Muon anomalous magnetic dipole moment in the minimal supersymmetric standard model

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(Received 21 December 1995)

The muon anomalous magnetic dipole moment (MDM) is calculated in the framework of the minimal supersymmetric standard model (MSSM). In this paper, we discuss how the muon MDM depends on the parameters in the MSSM in detail. We show that the contribution of the superparticle loop becomes significant especially when $\tan\beta$ is large. Numerically, it becomes of order 10^{-8}–10^{-9} in a wide parameter space, which is within the reach of the new Brookhaven E821 experiment. [S0556-2821(96)01411-7]

PACS number(s): 12.60.Jv, 13.40.Em, 14.60.Ef

I. INTRODUCTION

Supersymmetry (SUSY) [1] is one of the most attractive candidates of the new physics beyond the standard model. In SUSY models, quadratic divergences are automatically canceled out, and hence SUSY may be regarded as a solution to the naturalness problem [2]. In addition, precision measurements of the gauge coupling constants strongly suggest SUSY grand unified theory (GUT) [3]. Contrary to our theoretical interests, however, evidences of SUSY (especially, superpartners) have not been discovered yet, and hence superpartners are fascinating targets of the forthcoming high energy experiments like LEP II, LHC, and NLC.

Even if we do not have high energy colliders, we can constrain SUSY models by using precision measurements in low energy experiments. This is because superparticles contribute to low energy physics through radiative corrections. In particular, superparticles are assumed to have masses of the order of the electroweak scale, and hence their loop effects may become comparable to those of $W^{\pm}$- or Z-boson propagations. Therefore, low energy precision experiments are also very useful to obtain constraints on SUSY models.

One of the quantities which are measured in a great accuracy is the muon anomalous magnetic dipole moment (MDM), $a_{\mu}\equiv(1/2)(g-2)_{\mu}$. At present, the muon MDM is measured to be [4]

$$a_{\mu}^{\mathrm{expt}}=1\ 165\ 923(8.4)\times10^{-9}. \tag{1}$$

On the other hand, the standard model prediction on a_{μ} is given by [5]

$$a_{\mu}^{\mathrm{SM}}=116\ 591\ 802(153)\times10^{-11}, \tag{2}$$

which is completely consistent with experimental value. (For a review of the calculation of a_{μ}^{SM}, see also Ref. [6].)

Because of the great accuracy of a_{μ}^{expt} and a_{μ}^{SM} given above, we can derive a constraint on SUSY models from the muon MDM. Furthermore, the new Brookhaven E821 experiment [7] is supposed to reduce the error of the experimental value of a_{μ} to 0.4×10^{-9}, which is smaller than the present one by a factor ~20. The accuracy of the Brookhaven E821 experiment is of the order of the contribution of the $W^{\pm}$- and Z-boson loop, which means we may have a chance to measure the SUSY contribution to the muon MDM by that experiment.

In fact, there are several works in which the muon MDM is calculated in the framework of SUSY models [8–10]. In particular, Chattopadhyay and Nath recently pointed out that the muon MDM is a powerful probe of the models based on supergravity if $\tan\beta$ is large [10]. However, most of the recent works assume the boundary conditions on the SUSY breaking parameters based on the minimal supergravity, and/or radiative electroweak symmetry breaking scenario, and hence it is quite unclear to us how the SUSY contributions to the muon MDM depend on the parameters in MSSM. Thus, the aim of this paper is to clarify it, and to investigate the behavior of the muon MDM in the framework of MSSM. The mass matrices and mixing angles among the superparticles have model dependence even if we assume the boundary condition based on the minimal supergravity, and hence we believe that it is important to analyze the muon MDM in a more general framework of the SUSY standard model.

In this paper, we investigate the SUSY contribution to the muon MDM in the framework of MSSM as a low energy effective theory of SUSY GUT [11]. The organization of this paper is as follows. In the next section, we introduce a model we consider. In Sec. III, we show analytic forms of the SUSY contribution to the muon MDM, $\Delta a_{\mu}^{\mathrm{SUSY}}$. In Sec. IV, typical behavior of $\Delta a_{\mu}^{\mathrm{SUSY}}$ is discussed. In Sec. V, some numerical results are shown. Section VI is devoted to discussion.

II. MODEL

First of all, we would like to introduce a model we consider, i.e., MSSM as a low energy effective theory of SUSY GUT. All the fields we use in our analysis are

$$l_L=(\mu_L\nu),\quad \mu_R^c,\quad H_1=(H_1^-H_1^0),\quad H_2=\begin{pmatrix}H_2^+\\ H_2^0\end{pmatrix}, \tag{3}$$

where l_L ($\mathbf{2}^*$, $-1/2$) and μ_R^c ($\mathbf{1}$, 1) are left- and right-handed leptons in the second generation, while two Higgs doublets are represented as H_1 ($\mathbf{2}^*$, $-1/2$) and H_2 ($\mathbf{2}$, 1/2). [We denote the quantum numbers for the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$

*Electronic address: moroi@theor3.lbl.gov gauge group in the parentheses.] The Higgs doublets H_1 and H_2 are responsible for the electroweak symmetry breaking, and hence their vacuum-expectation values are constrained as $\langle H_1\rangle^2+\langle H_2\rangle^2\simeq(174\ \text{GeV})^2$ in order to give a correct value of the Fermi constant. On the other hand, the ratio of the vacuum-expectation values of two Higgs doublets is a free parameter in MSSM, which we define $\tan\beta\equiv\langle H_2\rangle/\langle H_1\rangle$.

The relevant part of the superpotential of MSSM is given by

$$W=y_\mu\epsilon^{\alpha\beta}\mu_R^c l_{L,\alpha}H_{1,\beta}+\mu_H H_{1,\alpha}H_2^\alpha, \tag{4}$$

where y_μ is the Yukawa coupling constant of muon, μ_H the SUSY invariant Higgs mass, and $\epsilon^{\alpha\beta}$ the antisymmetric tensor with $\epsilon^{12}=1$. Using the superpotential given above, the F-term contribution to the Lagrangian is obtained as

$$\mathcal{L}_F=-\int d^2\theta W+\text{H.c.} \tag{5}$$

Furthermore, soft SUSY breaking terms are given by

$$\mathcal{L}_{\text{soft}}=-m_L^2\tilde{l}_L^*\tilde{l}_L-m_R^2\tilde{\mu}_R^{c*}\tilde{\mu}_R^c-(A_\mu\epsilon^{\alpha\beta}\tilde{\mu}_R^c\tilde{l}_{L,\alpha}H_{2,\beta}+\text{H.c.})-\frac{1}{2}(m_{G2}\tilde{W}\tilde{W}+m_{G1}\tilde{B}\tilde{B}+\text{H.c.}). \tag{6}$$

Here, $\tilde{l}_L$, $\tilde{\mu}_R^c$, $\tilde{W}$, and $\tilde{B}$ represent left- and right-handed sleptons in second generation, and gauginos for $SU(2)_L$ and $U(1)_Y$ gauge group, respectively. Gaugino masses m_{G1} and m_{G2} are related by the GUT relation

$$\frac{m_{G2}}{g_2^2}=\frac{3}{5}\frac{m_{G1}}{g_1^2}, \tag{7}$$

where g_1 and g_2 are the gauge coupling constant of $SU(2)_L$ and $U(1)_Y$ gauge group, respectively.[1]

Here, we should comment on a flavor mixing in slepton mass matrices. If there are large flavor mixings in the slepton mass matrices, all the sleptons contribute to the muon MDM. However, flavor mixing in the slepton mass matrices may be dangerous, since it induces lepton flavor violation processes such as $\mu\to e\gamma$, $\tau\to\mu\gamma$, and so on. In particular, the mixing among the first and second generations is severely constrained from $\mu\to e\gamma$ especially when $\tan\beta$ is large [13]. On the other hand, the constraint on the mixing of the second and third generations is not so stringent. In this paper, for simplicity, we assume that the flavor mixing in the slepton mass matrix is not so large, and that it does not affect the following arguments. A comment on the case with the flavor mixing is given in Sec. VI.

