Dynamics of topological defects and inflation

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We study the dynamics of topological defects in the context of "topological inflation" proposed by Vilenkin and Linde independently. Analyzing the time evolution of planar domain walls and of global monopoles, we find that the defects undergo inflationary expansion if $\eta \stackrel{>}{\sim} 0.33 m_{Pl}$, where η is the vacuum expectation value of the Higgs field and m_{Pl} is the Planck mass. This result confirms the estimates by Vilenkin and Linde. The critical value of η is independent of the coupling constant λ and the initial size of the defect. Even for defects with an initial size much greater than the horizon scale, inflation does not occur at all if η is smaller than the critical value. We also examine the effect of gauge fields for static monopole solutions and find that the spacetime with a gauge monopole has an attractive nature, contrary to the spacetime with a global monopole. It suggests that gauge fields affect the onset of inflation.

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I. INTRODUCTION

Vilenkin [1] and Linde [2] independently pointed out that topological defects can be seeds for inflation. The basic idea of this "topological inflation" is as follows. Suppose that we have the Higgs field Φ^a ($a = 1, \ldots, N$), whose potential is

$$V(\Phi) = \frac{1}{4}\lambda(\Phi^2 - \eta^2)^2, \quad \Phi \equiv \sqrt{\Phi^a \Phi^a}, \tag{1.1}$$

where η is the vacuum expectation value and λ is a coupling constant. This model gives rise to different types of topological defects, i.e., domain walls for $N = 1$, cosmic strings for $N = 2$, and monopoles for $N = 3$. Because the center of a defect is in the false vacuum state ($\Phi = 0$), we can expect that inflation occurs near $\Phi = 0$ under some condition. Vilenkin and Linde claimed that topological defects expand exponentially if and only if

$$\eta > O(m_{Pl}) \tag{1.2}$$

for the following reasons.

(i) If the size of the false vacuum region is greater than the horizon size, it is natural to assume that this region undergoes inflationary expansion. The thickness of a domain wall in a flat spacetime is given by

$$\delta_0 = \frac{\sqrt{2}}{\sqrt{\lambda}\eta}, \tag{1.3}$$

and the horizon size corresponding to the vacuum energy $V(0)$ is

$$H_0^{-1} \equiv \left[\frac{8\pi}{3m_{Pl}^2}V(0)\right]^{-\frac{1}{2}} = \sqrt{\frac{3}{2\pi\lambda}}\frac{m_{Pl}}{\eta^2}. \tag{1.4}$$

Then the condition $\delta_0 > H_0^{-1}$ implies $\eta \stackrel{>}{\sim} m_{Pl}$ [1].

(ii) New inflation or chaotic inflation occurs in Friedmann-Robertson-Walker spacetime if the slow-rolling condition [$|\ddot{\Phi}| \ll 3H|\dot{\Phi}|$ and $\dot{\Phi}^2/2 \ll V(\Phi)$] is satisfied. In model (1.1) this condition leads to $\eta \gg m_{Pl}/\sqrt{6\pi}$ [1, 2].

(iii) Gravitational effect becomes important if the Schwarzschild radius $R_g = 2M/m_{Pl}^2$ is comparable to δ_0, where $M \approx (4\pi/3)\delta_0^3 V(0)$. This condition is reduced to $\eta \stackrel{>}{\sim} \sqrt{3/8\pi}m_{Pl}$ [1, 2].

(iv) In the case of a global string or a global monopole there is a deficit angle or a solid deficit angle in the static solution. For each defect, this exceeds 2π or 4π if $\eta > m_{Pl}$, which indicates no static solution with an asymptotically flat region [1].

(v) It was shown in [3–5] that nonsingular static gauge monopole solutions exist only if $\eta \stackrel{<}{\sim} m_{Pl}$; otherwise, the monopole becomes dynamic and may become a Reissner-Nordström black hole. Linde speculated that in this case the central region of the monopole expands exponentially because the condition, $\eta \stackrel{>}{\sim} m_{Pl}$, is simultaneously a condition of inflation [the slow-rolling condition in (ii)].

In this paper we study topological inflation in detail by numerical analysis. On this subject, we need to clarify the following points.

(1) Although the condition for inflation to occur, (1.2), looks plausible, it is based on discussions which did not show how the defects really inflate. It is important to verify these arguments by numerical analysis which comprehensively includes the gravitational effect.

(2) Vilenkin and Linde assumed the initial size of defects to be δ_0, which stems from the static solution without gravitational effect. We agree that δ_0 implies the typical size of the defects even in curved spacetime. In the beginning of the phase transition, however, the scalar field takes its phase randomly and global distribution may be chaotic. Therefore, it is worth studying the dynamics of the defects with various initial sizes. Particularly, our interest is the fate of the topological defect when the initial size is greater than the horizon scale but

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η is not so large. One may conceive that such defects also inflate.

