CP violation for electroweak baryogenesis from mixing of standard model and heavy vector quarks

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It is known that the CP violation in the minimal standard model is insufficient to explain the observed baryon asymmetry of the Universe in the context of electroweak baryogenesis. In this paper we consider the possibility that the additional CP violation required could originate in the mixing of the standard model quarks and heavy vector quark pairs. We consider the baryon asymmetry in the context of the spontaneous baryogenesis scenario. It is shown that, in general, the CP-violating phase entering the mass matrix of the standard model and heavy vector quarks must be space dependent in order to produce a baryon asymmetry, suggesting that the additional CP violation must be spontaneous in nature. This is true for the case of the simplest models which mix the standard model and heavy vector quarks. We derive a charge potential term for the model by diagonalizing the quark mass matrix in the presence of the electroweak bubble wall, which turns out to be quite different from the fermionic hypercharge potentials usually considered in spontaneous baryogenesis models, and obtain the rate of baryon number generation within the wall. We find, for the particular example where the standard model quarks mix with weak-isodoublet heavy vector quarks via the expectation value of a gauge singlet scalar, that we can account for the observed baryon asymmetry with conservative estimates for the uncertain parameters of electroweak baryogenesis, provided that the heavy vector quarks are not heavier than a few hundred GeV and that the coupling of the standard model quarks to the heavy vector quarks and gauge singlet scalars is not much smaller than order of 1, corresponding to a mixing angle of the heavy vector quarks and standard model quarks not much smaller than order of 10^{-1}.

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I. INTRODUCTION

The possibility that the baryon asymmetry of the Universe (BAU) could be generated during the electroweak phase transition has been extensively studied over the past decade [1–20]. It has become apparent that the electroweak standard model (SM) is unable to account for the observed magnitude of the BAU for two separate reasons: (i) there is not enough CP violation from the Kobayashi-Maskawa matrix to generate the B asymmetry, as it only contributes to electroweak baryogenesis via a seven-loop diagram [2] (finite-temperature effects do not alter this conclusion [3]) and (ii) for phenomenologically acceptable Higgs masses, the magnitude of the Higgs field at the end of the phase transition is not large enough to ensure that the anomalous $B+L$ violation due to sphaleron fluctuations is out of thermal equilibrium, resulting in the washout of any B asymmetry generated during the phase transition [2,10–12]. (We will refer to this as the Higgs mass problem of electroweak baryogenesis.) Various possible solutions, usually involving extensions of the SM, have been suggested for both these problems. The Higgs mass problem might be solved by extending the Higgs sector of the SM [13,14], or even by nonperturbative effects in the finite-temperature field theory describing the phase transition [15]. Additional CP violation might be introduced by extending the Higgs sector [16], by considering the supersymmetric (SUSY) extension of the SM [17], or by adding additional fermions, such as right-handed neutrinos in a Majoron model [18]. The possibility that spontaneous CP violation could play a role has been discussed for two-Higgs-doublet models [19] and for a model with an additional gauge singlet scalar [20].

In this paper we wish to consider the possibility that the additional CP violation could originate in the mixing of the SM quarks with heavy vector quark pairs—anomaly-free pairs of heavy quarks transforming as representations $R+\bar{R}$ of the gauge group. The effect of the mixing of the light SM quarks and heavy vector quarks is to introduce a CP-violating "charge potential" term [5] on diagonalizing the quark mass matrix in the presence of the electroweak bubble wall. In the presence of such a charge potential term the equilibrium asymmetry is nonzero as a result of the splitting of the energy of the quarks and antiquarks by the charge potential term, leading to the generation of a baryon asymmetry once B-violating sphaleron fluctuations are taken into account. In order to estimate the resulting baryon asymmetry we will follow the spontaneous baryogenesis scenario [5,6], in which the particle mean free paths are assumed small enough relative to the thickness of the electroweak bubble wall to allow a local thermal equilibrium to be established within the bubble wall. This appears to be consistent with most estimates of the bubble wall thickness and quark mean free path [8,9,12].