Once we have the MSSM Lagrangian, we can obtain mass eigenvalues and mixing matrices of the superparticles. The mass matrix for the smuon field is given by

$$M_{\tilde{\mu}}^2=\begin{pmatrix} m_{\tilde{\mu}L}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{\tilde{\mu}R}^2 \end{pmatrix}, \tag{8}$$

where

$$m_{\tilde{\mu}L}^2=m_L^2+m_Z^2\cos2\beta\left(\sin^2\theta_W-\frac{1}{2}\right), \tag{9}$$

$$m_{\tilde{\mu}R}^2=m_R^2-m_Z^2\cos2\beta\sin^2\theta_W, \tag{10}$$

$$m_{LR}^2=y_\mu\mu_H\langle H_2\rangle+A_\mu\langle H_1\rangle. \tag{11}$$

The mass matrix $M_{\tilde{\mu}}^2$ can be diagonalized by using a unitary matrix $U_{\tilde{\mu}}$ as

$$(U_{\tilde{\mu}}^\dagger M_{\tilde{\mu}}^2 U_{\tilde{\mu}})_{AB}=m_{\tilde{\mu}A}^2\delta_{AB}\quad(A,B=1,2), \tag{12}$$

where $m_{\tilde{\mu}A}$ is the mass eigenvalue of the smuon. Notice that, in our case, the off-diagonal element of the mass matrix given in Eq. (8) is substantially smaller than the diagonal elements, and hence $m_{\tilde{\mu}L}$ and $m_{\tilde{\mu}R}$ almost correspond to the mass eigenvalues. The mass of the sneutrino, $m_{\tilde{\nu}}$, is also easily obtained as

$$m_{\tilde{\nu}}^2=m_L^2+\frac{1}{2}m_Z^2\cos2\beta. \tag{13}$$

Next, we derive the mass matrices for neutralinos and charginos. For neutralinos, the mass terms are given by

$$\mathcal{L}_{\chi^0}=-\frac{1}{2}(i\tilde{B}\,i\tilde{W}^3\,\tilde{H}_1^0\,\tilde{H}_2^0)\begin{pmatrix} -m_{G1} & 0 & -\frac{1}{\sqrt{2}}g_1\langle H_1\rangle & \frac{1}{\sqrt{2}}g_1\langle H_2\rangle \\ 0 & -m_{G2} & \frac{1}{\sqrt{2}}g_2\langle H_1\rangle & -\frac{1}{\sqrt{2}}g_2\langle H_2\rangle \\ -\frac{1}{\sqrt{2}}g_1\langle H_1\rangle & \frac{1}{\sqrt{2}}g_2\langle H_1\rangle & 0 & \mu_H \\ \frac{1}{\sqrt{2}}g_1\langle H_2\rangle & -\frac{1}{\sqrt{2}}g_2\langle H_2\rangle & \mu_H & 0 \end{pmatrix}\begin{pmatrix} i\tilde{B} \\ i\tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}+\text{H.c.}, \tag{14}$$

[1]The GUT relation given in Eq. (7) holds in general if the gauge groups are unified in a larger group [12]. Therefore, we are not depending on a specific model of GUT.

where $\widetilde{H}_1$ and $\widetilde{H}_2$ represent the higgsino field. Then, we can find a unitary matrix U_{χ^0} which diagonalizes the mass matrix given above. Denoting the mass matrix given in Eq. (14) as M_{χ^0}, mass eigenvalues $m_{\chi^0 X}$ for the neutralino χ^0_X are given by

$$(U^{\dagger}_{\chi^0} M_{\chi^0} U_{\chi^0})_{XY} = m_{\chi^0 X} \delta_{XY} \quad (X,Y=1-4), \tag{15}$$

where $m_{\chi^0 X} \leqslant m_{\chi^0 Y}$ if $X<Y$.

Similarly, mass terms for the charginos are given by

$$\mathscr{L}_{\chi^{\pm}} = -(\widetilde{W}^{+} \widetilde{H}^{+}_2) \begin{pmatrix} -m_{G2} & g_2 \langle H_1 \rangle \\ -g_2 \langle H_2 \rangle & \mu_H \end{pmatrix} \begin{pmatrix} \widetilde{W}^{-} \\ \widetilde{H}^{-}_1 \end{pmatrix} + \text{H.c.}, \tag{16}$$

with $\widetilde{W}^{\pm} \equiv -(i/\sqrt{2})(\widetilde{W}^1 \mp i \widetilde{W}^2)$. The mass matrix given in Eq. (16), which we denote $M_{\chi^{\pm}}$, can be diagonalized by using two unitary matrices, U_{χ^+} and U_{χ^-}:

$$(U^{\dagger}_{\chi^+} M_{\chi^{\pm}} U_{\chi^-})_{XY} = m_{\chi^{\pm} X} \delta_{XY} \quad (X,Y=1,2), \tag{17}$$

where $m_{\chi^{\pm} X}$ represents the mass eigenvalue of the chargino field.

With the coupling constants and mixing matrices given above, we can write down muon-neutralino-smuon and muon-chargino-sneutrino vertices. Denoting the mass eigenstates of the smuon, neutralino, and chargino as $\widetilde{\mu}_A$, χ^0_X, and $\chi^{\pm}_X$, respectively, the interaction terms are given by

$$\mathscr{L}_{\text{int}} = \sum_{AX} \overline{\mu} (N^L_{AX} P_L + N^R_{AX} P_R) \chi^0_X \widetilde{\mu}_A + \sum_{X} \overline{\mu} (C^L_X P_L + C^R_X P_R) \chi^{\pm}_X \widetilde{\nu} + \text{H.c.}, \tag{18}$$

where $P_L = (1/2)(1-\gamma_5)$, $P_R = (1/2)(1+\gamma_5)$, and

$$N^L_{AX} = -y_{\mu} (U_{\chi^0})_{3X} (U_{\widetilde{\mu}})_{LA} - \sqrt{2} g_1 (U_{\chi^0})_{1X} (U_{\widetilde{\mu}})_{RA}, \tag{19}$$

$$N^R_{AX} = -y_{\mu} (U_{\chi^0})_{3X} (U_{\widetilde{\mu}})_{RA} - \frac{1}{\sqrt{2}} g_2 (U_{\chi^0})_{2X} (U_{\widetilde{\mu}})_{LA} - \frac{1}{\sqrt{2}} g_1 (U_{\chi^0})_{1X} (U_{\widetilde{\mu}})_{LA}, \tag{20}$$

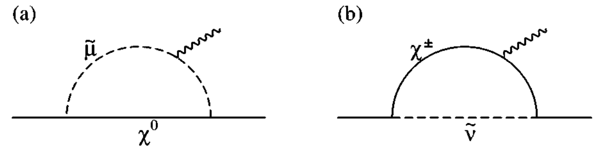

FIG. 1. Feynman diagrams which give rise to the muon MDM in the mass eigenstate basis. The external lines represent the muon (straight) and the photon (wavy).

$$C^L_X = y_{\mu} (U_{\chi^-})_{2X}, \tag{21}$$

$$C^R_X = -g_2 (U_{\chi^+})_{1X}. \tag{22}$$

By using the interaction terms given in Eq. (18), we calculate the SUSY contribution to the muon MDM.

III. ANALYTIC FORMULAS

Now, we are in position to calculate the SUSY contribution to the muon MDM. What we have to calculate is the ''magnetic moment type'' operator, which is given by

$$\mathscr{L}_{\text{MDM}} = \frac{e}{2m_{\mu}} F_2 \overline{\mu} \sigma_{\rho\lambda} \mu F^{\rho\lambda}. \tag{23}$$

Here, e is the electric charge, m_{μ} the muon mass, $\sigma_{\rho\lambda} = (i/2)[\gamma_{\rho}, \gamma_{\lambda}]$, $F_{\rho\lambda}$ the field strength of the photon field, and F_2 the magnetic form factor. The muon anomalous magnetic moment, a_{μ}, is related to F_2 as

$$a_{\mu} = F_2. \tag{24}$$

Thus, by calculating the magnetic form factor in the framework of MSSM, we can have SUSY contribution to the muon MDM.

In the SUSY model, there are essentially two types of diagrams which contribute to a_{μ}, i.e., one is the neutralino (χ^0)-smuon ($\widetilde{\mu}$) loop diagram [Fig. 1(a)] and the other is the chargino ($\chi^{\pm}$)-sneutrino ($\widetilde{\nu}$) loop diagram [Fig. 1(b)]:

$$\Delta a^{\text{SUSY}}_{\mu} = \Delta a^{\chi^0 \widetilde{\mu}}_{\mu} + \Delta a^{\chi^{\pm} \widetilde{\nu}}_{\mu}. \tag{25}$$