(3) The effect of gauge fields is unclear. Linde claimed that gauge fields do not affect inflation of strings or monopoles, because they exponentially decrease during inflation. We agree that, once inflation occurs, their effect gets smaller and smaller. However, it remains an unsettled question whether or not gauge fields affect the onset of inflation.

In order to answer the first and the second questions, we investigate the evolution of planar domain walls and global monopoles in Sec. II. As to the third question, we discuss static monopole solutions in Sec. III. Section IV is devoted to conclusions. In this paper we use the units $c = \hbar = 1$.

II. EVOLUTION OF PLANAR DOMAIN WALLS AND GLOBAL MONOPOLES

In what follows we numerically analyze the time evolution of domain walls ($N = 1$) in a plane symmetric spacetime and of global monopoles ($N = 3$) in a spher-

ically symmetric spacetime. The Einstein-Higgs system is described by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\rm Pl}^2}{16\pi} \mathcal{R} - \frac{1}{2}(\partial_\mu \Phi^a)^2 - V(\Phi) \right]. \quad (2.1)$$

The field equations derived from (2.1) are presented in the Appendix.

For the case of domain walls, we assume that the metric has a form,

$$ds^2 = -dt^2 + A^2(t,|x|)dx^2 + B^2(t,|x|)(dy^2 + dz^2).$$

$$(2.2)$$

As an initial configuration of the scalar field, it seems natural to use the same functional form as the static solution in a flat spacetime, $\Phi_{\rm flat}(x) \equiv \eta \tanh(x/\delta)$, where δ is a parameter of the initial width of the domain wall. In a curved spacetime, however, if we consider a single domain wall in a vacuum background spacetime, the spacetime becomes closed [6]. There is no far region from the wall. For this reason we adopt the periodic boundary condition and accordingly change the functional form for each section slightly into

$$\Phi(t=0,x) = \begin{cases} \eta\left[\frac{x}{\delta} - \frac{5}{4}\left(\frac{8}{15}\frac{x}{\delta}\right)^3 + \frac{3}{8}\left(\frac{8}{15}\frac{x}{\delta}\right)^5\right] & (0 \leq \frac{x}{\delta} \leq \frac{15}{8}), \\ \eta & (\frac{15}{8} \leq \frac{x}{\delta} \leq 2). \end{cases} \quad (2.3)$$

The above polynomial has been determined from the following assumptions: (1) fifth-order odd polynomial function of x; (2) the first term agrees with that of $\eta \tanh(x/\delta)$; (3) it connects the constant function $\Phi(x) = \eta$ at the point where $\Phi' \equiv \partial\Phi/\partial x = \Phi'' = 0$. For later convenience, we define a parameter of the initial width of a wall as $c \equiv \delta/\delta_0$.

One may wonder if our results really show generic properties of domain walls, because the global structure of a spacetime depends on how we choose boundary conditions. We will discuss this point in the final section.

For the case of global monopoles, we assume a spherically symmetric spacetime:

$$ds^2 = -dt^2 + A^2(t,r)dr^2 + B^2(t,r)r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

$$(2.4)$$

For the scalar field, we adopt the hedgehog ansatz:

$$\Phi^a = \Phi(t,r)\hat{r}^a \equiv \Phi(t,r)(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta).$$

$$(2.5)$$

As an initial configuration of the scalar field, we suppose

$$\Phi(t=0,r) = \Phi_{\rm flat}\left(\frac{r}{c}\right), \quad (2.6)$$

where $\Phi_{\rm flat}(r)$ is a static solution in a flat spacetime, which is obtained only numerically, and c is an initial size parameter of a monopole normalized by that of the

static solution.

As to the initial value of $\dot\Phi \equiv \partial\Phi/\partial t$, we suppose $\dot\Phi = 0$ in both the cases. Following the method shown in the Appendix, we solve the constraint equations for setting initial data and the dynamical equations for time evolution.

Our numerical results are summarized in Figs. 1–5. In Fig. 1 we show examples of the time evolution of the scalar field, which correspond to a stable domain wall [Fig. 1(a) $\eta = 0.2m_{\rm Pl}$] and an inflating wall [Fig. 1(b) $\eta = 0.6m_{\rm Pl}$], respectively. The abscissa, X, is defined as a proper distance along the x axis from the origin. The corresponding results for monopoles are presented in Fig. 2. In the case of Fig. 2 (b) we see that the bottleneck structure appears, but we do not find an apparent horizon of a black hole; this is not a wormhole. (As for the definition of an apparent horizon, see the last part of the Appendix.) In Fig. 3 we draw trajectories of the boundaries of a domain wall and a monopole. Here we define the boundary of the defect as $X_b(t) = X$ or $R_b(t) = Br$ at the position of $\Phi = \eta/2$. Figure 3 indicates that a domain wall or a monopole expands if $\eta \geq 0.4m_{\rm Pl}$, while it remains stable if $\eta \leq 0.3m_{\rm Pl}$. We survey for $0.3m_{\rm Pl} \leq \eta \leq 0.4m_{\rm Pl}$ and $10^{-4} \leq \lambda \leq 10$ closely, and find the critical value of η is around $0.33m_{\rm Pl}$, regardless of λ. This result supports the estimates of Vilenkin and Linde.

