Strong and electromagnetic interactions of heavy baryons

R. Delbourgo* and Dongsheng Liu†
Physics Department, University of Tasmania, GPO Box 252C Hobart, Australia 7001

(Received 5 December 1995)

It is possible to express all the strong and electromagnetic interactions of ground state hadrons in terms of a single coupling constant and the constituent quark masses $m_{ud} \simeq 0.34$ GeV, $m_s \simeq 0.43$ GeV, and $m_c \simeq 1.5$ GeV by using spin-flavor relativistic supermultiplet theory. We show that this produces results which are generally accurate to within 10%. We thereby predict widths and couplings of recently and soon-to-be discovered heavy hadrons. [S0556-2821(96)05111-3]

PACS number(s): 13.30.−a, 11.30.Hv, 13.40.Hq, 14.20.−c

I. INTRODUCTION

It has been almost 30 years since SU(6) theory [1] and its relativistic generalization [2] were conceived, before even the birth of quantum chromodynamics (QCD). Nowadays it is largely forgotten that, apart from weak interactions, it was spectacularly successful at predicting the strong and electromagnetic decays of hadrons. Further, it was realized in 1966 that the predictions could only be regarded as ''tree-level'' or effective interactions between the hadronic states rather than a fully fledged description, since unitarity provided definite corrections which broke the spin-flavor symmetry. However, thanks to the work of Isgur and Wise [3], today the symmetry is envisaged as applying to hadrons at equal velocity containing one heavy quark, since the QCD Lagrangian possesses such a symmetry in the heavy mass limit [4]. The current description popularly treats the light meson through chiral perturbation theory even though previous history indicates that they are equally well described by spin-flavor symmetry, weak interactions notwithstanding, provided that the quarks are accorded their constituent masses rather than the current quark values. In this paper we shall take these constituent or effective masses to be $m_{ud} \simeq 0.34$ GeV, $m_s \simeq 0.43$ GeV, and $m_c \simeq 1.5$ GeV, values which accord quite well with mass formulas and spin splittings.

Because a great deal of experimental data has become available since 1966 with which to test relativistic supermultiplet schemes, we shall reexamine some of these early predictions to test how well they pan out and, upon satisying ourselves that they generally lie within about 10% of the data, we will extrapolate to the heavy hadrons where they should be even more secure according to heavy quark lore. We intend to concentrate on processes and features that are amenable to experimental testing soon and will avoid weak decays: an area where understandably most of the recent research on heavy quarks is focused, because that is where the bulk of the data is to be found. The imminent arrival of b quark and charm factories promises an explosion of results every bit as impressive as the late 1960s and early 1970s proved to be for the strange hadronic states, and not purely in the $c\bar{c}$ and $b\bar{b}$ sector.

Instead of relying on tables of Clebsch-Gordan coefficients for the higher groups, we will base our analysis on a simple multispinor construction which produces the required symmetry relations from first principles. These states are in Tables I–III, listed in terms of the multispinors. It is very simple to read off the answers as needed or program them into algebraic computer packages such as MAPLE, to check or actually determine the requisite matrix elements. This procedure now goes under the name of the ''trace formula'' [5].

In the next section we shall set out the formalism. Our treatment of the quarks is deliberately naive as we wish to see how much one can learn simply by boosting up from rest the composite wave functions describing the hadrons, without taking account of any additional, finer effects. Our comparisons with the experimental data are given in the following three sections and the results indicate that subtler QCD corrections are rather minor, which is puzzling given our present knowledge of QCD.

II. MULTISPINOR STATES

We make the assumption, common to all quark models, that the hadrons are bound colorless S-wave states, of quark and antiquark for mesons, of three quarks for baryons. We take it that these hadrons consist of the various quarks moving in tandem, *with the same velocity* and, in keeping with our naive perspective, we shall neglect virtual gluons by supposing that their main function, apart from keeping the pieces together, is to give the quarks their composite (dynamical) masses. Neglecting the relative motion between quarks, which must of course average to zero, the states can be expressed as products of multispinors. We therefore represent the rest frame baryonic states by $\Psi_{(ABC)}$, with $2N(N+1)(2N+1)/3$ components, where N is the number of flavors and $A \equiv \alpha a$. a stands for the flavor index and α is the spinor index; α has only two effective components because of the on-shell spinor equation, which reads $(\gamma \cdot v - 1)u(v) = 0$.