Here, the contribution from the χ^0-$\widetilde{\mu}$ diagram, $\Delta a^{\chi^0 \widetilde{\mu}}_{\mu}$, is

$$\begin{aligned} a^{\chi^0 \widetilde{\mu}}_{\mu} &= m_{\mu} \sum_{AX} \{ -m_{\mu} (N^L_{AX} N^L_{AX} + N^R_{AX} N^R_{AX}) m^2_{\widetilde{\mu}A} J_5 (m^2_{\chi^0 X}, m^2_{\widetilde{\mu}A}, m^2_{\widetilde{\mu}A}, m^2_{\widetilde{\mu}A}, m^2_{\widetilde{\mu}A}) + m_{\chi^0 X} N^L_{AX} N^R_{AX} J_4 (m^2_{\chi^0 X}, m^2_{\chi^0 X}, m^2_{\widetilde{\mu}A}, m^2_{\widetilde{\mu}A}) \} \\ &= \frac{1}{16\pi^2} m_{\mu} \sum_{AX} \Bigg\{ -\frac{m_{\mu}}{6 m^2_{\widetilde{\mu}A} (1-x_{AX})^4} (N^L_{AX} N^L_{AX} + N^R_{AX} N^R_{AX}) (1 - 6x_{AX} + 3x^2_{AX} + 2x^3_{AX} - 6x^2_{AX} \ln x_{AX}) \\ &\quad - \frac{m_{\chi^0 X}}{m^2_{\widetilde{\mu}A} (1-x_{AX})^3} N^L_{AX} N^R_{AX} (1 - x^2_{AX} + 2x_{AX} \ln x_{AX}) \Bigg\}, \end{aligned} \tag{26}$$

where we are using mass eigenstate basis of χ^0 and $\tilde{\mu}$ [and that of $\chi^\pm$ in deriving Eq. (29)]. Here, $x_{AX}=m^2_{\chi^0 X}/m^2_{\tilde{\mu}A}$, and we define the functions I_N and J_N as

$$I_N(m_1^2,\ldots,m_N^2)=\int\frac{d^4k}{(2\pi)^4 i}\frac{1}{(k^2-m_1^2)\cdots(k^2-m_N^2)},\quad(27)$$

$$J_N(m_1^2,\ldots,m_N^2)=\int\frac{d^4k}{(2\pi)^4 i}\frac{k^2}{(k^2-m_1^2)\cdots(k^2-m_N^2)}.\quad(28)$$

Some useful formulas concerning the functions I_N and J_N are shown in the Appendix. The contribution from the $\chi^\pm$-$\tilde{\nu}$ loop diagram is also easily calculated, and the result is given by

$$\begin{aligned}\Delta a_\mu^{\chi^\pm\tilde{\nu}}=&m_\mu\sum_X\ [2m_\mu(C_X^LC_X^L+C_X^RC_X^R)\{J_4(m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\chi^\pm X})+m^2_{\tilde{\nu}}J_5(m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\tilde{\nu}})\\&-J_4(m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\tilde{\nu}})\}-2m_XC_X^LC_X^RJ_4(m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\chi^\pm X},m^2_{\tilde{\nu}})]\\=&\frac{1}{16\pi^2}m_\mu\sum_X\left\{\frac{m_\mu}{3m^2_{\tilde{\nu}}(1-x_X)^4}(C_X^LC_X^L+C_X^RC_X^R)\left(1+\frac{3}{2}x_X-3x_X^2+\frac{1}{2}x_X^3+3x_X\ln x_X\right)\right.\\&\left.-\frac{3m_{\chi^\pm X}}{m^2_{\tilde{\nu}}(1-x_X)^3}C_X^LC_X^R\left(1-\frac{4}{3}x_X+\frac{1}{3}x_X^2+\frac{2}{3}\ln x_X\right)\right\},\end{aligned}\quad(29)$$

where $x_X=m^2_{\chi^\pm X}/m^2_{\tilde{\nu}}$.

IV. BEHAVIOR OF THE SUSY CONTRIBUTION TO THE MUON MDM

Before evaluating the SUSY contribution to the muon MDM numerically, we would like to discuss the behavior of $\Delta a_\mu^{\mathrm{SUSY}}$, especially in the large $\tan\beta$ case. As we will soon see, $|\Delta a_\mu^{\mathrm{SUSY}}|$ becomes large as $\tan\beta$ increases. Thus, the discussion about the large $\tan\beta$ case will be helpful for us to understand the behavior of $\Delta a_\mu^{\mathrm{SUSY}}$.

For this purpose, it is more convenient to use the mass insertion method to calculate the penguin diagrams rather than working in the mass eigenstate basis of the superparticles which is used in the previous section. In the case where $\tan\beta$ is large, five diagrams dominantly contribute to $\Delta a_\mu^{\mathrm{SUSY}}$, which are shown in Fig. 2. Their contributions are given by

$$\Delta a_\mu^{N1}=g_1^2m_\mu^2m_{G1}\mu_H\tan\beta\{J_5(m^2_{G1},m^2_{G1},m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}R},m^2_{\tilde{\mu}R})+J_5(m^2_{G1},m^2_{G1},m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}R})\},\quad(30)$$

$$\Delta a_\mu^{N2}=-g_1^2m_\mu^2m_{G1}\mu_H\tan\beta\{J_5(m^2_{G1},m^2_{G1},\mu^2_H,m^2_{\tilde{\mu}R},m^2_{\tilde{\mu}R})+J_5(m^2_{G1},\mu^2_H,\mu^2_H,m^2_{\tilde{\mu}R},m^2_{\tilde{\mu}R})\},\quad(31)$$

$$\Delta a_\mu^{N3}=\tfrac{1}{2}g_1^2m_\mu^2m_{G1}\mu_H\tan\beta\{J_5(m^2_{G1},m^2_{G1},\mu^2_H,m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}L})+J_5(m^2_{G1},\mu^2_H,\mu^2_H,m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}L})\},\quad(32)$$

$$\Delta a_\mu^{N4}=-\tfrac{1}{2}g_2^2m_\mu^2m_{G2}\mu_H\tan\beta\{J_5(m^2_{G2},m^2_{G2},\mu^2_H,m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}L})+J_5(m^2_{G2},\mu^2_H,\mu^2_H,m^2_{\tilde{\mu}L},m^2_{\tilde{\mu}L})\},\quad(33)$$

$$\begin{aligned}\Delta a_\mu^{C}=&g_2^2m_\mu^2m_{G2}\mu_H\tan\beta\{2I_4(m^2_{G2},m^2_{G2},\mu^2_H,m^2_{\tilde{\nu}})-J_5(m^2_{G2},m^2_{G2},\mu^2_H,m^2_{\tilde{\nu}},m^2_{\tilde{\nu}})+2I_4(m^2_{G2},\mu^2_H,\mu^2_H,m^2_{\tilde{\nu}})\\&-J_5(m^2_{G2},\mu^2_H,\mu^2_H,m^2_{\tilde{\nu}},m^2_{\tilde{\nu}})\}.\end{aligned}\quad(34)$$

Here, Eqs. (30)–(33) are χ^0-$\tilde{\mu}$ loop contributions, while Eq. (34) represents the $\chi^\pm$-$\tilde{\nu}$ loop one. By using these expressions, the SUSY contribution to the muon MDM is approximately given by

$$\Delta a_\mu^{\chi^0\tilde{\mu}}\simeq\Delta a_\mu^{N1}+\Delta a_\mu^{N2}+\Delta a_\mu^{N3}+\Delta a_\mu^{N4},\quad(35)$$

$$\Delta a_\mu^{\chi^\pm\tilde{\nu}}\simeq\Delta a_\mu^{C}.\quad(36)$$

Notice that the SUSY contribution to the muon MDM given in Eqs. (30)–(34) approximately correspond to the terms which are proportional to N^LN^R or C^LC^R (i.e., terms which have a chirality flip in internal fermion line) in the exact formulas given in Eqs. (26) and (29).

The first thing we can learn from the above expressions is that all the terms given in Eqs. (30)–(34) are proportional to $\tan\beta$ [9,10]. This is due to the fact that the chirality is flipped not by hitting the mass of the external muon but by directly hitting the Yukawa coupling. This mechanism also occurs in the case of the lepton flavor violations [13]. Thus, $|\Delta a_\mu^{\mathrm{SUSY}}|$ becomes large as $\tan\beta$ increases, and we obtain a severe constraint on the parameter space as $\tan\beta$ gets larger.

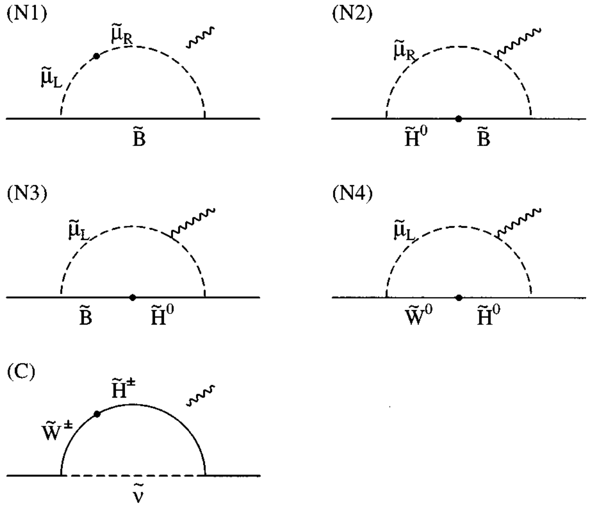

FIG. 2. Feynman diagrams which give rise to the muon MDM in the mass insertion method.