By varying c, we also investigate how the initial size of a domain wall or a monopole influences its dynamics.
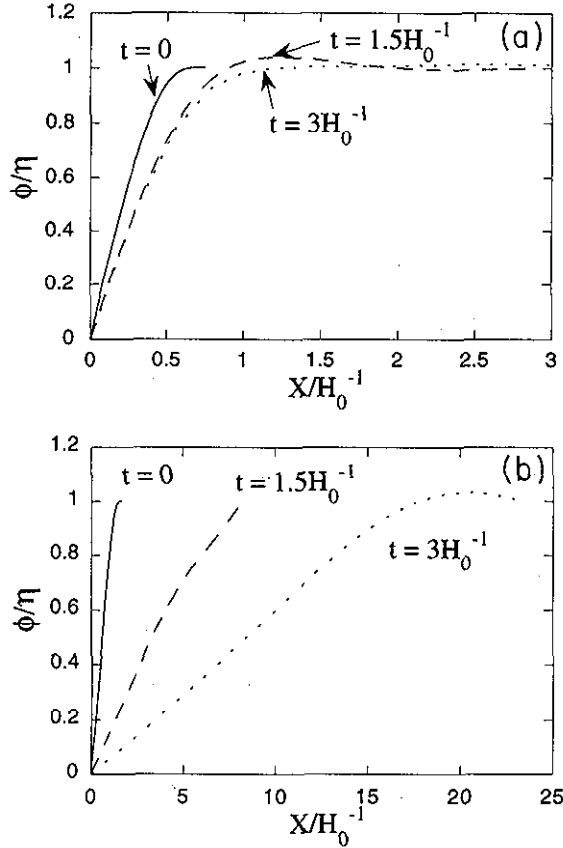
FIG. 1. Examples of the evolution of a domain wall. We set $\eta = 0.2m_{\rm Pl}$ in (a), $\eta = 0.6m_{\rm Pl}$ in (b), and $c = 1$ and $\lambda = 0.1$ in both cases. (a) expresses a stable wall and (b) expresses an inflating wall. The abscissa is a proper distance along the x axis from the origin.
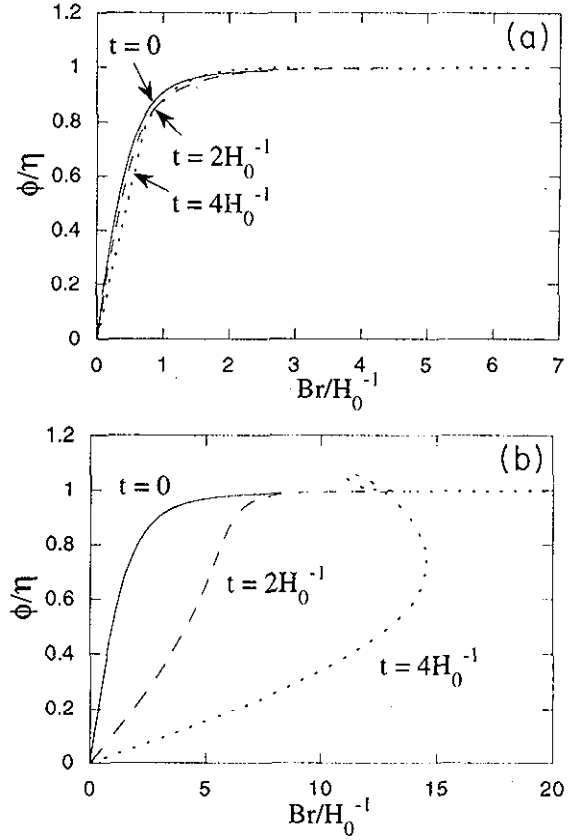

FIG. 2. Examples of the evolution of a global monopole. The parameters we choose are the same as in Fig. 1. In (b) we find that the bottleneck structure appears.