We can decompose the multispinor into SU(N)×SU(2) components in the traditional way:

$$\Psi_{(ABC)} = \psi_{(abc)(\alpha\beta\gamma)} + \frac{\sqrt{2}}{3}(\psi_{[ab]c[\alpha\beta]\gamma} + \psi_{[bc]a[\beta\gamma]\alpha} + \psi_{[ca]b[\gamma\alpha]\beta}). \tag{1}$$

*Electronic address: Bob.Delbourgo@phys.utas.edu.au
†Electronic address: liu@physvax.phys.utas.edu.au

TABLE I. Mixed symmetry (Λ-type) states $u_{[ab]c}$ associated with the spin-$1/2^+$ baryons. Multispinors are antisymmetric in $[ab]$ from which fact other states are immediately deduced.

$ab\downarrow$ $c\rightarrow$	1	2	3	4
12	$p/\sqrt{2}$	$n/\sqrt{2}$	$\Lambda/\sqrt{3}$	$\Lambda_c^+/\sqrt{3}$
13	$\Sigma^+/\sqrt{2}$	$\Sigma^0/2+\Lambda^0/2\sqrt{3}$	$-\Xi^0/\sqrt{2}$	$-\Xi_c^+/\sqrt{3}$
14	$\Sigma_c^{++}/\sqrt{2}$	$\Sigma_c^+/2+\Lambda_c^+/2\sqrt{3}$	$\Xi_c^{+\prime}/2-\Xi_c^+/2\sqrt{3}$	$-\Xi_{cc}^{++}/\sqrt{2}$
23	$\Sigma^0/2-\Lambda/2\sqrt{3}$	$\Sigma^-/\sqrt{2}$	$-\Xi^-/\sqrt{2}$	$-\Xi_c^0/\sqrt{3}$
24	$\Sigma_c^+/2-\Lambda_c^+/2\sqrt{3}$	$\Sigma_c^0/\sqrt{2}$	$\Xi_c^{\prime 0}/2-\Xi_c^0/2\sqrt{3}$	$-\Xi_{cc}^+/\sqrt{2}$
34	$\Xi_c^{\prime +}/2+\Xi_c^+/2\sqrt{3}$	$\Xi_c^{\prime 0}/2+\Xi_c^0/2\sqrt{3}$	$\Omega_c^{\prime 0}/\sqrt{2}$	$\Omega_{cc}^{++}/\sqrt{2}$

Our normalization is fixed by

$$\overline{\Psi}^{(ABC)}\Psi_{(ABC)}=\overline{\psi}^{(abc)(\alpha\beta\gamma)}\psi_{(abc)(\alpha\beta\gamma)}+\overline{\psi}^{[ab]c[\alpha\beta]\gamma}\psi_{[ab]c[\alpha\beta]\gamma},$$

and one may verify that the total number of components match up: there are the spin-3/2 SU(2) spinors, symmetric in flavor indices (abc), having $N(N+1)(N+2)/6$ components, as well as the spin-1/2 SU(2) spinors of mixed symmetry $[ab]c$, with $N(N^2-1)/3$ components. See Tables I–III for extra details, listing the multispinors relations to the particle states themselves. A similar treatment, when applied to the mesons, yields the vector-pseudoscalar supermultiplet:

$$\Phi_A^B=\delta_a^b\phi_{5\alpha}^\beta+\vec{\sigma}_a^b\cdot\vec{\phi}_\alpha^\beta.$$

Then, upon boosting up the quarks from rest, the wave functions assume their relativistic form (v denotes the incoming hadron four-velocity):

$$\begin{aligned}\Psi_{(ABC)}(v)=&[P_{+v}\gamma_\mu C]_{\alpha\beta}u^\mu_{\gamma(abc)}(v)\\&+\frac{\sqrt{2}}{3}\{[P_{+v}\gamma_5 C]_{\alpha\beta}u_{[ab]c\gamma}(v)\\&+[P_{+v}\gamma_5 C]_{\beta\gamma}u_{[bc]a\alpha}(v)\\&+[P_{+v}\gamma_5 C]_{\gamma\alpha}u_{[ca]b\beta}(v)\},\end{aligned}\tag{2}$$

$$\Phi_A^B(q)=[\mu P_{+v}(\gamma_5\phi_{5a}^b(q)-\gamma^\nu\phi_{\nu a}^b(q))],\quad q=\mu v,\tag{3}$$

where $P_{+v}\equiv(1+\not{v})/2$ is the positive energy projector. Of course the vector fields u_μ and ϕ_μ obey the constraints, $\gamma^\mu u_\mu=v^\mu u_\mu=v^\mu\phi_\mu=0$.

This much is a direct generalization from SU(6) to SU($2N$) of the old treatment. Now historically the quarks were given the same mass—this was one of the criticisms of the early work—but that assumption is quite unnecessary as we have learned from heavy quark theory. All one needs to appreciate is that the quarks have to be traveling with the same velocity, so that the formulas (2) and (3) apply perfectly well to unequal mass quarks [6]. Therefore one can readily substitute p/m for v, where p is the total four-momentum of the hadron and m is its total mass, without going wrong.