The second point we should mention is the relation between the sign of $\Delta a_\mu^{\rm SUSY}$ and those of parameters in MSSM. The dominant SUSY contribution given in Eqs. (30)–(34) are all proportional to $m_G \mu_H \tan\beta$ (with $m_G = m_{G1}, m_{G2}$ being gaugino mass). Thus, if we change the sign of this combination, $\Delta a_\mu^{\rm SUSY}$ also changes its sign. Furthermore, in the case where we assume GUT relation on the gaugino masses, we checked that Δa_μ^{N1} or Δa_μ^{C} dominates over other terms (Δa_μ^{N2}, Δa_μ^{N3}, Δa_μ^{N4}) in most of the parameter space. Here, both Δa_μ^{N1} and Δa_μ^{C} have the same sign as the combination $m_G \mu_H \tan\beta$. Therefore, $\Delta a_\mu^{\rm SUSY}$ becomes positive (negative) when the sign of the combination $m_G \mu_H \tan\beta$ is positive (negative).[2] In the next section, we will see that this relation really holds as a result of numerical calculations.

Furthermore, we comment here that the contribution of $\chi^\pm$-$\tilde{\nu}$ loop diagram dominates over that of the χ^0-$\tilde{\mu}$ loop ones if all the masses of the superparticles are almost degenerate. For example, let us consider the extreme case where all the masses for the superparticles (m_{G1}, m_{G2}, μ_H, $m_{\tilde{\mu}L}$, $m_{\tilde{\mu}R}$, $m_{\tilde{\nu}}$) are the same. Denoting the masses of the superparticles $m_{\rm SUSY}$, contributions of the χ^0-$\tilde{\mu}$ and $\chi^\pm$-$\tilde{\nu}$ loop diagrams to the muon MDM are given by

$$\begin{aligned}\Delta a_\mu^{\chi^0\tilde{\mu}} &\simeq \Delta a_\mu^{N1} + \Delta a_\mu^{N2} + \Delta a_\mu^{N3} + \Delta a_\mu^{N4} \\ &= \frac{1}{192\pi^2}\frac{m_\mu^2}{m_{\rm SUSY}^2}(g_1^2 - g_2^2)\tan\beta, \qquad (37)\end{aligned}$$

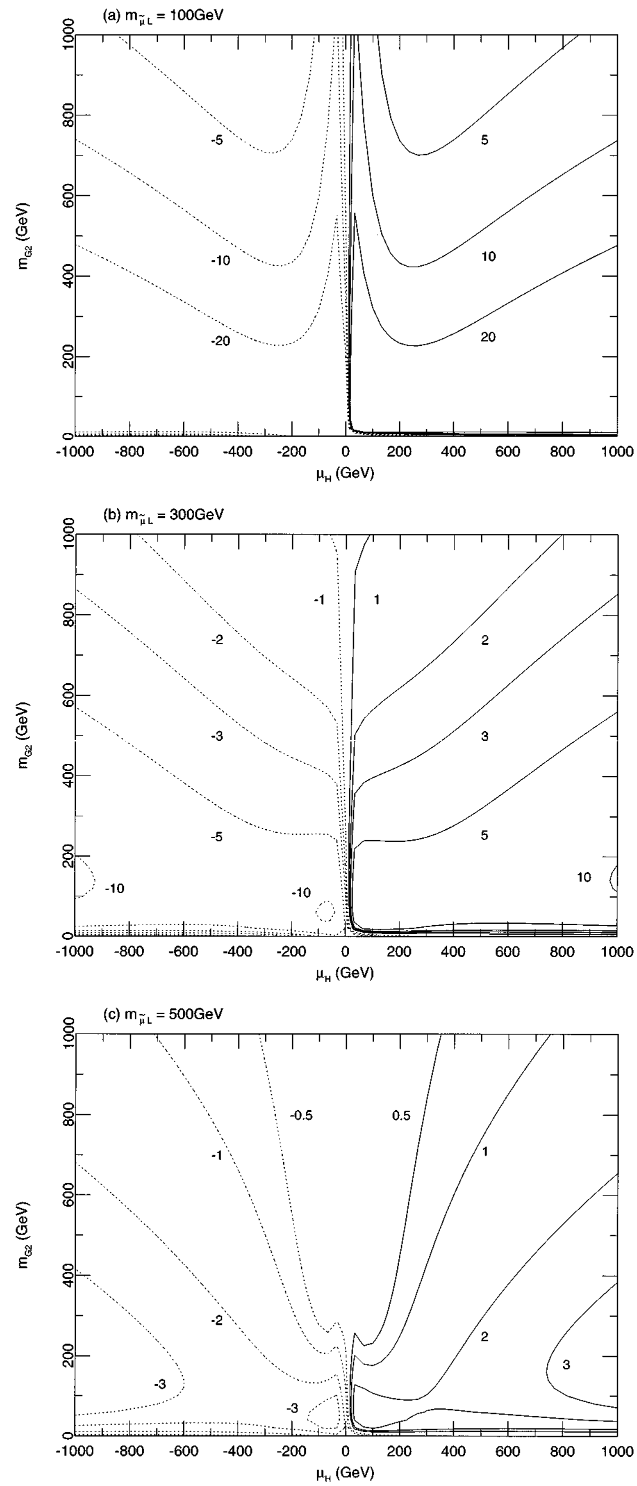

FIG. 3. The SUSY contribution to the muon MDM, $\Delta a_\mu^{\rm SUSY}$, in the μ_H-m_{G2} plane. The right-handed smuon mass is taken to be $m_{\tilde{\mu}R} = 100$ GeV. We take $\tan\beta = 30$, and the left-handed smuon mass $m_{\tilde{\mu}L}$ is (a) 100 GeV, (b) 300 GeV, and (c) 500 GeV. The numbers given in the figures represent the value of $\Delta a_\mu^{\rm SUSY}$ in units of 10^{-9}.

$$\Delta a_\mu^{\chi^\pm\tilde{\nu}} \simeq \Delta a_\mu^{C} = \frac{1}{32\pi^2}\frac{m_\mu^2}{m_{\rm SUSY}^2} g_2^2 \tan\beta. \qquad (38)$$

From the above expressions, we can see that the $\chi^\pm$-$\tilde{\nu}$ loop contribution is substantially larger than that of the χ^0-$\tilde{\mu}$

[2] If m_G or μ_H is small, this relation does not hold. This is mainly because the mass insertion method breaks down in such a case. Furthermore, in such a case, we cannot ignore Δa_μ^{N2} or terms which are not proportional to $\tan\beta$ [i.e., terms which are proportional to $N^L N^L$, $N^R N^R$, $C^L C^L$, and $C^R C^R$ in the exact formula given in Eqs. (26) and (29)]. In that case, the sign of $m_G \mu_H \tan\beta$ is not directly related to that of $\Delta a_\mu^{\rm SUSY}$.

loop. Thus, the $\chi^{\pm}$-$\tilde{\nu}$ loop gives a dominant contribution, as in the case of minimal SUSY GUT based on the minimal supergravity [10]. However, we should note here that the $\chi^{\pm}$-$\tilde{\nu}$ loop dominance does not hold in general. In the next section, we will see the SUSY contribution to $\Delta a_{\mu}^{\rm SUSY}$ significantly depends on the right-handed smuon mass $m_{\tilde{\mu}R}$ in certain parameter regions.

V. NUMERICAL RESULTS

In this section, we numerically estimate $\Delta a_{\mu}^{\rm SUSY}$ by using Eqs. (26) and (29). As we mentioned before, there are essentially six parameters on which $\Delta a_{\mu}^{\rm SUSY}$ depends, i.e., SU(2) gaugino mass m_{G2},[3] left- and right-handed smuon masses $m_{\tilde{\mu}L}$ and $m_{\tilde{\mu}R}$ (which essentially correspond to the soft SUSY breaking parameters m_L^2 and m_R^2, respectively), SUSY invariant Higgs mass μ_H, ratio of the vacuum-expectation values of the two Higgs doublets, $\tan\beta=\langle H_2\rangle/\langle H_1\rangle$, and the SUSY breaking A parameter for the smuon A_{μ}. However, especially in the large $\tan\beta$ region where SUSY contribution to $\Delta a_{\mu}^{\rm SUSY}$ may become significantly large, $\Delta a_{\mu}^{\rm SUSY}$ is not sensitive to A_{μ} if $A_{\mu}\sim O(y_{\mu}\mu_H)$. This is because A_{μ} always appears in expressions in the combination of $(A_{\mu}+y_{\mu}\mu_H\tan\beta)$, as shown in Eq. (11). Therefore, in our analysis, we take $A_{\mu}=0$.[4] Then, we take the other five parameters as free parameters and calculate the SUSY contribution to the muon MDM for a given set of parameters.