Figure 4 indicates that the final behavior of a domain wall or a monopole is determined only by η. In the case of Fig. 4(a) or 4(b) ($\eta = 0.2m_{\rm Pl}$), even if the initial size is much larger than H_0^{-1}, the configuration eventually approaches to $\Phi_{\rm flat}(X)$ or $\Phi_{\rm flat}(Br)$. We analyze whether or not the cosmological horizon appears or not by searching for an apparent horizon, because H_0^{-1} is not exactly the cosmological horizon. Figure 5 shows that an apparent horizon really appears and the surface of the shrinking monopole ($\eta = 0.2m_{\rm Pl}, c = 10$) crosses it. Thus we see that, even for defects with an initial size much greater than the horizon scale, inflation does not occur if η is smaller than the critical value.

III. EFFECT OF GAUGE FIELDS FOR STATIC MONOPOLE SOLUTIONS

In this section we describe the effect of gauge fields for static monopole solutions. We consider the SU(2) Einstein-Yang-Mills-Higgs system, which is described by

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\rm Pl}^2}{16\pi}\mathcal{R} - \frac{1}{4}(F_{\mu\nu}^a)^2 \right.$$
$$\left. -\frac{1}{2}(D_\mu \Phi^a)^2 - V(\Phi) \right], \qquad (3.1)$$

with

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\epsilon^{abc}A_\mu^b A_\nu^c,$$
$$D_\mu \Phi^a \equiv \partial_\mu \Phi^a + e\epsilon^{abc}A_\mu^b \Phi^c, \qquad (3.2)$$

where A_μ^a and $F_{\mu\nu}^a$ are the SU(2) Yang-Mills field potential and its field strength, respectively. D_μ is the covariant derivative and e is a gauge coupling constant. A static and spherically symmetric spacetime is described as

$$ds^2 = -\left(1 - \frac{2m_{\rm Pl}^{-2}M(R)}{R}\right)e^{-2\alpha(R)}dT^2$$
$$+ \left(1 - \frac{2m_{\rm Pl}^{-2}M(R)}{R}\right)^{-1}dR^2 + R^2(d\theta^2 + \sin^2\theta d\varphi^2).$$
$$(3.3)$$

Under the 't Hooft-Polyakov ansatz, the Yang-Mills potential is written as

$$A_i^a = \omega_i^c \epsilon^{cab}\hat{r}^b \frac{1 - w(R)}{eR}, \quad A_0^a = 0, \qquad (3.4)$$

where ω_i^a is a triad.

Breitenlohner et al. systematically surveyed regular monopole solutions in this system [3]. According to Fig. 6 in [3], static solutions cease to exist when η becomes larger than a critical value $\eta_{\rm cr}$, which has a little dependence on λ/e^2: $0.20m_{\rm Pl} < \eta_{\rm cr}(\lambda/e^2) < 0.39m_{\rm Pl}$ for $\infty \ge$
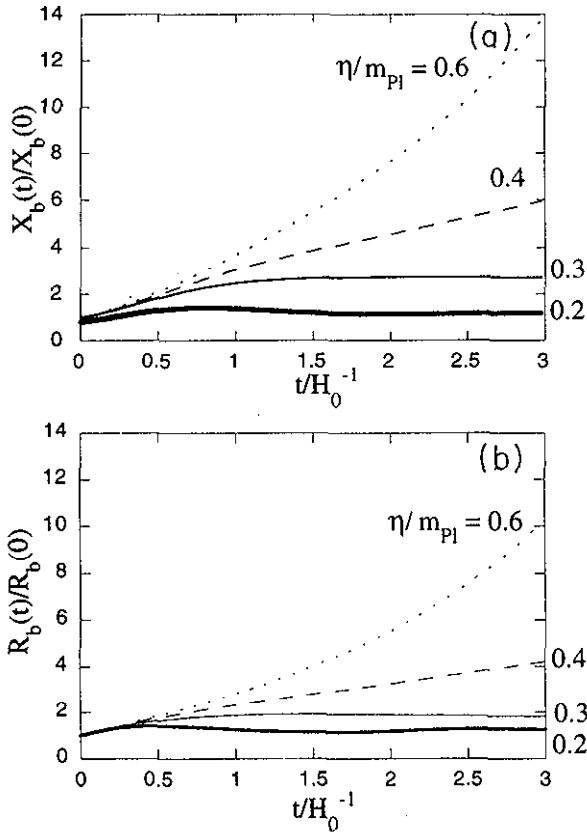
FIG. 3. Dependence of the evolution of a domain wall and a monopole on η. We plot trajectories of the positions of $\Phi = \eta/2$. We set $\lambda = 0.1$ and $c = 1$ in both cases. (a) and (b) express the dynamics of a domain wall and a global monopole, respectively. A domain wall or a monopole expands if $\eta \gtrsim 0.33 m_{\rm Pl}$.

$\lambda/e^2 \geq 0$. (The relations between the parameters in this paper and those in [3] are given by $\eta/m_{\rm Pl} = \alpha/\sqrt{4\pi}$ and $\lambda/e^2 = \beta^2/2$.) The exact value of $\eta_{\rm cr}$ for $\lambda/e^2 \to \infty$ was analytically obtained [5]: $\eta_{\rm cr}(\lambda/e^2 \to \infty) = m_{\rm Pl}/\sqrt{8\pi}$. We also examine this monopole system and confirm their results.

