Two-loop $O(\alpha_s G_F M_Q^2)$ heavy-quark corrections to the interactions between Higgs and intermediate bosons

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(Received 30 August 1995)

By means of a low-energy theorem, we analyze at $O(\alpha_s G_F M_Q^2)$ the shifts in the standard-model W^+W^-H and ZZH couplings induced by virtual high-mass quarks Q with $M_Q \gg M_Z, M_H$, which includes the top quark. Invoking the improved Born approximation, we then find the corresponding corrections to various four- and five-point Higgs-boson production and decay processes which involve the W^+W^-H and ZZH vertices with one or both of the gauge bosons being connected to light-fermion currents, respectively. This includes $e^+e^- \to f\bar{f}H$ via Higgs radiation, via W^+W^- fusion (with $f=\nu_e$), and via ZZ fusion (with $f=e$), as well as $H \to 2V \to 4f$ (with $V=W,Z$). [S0556-2821(96)04309-3]

PACS number(s): 12.15.Lk, 14.70.Fm, 14.70.Hp, 14.80.Bn

I. INTRODUCTION

The Higgs boson is the last missing link in the standard model (SM). The discovery of this particle and the study of its characteristics are among the prime objectives of present and future high-energy colliding-beam experiments. Following Bjorken's proposal [1], the Higgs boson is currently being searched for with the CERN Large Electron-Positron Collider (LEP 1) and the SLAC Linear Collider (SLC) via $e^+e^- \to Z \to f\bar{f}H$. At the present time, the failure of this search allows one to rule out the mass range $M_H \leq 64.3$ GeV at the 95% confidence level [2]. The quest for the Higgs boson will be continued with LEP 2 by exploiting the Higgs-radiation mechanism [3,4], $e^+e^- \to ZH \to f\bar{f}H$. In next-generation e^+e^- linear supercolliders (NLC), also $e^+e^- \to \nu_e \bar{\nu}_e H$ via W^+W^- fusion and, to a lesser extent, $e^+e^- \to e^+e^-H$ via ZZ fusion will provide copious sources of Higgs bosons.

The study of quantum corrections to the production and decay processes of the Higgs boson has received much attention in the literature; for a review, see Ref. [5]. Since the top quark, with pole mass $M_t = (180 \pm 12)$ GeV [6], is so much heavier than the intermediate bosons, the M_t-dependent corrections are particularly important. On the other hand, it is attractive to consider the extension of the SM by a fourth fermion generation, where such corrections may be even more significant. Some time ago, Hill and Paschos [7] proposed an interesting fourth-generation scenario with Majorana neutrinos, which exploits the seesaw mechanism to evade the LEP 1 or SLC constraint on the number of light neutrinos. The charged fermions of this model are assumed to be of Dirac type and to have standard couplings. Subsequently, this model was further elaborated, and the precise triviality bounds, renormalization-group fixed points, and related dynamical mechanisms were discussed [8]. In particular, it was demonstrated how this model is reconciled with the fermion-mass constraints established in Ref. [9]. In Ref. [10], it was shown that this model is compatible with precision data from low energies and LEP 1 or SLC. Very recently, it was noticed [11] that arguments favoring the presence of a fourth fermion generation may be adduced on the basis of the democratic mass-matrix approach [12]. The possible existence of a fourth fermion generation is also considered in the latest Particle Data Group book [13], where mass bounds are listed. For a recent model-independet analysis, see Ref. [14].

It is advantageous to trace such fourth-generation fermions via their loop effects in the Higgs sector, since these effects are also sensitive to mass-degenerate isodoublets via fermion-mass power corrections. This has originally been observed in Ref. [15] in connection with the $f\bar{f}H$, W^+W^-H, and ZZH couplings. Moreover, the ggH coupling may serve as a device to detect mass-degenerate isodoublets of ultraheavy quarks [16], although power corrections do not occur here. By contrast, in the gauge sector, power corrections only appear in connection with isospin breaking [17]. The influence of quarks is amplified relative to the one of leptons because they come in triplicate. The quark-induced corrections are greatly affected by QCD effects. The two-loop $O(\alpha_s G_F M_Q^2)$ corrections to $\Gamma(H \to f\bar{f})$ have recently been evaluated [18]. In this paper, we shall extend that analysis to processes involving the W^+W^-H and ZZH couplings. The simplest processes of this kind are $H \to W^+W^-$ and $H \to ZZ$. We shall also allow for one or both of the intermediate bosons to couple to light-fermion currents. Specifically, we shall consider $e^+e^- \to f\bar{f}H$ via Higgs radiation, via W^+W^- fusion (with $f=\nu_e$), and via ZZ fusion (with $f=e$), as well as the decays of the Higgs boson into four fermions via two intermediate bosons. In the case of five-point processes, we shall neglect interference terms with a single fermion trace, since these are suppressed by Γ_V/M_V, with $V=W,Z$. For simplicity, we ignore the possibility of Cabibbo-Kobayashi-Maskawa mixings between the external light fermions and the virtual high-mass quarks.

The hardest technical difficulty that needs to be tackled here is to solve the two-loop three-point integrals in connection with the W^+W^-H and ZZH vertex corrections in $O(\alpha_s G_F M_Q^2)$. Similarly to the analysis of the $O(\alpha_s G_F M_t^2)$

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corrections to $\Gamma(H\to b\bar{b})$ [19], $\Gamma(Z\to f\bar{f}H)$, and $\sigma(e^+e^-\to ZH)$ [20], we may take advantage of a particular low-energy theorem [3,21]. Generally speaking, this theorem relates the amplitudes of two processes which differ by the insertion of an external Higgs-boson line carrying zero four-momentum. It may be derived by observing the following two points. (1) The interactions of the Higgs boson with the massive particles in the SM emerge from their mass terms by substituting $M_i\to M_i(1+H/v)$, where M_i is the mass of the respective particle, H is the Higgs field, and v is the Higgs vacuum expectation value; (2) A Higgs boson with zero four-momentum is represented by a constant field.