First, we show the SUSY contribution to the muon MDM for fixed values of $m_{\tilde{\mu}R}$ and $m_{\tilde{\mu}L}$ in the μ_H-m_{G2} plane. In Fig. 3, we plotted the results for $m_{\tilde{\mu}R}=100$ GeV and $\tan\beta=30$. Here, the left-handed smuon mass is taken to be 100 GeV [Fig. 3(a)], 300 GeV [Fig. 3(b)], and 500 GeV [Fig. 3(c)]. The results for the cases of $m_{\tilde{\mu}R}=300$ GeV and $m_{\tilde{\mu}R}=1$ TeV are also shown in Figs. 4 and 5, respectively. As we can see, if we take a smaller value of $m_{\tilde{\mu}R}$, the SUSY contribution to the muon MDM is enhanced in the large μ_H region. This can be easily understood if we think of the fact that Δa_{μ}^{N1} gives a large contribution in such a parameter region.

Furthermore, by choosing the right-handed smuon mass $m_{\tilde{\mu}R}$ so that $|\Delta a_{\mu}^{\rm SUSY}|$ is minimized, we obtain the lower bound on the SUSY contribution to the muon MDM. The results are shown in Fig. 6. Here, we assume $45\,{\rm GeV}\leq m_{\tilde{\mu}R}\leq 1\,{\rm TeV}$. The lower bound is obtained from the negative search for the smuon [4], while the upper bound is due to the naturalness point of view. In fact, the results are insensitive to the upper bound if we take the upper bound larger than about 1 TeV, since the effects of the right-handed smuon decouple when we take $m_{\tilde{\mu}R}\to\infty$.

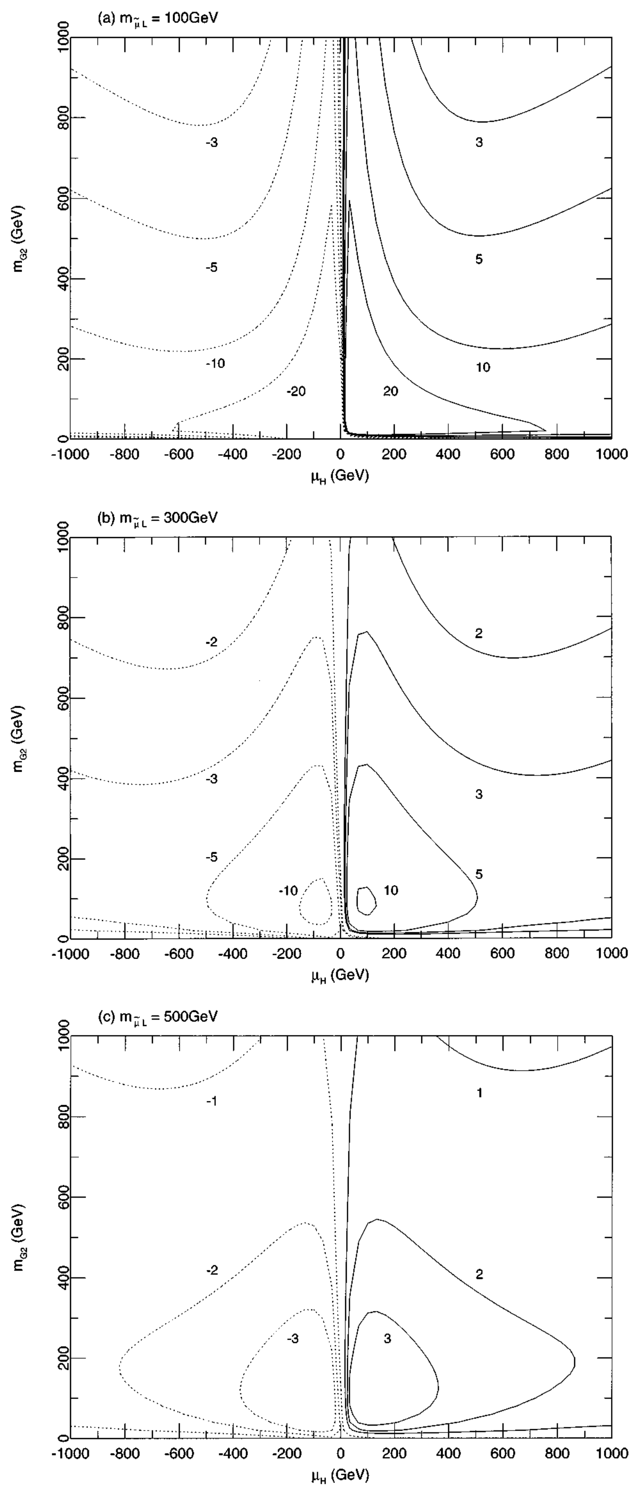

FIG. 4. Same as Fig. 3 except for $m_{\tilde{\mu}R}=300$ GeV.

Here, we would like to discuss the behavior of $\Delta a_{\mu}^{\rm SUSY}$ shown in Fig. 6. As can be seen, $\Delta a_{\mu}^{\rm SUSY}$ changes its behavior at around $|\mu_H|\sim m_{\tilde{\mu}L}$. This can be understood in the following way. In the case of $|\mu_H|\sim m_{\tilde{\mu}L}$, Δa_{μ}^{N1} and Δa_{μ}^{N2} almost cancel out and $\Delta a_{\mu}^{\rm SUSY}$ becomes insensitive to $m_{\tilde{\mu}R}$. In the case of $|\mu_H|\gtrsim m_{\tilde{\mu}L}$, the $m_{\tilde{\mu}R}$ dependence of $\Delta a_{\mu}^{\rm SUSY}$ is almost determined by that of Δa_{μ}^{N1}. Then, $|\Delta a_{\mu}^{\rm SUSY}|$ be-

[3]Gaugino mass for the $U(1)_Y$ gauge group is determined by the GUT relation (7).

[4]The supergravity model suggests $A_{\mu}\sim O(y_{\mu}m_{\tilde{\mu}})$ [14]. Furthermore, it was pointed out that some unwanted minimum appears in the potential of the smuon when $|A_{\mu}|>O(1)\times y_{\mu}m_{\tilde{\mu}}$ [15], which may cause cosmological difficulties. We checked that the results are almost unchanged even if we take $A_{\mu}=3y_{\mu}m_{\tilde{\mu}L}$.

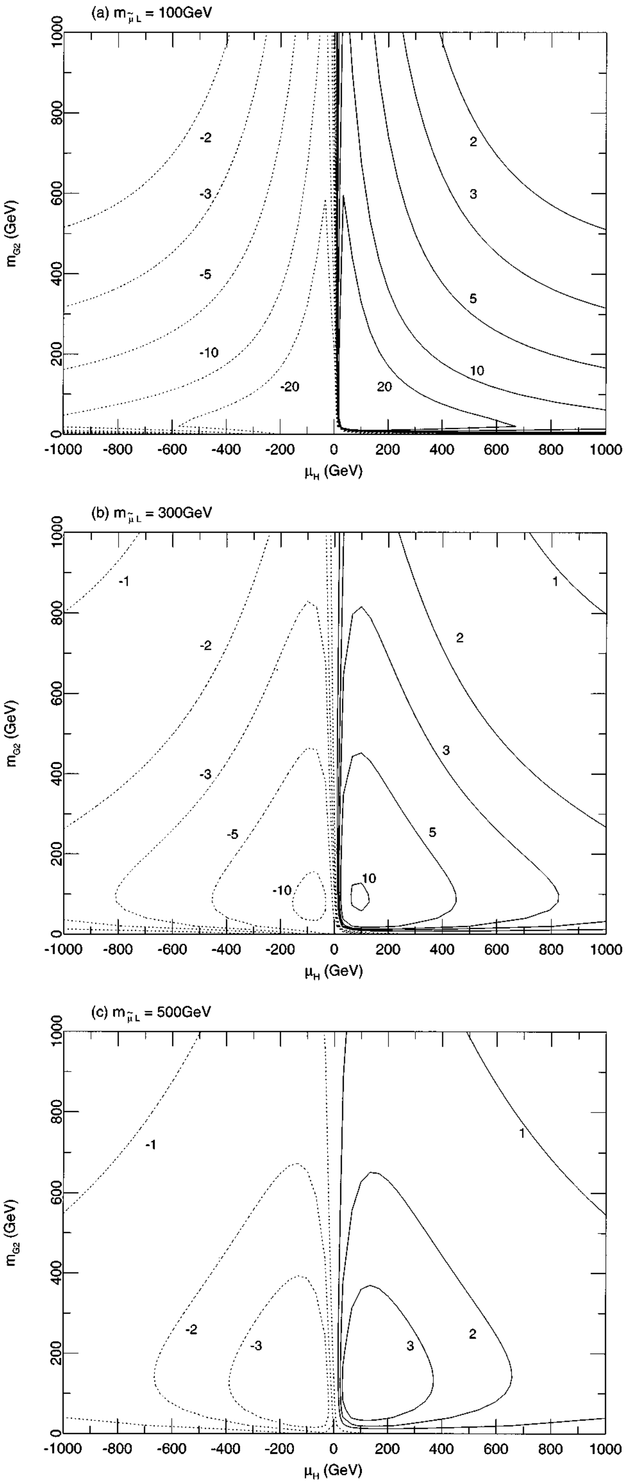

FIG. 5. Same as Fig. 3 except for $m_{\tilde{\mu}R}=1$ TeV.