The processes which we shall examine, including the charmed and bottom hadrons, have their origin in the strong three-point vertices

$$\mathcal{L}=F\Phi(q_1)\Phi(q_2)\Phi(q_3)+G\Psi(p')\Phi(q)\Psi(p),\tag{4}$$

where F and G are "universal" coupling constants. With our convention, Φ has mass dimensions $[M]^2$ and Ψ has dimension $[M]^{3/2}$, because the component fields possess the conventional dimensions of Fermi and Bose fields. Therefore $G\sim[M]^{-1}$ and $F\sim[M]^{-2}$ are dimensionful couplings and we will be faced with interpreting them before comparing our results with *physical* amplitudes and decay rates. The point is that the naive view which we are adopting takes the hadron mass as the sum of the constituent masses (spin splitting being neglected in the first instance); this is sometimes a far cry from the physical mass and we cannot gloss over this problem.

The electromagnetic interactions in Sec. V will be handled through the vector dominance model—albeit with some finesse—and thus follow from the strong vertices above. Whether we are dealing with pseudoscalar or vector mesons, the subsidiary conditions ensure that there is an overall factor of the sum of the participating hadron masses

TABLE II. Alternative mixed symmetry (Σ-type) states $U_{(ab)c}$ associated with the spin-$1/2^+$ baryons. Multispinors are now symmetric in (ab) whereupon other states are immediately deduced. Multispinors with equicomponent indices $U_{(aa)c}=2u_{[ca]a}$ can be read off from Table I.

$ab\downarrow$ $c\rightarrow$	1	2	3	4
12	$p/\sqrt{2}$	$-n/\sqrt{2}$	$-\Sigma^0$	$-\Sigma_c$
13	$\Sigma^+/\sqrt{2}$	$\Sigma^0/2-\sqrt{3}\Lambda^0/2$	$\Xi^0/\sqrt{2}$	$\Xi_c^{\prime +}$
14	$\Sigma_c^{++}/\sqrt{2}$	$\Sigma_c^+/2+\sqrt{3}\Lambda_c^+/2$	$\Xi_c^{+\prime}/2+\sqrt{3}\Xi_c^+/2$	$\Xi_{cc}^{++}/\sqrt{2}$
23	$\Sigma^0/2+\sqrt{3}\Lambda/2$	$\Sigma^-/\sqrt{2}$	$-\Xi^-/\sqrt{2}$	$-\Xi_c^{\prime 0}$
24	$\Sigma_c^+/2+\sqrt{3}\Lambda_c^+/2$	$\Sigma_c^0/\sqrt{2}$	$\Xi_c^{\prime 0}/2+\sqrt{3}\Xi_c^0/2$	$\Xi_{cc}^+/\sqrt{2}$
34	$\Xi_c^{\prime +}/2-\sqrt{3}\Xi_c^+/2$	$\Xi_c^{\prime 0}/2-\sqrt{3}\Xi_c^0/2$	$\Omega_c^{\prime 0}/\sqrt{2}$	$-\Omega_{cc}^{++}/\sqrt{2}$

TABLE III. Symmetric states $u_{(abc)}$ associated with the spin-$3/2^+$ baryons. Asterisked states in the table are obviously obtainable from the other entries via the complete symmetry in flavor indices.

$ab\downarrow$ $c\rightarrow$	1	2	3	4
11	Δ^{++}	$\Delta^{+}/\sqrt{3}$	$\Sigma^{*+}/\sqrt{3}$	$\Sigma_c^{*++}/\sqrt{3}$
12	*	$\Delta^{0}/\sqrt{3}$	$\Sigma^{*0}/\sqrt{6}$	$\Sigma_c^{*+}/\sqrt{6}$
13	*	*	$\Xi^{*0}/\sqrt{3}$	$-\Xi_c^{*+}/\sqrt{6}$
14	*	*	*	$\Xi_{cc}^{*++}/\sqrt{3}$
22	*	Δ^{-}	$\Sigma^{*-}/\sqrt{3}$	$\Sigma_c^{*0}/\sqrt{3}$
23	*	*	$\Xi^{*-}/\sqrt{3}$	$\Xi_c^{*0}/\sqrt{6}$
24	*	*	*	$\Xi_{cc}^{*+}/\sqrt{3}$
33	*	*	Ω^{-}	$\Omega_c^{*0}/\sqrt{3}$
34	*	*	*	$\Omega_{cc}^{*+}/\sqrt{3}$
44	*	*	*	Ω_{ccc}^{++}

multiplying the couplings F and G. Consequently we shall regard dimensionless $g=3G\Sigma/4$, where Σ is the sum of the masses as the proper universal meson-baryon coupling and $f=F\mu\Sigma$ as the proper universal meson-meson coupling, from the point of view of the rest frame $\mathrm{SU}(2N)\times\mathrm{SU}(2N)$ symmetry. The consequences of this are explained shortly.