This immediately implies that a zero-momentum Higgs boson may be attached to an amplitude, $\mathscr{M}(A\to B)$, by carrying out the operation

$$\lim_{p_H\to 0}\mathscr{M}(A\to B+H)=\frac{1}{v}\sum_i \frac{M_i\partial}{\partial M_i}\mathscr{M}(A\to B), \qquad (1)$$

where i runs over all massive particles which are involved in the transition $A\to B$. This low-energy theorem comes with two caveats. (1) The differential operator in Eq. (1) does not act on the M_i appearing in coupling constants, since this would generate tree-level vertices involving the Higgs boson that do not exist in the SM; (2) Eq. (1) must be formulated for bare quantities if it is to be applied beyond the leading order.

This paper is organized as follows. In Sec. II, we review the $O(G_FM_Q^2)$ corrections to $\Gamma(H\to f\bar{f})$, $\Gamma(H\to W^+W^-)$, and $\Gamma(H\to ZZ)$ and derive those to $\sigma(e^+e^-\to f\bar{f}H)$ and $\Gamma(H\to 2V\to 4f)$ by invoking the so-called improved Born approximation (IBA) [22]. In Sec. III, we construct, by means of the low-energy theorem (1), a heavy-quark effective Lagrangian for the W^+W^-H and ZZH interactions which accommodates the $O(G_FM_Q^2)$ and $O(\alpha_sG_FM_Q^2)$ corrections, and apply it along with the IBA to the processes discussed in Sec. II. In Sec. IV, we numerically analyze the phenomenological consequences of our results. Section V contains our conclusions.

II. ONE-LOOP RESULTS

In this section, we review the leading one-loop effects on processes involving a W^+W^-H or ZZH coupling in the presence of a generic doublet of fourth-generation flavors, (U,D), with masses $M_U,M_D\gg M_Z,M_H$ and quantum-number assignments as in the first three fermion generations. For completeness, we also consider the implications for the $f\bar{f}H$ Yukawa couplings, assuming that f does not mix with U or D. Throughout this paper, we employ dimensional regularization with $n=4-2\epsilon$ space-time dimensions and a 't Hooft mass, μ, to keep the coupling constants dimensionless. As usual, we take γ_5 to be anticommuting. We work in the on-mass-shell renormalization scheme [23], with G_F as a basic parameter and the definition $c_w^2=1-s_w^2=M_W^2/M_Z^2$.

First, we recall that, at one loop, an additional (U,D) doublet contributes to the deviation of the ρ parameter from unity, $\Delta\rho=1-1/\rho$, the amount [17]

$$\Delta\rho_1=\frac{N_c}{2}G\left(\frac{M_U^2+M_D^2}{2}-\frac{M_U^2M_D^2}{M_U^2-M_D^2}\ln\frac{M_U^2}{M_D^2}\right)\geqslant 0, \qquad (2)$$

where $N_c=1$ (3) for leptons (quarks) and $G=(G_F/2\pi^2\sqrt{2})$. Here and in the following, the subscripts 1 and 2 mark $O(G_FM_Q^2)$ and $O(\alpha_sG_FM_Q^2)$ contributions, respectively. Equation (2) is valid for M_U and M_D arbitrary. It is well known that $\Delta\rho$ measures the isospin breaking in the fermion sector; $\Delta\rho_1$ vanishes for $M_U=M_D$. By contrast, the corresponding shifts, δ, in the tree-level couplings of the Higgs boson to physical particles are not quenched for $M_U=M_D$. They have been calculated, in the one-loop approximation, for $\Gamma(H\to f\bar{f})$ in Ref. [24], for $\Gamma(H\to W^+W^-)$ in Refs. [25,26], and for $\Gamma(H\to ZZ)$ in Refs. [25,27]. Writing the corrections to these observables in the form $K=(1+\delta)^2$ and considering the limit $M_U,M_D\gg M_Z,M_H$, one has [15]

$$\delta_1^u=\frac{\Delta\rho_1}{2}+\frac{N_c}{6}G(M_U^2+M_D^2)>0, \qquad (3)$$

$$\delta_1^{WWH}=\delta_1^{ZZH}=\frac{\Delta\rho_1}{2}-\frac{N_c}{3}G(M_U^2+M_D^2)<0. \qquad (4)$$

Equation (3) refers to $\Gamma(H\to f\bar{f})$, where f does not mix with U or D, so that only the renormalizations of the Higgs-boson wave function and vacuum expectation value contribute. The superscript u is to indicate that this is a *universal* correction, which occurs as a building block in the renormalization of any Higgs-boson production and decay process. On the other hand, Eq. (4) also contains genuine vertex corrections. The equality of δ_1^{WWH} and δ_1^{ZZH} is broken by subleading one-loop terms, of $O(G_FM_H^2)$. We anticipate that it is also spoiled by the leading two-loop QCD corrections, of $O(\alpha_sG_FM_Q^2)$, to be calculated in Sec. III, unless U and D are mass degenerate.

In order to describe the production of the Higgs boson in high-energy colliding-beam experiments, we have to consider the Feynman diagrams which emerge from the W^+W^-H and ZZH vertices by linking the intermediate-boson legs to light-fermion lines. Then, Eq. (4) must be complemented by the $O(G_FM_Q^2)$ corrections which arise from the gauge-boson propagators; the gauge-boson wave-function renormalizations, which appear in connection with $\Gamma(H\to W^+W^-)$ and $\Gamma(H\to ZZ)$, do not receive such corrections. This may be achieved by invoking the IBA. The IBA provides a systematic and convenient method to incorporate the dominant corrections of fermionic origin to processes within the gauge sector of the SM. These are contained in $\Delta\rho$ and $\Delta\alpha=1-\alpha/\bar{\alpha}$, which parametrizes the running of the fine-structure constant from its value, α, defined in Thomson scattering to its value, $\bar{\alpha}$, measured at the Z-boson scale. The recipe is as follows. Starting from the Born formula expressed in terms of c_w, s_w, and α, one substitutes

$$\alpha\to\bar{\alpha}=\frac{\alpha}{1-\Delta\alpha}, \quad c_w^2\to\bar{c}_w^2=1-\bar{s}_w^2=c_w^2(1-\Delta\rho). \quad (5)$$

To eliminate $\bar{\alpha}$ in favor of G_F, one exploits the relation

$$\frac{\sqrt{2}}{\pi}G_F=\frac{\overline{\alpha}}{\overline{s}_w^2 M_W^2}=\frac{\overline{\alpha}}{\overline{c}_w^2\overline{s}_w^2 M_Z^2}(1-\Delta\rho), \tag{6}$$

which correctly accounts for the leading fermionic corrections.