The paper is organized as follows. In Sec. II we discuss the introduction of CP violation via mixing of SM quarks and heavy vector quarks and calculate the baryon asymmetry generation rate due to the resulting charge potential within the electroweak bubble wall. In Sec. III we give an explicit example of the generation of a baryon asymmetry in which the CP-violating phase and the mixing of the SM and vector quarks is due to the expectation value of a complex gauge singlet scalar. In Sec. IV we present our conclusions. Some details of the local thermal equilibrium of interactions within the bubble wall are discussed in the Appendix.

II. ADDITIONAL CP VIOLATION VIA MIXING OF SM QUARKS AND HEAVY VECTOR QUARKS

In this section we discuss the mixing of light SM quarks and heavy vector quarks and the resulting rate of B generation within the electroweak bubble wall, applying the results to a specific model of electroweak baryogenesis in the next section.

We focus throughout on the case of a pair of heavy isodoublet vector quarks $V_L + V_R$, where V_L and V_R have the same transformation properties under the standard model gauge group $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_w\times\mathrm{U}(1)_y$, and where V_L transforms in the same way as a standard model quark doublet Q_L. Analogous results may be obtained by considering isosinglet vector quarks transforming as either u_R or d_R. The mass terms of the model, in the presence of the space- and time-dependent Higgs field in the bubble wall, are assumed to be given by

$$\tilde{m}(x)e^{i\theta(x)}\bar{V}_R Q_L + m_V \bar{V}_R V_L + \text{H.c.}\,, \tag{2.1}$$

where $\tilde{m}(x)$ is x dependent within the electroweak bubble wall. We are considering the case where the SM quark mass within the wall is small compared with the mass terms in (2.1), which is plausible as the magnitude of the Higgs field within the wall at the electroweak phase transition is small compared with its $T = 0$ value [10]. In general, one can have couplings of all three generations of SM quarks to V_R, but since only one linear combination of these will actually mix with V_L we can focus on (2.1) with just one Q_L. The first point to note is that it is essential for the phase θ to be x dependent, in order for it to have any physical consequences. This is because any constant explicit phase on $\tilde{m}(x)$ can simply be rotated away by redefinitions of the phases of V_L and V_R. This suggests that the CP violation must be *spontaneous*, with a space-dependent phase within the bubble wall region. This is true of the simplest models which give rise to a mass term of the form (2.1), as discussed in the next section. It also means that no additional CP violation is introduced in the $T = 0$ theory, where θ is constant; the additional CP violation is only active during the electroweak phase transition when θ is x dependent. [This is true of the case where one considers only the mass terms (2.1); however, there can be $T = 0$ CP violation associated with the additional scalars which are needed in order to produce a mass term of the form (2.1). We will discuss this point in the context of an explicit model given in the next section.] On diagonalizing the mass terms, we obtain mass eigenstates Q_L' and V_L', where

$$\begin{pmatrix} Q_L' \\ V_L' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} Q_L \\ V_L \end{pmatrix} \tag{2.2}$$

with $\alpha = [1 + (\tilde{m}/m_V)^2]^{-1/2}$ and $\beta = -(\tilde{m}/m_V)[1 + (\tilde{m}/m_V)^2]^{-1/2}e^{-i\theta}$. There is a zero-mass eigenstate and a massive eigenstate with mass $\tilde{m}_V = m_V/\alpha$. It is important to check that the field redefinition (2.2) does not introduce any anomalous coupling to the gauge fields [5]. In fact, one can see that the redefinition (2.2) is anomaly-free, since it is equivalent to an anomaly-free rotation $V_{L,R} \rightarrow e^{i\theta(x)}V_{L,R}$, which removes the complex phase from (2.1), followed by a diagonalization of the now real mass matrix, followed by a second anomaly-free rotation of the heavy quark mass eigenstates $V_{L,R}' \rightarrow e^{-i\theta(x)}V_{L,R}'$.