III. RELATING THE STRONG INTERACTIONS

To uncover the relations between the strong interactions of the spin components, one only needs to insert the expansions (2) and (3) into (4) and take traces as required by the spinor algebra. This mechanical process leads to the effective interactions

$1^-\rightarrow 0^- + 0^-$

$$\mathcal{L}_{311}=\tfrac{1}{2} f\{(q_2-q_3)^\lambda[\phi_\lambda(q_1)\phi_5(q_2)\phi_5(q_3)]_- + 2 \text{ cyclic perms in } q\}, \tag{5}$$

where $[XYZ]_-\equiv X^b_a[Y^c_b Z^a_c - Z^c_b Y^a_c]$ is the antisymmetric flavor combination, consistent with Bose statistics.

$0^-\rightarrow 1^- + 1^-$

$$\mathcal{L}_{133}=f\{\epsilon_{\mu\nu\rho\sigma}q_2^\rho q_3^\sigma[\phi_5(q_1)\phi^\mu(q_2)\phi^\nu(q_3)]_+/\mu + 2 \text{ cyclic perms in } q\}, \tag{6}$$

where $[XYZ]_+\equiv X^b_a[Y^c_b Z^a_c + Z^c_b Y^a_c]$ is the symmetric flavor combination; this also is in keeping with Bose symmetry.

$1^-\rightarrow 1^- + 1^-$

$$\begin{aligned}\mathcal{L}_{333}=\tfrac{1}{2} f\{&[(q_2-q_3)_\lambda\eta_{\mu\nu}+(q_3-q_1)_\mu\eta_{\nu\lambda}+(q_1-q_2)_\nu\eta_{\lambda\mu}\\&+(q_2-q_3)_\lambda(q_3-q_1)_\mu(q_1-q_2)_\nu/6\mu^2]\\&\times[\phi^\lambda(q_1)\phi^\mu(q_2)\phi^\nu(q_3)]_+ + q \text{ perms}\},\end{aligned} \tag{7}$$

where we have taken the vectors to possess common mass μ. Notice the similarity of the first part of this expression to the Yang-Mills vertex.

$1/2^+\rightarrow 1/2^+ + 0^-$

$$\mathcal{L}_{221}=\tfrac{1}{2} g(1+v\cdot v')[\overline{u(p')}\gamma_5\phi_5(q)u(p)]_{D-S+2F/3}, \tag{8}$$

where the F,D,S combinations correspond to the internal symmetry combinations

$$F+3S\equiv[3\overline{u^{[bc]a}}\phi^d_a u_{[bc]d}+\overline{U^{(bc)a}}\phi^d_a U_{(bc)d}]/4, \tag{9}$$

$$D-3S\equiv[\overline{u^{[bc]a}}\phi^d_a u_{[bc]d}-\overline{U^{(bc)a}}\phi^d_a U_{(bc)d}]/4, \tag{10}$$

and

$$U_{(bc)a}\equiv u_{[ab]c}+u_{[ac]b}, \tag{11}$$

hailing from SU(3) days. The multispinor U possesses mixed symmetry too; instead of being antisymmetric in its first two indices like u, it is symmetric in them. Just like u, U obeys the cyclicity relation

$$U_{(ab)c}+U_{(bc)a}+U_{(ca)b}=0.$$

$1/2^+\rightarrow 1/2^+ + 1^-$

Here we express the interactions in terms of the electric and magnetic form factor combinations, which multiply the vectors $E_\lambda\equiv(v+v')_\lambda/2$ and $M_\lambda\equiv\epsilon_{\lambda\kappa\mu\nu}\gamma^\kappa\gamma_5 v^\mu v'^\nu/2$, respectively,

$$\mathcal{L}_{223}=g\left(\frac{\mu}{2m}[\overline{u(p')}E_\lambda\phi^\lambda u(p)]_{F+3S} + [\overline{u(p')}M_\lambda\phi^\lambda u(p)]_{D-S+2F/3}\right). \tag{12}$$

TABLE IV. Magnetic moments of spin-1/2 baryons, compared with experimentally found values. The quantities are theoretically determined by the constituent quark mass ratios, $m_n/m_s\simeq 0.79$, $m_n/m_c\simeq 0.23$. We have included a few charmed states although the magnetic moment data for them are not yet available—denoted by ?. When no errors are quoted they are very small.