We shall first concentrate on the processes with a ZZH coupling. Combining specific knowledge of δ^{ZZH} with the IBA, we obtain the correction factors for $\sigma(f\bar{f}\to ZH)$, $\Gamma(Z\to f\bar{f}H)$, and $\Gamma(H\to f\bar{f}Z)$ in the form [20,28]

$$K_1^{(f)}=\frac{(1+\delta^{ZZH})^2}{1-\Delta\rho}\frac{\overline{v}_f^2+a_f^2}{v_f^2+a_f^2}$$
$$=1+2\delta^{ZZH}+\left(1-8c_w^2\frac{Q_f v_f}{v_f^2+a_f^2}\right)\Delta\rho, \tag{7}$$

where $v_f=2I_f-4s_w^2Q_f$, $\overline{v}_f=2I_f-4\overline{s}_w^2Q_f$, $a_f=2I_f$, Q_f is the electric charge of f in units of the positron charge, I_f is the third component of weak isospin of the left-handed component of f, and we have omitted terms of $O(G_F^2M_Q^4)$ in the second line. The corresponding Born formulas may be found in Refs. [29,28,30], respectively. Furthermore, the correction factors for $\sigma(f_1\bar{f}_1\to f_2\bar{f}_2H)$ (via $f_1\bar{f}_1$ annihilation) and $\Gamma(H\to f_1\bar{f}_1f_2\bar{f}_2)$ (via a ZZ intermediate state) read [31]

$$K_2^{(f_1f_2)}=\frac{(1+\delta^{ZZH})^2}{(1-\Delta\rho)^2}\frac{\overline{v}_{f_1}^2+a_{f_1}^2}{v_{f_1}^2+a_{f_1}^2}\frac{\overline{v}_{f_2}^2+a_{f_2}^2}{v_{f_2}^2+a_{f_2}^2}$$
$$=1+2\delta^{ZZH}$$
$$+2\left[1-4c_w^2\left(\frac{Q_{f_1}v_{f_1}}{v_{f_1}^2+a_{f_1}^2}+\frac{Q_{f_2}v_{f_2}}{v_{f_2}^2+a_{f_2}^2}\right)\right]\Delta\rho. \tag{8}$$

The corresponding tree-level results are listed in Refs. [32,33], respectively. Here and in the following, we neglect interference terms of five-point amplitudes with a single fermion trace, since these are strongly suppressed, by Γ_V/M_V, with $V=W,Z$. Such terms have recently been included in a tree-level calculation of $\Gamma(H\to 2V\to 4f)$ for $M_H\ll M_W$ [34]. The formulas become slightly more complicated if a fermion line runs from the initial state to the final state, e.g., in the case of ZZ fusion. In the latter case, the Born cross section may be evaluated from

$$\sigma(f_1f_2\to f_1f_2H)=\frac{G_F^3M_Z^8}{64\pi^3\sqrt{2}s^2}[(v_{f_1}^2+a_{f_1}^2)(v_{f_2}^2+a_{f_2}^2)A$$
$$\pm 4v_{f_1}a_{f_1}v_{f_2}a_{f_2}B], \tag{9}$$

where

$$A=\int_{M_H^2/s}^1 da f(a),\quad B=\int_{M_H^2/s}^1 da g(a), \tag{10}$$

$\sqrt{s}$ is the center-of-mass energy, $f(a)$ and $g(a)$ are listed in Eq. (A9) of Ref. [29], and the plus/minus sign refers to an odd/even number of antifermions in the initial state. For example, $e^+e^-\to e^+e^-H$ requires the plus sign. From the IBA it follows on that the correction factor for Eq. (9) is given by

$$K_3^{(f_1f_2)}=\frac{(1+\delta^{ZZH})^2}{(1-\Delta\rho)^2}$$
$$\times\frac{(\overline{v}_{f_1}^2+a_{f_1}^2)(\overline{v}_{f_2}^2+a_{f_2}^2)A\pm 4\overline{v}_{f_1}a_{f_1}\overline{v}_{f_2}a_{f_2}B}{(v_{f_1}^2+a_{f_1}^2)(v_{f_2}^2+a_{f_2}^2)A\pm 4v_{f_1}a_{f_1}v_{f_2}a_{f_2}B}$$
$$=1+2\delta^{ZZH}+2\left[1-\frac{4c_w^2}{1+r}\left(\frac{Q_{f_1}v_{f_1}}{v_{f_1}^2+a_{f_1}^2}+\frac{Q_{f_2}v_{f_2}}{v_{f_2}^2+a_{f_2}^2}\right)\right.$$
$$\left.-\frac{2c_w^2}{1+1/r}\left(\frac{Q_{f_1}}{v_{f_1}}+\frac{Q_{f_2}}{v_{f_2}}\right)\right]\Delta\rho, \tag{11}$$

where

$$r=\frac{\pm 4v_{f_1}a_{f_1}v_{f_2}a_{f_2}B}{(v_{f_1}^2+a_{f_1}^2)(v_{f_2}^2+a_{f_2}^2)A}. \tag{12}$$

In practice, one has $|r|\ll 1$ (see Table II), so that Eq. (8) is approximately recovered.

The processes which correspond to a W^+W^-H vertex with one or both of the W bosons coupled to light-fermion currents do not receive additional dominant fermionic corrections beyond the factor

$$K_{WWH}=(1+\delta^{WWH})^2, \tag{13}$$

which already corrects $\Gamma(H\to W^+W^-)$. This may be understood by observing that G_F is *defined* through the radiative correction to a four-fermion charged-current process, namely the muon decay. The tree-level formulas for $\sigma(f\bar{f}'\to W^\pm H)$, $\Gamma(H\to f\bar{f}'W^\pm)$, and $\Gamma(H\to f_1\bar{f}_1'f_2'\bar{f}_2)$ (with a W^+W^- intermediate state) may be found in Refs. [35,30,33], respectively. Here and in the following, f' denotes the isopartner of f. The lowest-order cross section of $f_1f_2\to f_1'f_2'H$ via W^+W^- fusion is described by Eqs. (9) and (10), with $v_f=a_f=\sqrt{2}$ and M_Z replaced by M_W.