From the two facts of the disappearance of static solutions for large η and of the inflation of a global monopole for large η, we might expect that gauge monopoles also inflate for large η. In order to see the effect of gauge fields, we calculate the acceleration of test particles, which is governed by the geodesic equation in the coordinate system (3.3):

$$\frac{d^2 R}{d\tau^2} = -\frac{e^{2\alpha(R)}}{2}\frac{d}{dR}\left[e^{-2\alpha(R)}\left\{1 - \frac{2m_{\rm Pl}^{-2}M(R)}{R}\right\}\right],$$

$$(3.5)$$

where τ is the proper time and we have assumed $dR/d\tau = d\theta/d\tau = d\varphi/d\tau = 0$. Although we cannot discuss the dynamics of a static spacetime by its metric itself, the acceleration of test particles gives us some information about the spacetime structure. For example, in de Sitter spacetime $[M(R) = m_{\rm Pl}^2 \bar{H}^2 R^3/2,\ \bar{H} = {\rm const},\ \alpha(R) = 0]$
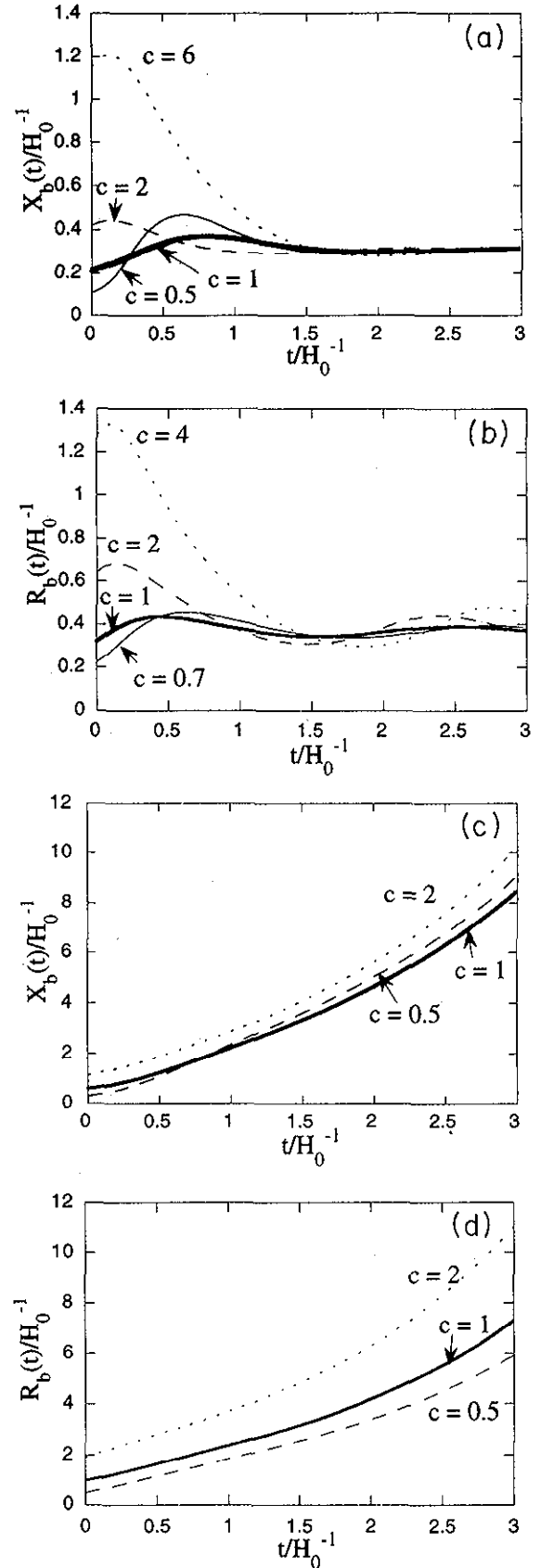