comes smaller as $m_{\tilde{\mu}R}$ becomes larger. On the other hand, if $|\mu_H|\lesssim m_{\tilde{\mu}L}$, Δa_μ^{N2} determines the $m_{\tilde{\mu}R}$ dependence of $\Delta a_\mu^{\mathrm{SUSY}}$. The important point is that the sign of Δa_μ^{N2} is opposite to that of Δa_μ^{C} which gives the dominant contribution. Thus, $|\Delta a_\mu^{\mathrm{SUSY}}|$ gets smaller as $m_{\tilde{\mu}R}$ decreases. In summary, in the case of $|\mu_H|\gtrsim m_{\tilde{\mu}L}$, $|\Delta a_\mu^{\mathrm{SUSY}}|$ increases as $m_{\tilde{\mu}R}$ decreases, while in the case of $|\mu_H|\lesssim m_{\tilde{\mu}L}$, $|\Delta a_\mu^{\mathrm{SUSY}}|$ decreases as $m_{\tilde{\mu}R}$ gets smaller.

Notice that some regions of the μ_H-m_{G2} plane are excluded by the negative search for signals of neutralinos or charginos [16,17]. In Fig. 7, we show the excluded region for $\tan\beta=30$, i.e., for the large $\tan\beta$ case.[5] Here, we adopt the following constraints [17]:

$$\Delta\Gamma_Z<23.1 \quad \mathrm{MeV}, \tag{39}$$

$$\Delta\Gamma_{\mathrm{inv}}<8.4 \quad \mathrm{MeV}, \tag{40}$$

$$Br(Z\to\chi_1^0\chi_2^0)<5\times10^{-5}, \tag{41}$$

$$Br(Z\to\chi_2^0\chi_2^0)<5\times10^{-5}, \tag{42}$$

where $\Delta\Gamma_Z$ is the partial width of the Z boson decaying into charginos or neutralinos, while $\Delta\Gamma_{\mathrm{inv}}=\Gamma_Z(Z\to\chi_1^0\chi_1^0)$ represents the neutralino contribution to the invisible width. For the constraint on the chargino mass, we consider several cases where the lower bound on the chargino mass is given by 45 GeV (LEP), 90 GeV (LEP II), and 250 GeV (NLC with $\sqrt{s}=500$ GeV). Comparing Fig. 7 with Figs. 3–6, we can see that the muon MDM has a better sensitivity to MSSM than colliders in some parameter space.

Remember that the SUSY contribution to the muon MDM is approximately proportional to $\tan\beta$. Therefore, even for the case of $\tan\beta\neq30$, we can read off the approximate value of $\Delta a_\mu^{\mathrm{SUSY}}$ from Figs. 3–6. For example, the contours for $\Delta a_\mu^{\mathrm{SUSY}}=2\times10^{-9}$ in these figures correspond to $\Delta a_\mu^{\mathrm{SUSY}}\simeq4\times10^{-9}$ for the case of $\tan\beta=60$.

If the new Brookhaven E821 experiment measures the muon MDM with the accuracy of their proposal, it will have a great impact on MSSM. In a large parameter space, $|\Delta a_\mu^{\mathrm{SUSY}}|$ becomes $O(10^{-9})$, which is within the reach of the new Brookhaven E821 experiment. Furthermore, the theoretical uncertainty, which is almost originate to the hadronic uncertainty, is also expected to be decreased due to better measurements of the cross section of $e^++e^-\to$hadrons at low energies. Thus, the muon MDM should be regarded as a good probe of MSSM. In particular, the Brookhaven E821 may be able to see the signal of MSSM even in the case where we cannot find any superparticle by NLC with $\sqrt{s}=500$ GeV.

The SUSY contribution should be compared with the present constraints on the muon MDM from experiment and theoretical calculations, which are given in Eqs. (1) and (2). Combining them, we obtain a constraint on the SUSY contribution to the muon MDM, $\Delta a_\mu^{\mathrm{SUSY}}$, which is given by

$$-9.0\times10^{-9}\leqslant\Delta a_\mu^{\mathrm{SUSY}}\leqslant19.0\times10^{-9} \quad (90\%\ \mathrm{C.L.}). \tag{43}$$

In Fig. 8, we show the contour of $\tan\beta$ which gives the threshold value of the present constraint on $\Delta a_\mu^{\mathrm{SUSY}}$ given above (i.e., $\Delta a_\mu^{\mathrm{SUSY}}=-9.0\times10^{-9}$ and

[5]If $\tan\beta$ is fairly large ($\tan\beta\gtrsim5$), mass matrices of the charginos and neutralinos become almost independent of $\tan\beta$. In this case, the constraint is insensitive to $\tan\beta$. We would like to note here that if $\tan\beta$ is not so large, in our convention, the constraint becomes severer for the case of $\mu_Hm_{G2}>0$ rather than $\mu_Hm_{G2}<0$, as shown in Refs. [16,17].

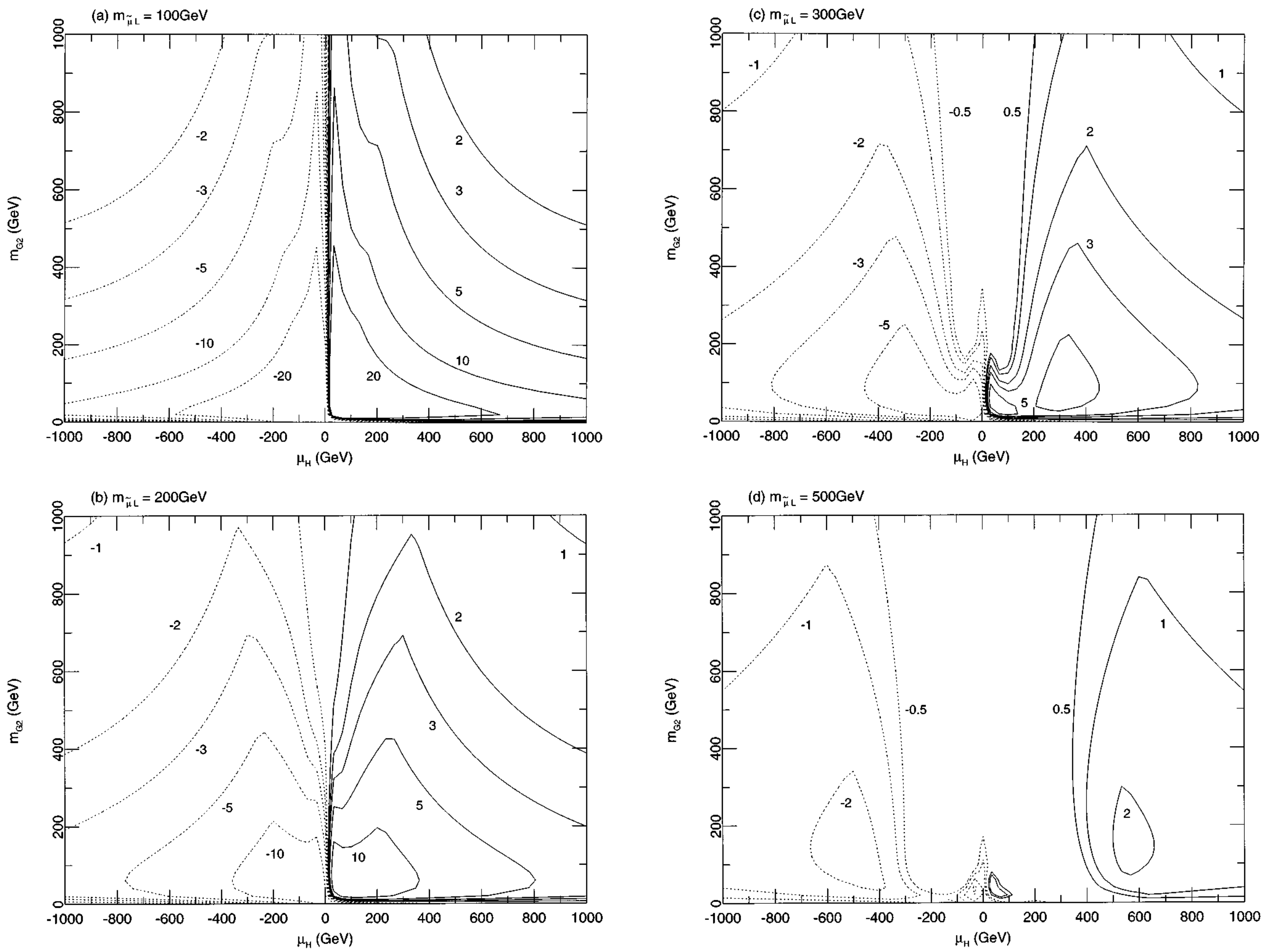

FIG. 6. The SUSY contribution to the muon MDM, $\Delta a_\mu^{\rm SUSY}$, in the μ_H-m_{G2} plane. The right-handed smuon mass $m_{\tilde{\mu}R}$ is determined so that $\Delta a_\mu^{\rm SUSY}$ takes its minimal value. We take $\tan\beta=30$, and the left-handed smuon mass $m_{\tilde{\mu}L}$ is taken to be (a) 100 GeV, (b) 200 GeV, (c) 300 GeV, and (d) 500 GeV. The numbers given in the figures represent the value of $\Delta a_\mu^{\rm SUSY}$ in units of 10^{-9}.