The aim of this paper is to complete the knowledge of the $O(\alpha_sG_FM_Q^2)$ corrections to the Higgs-boson production and decay rates. In the remainder of this section, we shall collect the results which are already known. In the case of $\Delta\rho$, we have [36]

$$\Delta\rho_2=-\frac{N_c}{4}C_FaG\left[\frac{M_U^2+M_D^2}{2}+F(M_U^2,M_D^2)\right]\leqslant 0, \tag{14}$$

where $N_c=3$, $C_F=(N_c^2-1)/(2N_c)=4/3$, $a=\alpha_s(\mu)/\pi$, and

$$F(u,d)=(u-d)\mathrm{Li}_2\left(1-\frac{d}{u}\right)+\frac{d}{u-d}\ln\frac{u}{d}\left[u-\frac{3u^2+d^2}{2(u-d)}\ln\frac{u}{d}\right]. \tag{15}$$

Note that $F(u,d)=F(d,u)$. From Eq. (15), we may read off the properties $F(u,u)=-u$ and $F(u,0)=\zeta(2)u$. For $M_U=M_t$ and $M_D=0$, Eq. (14) reproduces the well-known $O(\alpha_sG_FM_t^2)$ result [37]. For later use, we observe that

$$\sum_{Q=U,D}\frac{M_Q^2\partial}{\partial M_Q^2}\Delta\rho_{1,2}=\Delta\rho_{1,2}. \tag{16}$$

The QCD correction to Eq. (3) reads [18]

$$\delta_2^u = \frac{\Delta\rho_2}{2} - \frac{N_c}{8} C_F a G(M_U^2 + M_D^2) < 0. \tag{17}$$

In the next section, we shall derive the $O(\alpha_s G_F M_Q^2)$ corrections to δ^{WWH} and δ^{ZZH} by means of the low-energy theorem (1). The formalism developed in this section to find the $O(G_F M_Q^2)$ corrections to the four- and five-point processes with a ZZH coupling readily carries over to $O(\alpha_s G_F M_Q^2)$. If the external fermions are leptons, we just need to include in Eqs. (7), (8), and (11) the corresponding terms of δ^{ZZH} and $\Delta\rho$. Similarly, the four- and five-point processes with a W^+W^-H coupling are then simply corrected by K_{WWH} given in Eq. (13), with the $O(\alpha_s G_F M_Q^2)$ term included.

III. EFFECTIVE LAGRANGIAN

In the following, we shall proceed along the lines of Ref. [20], where the $O(\alpha_s G_F M_t^2)$ correction to the ZZH vertex was found by means of the low-energy theorem (1), assuming that $m_b = 0$. We extend that analysis by keeping the quark masses arbitrary and by considering also the W^+W^-H coupling. We shall explicitly work out the W^+W^-H case, which is more involved technically. The ZZH results will then be listed without derivation.

The starting point of our analysis is the amplitude characterizing the propagation of an on-shell W boson in the presence of quantum effects due a doublet (U,D) of high-mass quarks,

$$\mathscr{M}(W\to W) = (M_W^0)^2 - \Pi_{WW}(q^2)\big|_{q^2=(M_W^0)^2}, \tag{18}$$

where $\Pi_{WW}(q^2)$ is the transverse W-boson self-energy, at four-momentum q, written in terms of bare parameters. Here and in the following, bare parameters are marked by the superscript 0. In the G_F representation, $\Pi_{WW}(q^2)$ is proportional to $(M_W^0)^2$, which originates from the two UDW gauge couplings. Apart from this prefactor, we may put $q^2 = 0$ in Eq. (18), since we are working in the high-M_Q approximation. The low-energy theorem (1) now tells us that we may attach a zero-momentum Higgs boson to the $W\to W$ transition amplitude by carrying out the operation

$$\lim_{p_H\to 0} \mathscr{M}(W\to W+H) = \frac{1}{v^0}\left(\sum_{Q=U,D} \frac{M_Q^0 \partial}{\partial M_Q^0} + \frac{M_W^0 \partial}{\partial M_W^0}\right)\mathscr{M}(W\to W), \tag{19}$$

where we must treat the overall factor $(M_W^0)^2$ of $\Pi_{WW}(0)$ in Eq. (18) as a constant. This leads us to

$$\lim_{p_H\to 0} \mathscr{M}(W\to W+H) = \frac{2(M_W^0)^2}{v^0}(1+E), \tag{20}$$

with

$$E = -\sum_{Q=U,D} \frac{(M_Q^0)^2 \partial}{\partial (M_Q^0)^2} \frac{\Pi_{WW}(0)}{(M_W^0)^2}. \tag{21}$$

We are now in the position to write down the heavy-quark effective W^+W^-H interaction Lagrangian:

$$\mathscr{L}_{WWH} = 2(M_W^0)^2 (W_\mu^+)^0 (W^{-\mu})^0 \frac{H^0}{v^0}(1+E). \tag{22}$$

Then, we have to carry out the renormalization procedure, i.e., we have to split the bare parameters into renormalized ones and counterterms. We fix the counterterms according to the on-shell scheme. In the case of the W-boson mass and wave function, we have

$$(M_W^0)^2 = M_W^2 + \delta M_W^2, \quad (W_\mu^\pm)^0 = (1+\delta Z_W)^{1/2} W_\mu^\pm, \tag{23}$$

with

$$\delta M_W^2 = \Pi_{WW}(0), \quad \delta Z_W = -\Pi'_{WW}(0), \tag{24}$$

where we have neglected M_W against M_Q in the loop amplitudes. For dimensional reasons, δZ_W does not receive corrections in $O(G_F M_Q^2)$ and $O(\alpha_s G_F M_Q^2)$. Furthermore, we have [18]

$$\frac{H^0}{v^0} = 2^{1/4} G_F^{1/2} H(1+\delta^u), \tag{25}$$

where the $O(G_F M_Q^2)$ and $O(\alpha_s G_F M_Q^2)$ terms of δ^u are given in Eqs. (3) and (17), respectively. Putting everything together, we obtain the renormalized version of Eq. (22):

$$\mathscr{L}_{WWH} = 2^{5/4} G_F^{1/2} M_W^2 W_\mu^+ W^{-\mu} H(1+\delta^{WWH}), \tag{26}$$

with

$$\delta^{WWH} = \delta^u + \frac{\delta M_W^2}{M_W^2} + E. \tag{27}$$

In order for δ^{WWH} to be finite through $O(\alpha_s G_F M_Q^2)$, we still need to renormalize the masses of the U and D quarks in the $O(G_F M_Q^2)$ expressions for $\delta M_W^2/M_W^2$ and E; i.e., we need to substitute

$$M_Q^0 = M_Q + \delta M_Q, \tag{28}$$

with [38]

$$\frac{\delta M_Q}{M_Q} = -\frac{a}{4} C_F \left(\frac{4\pi\mu^2}{M_Q^2}\right)^\epsilon \Gamma(1+\epsilon) \frac{3-2\epsilon}{\epsilon(1-2\epsilon)}, \tag{29}$$

where Γ is Euler's gamma function.