Baryon	Theory	Experiment
p	2.75	2.79
n	-1.84	-1.91
Λ	-0.72	-0.61 ± 0.01
Σ^+	2.69	2.46 ± 0.01
$\Sigma^0-\Lambda$	-1.59	-1.61 ± 0.08
Σ^-	-0.98	-1.16 ± 0.02
Ξ^0	-1.58	-1.25 ± 0.01
Ξ^-	-0.66	-0.65
Λ_c^+	0.20	?
Σ_c^{++}	2.38	?
$\Sigma_c^+-\Lambda_c^+$	-1.59	?
Ξ_c^+	0.20	?

The significant point is that the two form factors (electric and magnetic, directly associated with helicity amplitudes) are related and the overall coupling is connected to the pseudoscalar interaction.

3/2⁺→1/2⁺+0⁻

There is but one possible internal index contraction and one gets the interaction

$$\mathcal{L}_{421}=g\overline{u^{[ab]c}}(p')v'^{\nu}\phi^{d}_{5a}u_{(bcd)\nu}(p)/\sqrt{2}, \tag{13}$$

where the incoming spin-3/2 particle is a Rarita-Schwinger spinor carrying momentum p, *symmetric* in its internal indices.

3/2⁺→1/2⁺+1⁻

In general there would be three independent transition amplitudes here but the spin-flavor symmetry relates them all via the effective coupling

$$\mathcal{L}_{423}=g\epsilon_{\kappa\lambda\mu\nu}v^{\mu}v'^{\nu}\overline{u^{[ab]c}}(p')\phi^{\kappa d}_{a}u^{\lambda}_{(bcd)}/\sqrt{2}. \tag{14}$$

The significance of this will become apparent when we study the radiative decays of the excited baryons.

3/2⁺→3/2⁺+0⁻

In this case we would normally expect two independent couplings but they become united in

$$\mathcal{L}_{441}=\tfrac{3}{4}g(\eta_{\mu\nu}(1+v\cdot v')-v_{\mu}v'_{\nu})\times\overline{u(p')}^{(abc)\mu}\gamma_5\phi^{d}_{5a}u^{\nu}_{(bcd)}(p). \tag{15}$$

It is much harder to obtain data that tests this relation between the couplings. However, the internal index contraction is at least unique.

3/2⁺→3/2⁺+1⁻

In this case we should expect five independent form factors but they all collapse into

$$\mathcal{L}_{443}=\tfrac{3}{2}g(\eta_{\mu\nu}-v_{\mu}v'_{\nu}/(1+v\cdot v'))\overline{u^{(abc)}}(p')[(\mu/2m)E_{\lambda}+M_{\lambda}]u_{(bcd)}(p)\phi^{d\lambda}_{a}. \tag{16}$$

Fortunately there is some experimental data with which to check this interaction.

IV. TESTING THE STRONG INTERACTIONS

Because our interactions (5)–(16) apply purely to strong interactions, the data for checking them out is somewhat limited. We need to look at processes where the couplings are readily extracted either directly from strong decays or else from residues of dominant poles in scattering processes. If we concentrate first on the strong decays, there is considerable data on the widths of the vector mesons and on the strange baryonic excitations. However there is little information about the charmed mesons and baryons and what exists is rather sensitive to the masses of the charmed and bottom excited states [7]. In some instances the masses are not yet well determined so we shall provide a range of predictions, depending on what we assume for the masses, with a little nous from mass formulas.

The results concerning purely mesonic processes have been published elsewhere [8] so we shall only summarize the findings here. We make the simplifying approximation that

$$\phi\simeq s\overline{s},\ \ \omega\simeq(u\overline{u}+d\overline{d})/\sqrt{2},\ \ \psi\simeq c\overline{c}$$

for 1^- mesons, but pay proper heed to the mixing angles for 0^- states. Vector meson decays into two pseudoscalars indicate that the corresponding coupling constant $g_{\rm VPP}=f$ varies slowly with the mass. This is not altogether surprising from the point of view of heavy quark symmetry, since f multiplies a momentum factor, according to (5). Rewriting in terms of velocities, we anticipate some mass dependence, via a quark loop for instance; since this is typically governed by the sum of the masses as we have seen, it suggests we should divide out the mass factor and look for the constancy of the ratio $g_{\rm VPP}/\Sigma\mu$ in those processes. The data seems to bear out this guess fairly well: for $\rho\pi\pi$, $K^*K\pi$, $\phi K\overline{K}$ decays, $g_{\rm VPP}$ equals 4.25, 4.57, and 4.90, respectively. Correspondingly, the mass sum ratios $3m_{ud}, 2m_{ud}+m_s, m_{ud}+2m_s$ provide the ratios 1.02, 1.11, and 1.20 (using the constituent quark masses mentioned in the Introduction) and seem to account for the SU(3) variation of $g_{\rm VPP}$. Extrapolating to the charmed decays $D^*D\pi$, we would expect $g_{\rm VPP}$ here to equal something like $4.25\times(m_c+2m_{ud})/3m_{ud}\simeq 8.9$, which lies below the experimental bound of 10.2 but will surely be tested before very long.