Unlike the case of a constant $\tilde{m}(x)e^{i\theta(x)}$, diagonalizing the mass terms with a spacetime-dependent $\tilde{m}(x)e^{i\theta(x)}$ results in nondiagonal terms coming from the kinetic terms,

$$\begin{aligned} &\bar{Q}_L i\gamma^\mu\partial_\mu Q_L + \bar{V}_L i\gamma^\mu\partial_\mu V_L \\ &\quad\rightarrow \bar{Q}_L' i\gamma^\mu\partial_\mu Q_L' + \bar{V}_L' i\gamma^\mu\partial_\mu V_L' \\ &\quad\quad+\frac{i}{2}(\beta\partial_\mu\beta^* - \beta^*\partial_\mu\beta)(\bar{Q}_L'\gamma^\mu Q_L' - \bar{V}_L'\gamma^\mu V_L') \\ &\quad\quad+[i(\beta\partial_\mu\alpha - \alpha\partial_\mu\beta)\bar{Q}_L'\gamma^\mu V_L' + \text{H.c.}]\,. \end{aligned} \tag{2.3}$$

The terms proportional to the time derivatives of α and β cause a splitting in the energy of the particles and antiparticles within the bubble wall, resulting locally in a nonzero equilibrium baryon asymmetry [5,6]. To leading order, the spatial derivatives do not affect the local equilibrium particle number asymmetry, but instead introduce small corrections to the diffusion equations for the particle asymmetries [8]. Thus, to discuss the baryon asymmetry in the original spontaneous baryogenesis scenario (in which diffusion is neglected), we need only consider the time derivative terms from (2.3), which give a "charge potential"-type term [5]

$$\begin{aligned} &\frac{i}{2}(\beta\partial_0\beta^* - \beta^*\partial_0\beta)(\bar{Q}_L'\gamma^0 Q_L' - \bar{V}_L'\gamma^0 V_L') \\ &\quad\quad+[i(\beta\partial_0\alpha - \alpha\partial_0\beta)\bar{Q}_L'\gamma^0 V_L' + \text{H.c.}]\,. \end{aligned} \tag{2.4}$$

In discussing the baryon number generated by this charge potential term, a fundamental point is whether or not the Yukawa coupling of the V_L' quark to the top quark and Higgs boson is in thermal equilibrium within the bubble wall. On redefining the mass eigenstates, the t quark Yukawa coupling becomes

$$\lambda_t \bar{t}_R H Q_L \rightarrow \lambda_t \bar{t}_R H(\alpha Q_L' - \beta V_L')\,. \tag{2.5}$$

In the Appendix we discuss the question of the conditions under which the Yukawa couplings are in thermal equilibrium within the wall. We find that for $|\beta| \lesssim 0.3$ the V_L' quark Yukawa coupling will be out of equilibrium within the bubble wall. In this case the charge potential (2.4) applies. If, on the other hand, $|\beta| \gtrsim 0.3$, then it is necessary to redefine the phase of V_L' via an anomaly-free rotation $(V_L', V_R) \to e^{i\theta(x)}(V_L', V_R)$ in order to remove the phase from the complex Yukawa coupling $\beta\lambda_t$. We first consider the case where (2.4) applies, $|\beta| \lesssim 0.3$. (We will drop the primes on mass eigenstates from now on.)

A. V_L Yukawa coupling out of thermal equilibrium within the bubble wall

We can use the fact that the phase θ can be shifted by a constant without any physical consequences to show that the cross-term $\bar{Q}_L\gamma^0 V_L$ in (2.4) will not contribute to the CP-violating part of the free energy, the part responsible for driving the generation of the baryon asymmetry. The coefficient of the cross-term is

$$i(\beta\partial_0\alpha - \alpha\partial_0\beta) = i\alpha\frac{\tilde{m}}{\tilde{m}_V}e^{-i\theta}\left[\frac{\partial_0\tilde{m}}{\tilde{m}} - i\partial_0\theta\right] . \quad (2.6)$$

Since this is proportional to $e^{-i\theta}$, in order to be independent of a constant phase shift, any contribution of the cross-term to the free energy must depend only on the magnitude of (2.6) and so must be proportional to $[|\partial_0\tilde{m}/\tilde{m}|^2 + |\partial_0\theta|^2]^{n/2}$, for some power n. (The $e^{-i\theta}$ factor appears in the Lagrangian only in the coefficient of the cross-term, since, as shown below, the coefficient of the diagonal terms in the charge potential are real, and the phase-dependent V_L Yukawa coupling is out of equilibrium within the bubble wall.) Since any CP-violating contribution should vanish in the limit of constant θ, it then follows that any such term must generally be zero. Thus we can drop the cross-terms from the charge potential when discussing the baryon asymmetry, and consider just the term

$$\frac{i}{2}(\beta\dot{\beta}^* - \beta^*\dot{\beta})(\bar{Q}_L\gamma^0 Q_L - \bar{V}_L\gamma^0 V_L). \quad (2.7)$$