For convenience, we introduce the shorthand notations $q = M_Q^2$ and $W = \delta M_W^2/M_W^2$. Quantities with (without) the superscript 0 are written in terms of M_Q^0 (M_Q). First, we shall check our formalism in $O(G_F M_Q^2)$. We extract from Refs. [15,26] the $O(G_F M_Q^2)$ amplitudes:

$$W_1 = -\Delta\rho_1 - \frac{N_c}{2} G \sum_Q M_Q^2 \left(\frac{4\pi\mu^2}{M_Q^2}\right)^\epsilon \Gamma(1+\epsilon)\left[\frac{1}{\epsilon} + O(\epsilon)\right], \tag{30}$$

$$E_1=\Delta\rho_1+\frac{N_c}{2}G\sum_Q M_Q^2\left(\frac{4\pi\mu^2}{M_Q^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left[\frac{1}{\epsilon}-1+O(\epsilon)\right]. \tag{31}$$

Substituting Eqs. (3), (30), and (31) into Eq. (27), we reproduce Eq. (4). Furthermore, with the help of Eq. (16), we immediately verify Eq. (21) at one loop: viz.,

$$E_1=-\sum\frac{q\partial}{\partial q}W_1. \tag{32}$$

Now, we shall proceed to two loops. We have [36]

$$W_2=-\Delta\rho_2+\frac{N_c}{8}C_F aG\sum_{Q=U,D}M_Q^2\left(\frac{4\pi\mu^2}{M_Q^2}\right)^{2\epsilon}\Gamma^2(1+\epsilon)\times\left[\frac{3}{\epsilon^2}+\frac{11}{2\epsilon}+\frac{31}{4}+O(\epsilon)\right], \tag{33}$$

where $\Delta\rho_2$ is given in Eq. (14). Notice that Eq. (33) already contains the contributions proportional to δM_Q which emerge from the renormalization of the quark masses in Eq. (30). We wish to compute E_2. According to Eq. (21), we have

$$E_2^0=-\sum\frac{q\partial}{\partial q}W_2^0. \tag{34}$$

Furthermore, we have

$$W_2=W_2^0+\delta W_2,\quad E_2=E_2^0+\delta E_2, \tag{35}$$

where the counterterms are obtained by scaling the one-loop results:

$$\delta W_2=\sum\frac{\delta q}{q}\frac{q\partial}{\partial q}W_1,\quad \delta E_2=\sum\frac{\delta q}{q}\frac{q\partial}{\partial q}E_1. \tag{36}$$

Using Eqs. (32), (34), (35), and (36) along with

$$\frac{q\partial}{\partial q}\frac{\delta q}{q}=-\epsilon\frac{\delta q}{q}, \tag{37}$$

which may be gleaned from Eq. (29), we find

$$E_2=-\sum\left(\frac{q\partial}{\partial q}W_2+\epsilon\frac{\delta q}{q}\frac{q\partial}{\partial q}W_1\right). \tag{38}$$

Obviously, knowledge of the $O(\epsilon)$ term of W_1 is not necessary for our purposes. Inserting Eqs. (29), (30), and (33) into Eq. (38) and employing Eq. (16), we obtain the desired two-loop three-point amplitude:

$$E_2=\Delta\rho_2-\frac{3}{2}C_F a\Delta\rho_1-\frac{N_c}{8}C_F aG\sum_{Q=U,D}M_Q^2\left(\frac{4\pi\mu^2}{M_Q^2}\right)^{2\epsilon}\times\Gamma^2(1+\epsilon)\left[\frac{3}{\epsilon^2}+\frac{11}{2\epsilon}-\frac{5}{4}+O(\epsilon)\right], \tag{39}$$

where $\Delta\rho_1$ and $\Delta\rho_2$ are given in Eqs. (2) and (14), respectively. The sum of Eqs. (17), (33), and (39) is devoid of ultraviolet divergences and reads

$$\delta_2^{WWH}=\frac{\Delta\rho_2}{2}-\frac{3}{2}C_F a\Delta\rho_1+N_cC_F aG(M_U^2+M_D^2)=\frac{\Delta\rho_2}{2}-3C_F a\,\delta_1^{WWH}>0, \tag{40}$$

where δ_1^{WWH} is given in Eq. (4). This completes the derivation of the effective W^+W^-H interaction Lagrangian (26). We observe that δ_2^{WWH} weakens the negative effect of δ_1^{WWH}. In the cases of no ($M_U=M_D$) and maximum ($M_U\gg M_D$) isospin breaking, we have

$$\delta^{WWH}=-\frac{2}{3}N_cGM_U^2(1-3C_F a)\approx-2GM_U^2(1-1.27324\alpha_s),$$
$$\delta^{WWH}=-\frac{5}{24}N_cGM_U^2\left\{1+\frac{3}{5}C_F a\left[\zeta(2)-\frac{9}{2}\right]\right\}\approx-\frac{5}{8}GM_U^2(1-0.72704\alpha_s), \tag{41}$$

respectively.