Electromagnetic decays offer more clues if one is prepared to apply vector dominance concepts; we shall discuss those processes presently. Meanwhile, turning to strong baryon decays, there is a wealth of information from the spin-3/2 sector. Aside from Clebsch-Gordan coefficients, which can be read off from Tables I–III, an interaction such as (13) leads to a decay width:

$$\Gamma=\Delta^3g^2(1+v\cdot v')/96\pi m^4m', \tag{17}$$

where $\Delta(m',m,\mu)\equiv\sqrt{[m^2-(m'+\mu)^2][m^2-(m'-\mu)^2]}$ is the standard triangle function, proportional to the magnitude of the decay product three-momentum in the rest frame of the decaying particle (mass m). After extracting the physical phase space factors from (17) we may determine the coupling g for a variety of decays. The results are amazingly constant: all of the decays $\Delta\to N\pi$, $\Sigma^*\to\Lambda\pi$, $\Sigma^*\to\Sigma\pi$, and $\Xi^*\to\Xi\pi$, yielding $g\simeq 21$, to within 1%. This encourages us to predict the widths for the charmed counterparts, Σ_c^* and Ξ_c^{*0}, provided the participating masses are precisely known, which they are not. As $m(\Sigma_c^*)$ varies from 2.50 GeV to 2.54 GeV the width $\Gamma(\Sigma_c^*\to\Lambda_c\pi)\sim 4.5$ to 8.5 MeV, is what we would predict; the favored mass and width are 2.53 GeV and 7.1 MeV. Similarly, as $m(\Xi_c^{*0})$ runs from 2.62 to 2.65 GeV, we predict that $\Gamma(\Xi_c^{*0}\to\Xi_c\pi)$ will vary between 0.10 MeV and 0.85 MeV, the most likely value being about 0.68 MeV, corresponding to $m(\Xi_c^{*0})=2.645$ GeV.

One other strong charmed decay is that of the spin-1/2 particle, $\Sigma_c\to\Lambda_c\pi$, but before we consider that, let us examine some better known couplings that follow from pole dominance or dispersion relations in strong scattering processes. First and foremost there is the on-shell pion nucleon coupling ($g_{\pi^0 pp}$) which is predicted to equal

$$g_{\pi NN}=g(1-m_\pi^2/4m_N^2)5/6\sqrt{2}\simeq 12.4,$$

which can be compared with the known value 13.4: a 10% error seems quite reasonable considering the extrapolation involved here. Similarly the kaon couplings are predicted to be

$$g_{KN\Sigma}=g\frac{(m_N+m_\Sigma)^2-m_K^2}{4m_N m_\Sigma}\frac{1}{6\sqrt{2}}\simeq 2.4,$$

$$g_{KN\Lambda}=g\frac{(m_N+m_\Lambda)^2-m_K^2}{4m_N m_\Lambda}\frac{\sqrt{6}}{4}\simeq 12.2.$$

The information from KN scattering (which is very sensitive to how the dispersion integrals are evaluated) concentrates on the quantity $(g_{KN\Lambda}^2+0.85g_{KN\Sigma}^2)/4\pi$ and gives the range 9–17 for its value. Our prediction of 12.3 lies comfortably within that range.

Moving up to the Σ_c, the model predicts

$$g_{\pi\Lambda_c\Sigma_c}=g\frac{(m_{\Lambda_c}+m_{\Sigma_c})^2-m_\pi^2}{4m_{\Lambda_c}m_{\Sigma_c}\sqrt{6}}\simeq 8.6$$

and in turn leads to a strong decay width prediction,

$$\Gamma(\Sigma_c\to\pi\Lambda_c)\simeq 28\ \text{keV}.$$

Unfortunately the present data tables do not quote a reliable value for that. The situation is much worse for the bottom mesons and it will probably be a good while before any sensible numbers are forthcoming for those states.

Before leaving strong interactions, it is worth making some brief remarks about the vector meson couplings to the baryons. These are obtained from (12) and include the ρ meson charge coupling. At zero momentum transfer, g_ρ is related to the pion coupling through

$$\frac{g_{\pi NN}}{g_{\rho NN}}=\frac{5}{3}\left(\frac{2m}{2\mu}\right)\left(1-\frac{m_\pi^2}{4m_N^2}\right)\simeq 5,$$

upon substituting $m=3m_{ud}$ and $\mu=2m_{ud}$. This gives approximately $g_{\rho NN}\simeq g_{\pi NN}/5\simeq 2.7$, agreeing fairly well with isospin universality of ρ couplings, which requires that $g_{\rho pp}=g_{\rho\pi\pi}/2\simeq 3$. Although we have little direct evidence for other strong vector couplings to other baryonic states, we do have a large pool of data on electromagnetic interactions. So we turn to this next.