The importance of this is that it allows for a simple chemical potential analysis of the resulting baryon asymmetry, which is not the case if the cross-terms contribute to the CP-violating part of the free energy. Introducing $\beta = -(\tilde{m}/m_V)[1 + (\tilde{m}/m_V)^2]^{-1/2}e^{-i\theta}$, (2.7) becomes

$$\phi_q(\bar{Q}_L\gamma^0 Q_L - \bar{V}_L\gamma^0 V_L) , \quad (2.8)$$

where

$$\phi_q = -|\beta|^2\dot{\theta} . \quad (2.9)$$

We next consider the effect of this charge potential on B-violating processes within the electroweak bubble wall. To discuss this, we first consider the original spontaneous baryogenesis scenario [5,6], neglecting diffusion effects [8,9]. Later we will comment on how diffusion may alter the baryon asymmetry thus obtained.

We will consider throughout the case where $m_V \gtrsim T$ at the electroweak phase transition. This is likely to be necessary phenomenologically, since T is typically around 100 GeV for Higgs masses less than or of the order of 100 GeV [10]. Such small Higgs masses are probably necessary in electroweak baryogenesis models in order that the baryon asymmetry is not washed out once the phase transition is complete [2,10–12]. Within the region of the bubble wall where baryon asymmetry generation is occurring at a significant rate, the sphaleron fluctuations are not Boltzmann suppressed, having energy $\lesssim T$ [6]. Therefore, in the case where $m_V \gtrsim T$, the sphaleron fluctuations cannot produce V quark pairs.

Let n_i and μ_i be the number density and chemical potential within the electroweak bubble wall of a particle species i, taken to be in thermal equilibrium within the bubble wall. The rate at which a particle species A_i reaches thermal equilibrium, via interactions of the form $\sum_i \alpha_i A_i \leftrightarrow 0$, is given by [21]

$$\frac{dn_i}{dt} = -\frac{\Gamma_{\rm eq}}{T}\delta\mathcal{F}\alpha_i = -\frac{\Gamma_{\rm eq}}{T}\left(\sum_j \alpha_j\mu_j\right)\alpha_i , \quad (2.10)$$

where $\delta\mathcal{F}$ is small compared with T. $\Gamma_{\rm eq}$ is the equilibrium rate for the interaction at temperature T and $\delta\mathcal{F}$ is the change in free energy density in each such interaction.

The sphaleron process for the SM with N_G generations introduces an effective $4N_G$ fermion interaction of the form [1,2,6]

$$\prod_{i=1}^{N_G}(QQQL)^i , \quad (2.11)$$

where i is a generation index. For each generation, the $(B+L)$-violating processes are of the form [1,6]

$$\text{(i)}\ \ uudl \leftrightarrow 0 \quad (2.12\text{a})$$

and

$$\text{(ii)}\ \ udd\nu \leftrightarrow 0 \quad (2.12\text{b})$$

where color indices are suppressed. Suppose that there are N total separate possible processes, with $N/2$ corresponding to (i) and $N/2$ corresponding to (ii). Then the rate for each separate process is $\Gamma_{\rm sp}/N$. Thus, if we consider the rate of change of the asymmetry in u_L and d_L quarks of generation i and a given color, we obtain from processes (i) and (ii), respectively,

$$\frac{dn^i_{u_L}}{dt} = -2(2\mu^i_{u_L} + \mu^i_{d_L} + \mu^i_{l_L})\frac{\Gamma_{\rm sp}}{NT} , \quad (2.13\text{a})$$

$$\frac{dn^i_{u_L}}{dt} = -(\mu^i_{u_L} + 2\mu^i_{d_L} + \mu^i_{\nu_L})\frac{\Gamma_{\rm sp}}{NT} , \quad (2.13\text{b})$$

and

$$\frac{dn^i_{d_L}}{dt} = -(2\mu^i_{u_L} + \mu^i_{d_L} + \mu^i_{l_L})\frac{\Gamma_{\rm sp}}{NT} , \quad (2.14\text{a})$$