The derivation of the effective ZZH interaction Lagrangian proceeds in close analogy to the W^+W^-H case and leads to

$$\mathscr{L}_{ZZH}=2^{1/4}G_F^{1/2}M_Z^2Z_\mu Z^\mu H(1+\delta^{ZZH}), \tag{42}$$

with

$$\delta^{ZZH}=\frac{\Delta\rho}{2}+\left(\delta_1^{ZZH}-\frac{\Delta\rho_1}{2}\right)(1-3C_F a)=\delta^{WWH}+\frac{3}{2}C_F a\Delta\rho_1. \tag{43}$$

Again, we have $\delta_1^{ZZH}<0<\delta_2^{ZZH}$, i.e., the $O(G_FM_Q^2)$ term is partly compensated by its QCD correction. For $M_U=M_D$, δ^{ZZH} coincides with δ^{WWH}. For $M_U\gg M_D$, we recover the result of Ref. [20]:

$$\delta^{ZZH}=-\frac{5}{24}N_cGM_U^2\left\{1+3C_F a\left[\frac{\zeta(2)}{5}-\frac{3}{2}\right]\right\}\approx-\frac{5}{8}GM_U^2(1-1.49098\alpha_s). \tag{44}$$

It is interesting to observe that δ^u may be written in a form similar to the first line of Eq. (43): namely,

$$\delta^u=\frac{\Delta\rho}{2}+\left(\delta_1^u-\frac{\Delta\rho_1}{2}\right)\left(1-\frac{3}{4}C_F a\right). \tag{45}$$

As a corollary, we note that $\delta_{1,2}^u$, $\delta_{1,2}^{WWH}$, and $\delta_{1,2}^{ZZH}$ also satisfy identities similar to Eq. (16).

If the external fermions are leptons, then we may implement the $O(\alpha_sG_FM_Q^2)$ corrections to the four- and five-point processes considered in Sec. II by evaluating the K factors in Eqs. (7), (8), (11), and (13) with the QCD-corrected expressions for δ^{WWH}, δ^{ZZH}, and $\Delta\rho$. In the case of external quarks, we also need to include the leading-order QCD corrections to their couplings with the intermediate bosons, since these will combine with the $O(G_F M_Q^2)$ corrections to give additional $O(\alpha_s G_F M_Q^2)$ terms. In the case of jet production, we have to include an additional factor $[1+(3C_F a/4)]$ for each quark pair in the final state. Our formalism is also applicable to Higgs-boson production via quark-pair annihilation at hadron colliders and via intermediate-boson fusion at hadron and ep colliders. Then, the pure QCD corrections to these processes may be conveniently incorporated by using the appropriate hadronic structure functions [39].

If we set $M_U = M_t$ and $M_D = 0$, our formulas may also be used to describe the loop corrections induced by the top quark. Then, however, special care must be exercised if there is beauty in the external legs. Specifically, if a $b\bar{b}$ pair is produced via a virtual Z boson, e.g., by $Z\rightarrow b\bar{b}H$, $H\rightarrow b\bar{b}Z$, and $e^+e^-\rightarrow b\bar{b}H$, then we must substitute $\overline{v_b} = 2I_b - 4\overline{s_w^2}Q_b/(1+\tau)$ in the corresponding K factor and include an overall factor $(1+\tau)^2$, where [40]

$$\tau = -\frac{G}{2}M_t^2\left[1-\frac{3}{2}\zeta(2)C_F a\right]. \tag{46}$$

Consequently, the relevant K factors in Eqs. (7) and (8) become

$$\begin{aligned} K_1^{(b)} &= \left(1+\frac{3}{4}C_F a\right)(1+\delta^{ZZH})^2\,\frac{(1+\tau)^2}{1-\Delta\rho}\,\frac{\overline{v_b^2}+a_b^2}{v_b^2+a_b^2} \\ &= \left(1+\frac{3}{4}C_F a\right)\left[1+2\,\delta^{ZZH}+\left(1-8c_w^2\frac{Q_b v_b}{v_b^2+a_b^2}\right)\Delta\rho+2 \right.\\ &\quad \left.\times\left(1+4s_w^2\frac{Q_b v_b}{v_b^2+a_b^2}\right)\tau\right], \end{aligned}$$

$$\begin{aligned} K_2^{(\ell b)} &= \left(1+\frac{3}{4}C_F a\right)(1+\delta^{ZZH})^2\,\frac{(1+\tau)^2}{(1-\Delta\rho)^2}\,\frac{\overline{v_\ell^2}+a_\ell^2}{v_\ell^2+a_\ell^2}\,\frac{\overline{v_b^2}+a_b^2}{v_b^2+a_b^2} \\ &= \left(1+\frac{3}{4}C_F a\right)\left\{1+2\,\delta^{ZZH}+2\left[1-4c_w^2\left(\frac{Q_\ell v_\ell}{v_\ell^2+a_\ell^2}\right.\right.\right.\\ &\quad \left.\left.\left.+\frac{Q_b v_b}{v_b^2+a_b^2}\right)\right]\Delta\rho+2\left(1+4s_w^2\frac{Q_b v_b}{v_b^2+a_b^2}\right)\tau\right\}, \end{aligned} \tag{47}$$

respectively.

IV. NUMERICAL RESULTS

We are now in a position to explore the phenomenological implications of our results. We take the values of our input parameters to be $M_W = 80.26$ GeV and $M_Z = 91.1887$ GeV [41], so that $s_w^2 = 0.2253$.

In Eqs. (7), (8), and (11), we have presented correction factors for various four- and five-point Higgs-boson production and decay processes with a ZZH coupling in terms of δ_{ZZH} and $\Delta\rho$. It is instructive to cast these correction factors into the generic form

$$K = 1+\frac{N_c}{4}GM_U^2 C_1\left(\frac{M_D}{M_U}\right)\left[1+\frac{3}{4}C_F a C_2\left(\frac{M_D}{M_U}\right)\right], \tag{48}$$

