave functions cannot describe the experimental data at momentum transfer of a few GeV$^2$ explicitly. The solid curves are obtained from the complete expression of $T_H$ [see Eq. (10)] with the transverse momentum corrections. As expected, the higher twist correction are suppressed by $1/Q^2$ and the prediction approaches to a constant which depends on the wave function at large $Q$ region. The perturbative predictions are smaller than the experimental data, especially for $Q^2$ of 1–3 GeV$^2$, which supports the suggestion that the higher order effects should provide some contributions at experimental accessible momentum transfer and become more important with $Q^2$ decreasing. Although the asymptotic behaviors of $F_{\pi\gamma}$ predicted from the BHL model and CZ-like model of wave function are quite different, their predictions at finite $Q^2$ obtained with transverse momentum corrections are consistent with the experimental data. The reason is as follows: There are two factors to affect the prediction with the CZ-like wave function. First, the CZ-like model emphasizes the end-point region in a strong way, which enhances its prediction of $F_{\pi\gamma}$. Second, the transverse momentum corrections become more important in the end-point region, which make its prediction decrease. Combining these two factors, the CZ-like model gives a very similar prediction as the BHL model in the finite momentum transfer region. Thus, neither of the two models of wave function can be excluded by the available data of this exclusive process.

The results of $F_{\pi\gamma^*}$ calculated with $\psi^{\rm BHL}$ and $\psi^{\rm CZ}$ are plotted in Fig. 3. Once again, the higher twist corrections are suppressed by $1/Q^2$ and provide more contributions as $Q^2$ decreasing. The predictions of the two models are not dramatically different, whether the transverse momentum corrections are taken into account or not, since the energy scale $Q'$ coming from the other virtual photon makes the hard scattering amplitude not as singular as that in the case of $F_{\pi\gamma}$. It is also difficult to exclude one of the two models of wave function based on $F_{\pi\gamma^*}$. At present, the lack of experimental data makes the examination of higher twist effects in $F_{\pi\gamma^*}$ more complex than that in $F_{\pi\gamma}$. But the future high-luminosity $e^+e^-$ colliders in the ''$\tau$-charm factory'' or ''$B$ factory'' will make this examination feasible.

## IV. SUMMARY

In summary, we emphasize again that the light-cone perturbative QCD is a natural framework to calculate the large-

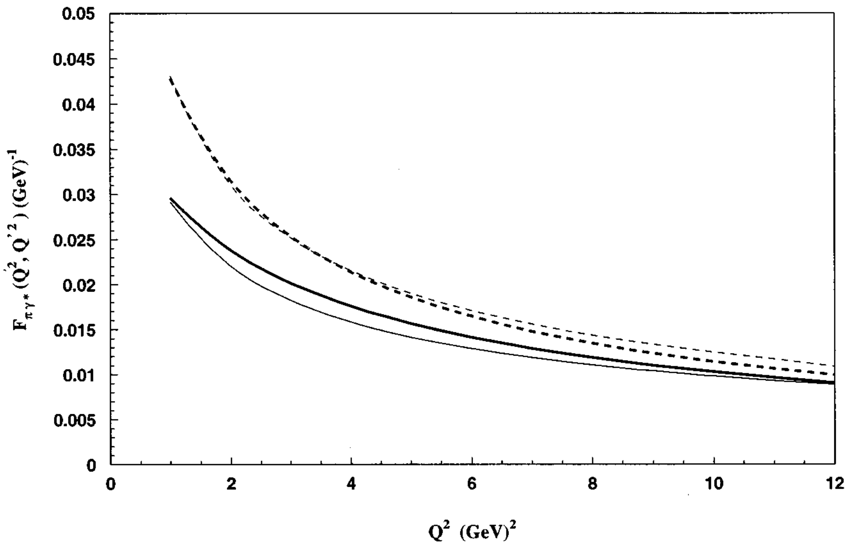


FIG. 3. The $\pi$-$\gamma^*$ form factor at $Q'^2=2$ GeV$^2$. The explanation of the curves is similar to Fig. 2.

momentum-transfer exclusive processes. It is reasonable to get the higher twist corrections by taking into account the quark transverse momentum dependence. As $Q^2\to\infty$, these corrections become negligible. After taking into account the transverse momentum dependence, PQCD may give the correct prediction for the pion-photon transition form factor which is consistent with the experimental data. The transverse momentum dependence in both the numerator and the denominator of the hard scattering amplitude $T_H$ is of the same importance and should be considered consistently. Neither the BHL model nor the CZ-like model, the two typical models of wave functions, can be excluded by the available data of the pion-photon transition form factors. The future "$\tau$-charm factory" as well as "$B$ factory" will provide the opportunity to examine the higher twist effects in the perturbative calculation of $F_{\pi\gamma*}$ and to test the validity of the perturbative analysis.

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