$$\frac{dn^i_{d_L}}{dt} = -2(\mu^i_{u_L} + 2\mu^i_{d_L} + \mu^i_{\nu_L})\frac{\Gamma_{\rm sp}}{NT} , \quad (2.14\text{b})$$

where we have used the fact that the chemical potentials of quarks differing only in color will be the same and have dropped the color indices on the quark chemical potentials. Then summing over all $N/2$ processes for both (i) and (ii), and with the baryon number density given by $n_B = \frac{1}{3}\sum(n_{u_L} + n_{d_L})$, where the sum is over all generations and colors, we obtain

$$\frac{dn_B}{dt} = -\sum_{i=1}^{N_G}(3\mu_{u^i_L} + 3\mu_{d^i_L} + \mu_{l^i_L} + \mu_{\nu^i_L})\frac{\Gamma_{\rm sp}}{2T} \ . \quad (2.15)$$

[We could include V quarks in the definition of n_B, but since we are considering the case where they are not produced in sphaleron processes this does not alter (2.15).]

In order to discuss the rate of baryon number generation inside the bubble wall, we need to consider what are the conserved quantities inside the bubble wall and then obtain the μ_i via a standard chemical potential analysis [5]. For simplicity, we will ignore the small Higgs expectation value within the bubble wall region when discussing the chemical potential analysis and work in the unbroken $\mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ phase. This is reasonable, since the Higgs expectation value within the bubble wall during the electroweak phase transition will be small compared with the thermal average value of the Higgs field due to thermal fluctuations $\langle h^2 \rangle_T^{1/2}$, and so will add only small corrections to the free energy [22]. Since the chemical potentials of all gauge bosons are zero, the chemical potentials of all particles differing only by a color or $\mathrm{SU}(2)_w$ index will be the same. Therefore we can write $\mu_{u^i_L} = \mu_{d^i_L} \equiv \mu_{Q^i}$, and $\mu_{\nu^i_L} = \mu_{l^i_L} \equiv \mu_{L^i}$. Within the bubble wall only the t quark Yukawa coupling will come into thermal equilibrium, the mean free path of the other Yukawa couplings being too long relative to the thickness of the electroweak bubble wall to allow them to achieve equilibrium within the wall [5]. (We discuss the question of local thermal equilibrium of the Yukawa interactions within the bubble wall in the Appendix.) Thus the only perturbative interaction which is in equilibrium within the bubble wall and which relates different chemical potentials is the t quark–Higgs Yukawa interaction. This interaction implies that

$$\mu_{t_R} = \mu_{Q^3} + \mu_H \quad (2.16)$$

where μ_H is the chemical potential for the Higgs doublet. We see that we have as independent chemical potentials within the bubble wall $\mu_{Q^i}, \mu_{u^i_R}$ $(i = 1, 2)$, $\mu_{L^i}, \mu_{l^i_R}, \mu_{d^i_R}$ $(i = 1, 2, 3)$, $\mu_{Q^3}, \mu_{t_R}, \mu_{V_L}$, and μ_{V_R}. We can then obtain the value of these chemical potentials by fixing the value of the conserved charges within the wall, which for the case of unbroken $\mathrm{SU}(2)_w \times \mathrm{U}(1)_Y$ may be taken to be B and Y.

In general, the charge potential term modifies the SM Lagrangian according to

$$\mathcal{L}_{\rm SM} \to \mathcal{L}_{\rm SM} - n_i\phi_i \ , \quad (2.17)$$

where $n_i = \bar{f}_i\gamma^0 f_i$, for each fermion i. In order to find the rate of baryon number production from (2.15) we need to find the chemical potential of the particles in the presence of this charge potential. It is useful to define the chemical potentials in the absence of the charge potential by μ_{i0}, where

$$\mu_{i0} = \frac{\partial \mathcal{F}_0}{\partial n_i} \quad (2.18)$$

and $\mathcal{F} = \mathcal{F}_0 + n_i\phi_i$, where $\mathcal{F}_0$ is the free energy density without the charge potential, and $\mu_i = \partial\mathcal{F}/\partial n_i$ is the chemical potential in the presence of the charge potential. This is useful because we can use the relation between μ_{i0} and the number density of fermion species i, as given by the Fermi-Dirac distribution [23]