V. RELATING AND TESTING THE ELECTROMAGNETIC INTERACTIONS

As mentioned in the Introducton, we shall use the vector dominance model when coupling the photon to the hadrons. In principle we must couple the photon to all possible 1^{--} vector mesons, and this could include the $\ell=2$ excitations of the ground state mesons, not to mention radial excitations. However, as these have considerably higher mass than the ground state particles, it is sufficient for our purpose to mediate the electromagnetic interaction by the $\ell=0$ states, namely the meson supermultiplet itself. We believe that it will not greatly damage the accuracy of our evaluations which are relatively crude anyhow. Now, the normal procedure is to take the matrix element of the electromagnetic current J to be

$$\langle V(k)|J_\lambda^{\mathrm{em}}|0\rangle=e\epsilon_\lambda^*(k)\mu_V^2/g_V,$$

where g_V is the strong coupling of the vector meson V to the hadrons. Of course, because we are assuming flavor symmetry, we have $3g_\rho=g_\omega=-g_\phi=2g_\psi$ for any hadron.

The strong current is a matrix in flavor space J_a^d and we need only select the charge projection $(2J_1^1-J_2^2-J_3^3+2J_4^4)/3$ to ascertain the relevant part of the strong interaction. However, there is one subtle point about our application of the vector meson dominance (VMD) model which is worth pointing out. It has to do with the question of which form factors are dominated by the vector meson pole, because that choice can make a substantial difference to the results.

Suppose for instance that we write the strong vector current element in the traditional manner:

$$X_\lambda=g\overline{u(p')}[\gamma_\mu F_1+i\sigma_{\lambda\kappa}q^\kappa F_2]u(p).$$

Then if we were to apply VMD blindly, the electromagnetic current would be $eX_\lambda/(1-q^2/\mu^2)$, where F_1,F_2 are evaluated on the meson mass shell ($q^2=\mu^2$). However, if one expresses the strong vector current element in the alternative way,

$$Y_\lambda=g\overline{u(p')}[E_\lambda F_E+M_\lambda F_M]u(p),$$

then one may contemplate another VMD version for the electromagnetic current at nonvanishing momentum transfer, viz. $Y_\lambda/(1-q^2/\mu^2)$, where F_E,F_M are worked out on the meson shell. To appreciate the difference, consider the identity

$$i\overline{u'}\sigma_{\lambda\kappa}q^\kappa u=\overline{u'}[q^2E_\lambda+4m^2M_\lambda]u\frac{2m}{4m^2-q^2}.$$

There is substantial difference between applying VMD to the left-hand side [i.e., multiplying by $\mu^2/(\mu^2-q^2)$] and doing the same *at the meson pole* on the right-hand side. Therefore we must declare how we propose to handle this. Because the Sachs form factors F_E,F_M are directly related to helicity amplitudes and are physically proportional to one another, we will apply VMD to the electric-magnetic decomposition. This choice then dictates that the isovector electromagnetic interaction between equal mass fermions, say, is

$$\langle v'|J_\lambda|v\rangle=\frac{1}{2}e\overline{u'}[E_\lambda+(2m/\mu_V)M_\lambda]u\frac{1}{1-q^2/\mu_V^2}. \tag{18}$$

Similarly for the isoscalar contribution. The method predicts that the magnetic moment is $2m/\mu$ in magnetons corresponding to that particle, times a characteristic Clebsch-Gordan coefficient. Since it is measured in quark magnetons e/m, we can say that the magnetic moment is given as e/μ

magnetons, where μ will vary with the mediating meson mass (namely the sum of its quark constituents). One of the immediate consequences is that the proton magnetic moment, in nucleon magnetons, equals $m_{\text{proton}}/m_{ud} \simeq 2.75$. More generally we may calculate the magnetic moment of the spin-1/2 baryons through the linear combination $D-S+2F/3$ arising in the sum of the components $[2J_1^1 - J_2^2 - (m_{ud}/m_s)J_3^3 + 2(m_{ud}/m_c)J_4^4]/3$, multiplied by the proton magnetic moment. We have collected these results in Table IV and also listed the experimental values for comparison. All in all, the fit is reasonable, bearing in mind that calculating magnetic moments is a delicate business and that we have no parameters apart from constituent quark masses, which are already fixed. The worst prediction is for Ξ^0 which is out by 20%. The future will produce determinations of moments for charmed and maybe even bottom baryons, but for the present we must remain ignorant about the validity of our predictions for them.