FIG. 4. Dependence of the evolution on the initial size. We set $\lambda = 0.1$ in all cases. (a) domain wall, $\eta = 0.2 m_{\rm Pl}$. (b) global monopole, $\eta = 0.2 m_{\rm Pl}$. (c) domain wall, $\eta = 0.6 m_{\rm Pl}$. (d) global monopole, $\eta = 0.6 m_{\rm Pl}$. Only η determines whether or not inflation occurs, regardless of the initial size.
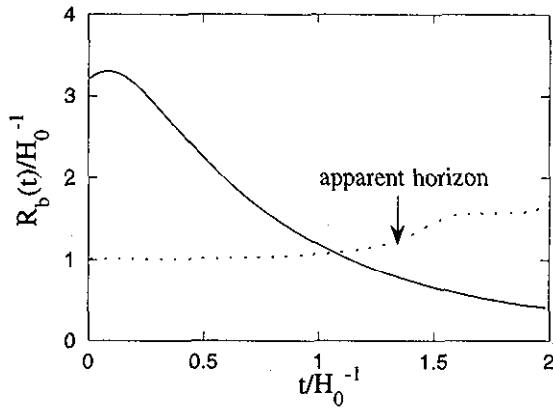
FIG. 5. Trajectories of the apparent (cosmological) horizon and of the boundary of a shrinking monopole. We set $c = 10$, $\eta = 0.2m_{\rm Pl}$, and $\lambda = 0.1$. The boundary of the shrinking monopole crosses the horizon.

and in Schwarzschild spacetime [$M(R) = \bar{M} = $ const, $\alpha(R) = 0$], (3.5) is reduced to $d^2R/d\tau^2 = \bar{H}^2 R > 0$ and $d^2R/d\tau^2 = -m_{\rm Pl}^{-2}\bar{M}/R^2 < 0$, respectively. The sign of acceleration indicates whether the spacetime is repulsive (+) or attractive (−). In Fig. 6 we present examples of the acceleration in spacetimes with a global monopole
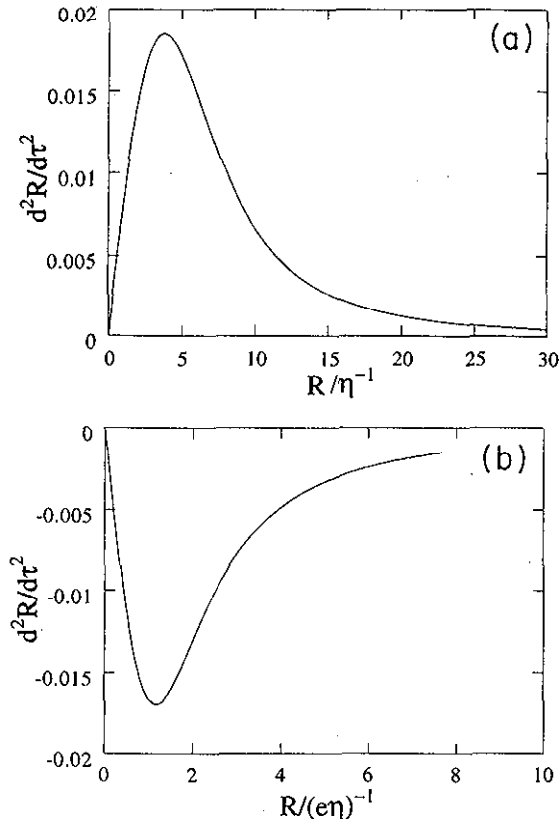

FIG. 6. Acceleration of test particles in spacetimes (a) with a global monopole and (b) with a gauge monopole. We set $\lambda = 0.1$ in (a), $\lambda/e^2 = 0.1$ in (b), and $\eta = 0.2m_{\rm Pl}$ in both cases. We find that the spacetime with a global monopole has a repulsive nature, while the spacetime with a gauge monopole has an attractive nature.

and with a gauge monopole. The figures show that the two spacetimes have different properties: the spacetime with a global monopole has a repulsive nature [7], while the spacetime with a gauge monopole has an attractive nature. Neither sign of acceleration changes even for large η. Although we have analyzed only static spacetimes, our results indicate that there may be another possibility that the monopole collapses and becomes a black hole.

IV. DISCUSSIONS

We have studied the dynamics of topological defects numerically. We have examined three questions on "topological inflation" and found the following results.

The first subject was simply verifying the arguments of Vilenkin and Linde. Our results support their discussions completely and we obtain more exactly the critical value of η which determines whether defects inflate or not. We found that, if $\eta \gtrsim 0.33m_{\rm Pl}$ is satisfied, planar domain walls and global monopoles inflate.

The second was to examine the dynamics of defects of various initial size. Our results show that in the case of planar domain walls and global monopoles, only η determines whether or not they expand. Even if the initial size of the defect is much greater than the horizon scale, it shrinks and approaches a stable configuration if η is less than the critical value. Some readers may feel this result is surprising, but we can reasonably interpret it as follows. First, we found in our analysis that the boundary of a defect [$X_b(t)$ or $R_b(t)$] can be a spacelike hypersurface, which was also pointed out by Vilenkin [1]. Thus there is no reason that the "horizon scale" prevents a defect from shrinking. Second, if $\eta \ll m_{\rm Pl}$, the slow-rolling condition is not satisfied. This condition is expressed only by η, regardless of λ and the horizon scale. We therefore conclude that, although the statement "*if the size of a defect is greater than the horizon scale, it inflates*" sounds true, in a strict sense it is not.