$$n_i = \frac{g_i}{2\pi^2}\int_{m_i}^{\infty} E(E^2 - \mu_{i0}^2)^{1/2} dE \times \left[\frac{1}{(1 + e^{(E-\mu_{i0})/T})} - \frac{1}{(1 + e^{(E+\mu_{i0})/T})}\right] \quad (2.19)$$

which for $\mu_{0i}/T \ll 1$ becomes

$$n_i = \frac{g_i\mu_{i0}T^2}{6} f_i \quad (2.20)$$

where

$$f_i = \frac{6}{\pi^2}\int_{x_{m_i}}^{\infty} x(x^2 - x_{m_i}^2)^{1/2}\frac{e^{-x}}{(1+e^{-x})^2}dx \quad (2.21)$$

with $x_{m_i} = m_i/T$. g_i gives the number of degrees of freedom of the fermion of species i ($g_i = 4$ for Dirac fermions). $f_i \approx 1$ for $m_i/T \lesssim 1$. We give some values for f_i in Table I.

If we have a conserved charge density Q, then we can write

$$Q = q_i^Q n_i \propto q_i^Q g_i \mu_{i0} f_i \ , \quad (2.22)$$

where q_i^Q is the charge of particle i with respect to Q. Then with $\mathcal{F} = \mathcal{F}_0 + n_i\phi_i$ we find

$$\mu_i = \frac{\partial\mathcal{F}}{\partial n_i} = \mu_{i0} + \phi_i \quad (2.23)$$

and so

$$Q \propto q_i^Q g_i(\mu_i - \phi_i) f_i \ . \quad (2.24)$$

In the case of the electroweak bubble walls, we are for now interested in the original spontaneous baryogenesis scenario [5], where the baryon density is initially zero and can only increase due to sphaleron processes, with the transport of charges into the bubble wall [8,9] being

TABLE I. Values of the Boltzmann suppression factor f_i.

x_{m_i}	f_i
0.1	0.998
0.5	0.963
1.0	0.863
1.5	0.725
2.0	0.578
2.5	0.442
3.0	0.328
5.0	0.080
10.0	0.001

neglected. The number density of all quarks and leptons except Q_L^3 and t_R is effectively conserved within the bubble wall, due to the lack of Yukawa interactions in thermal equilibrium. [The V_L and V_R quarks are conserved separately, since, as discussed in the Appendix, $V_L \leftrightarrow V_R$ transitions are out of equilibrium within the bubble wall.] Thus setting these number densities to zero (corresponding to the lack of an asymmetry outside the bubble wall) implies, from (2.24), that $\mu_i = \phi_i$ for these particles. To fix the remaining chemical potentials μ_{Q^3} and μ_{t_R}, we impose the conditions $B = 0$ and $Y = 0$ inside the wall. From (2.24) these imply that

$$B \propto 4\mu_{Q^3} + 2\mu_{t_R} - (4\phi_{Q^3} + 2\phi_{t_R}) = 0 \tag{2.25}$$

and

$$Y \propto 6\mu_{t_R} - (2\phi_{Q^3} + 4\phi_{t_R} + 2\phi_H) = 0 \ . \tag{2.26}$$

Therefore we find

$$\mu_{t_R} = \tfrac{1}{3}(\phi_{Q^3} + 2\phi_{t_R} + \phi_H) \tag{2.27a}$$

and

$$\mu_{Q^3} = \tfrac{1}{4}(\tfrac{10}{3}\phi_{Q^3} + \tfrac{2}{3}\phi_{t_R} - 2\phi_H) \ . \tag{2.27b}$$

This specifies all the chemical potentials inside the bubble wall. Thus, for a given set of ϕ_i, we can obtain from (2.15) the rate of B generation within the bubble wall.

For the charge potential (2.8), only ϕ_Q is nonzero. For simplicity, let us assume that Q in (2.8) corresponds to Q^3, although in general it would correspond to a linear combination of all three generations of SM quark doublets. From (2.27b) we then obtain

$$\mu_{Q^3} = \tfrac{5}{6}|\beta|^2\dot{\theta} \tag{2.28}$$

and so from (2.15)

$$\frac{dn_B}{dt} = -\frac{5}{2}\frac{\Gamma_{\rm sp}}{T}|\beta|^2\dot{\theta} \ . \tag{2.29}$$

The B asymmetry resulting from the passage of the bubble wall is then simply obtained by integrating (2.29) until the time when the Higgs field is large enough to put the sphaleron processes out of thermal equilibrium [5,6]. We will illustrate this by considering an explicit model for light-heavy quark mixing in the next section. Before doing so, we first comment on the case where the V_L quark Yukawa coupling is in thermal equilibrium within the bubble wall.