$\Delta a_\mu^{\rm SUSY}=19.0\times10^{-9}$). Here, we choose $m_{\tilde{\mu}R}$ so that $|\Delta_\mu^{\rm SUSY}-5.0\times10^{-9}|$ is minimized [where 5.0×10^{-9} is the center value of the constraint (43)]. Thus, Fig. 8 should be regarded as a constraint on the μ_H-m_{G2} plane for a fixed value of $m_{\tilde{\mu}L}$ and $\tan\beta$. Notice that if we assume a larger value of $\tan\beta$, SUSY contribution exceeds the present limit on the muon MDM in wider regions.

Before closing this section, we point out the fact that the contour in Fig. 8 is not symmetric under $\mu_H\rightarrow-\mu_H$. This is because the center value of the constraint given in inequality (43) is 5.0×10^{-9}, which deviates from zero. Therefore, constraint (43) prefers positive value of $\Delta a_\mu^{\rm SUSY}$, and hence we have severe constraint for $\mu_H<0$.

VI. DISCUSSION

In this paper, we have investigated the SUSY contribution to the muon MDM by regarding all the parameters in MSSM as free parameters. Especially when $\tan\beta$ is large, the SUSY contribution is enhanced, and some parameter region of MSSM is excluded not to conflict with the present constraint on the muon MDM. Furthermore, even in the case where $\tan\beta$ is not so large ($\tan\beta\lesssim10$), $\Delta a_\mu^{\rm SUSY}$ may become comparable to the present limit on the muon MDM, if the masses of the superparticles are quite light [see Fig. 8(a)].

In MSSM, the large $\tan\beta$ scenario is an interesting issue that has received attention in recent years. One of the motivations of large $\tan\beta$ is the unification of the masses of bottom and tau in SUSY GUT [20]. That is, in SUSY GUT where the Yukawa coupling constants for bottom, y_b and tau, y_τ are unified at the GUT scale, the Yukawa coupling constant of bottom (or top) is claimed to be significantly large in order to have the observed value of the bottom mass. Thus, for the successful unification of y_b and y_τ, large value of $\tan\beta$ is preferred. (Another solution is to assume $\tan\beta\sim1$ so that the Yukawa coupling constant for top, y_t, becomes large.) SUSY GUT based on SO(10) may give us another motivation of large $\tan\beta$ [21]. In a simple SO(10) GUT, all the Yukawa coupling constants (especially, y_b and y_t) are unified at the GUT scale. In this case, $\tan\beta$ as large as $m_t/m_b\sim50$ is required in order to make the hierarchy between the top and bottom masses. Furthermore, in some model in which the masses of the light fermions are generated radiatively, we need large value of $\tan\beta$ [22]. The new Brookhaven E821 experiment will be a powerful test for such types of large $\tan\beta$ scenarios.

Due to the fact that the SUSY contribution to the muon MDM strongly depends on $\tan\beta$, we may be able to use the

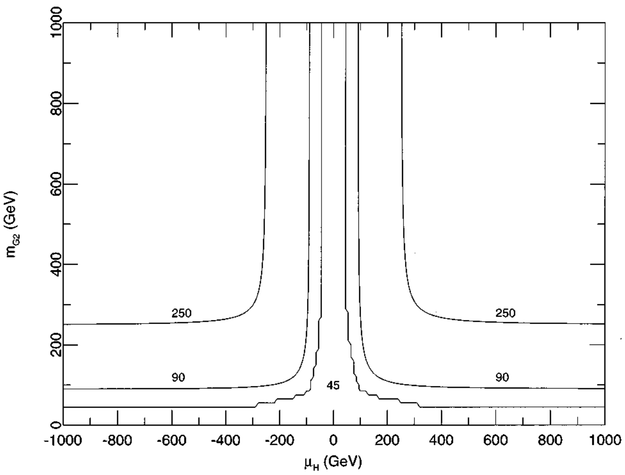

FIG. 7. Constraints on the μ_H-m_{G2} plane for $\tan\beta = 30$ from the negative searches for the neutralinos and the charginos. The numbers on the figure represent the lower bound on the chargino mass in units of GeV. The contour with $m_{G2} \leqslant 45$GeV corresponds to the constraint from LEP [16,17].

muon MDM for the determination of $\tan\beta$, especially for the large $\tan\beta$ case. That is, by future experiments, in particular by NLC, we will be able to measure the masses of the superparticles accurately, and it can hopefully fix most of the parameters on which the muon MDM depends. Then, precise measurement of the muon MDM will give us useful information about $\tan\beta$.

Comparison of our results with those based on minimal supergravity [10] may be interesting. In both cases, the SUSY contribution to the muon MDM may become of order $10^{-8}-10^{-9}$ if $\tan\beta$ is large. However, in our result, we can see several interesting behaviors which hardly occur in the case of minimal supergravity. That is, if we go away from the assumption of the universal scalar mass, a cancellation may occur among several diagrams when the mass splitting of left- and right-handed smuon is large. Furthermore, in the case where the SUSY invariant Higgs mass μ_H is quite larger than the SUSY breaking parameters, diagram $(N1)$ in Fig. 2 becomes significant, resulting in the enhancement of $\Delta a_\mu^{\mathrm{SUSY}}$.

Finally, we would like to comment on the case with the flavor mixing in the slepton mass matrices. In particular, even in the case of minimal supergravity, the sfermion mass matrices receive renormalization effects from the physics much above the electroweak scale, such as the right-handed neutrino multiplets [18,13] or GUT [19], resulting in non-vanishing off-diagonal elements of the slepton mass matrix. If the off-diagonal elements of the slepton mass matrices are substantially large, all the sleptons contribute to the muon MDM, as we mentioned before. However, for the case where the flavor mixing exists only in the left- or right-handed lepton mass matrix, the previous arguments are almost unchanged. If both left- and right-handed slepton mass matrices have large off-diagonal elements, the situation changes. In particular, in this case, the Yukawa coupling constant of tau can contribute to the muon MDM through the Feynman diagram like $(N1)$ in Fig. 2, and hence the muon MDM may be enhanced.