where C_1 and C_2 are dimensionless functions of M_D/M_U. Since, in the high-M_Q limit, $\Delta\rho$, δ^u, δ^{WWH}, and δ^{ZZH} are symmetric in M_U and M_D, we may, without loss of generality, assume that $M_D/M_U \leq 1$. The specific forms of the prefactors are chosen in such a way that, in the case of the leading M_t-dependent contribution to $\Delta\rho$, the familiar values $C_1(0)=1$ [17] and $C_2(0)=(2/3)[2\zeta(2)+1]\approx 2.860$ [37] are recovered. Relative to $M_t = 180$ GeV, we have $(N_c/4)GM_U^2 \approx 1.015\% \times (M_U/M_t)^2$. The outcome of this decomposition is displayed in Table I, where C_1 and C_2 are listed as functions of M_D/M_U for various classes of processes with a ZZH coupling. Specifically, $K_1^{(f)}$ ($f=\nu,\ell,u,d$) refers to $Z\rightarrow f\bar{f}H$ and $H\rightarrow f\bar{f}Z$, $K_2^{(\nu\nu)}$ to $H\rightarrow \nu\bar{\nu}\nu'\bar{\nu}'$, $K_2^{(\ell f)}$ to $e^+e^-\rightarrow f\bar{f}H$ via Higgs radiation, and $K_3^{(\ell\ell)}$ to $e^+e^-\rightarrow e^+e^-H$ via ZZ fusion. For completeness, also $\Delta\rho$ and the correction factors K_{ffH}, K_{WWH}, and K_{ZZH} for $\Gamma(H\rightarrow f\bar{f})$, $\Gamma(H\rightarrow W^+W^-)$, and $\Gamma(H\rightarrow ZZ)$, respectively, are considered. As explained in Sec. II, K_{WWH} also applies to four- and five-point processes with a W^+W^-H coupling, such as $H\rightarrow f\bar{f}'W^\pm$, $H\rightarrow f_1\bar{f}_1' f_2' \bar{f}_2$, and $e^+e^-\rightarrow \nu_e\bar{\nu}_e H$ via W^+W^- fusion. Notice that there are additional QCD corrections beyond Eq. (48) if external quarks are involved. As discussed in Sec. III, in the case of dijet production via an intermediate boson, these give rise to an overall factor $[1+(3C_F a/4)]$ on the right-hand side of Eq. (48). Such QCD corrections are not included in Table I.

In the case of $K_3^{(\ell\ell)}$, we have treated $x=B/A$, where A and B are defined in Eq. (10), as an additional expansion parameter and discarded terms of $O(x^2)$. This is justified because, in practice, $|x| \ll 1$, e.g., for $\sqrt{s}=300$ GeV and $M_H = 100$ GeV, we find $x\approx -5.233\%$. In Table II, we list $-x$ (in %) and $\sigma(e^+e^-\rightarrow e^+e^-H)$ (in fb) as functions of $M_H/\sqrt{s}$ for LEP 2 energy and various envisaged Next-Linear Collider (NLC) energies. We observe that $|x|$ decreases with σ increasing and is at the few-% level or below whenever $e^+e^-\rightarrow e^+e^-H$ is phenomenologically interesting.

Looking at Table I, we see that, for all quantities except $\Delta\rho$, C_1 grows in magnitude as M_D/M_U approaches unity. As is well known, $\Delta\rho$ is quenched in this limit. Moreover, the majority of the Higgs-related K factors have $|C_1(0)|>1$, i.e., the corresponding observables are more sensitive to the existence of fourth-generation fermion doublets than the ρ parameter itself, even if isospin is badly broken. While $C_1>0$ for K_{ffH}, $C_1<0$ for all other Higgs-boson observables, with the exception of $K_2^{(\nu\nu)}$. The case of $K_2^{(\nu\nu)}$ is special, since there C_1 changes sign, at $M_D/M_U \approx 0.113$. Except for $K_2^{(\nu\nu)}$ with M_D/M_U below this value, we always have $C_2<0$, i.e., the QCD corrections generally reduce the leading one-loop terms in size. In the presence of a ZZH coupling, this screening effect is considerably stronger than in the case of $\Delta\rho$. In fact, for the ZZH-type processes, we throughout have $C_2(M_D/M_U)\leq C_2(1)=-4$. Except in the small range $M_D/M_U \lesssim 0.095$, also K_{WWH} exhibits a stronger QCD screening than $\Delta\rho$. K_{WWH} and all ZZH-type K factors coincide if $M_D/M_U=1$, since then $\delta^{WWH}=\delta^{ZZH}$ and $\Delta\rho=0$.

TABLE I. Coefficients C_1 (upper entries) and C_2 (lower entries) in Eq. (48) as functions of M_D/M_U for the various Higgs-boson decay rates and production cross sections discussed in the text. In the last line, $x=B/A$, where A and B are given by Eq. (10), and terms of $O(x^2)$ have been neglected.

M_D/M_U	0	0.2	0.4	0.6	0.8	1
$\Delta\rho$	1	0.772	0.462	0.211	0.053	0
	−2.860	−2.407	−2.166	−2.056	−2.011	−2
K_{ffH}	7/3	2.158	2.009	2.024	2.240	8/3
	−1.797	−1.503	−1.268	−1.110	−1.024	−1
K_{WWH}	−5/3	−2.002	−2.631	−3.416	−4.320	−16/3
	−2.284	−3.072	−3.620	−3.873	−3.975	−4
K_{ZZH}	−5/3	−2.002	−2.631	−3.416	−4.320	−16/3
	−4.684	−4.614	−4.322	−4.120	−4.024	−4
$K_1^{(\nu)}$	−2/3	−1.230	−2.170	−3.205	−4.267	−16/3
	−7.420	−5.999	−4.781	−4.256	−4.050	−4
$K_1^{(\ell)}$	−1.272	−1.697	−2.449	−3.333	−4.299	−16/3
	−5.249	−5.010	−4.482	−4.171	−4.034	−4
$K_1^{(u)}$	−2.089	−2.328	−2.827	−3.505	−4.343	−16/3
	−4.315	−4.305	−4.173	−4.067	−4.014	−4
$K_1^{(d)}$	−1.637	−1.979	−2.618	−3.410	−4.319	−16/3
	−4.717	−4.640	−4.333	−4.124	−4.025	−4
$K_2^{(\nu\nu)}$	1/3	−0.458	−1.708	−2.995	−4.214	−16/3
	6.261	−12.050	−5.488	−4.410	−4.075	−4
$K_2^{(\nu\ell)}$	−0.272	−0.925	−1.987	−3.122	−4.246	−16/3
	−14.025	−7.180	−5.021	−4.314	−4.060	−4
$K_2^{(\ell\ell)}$	−0.878	−1.393	−2.267	−3.250	−4.278	−16/3
	−6.323	−5.579	−4.668	−4.225	−4.044	−4
$K_2^{(\ell u)}$	−1.695	−2.023	−2.644	−3.422	−4.322	−16/3
	−4.654	−4.590	−4.311	−4.116	−4.024	−4
$K_2^{(\ell d)}$	−1.243	−1.674	−2.436	−3.327	−4.298	−16/3
	−5.307	−5.045	−4.495	−4.175	−4.035	−4
$K_3^{(\ell\ell)}$	$(-0.878 - 2.353x)$	$(-1.393 - 1.816x)$	$(-2.267 - 1.087x)$	$(-3.250 - 0.496x)$	$(-4.278 - 0.125x)$	−16/3
	$(-6.323 + 9.281x)$	$(-5.579 + 4.134x)$	$(-4.668 + 1.200x)$	$(-4.225 + 0.331x)$	$(-4.044 + 0.059x)$	−4