Let us summarize the main points: first, the condition of inflation for domain walls and for global monopoles are the same; second, the condition does not depend on the initial size. From these two facts, we may understand that whether inflation occurs or not is determined not by the global structure of a spacetime but by the local "slow-rolling" condition; our results indicate that a slow-rolling condition is a necessary and sufficient condition for topological inflation. Therefore, although we have assumed a plane symmetric spacetime and the periodic boundary condition for a domain wall system, it seems reasonable to conclude that our results show generic properties of domain walls. Similarly, we may extend our results to the case of global strings.

The final subject we have investigated is the effect of gauge fields. Comparing the gauge monopole static solution with the global monopole one, we find that these spacetimes have different characters: one is attractive and the other is repulsive. Therefore, gauge fields work to obstruct inflation and their effect cannot be ignored when we discuss the onset of inflation. It may be inter-

esting to study time-dependent gauge monopoles, which would give us a definite answer to this issue.

ACKNOWLEDGMENTS

APPENDIX: FIELD EQUATIONS AND NUMERICAL METHOD

In this appendix we explain how we solve the field equations for planar domain walls and for global monopoles. At the end we also summarize how we search for an apparent horizon in a global monopole system.

The variation of (2.1) with respect to $g_{\mu\nu}$ and Φ^a yield the Einstein equations

$$G_{\mu\nu} \equiv \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \frac{8\pi}{m_{\mathrm{Pl}}^2}T_{\mu\nu},$$

$$T_{\mu\nu} = \partial_\mu\Phi^a\partial_\nu\Phi^a - g_{\mu\nu}\left[\frac{1}{2}(\partial_\sigma\Phi^a)^2 + V(\Phi)\right], \quad \text{(A1)}$$

and the scalar field equations,

$$\Box\Phi^a = \frac{\partial V(\Phi)}{\partial\Phi^a}. \quad \text{(A2)}$$

(1) Planar domain wall system. Using the metric (2.2), we write down the field equations (A1) and (A2) as

$$-G_0^0 \equiv K_2^2(2K - 3K_2^2) - \frac{2B''}{A^2B} - \frac{B'^2}{A^2B^2} + \frac{2A'B'}{A^3B}$$
$$= \frac{8\pi}{m_{\mathrm{Pl}}^2}\left(\frac{\dot\Phi^2}{2} + \frac{\Phi'^2}{2A^2} + V\right), \quad \text{(A3)}$$

$$\frac{1}{2}G_{01} \equiv K_2^{2\prime} + \frac{B'}{B}(3K_2^2 - K) = \frac{4\pi}{m_{\mathrm{Pl}}^2}\dot\Phi\Phi', \quad \text{(A4)}$$

$$\frac{1}{2}(G_1^1 + G_2^2 + G_3^3 - G_0^0) \equiv \dot K - (K_1^1)^2 - 2(K_2^2)^2$$
$$= \frac{8\pi}{m_{\mathrm{Pl}}^2}(\dot\Phi^2 - V), \quad \text{(A5)}$$

$$-\mathcal{R}_2^2 - G_0^0 \equiv \dot K_2^2 + \frac{B'^2}{2A^2B^2} - \frac{3}{2}K_2^2$$
$$= \frac{4\pi}{m_{\mathrm{Pl}}^2}\left(\frac{\dot\Phi^2}{2} + \frac{\Phi'^2}{2A^2} - V\right), \quad \text{(A6)}$$

$$\ddot\Phi - K\dot\Phi - \frac{\Phi''}{A^2} - \left(-\frac{A'}{A} + \frac{2B'}{B}\right)\frac{\Phi'}{A^2} + \frac{dV}{d\Phi} = 0, \quad \text{(A7)}$$

where we have introduced the extrinsic curvature tensor of $t = $ constant hypersurface, K_{ij}, whose components are given by

$$K_1^1 = -\frac{\dot A}{A}, \quad K_2^2 \ (= K_3^3) = -\frac{\dot B}{B}, \quad \text{(A8)}$$

and denoted its trace by $K \equiv K_i^i$.

In order to set up initial data, we have to solve the Hamiltonian constraint equation (A3) and the momentum constraint equation (A4). We assume the homogeneous and isotropic curvature,

$$\frac{K}{3} = K_1^1 = K_2^2 = \text{const}, \quad \text{(A9)}$$

which makes (A4) trivial, and the conformally flat spatial gauge, $A = B$. Because we have adopted the periodic boundary condition such that the period is 4δ, and assumed reflection symmetry, we must keep the condition $B'(t, x = 0) = B'(t, x = 2\delta) = 0$. By changing K as a shooting parameter, we iteratively integrate (A3) until the above condition is satisfied.