Detailed analysis of this case is quite complicated since

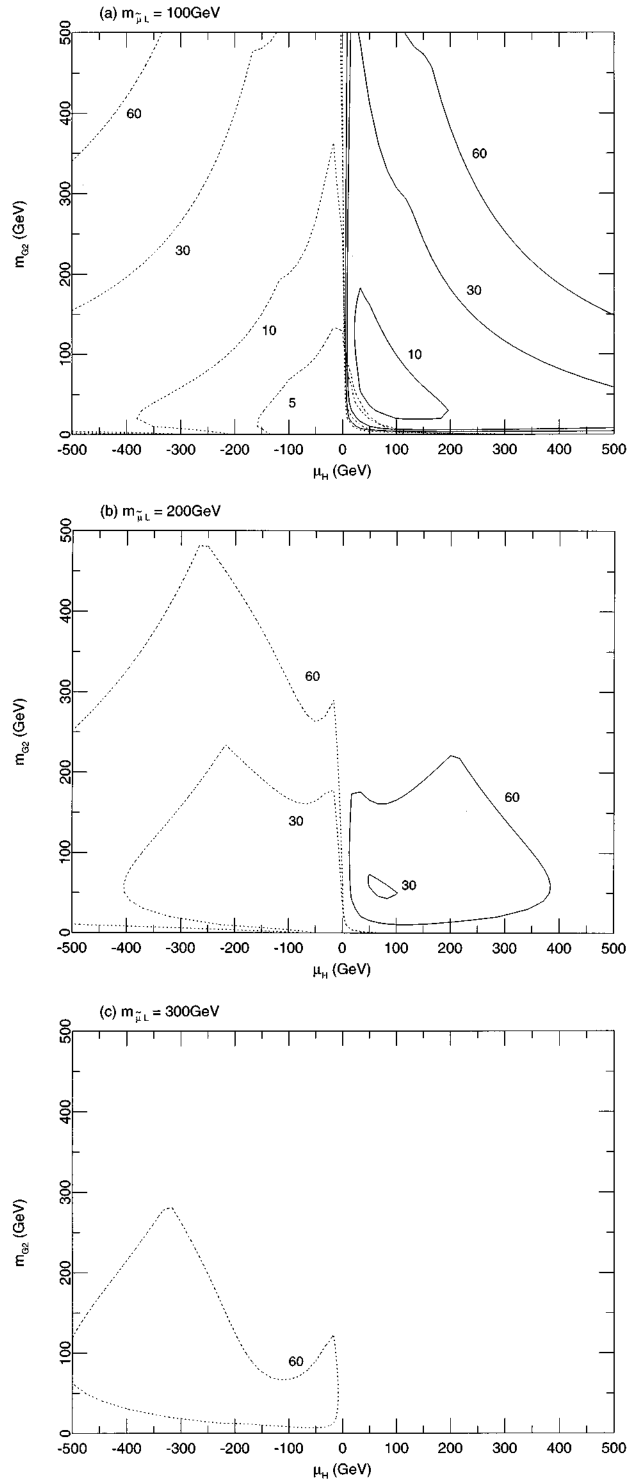

FIG. 8. Contours which gives the threshold value, i.e., $\Delta a_\mu^{\mathrm{SUSY}} = -9.0\times 10^{-9}$ (dotted line) and $\Delta a_\mu^{\mathrm{SUSY}} = 19.0\times 10^{-9}$ (solid line). The right-handed smuon mass $m_{\tilde{\mu}R}$ is determined so that $|\Delta a_\mu^{\mathrm{SUSY}} - 5.0\times 10^{-9}|$ is minimized. The values shown in the figures represent those of $\tan\beta$, and we take the left-handed smuon mass $m_{\tilde{\mu}L}$ to be (a) 100 GeV, (b) 200 GeV, and (c) 300 GeV.

the muon MDM depends on a large number of parameters. Thus, we only discuss the case where the diagonal element of the left- and right-handed sleptons, $\hat{m}_L^2$ and $\hat{m}_R^2$, are proportional to unit matrix $\hat{m}_{L,ii}^2 = m_L^2$, $\hat{m}_{R,ii}^2 = m_R^2$ (i, not summed). First, we consider the case where $\hat{m}_L^2$ or $\hat{m}_R^2$ has an off-diagonal element. In this case, the results of the previous analysis are almost unaffected. For example, even if $\hat{m}_{L,23}^2/\hat{m}_{L,22}^2=0.5$ (or $\hat{m}_{R,23}^2/\hat{m}_{R,22}^2=0.5$), the correction to $\Delta a_\mu^{\rm SUSY}$ is less than $\sim 10\%$. If both $\hat{m}_L^2$ and $\hat{m}_R^2$ have large off-diagonal elements, $\Delta a_\mu^{\rm SUSY}$ may receive a large correction. Numerically, when $\hat{m}_{L,23}^2/\hat{m}_{L,22}^2 \sim \hat{m}_{R,23}^2/\hat{m}_{R,22}^2 \sim 0.2$, the correction is $\sim 10\%$. The correction gets larger as the off-diagonal elements increase.

The new Brookhaven E821 experiment will have a strong impact on SUSY models. By the experiment, the muon MDM is expected to be measured with accuracy about 0.4×10^{-9}. Furthermore, the uncertainty in the theoretical prediction, which mainly comes from hadronic contributions, is hoped to be reduced by several experiments like VEPP-2M, DAΦNE, and so on. On the contrary, we may have the SUSY contribution to the muon MDM to be of order $\sim 10^{-9}$ even if all the superparticles are heavier than, say, 300 GeV [see Fig. 4(b)] in which case we cannot detect the superparticles even by NLC with $\sqrt{s}=500$ GeV. Therefore, we may be able to have a signal of the superparticles by using the muon MDM even if the superparticles are out of the reach of the forthcoming high energy colliders.

ACKNOWLEDGMENTS

The author would like to thank J. Hisano and H. Murayama for useful discussions, and C.D. Carone for reading of the manuscript. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

APPENDIX: FUNCTIONS I_N AND J_N

In this appendix, we show some useful formulas for the functions I_N and J_N, which are defined as

$$I_N(m_1^2,\ldots,m_N^2)=\int\frac{d^4k}{(2\pi)^4 i}\frac{1}{(k^2-m_1^2)\cdots(k^2-m_N^2)}, \tag{A1}$$

$$J_N(m_1^2,\ldots,m_N^2)=\int\frac{d^4k}{(2\pi)^4 i}\frac{k^2}{(k^2-m_1^2)\cdots(k^2-m_N^2)}. \tag{A2}$$

The signs of the functions I_N and J_N are given by

$$(-1)^N I_N(m_1^2,\ldots,m_N^2)>0, \tag{A3}$$

$$(-1)^{N+1} J_N(m_1^2,\ldots,m_N^2)>0. \tag{A4}$$

The functions I_N and I_{N-1} are related as

$$I_N(m_1^2,\ldots,m_N^2)=\frac{1}{m_1^2-m_N^2}\{I_{N-1}(m_1^2,\ldots,m_{N-1}^2) - I_{N-1}(m_2^2,\ldots,m_N^2)\}, \tag{A5}$$

and the explicit form of I_2 is given by

$$I_2(m_1^2,m_2^2)=-\frac{1}{16\pi^2}\left\{\frac{m_1^2}{m_1^2-m_2^2}\ln\left(\frac{m_1^2}{\Lambda^2}\right) + \frac{m_2^2}{m_2^2-m_1^2}\ln\left(\frac{m_2^2}{\Lambda^2}\right)\right\}. \tag{A6}$$

Notice that the function I_2 is logarithmically divergent, and hence I_2 defined in Eq. (A6) depends on a cutoff parameter Λ. However, I_N $(N\geqslant 3)$ which is iteratively defined by using Eq. (A5) is independent of Λ, as it should be. In addition, J_N is related to I_N and I_{N-1} as

$$J_N(m_1^2,\ldots,m_N^2)=I_{N-1}(m_1^2,\ldots,m_{N-1}^2) + m_N^2 I_N(m_1^2,\ldots,m_N^2). \tag{A7}$$

In the case where all the masses m_1-m_N are almost degenerate, it is convenient to use the Taylar expansion of I_N. Define

$$\epsilon_i \equiv \frac{\overline{m^2}-m_i^2}{\overline{m^2}} \quad (i=1-N), \tag{A8}$$

with $\overline{m}$ being an arbitrary mass scale, then I_N is expanded as

$$I_N(m_1^2,\ldots,m_N^2)=\frac{(-1)^N}{16\pi^2}\frac{1}{\overline{m^{2(N-2)}}} \times\sum_{p=0}^{\infty}\frac{1}{(N+p-2)(N+p-1)} \times\sum_{j_1+\cdots+j_N=p}\epsilon_1^{j_1}\cdots\epsilon_N^{j_N} \quad (N\geqslant 3), \tag{A9}$$

and for $N=2$,

$$I_2(m_1^2,m_2^2)=-\frac{1}{16\pi^2}\left\{\ln\left(\frac{\overline{m^2}}{\Lambda^2}\right)+1\right\} + \frac{1}{16\pi^2}\sum_{p=1}^{\infty}\frac{1}{p(p+1)}\sum_{j_1+j_2=p}\epsilon_1^{j_1}\epsilon_2^{j_2}. \tag{A10}$$

Notice that Eqs. (A5)–(A10) are useful for numerical calculations.

Furthermore, the function I_N has mass dimension $(4-2N)$. Therefore, we obtain

$$\frac{d}{d\lambda}\{\lambda^{2-N} I_N(\lambda m_1^2,\ldots,\lambda m_N^2)\}=0, \tag{A11}$$

which reduces to

$$(2-N)I_N(m_1^2,\ldots,m_N^2)+\sum_{i=1}^{N} m_i^2 I_{N+1}(m_1^2,\ldots,m_i^2,m_i^2,\ldots,m_N^2)=0. \quad \text{(A12)}$$

A similar formula can be obtained for J_N:

$$(3-N)J_N(m_1^2,\ldots,m_N^2)+\sum_{i=1}^{N} m_i^2 J_{N+1}(m_1^2,\ldots,m_i^2,m_i^2,\ldots,m_N^2)=0. \quad \text{(A13)}$$

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