V. CONCLUSIONS

The implications of the possible existence of a fourth fermion generation for electroweak physics have been extensively studied at one loop [5,11,15–17, 24–27, 29]. Recently, this study has been extended to the two-loop level by analyzing the virtual QCD effects of $O(\alpha_s G_F M_Q^2)$ due to a quark doublet, (U,D), with arbitrary masses, in the gauge sector [36] and in the ffH Yukawa couplings of the first three generations [18]. In the present paper, this research program has been continued by investigating the $O(\alpha_s G_F M_Q^2)$ corrections to the W^+W^-H and ZZH couplings. In contrast to the vacuum-polarization analyses of Refs. [18,36], this involves two-loop three-point amplitudes, which are usually much harder to compute. To simplify matters, we assumed that M_U and M_D are large against the physical (invariant) masses of the on-shell (off-shell) W, Z, and Higgs bosons, which allowed us to take advantage of the low-energy theorem (1) [3,21]. The range of validity of this heavy-quark approximation may be defined more accurately by considering the thresholds in the relevant self-energy and vertex diagrams. Then, it becomes apparent that the leading $O(G_F M_Q^2)$ terms and their QCD corrections are expected to provide useful approximations to the full M_Q-dependent expressions as long as $\min(M_U^2, M_D^2) \gg \max(p_{V_1}^2, p_{V_2}^2, p_H^2)/4$ is satisfied, where p_{V_1}, p_{V_2}, and p_H are the four-momenta flowing into the V_1V_2H vertex of the considered process. Assuming $M_D \leq M_U$, this implies $M_D \gg \sqrt{s}/2$ for Higgs radiation, $M_D \gg \max(M_H, \sqrt{s-M_H^2})/2$ for intermediate-boson fusion, and $M_D \gg M_H/2$ for Higgs-boson decay. In the case of Higgs-boson production, we have $\sqrt{s}=M_Z$ for $Z\rightarrow f\bar{f}H$, $\sqrt{s}>M_H$ for $e^+e^-\rightarrow f\bar{f}H$, and $\sqrt{s}>M_Z+M_H$ for $e^+e^-\rightarrow ZH$, while, in the case of Higgs-boson decay, we have $M_H>0$ for $H\rightarrow 4f$, $M_H>M_V$ for $H\rightarrow V+2f$, and $M_H>2M_V$ for $H\rightarrow 2V$. We recovered the notion, established in Refs. [18,36], that, in the on-shell scheme implemented with G_F, the leading $O(G_F M_Q^2)$ terms get reduced in magnitude by their QCD corrections. It turned out that, in general, this screening effect is considerably more pronounced in the W^+W^-H and ZZH observables than in the electroweak parameters [36] and Yukawa couplings [18]. Nevertheless, the observables in the Higgs sector tend to be more sensitive to the presence of fourth-generation fermion doublets, especially if isospin is only mildly broken, in

TABLE II. Values of $-x$ in % (upper entries) and $\sigma(e^+e^-\to e^+e^-H)$ in fb (lower entries) as functions of $M_H/s^{1/2}$ for selected values of $s^{1/2}$.

$M_H/\sqrt{s}$	$\sqrt{s}$ [GeV]					
	175	300	500	1000	1500	2000
0.3	13.623	5.103	1.603	0.271	0.088	0.039
	1.283	3.160	5.627	8.869	10.249	10.956
0.4	14.417	5.550	1.754	0.292	0.094	0.040
	0.743	1.852	3.282	5.079	5.806	6.168
0.5	15.490	6.210	1.997	0.332	0.106	0.045
	0.378	0.970	1.736	2.683	3.058	3.242
0.6	16.946	7.206	2.392	0.403	0.128	0.055
	0.161	0.433	0.797	1.253	1.433	1.521
0.7	18.958	8.786	3.083	0.537	0.173	0.074
	0.051	0.150	0.291	0.477	0.553	0.590
0.8	21.837	11.535	4.473	0.838	0.276	0.120
	0.010	0.032	0.069	0.124	0.148	0.160

which case the ρ parameter fails to serve as a useful probe.

In the discussion of fourth-generation scenarios, one has to bear in mind that the fermion masses must not exceed the vacuum-stability bound, which follows from the requirement that the running Higgs quartic coupling, $\lambda(\mu)$, must not turn negative for renormalization scales $\mu<\Lambda$, where Λ is the assumed mass scale of some new interaction [9]. This bound may be stringent, of order $v=246$ GeV, should there be a grand dessert up to $\Lambda=\Lambda_{\rm GUT}\approx10^{16}$ GeV, but it may be considerably relaxed for Λ in the few-TeV range. Another theoretical difficulty related to M_Q values in excess of v is that the $Q\bar{Q}H$ Yukawa coupling then becomes strong so that the Higgs-exchange corrections may not be negligible anymore. Without explicit calculation, it is very difficult to predict above which values of M_Q these corrections will surpass the QCD ones. In the case of Higgs-boson production via gluon fusion, $gg\to H$, at the CERN Large Hadron Collider, the QCD correction increases the lowest-order cross section by approximately 70%, while, even for $M_Q=500$ GeV, the Higgs-related correction amounts to just 5% [42]. By analogy, this suggests that the two-loop Higgs-exchange contributions of $O(G_F^2M_Q^4)$ to the W^+W^-H and ZZH observables are also likely to be small as long as the vacuum-stability constraint is satisfied. However, final clarity concerning this point can only come from a complete $\mathcal{O}(G_F^2M_Q^4)$ calculation, which is a separate issue and lies beyond the scope of the present work.

ACKNOWLEDGMENTS

The author would like to thank Manfred Lindner for useful information on vacuum-stability bounds in fourth-generation scenarios and Michael Spira for fruitful discussions concerning the extension of the soft-Higgs theorem beyond the leading order. He is indebted to the National Institute for Nuclear Theory in Seattle for its great hospitality during a visit when this manuscript was finalized, and to the Department of Energy for partial support during the completion of this work.

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