Now we have five dynamical variables: A, B, K, K_2^2, and Φ. Equations (A8), (A5), and (A7) provide the next time step of A, B, K, and Φ. For the time evolution of K_2^2, we use (A6) only at $x = 0$, and then integrate (A4) in the x direction to obtain other values of K_2^2. In this way we have reduced spatial derivatives appearing in the equations, which may become seeds for numerical instability.

(2) Global monopole system. In a method similar to that applied for the domain wall system, we solve the field equations for the monopole system. Here we present the equations and only give comments on some differences. Under the assumption of the metric (2.4) and the hedgehog ansatz (2.5), we write down the field equations (A1) and (A2) as

$$-G_0^0 \equiv K_2^2(2K - 3K_2^2) - \frac{2B''}{A^2B} - \frac{B'^2}{A^2B^2} + \frac{2A'B'}{A^3B}$$
$$- \frac{6B'}{A^2Br} + \frac{2A'}{A^3r} - \frac{1}{A^2r^2} + \frac{1}{B^2r^2}$$
$$= \frac{8\pi}{m_{\mathrm{Pl}}^2}\left(\frac{\dot\Phi^2}{2} + \frac{\Phi'^2}{2A^2} + \frac{\Phi^2}{B^2r^2} + V\right), \quad \text{(A10)}$$

$$\frac{1}{2}G_{01} \equiv K_2^{2\prime} + \left(\frac{B'}{B} + \frac{1}{r}\right)(3K_2^2 - K) = \frac{4\pi}{m_{\mathrm{Pl}}^2}\dot\Phi\Phi', \quad \text{(A11)}$$

$$\frac{1}{2}(G_1^1 + G_2^2 + G_3^3 - G_0^0) \equiv \dot K - (K_1^1)^2 - 2(K_2^2)^2$$
$$= \frac{8\pi}{m_{\mathrm{Pl}}^2}(\dot\Phi^2 - V), \quad \text{(A12)}$$

$$\ddot\Phi - K\dot\Phi - \frac{\Phi''}{A^2} - \left(-\frac{A'}{A} + \frac{2B'}{B} + \frac{2}{r}\right)\frac{\Phi'}{A^2}$$
$$+ \frac{2\Phi}{B^2r^2} + \frac{dV}{d\Phi} = 0. \quad \text{(A13)}$$

In this system the homogeneous curvature initial condition (A9) is not appropriate because the far region is asymptotically flat. We thereby suppose $A(t = 0, r) = B(t = 0, r) = 1$ and solve the constraint equations (A10) and (A11) to determine $K(t = 0, r)$ and $K_2^2(t = 0, r)$. This treatment is not usually adopted, but it is suitable for this system because we obtain

$$-\frac{K}{3} \approx -K_2^2 \approx \sqrt{\frac{8\pi}{3m_{\rm Pl}^2}\left(\frac{\Phi'^2}{2} + \frac{\Phi^2}{r^2} + V\right)}, \quad (A14)$$

which approaches zero as r increases. The numerical boundary is fixed at $r = 10\delta$ ($\delta \equiv c\delta_0$).

In regard to the time evolution of K_2^2, we do not have to solve the equation at the origin which corresponds to (A6), because in this case we have the relation $K_2^2(t, r = 0) = K(t, r = 0)/3$ from the regularity condition.

In both cases (1) and (2), we use a finite difference method with 1000 or 2000 meshes. The Hamiltonian constraint equation (A3) or (A10) remains unsolved during the evolution and is used for checking the numerical accuracy. Through all the calculations the errors are always less than a few percent.

(3) Apparent horizon. In our analysis for a global monopole system, we should investigate the global structure of the spacetime, such as the existence of an event horizon of a black hole, or the existence of a cosmological horizon. In numerical relativity, one practical method to see the spacetime structure is to search an apparent horizon. The apparent horizon is defined as the outermost/innermost closed two-surface where the expansion of a null geodesic congruence vanishes. For the metric (2.4), the expansion, $\Theta_{\pm}$, is written as

$$\Theta_{\pm} = k_{\pm;2}^2 + k_{\pm;3}^3 = 2\left[-K_2^2 \pm \frac{(Br)'}{ABr}\right], \quad (A15)$$

where $k_{\pm}^{\mu} = (-1, \pm A^{-1}, 0, 0)$ is an outgoing (+) or ingoing (−) null vector.

We observe the signs of $\Theta_{\pm}$ at all points in the numerical spacetime, and see if there is a two-surface where Θ_+ or Θ_- vanishes. In a black hole system, it is proved that, if an apparent horizon exists, an event horizon also exists outside (or coincides with) it. Here, as well as we search for a black hole horizon, we use an apparent horizon as a tool for finding a cosmological horizon; a cosmological horizon exists inside (or coincides with) the apparent horizon.

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