Of course we also have predictions for the spin-3/2 baryons and for electromagnetic transition elements (3/2 to 1/2), but the data are limited. Of the excited baryons the only estimated magnetic moment is for the Δ resonance. The Particle Data Group [9] state that the Δ^{++} moment lies between about 4 and 7, while we [really SU(4)] predict that it equals 5.5; not a very stringent test. However, a lot more is known about the electromagnetic Δ^+-p transition: here one finds the decay rates expressed in terms of 3/2 and 1/2 helicity amplitudes. The absolute magnitude of the width $\Gamma_{\Delta^+ p\gamma} = 0.78$ MeV, implies $g_{\Delta p \gamma} \simeq 0.69$ while the supermultiplet prediction is $\sqrt{6}e \simeq 0.73 \pm 0.04$, which is satisfactory. Furthermore, from (16) one may work out the ratio between the two helicity amplitudes to be $S^{3/2}/S^{1/2} = \sqrt{3}:1$. The experimental ratio being 1.82 ± 0.10, this is another good prediction. Unfortunately there is a dearth of data for transition elements between the strange baryons, except for the transition moment $\Sigma - \Lambda$ which is quoted in Table IV. But the situation is sure to change with time.

VI. CONCLUSIONS

We have seen that all the main features of strong and electromagnetic interactions can be understood by relativistically boosting up from rest spin-flavor symmetric vertices. Apart from the very odd case, all the results can be described by just *one* coupling constant g and three effective constituent masses for the quarks. They are generally correct to within 10%, and often they are better than that. This puts the lie to the claim that the light meson sector should be handled differently from the heavy quark sector, although we would be the first to admit that it is not easy to understand why. After all, the nonstrange and quark dynamical masses ~ 300 to 450 MeV are comparable to the QCD mass scale Λ.

We have stayed away from weak interactions, because it is necessary to comprehend how the weak bosons Z and W link with the strong supermultiplets. While one can see how the vector components of the weak current can be dominated by the $\ell = 0$ mesons, the axial component should couple to the excited $\ell = 1$ meson supermultiplet; this brings in a new, independent coupling constant. (A proper quark model will relate this to the ground state coupling of course.) Thus g_V and g_A are *distinct* couplings according to our perspective and their ratio is not given by 5/3 via the axial-pseudoscalar D/F ratio, as is commonly stated. The bulk of the recent research activity has naturally been focused on weak decays, because these channels predominate, not strong nor electromagnetic channels. We therefore intend to generalize the work presented in this paper to those processes, as the next logical step and see how far we can go with only one extra strong vertex associated with the first orbital excitation of the meson supermultiplet.

ACKNOWLEDGMENTS

We would like to express our thanks to the Australian Research Council who have supported this research through a grant. We also are indebted to Dr. Thompson for helpful feedback on our manuscript.

[1] F. Gursey and L. Radicati, Phys. Rev. Lett. **299**, 13 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964); A. Pais, Phys. Rev. Lett. **13**, 175 (1964).

[2] A. Salam, R. Delbourgo, and J. Strathdee, Proc. R. Soc. London Ser. A **284**, 146 (1965); M. A. Beg and A. Pais, Phys. Rev. Lett. **14**, 264 (1965); B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965); R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, *The U(12) Symmetry* (IAEA, Vienna, 1965).

[3] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).

[4] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)]; H. D. Politzer and M. B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988); E. Eichten and B. Hill, *ibid.* **234**, 511 (1990); H. Georgi, *ibid.* **240**, 447 (1990); J. G. Körner and G. Thompson, *ibid.* **264**, 185 (1991).

[5] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990); F. Hussain, J. G. Körner, K. Schilcher, G. Thompson, and Y. L. Wu, Phys. Lett. B **249**, 295 (1990); H. Georgi, Nucl. Phys. **B348**, 293 (1990); F. Hussain, J. G. Körner, M. Kramer, and G. Thompson, Z. Phys. C **51**, 321 (1991); F. Hussain, Dongsheng Liu, M. Kramer, J. G. Korner, and S. Tawfiq, Nucl. Phys. **B370**, 259(1992); A. Falk, *ibid.* **B378**, 79 (1992); R. Delbourgo and Dongsheng Liu, Phys. Rev. D **49**, 5979 (1994).

[6] F. Hussain, J. G. Körner, and G. Thompson, Ann. Phys. (N.Y.) **206**, 334 (1991).

[7] J. G. Körner, D. Pirjol, and M. Kramer, Prog. Part. Nucl. Phys. **33**, 787 (1994).

[8] N. R. Jones and R. Delbourgo, Aust. J. Phys. **48**, 55 (1995).

[9] Particle Data Group, L. Montanet *et al.*, Phys. Rev. **50**, 1173 (1994).