B. V_L Yukawa coupling in thermal equilibrium within the bubble wall

In this case, we cannot use the charge potential term (2.4) coming from simply diagonalizing the mass term (2.1) in the presence of the bubble wall. This is because the Yukawa coupling of the V_L mass eigenstate is given by $-\beta\lambda_t\bar{t}_R H V_L$, where β is complex. Thus we must perform an additional anomaly-free redefinition $(V_L, V_R) \to e^{i\theta}(V_L, V_R)$ in order to remove this phase, which gives a new charge potential term,

$$-\beta^2\partial_0\theta(\bar{Q}_L\gamma^0 Q_L - \bar{V}_L\gamma^0 V_L) - \partial_0\theta(\bar{V}_L\gamma^0 V_L + \bar{V}_R\gamma^0 V_R)$$
$$+\{[i(\beta\partial_0\alpha - \alpha\partial_0\beta) - \alpha\beta\partial_0\theta]\bar{Q}_L\gamma^0 V_L + \text{H.c.}\} \ , \tag{2.30}$$

where the fields correspond to the redefined fields and β here corresponds to β in (2.2) without the $e^{-i\theta}$ phase factor. In this case we see that, since there is no overall $e^{-i\theta}$ phase factor for the coefficient of the cross-term, we cannot use the constant phase shift argument to show that the cross-term does not contribute to the CP-violating part of the free energy. However, there is a second argument which suggests that the cross-terms may be neglected when calculating the free energy.

The free energy F can be written in terms of the thermal expectation value of the energy $\bar{E} = \langle H\rangle_T$ according to the standard relation [21]

$$F = \frac{1}{\beta}\int d\beta\, \bar{E} + K \ , \tag{2.31}$$

where $\beta = 1/T$ and K is a T-independent constant. The contribution of the charge potential term to the thermal average energy is then

$$-\int d^3x \langle \mathcal{L}_{\rm cp}\rangle_T \tag{2.32}$$

where $\langle \mathcal{L}_{\rm cp}\rangle_T$ denotes the thermal average of the charge potential contribution to the Lagrangian density, with $\mathcal{L}_{\rm cp}$ given by (2.30). With $\bar{Q}_L\gamma^0 V_L \equiv Q_L^\dagger V_L$, we see that the contribution of the $\bar{Q}_L\gamma^0 Q_L$ term to the free energy is proportional to $\langle Q_L^\dagger Q_L\rangle_T$, whereas the contribution of the $\bar{Q}_L\gamma^0 V_L$ term is proportional to $\langle Q_L^\dagger V_L\rangle_T$. In the absence of interactions between the Q_L and V_L quarks, since the Q_L and V_L fields will be uncorrelated, $\langle Q_L^\dagger V_L\rangle_T$ is zero, unlike $\langle Q_L^\dagger Q_L\rangle_T$ or $\langle V_L^\dagger V_L\rangle_T$. The charge potential term (2.30) will itself correspond to an interaction between Q_L and V_L quarks, but, since we will be considering the case where the coefficients of the charge potential terms are small compared with T, this will lead to a contribution from $\langle Q_L^\dagger V_L\rangle_T$ to the free energy of higher order in the small coefficients and so negligible compared with that from the "diagonal" terms in the charge potential. Thus by this argument the important terms in the charge potential from the point of view of the free energy are

$$-\beta^2\partial_0\theta\bar{Q}_L\gamma^0 Q_L - \partial_0\theta(\alpha^2\bar{V}_L\gamma^0 V_L + \bar{V}_R\gamma^0 V_R) \ . \tag{2.33}$$

The effect of the V_L quark Yukawa coupling being in thermal equilibrium is to impose the relation $\mu_{t_R} = \mu_{V_L} + \mu_H$, which implies, from (2.16), that $\mu_{V_L} = \mu_{Q^3}$. Then, following a chemical equilibrium analysis analogous to that given above, we obtain the following relations for $B = Y = 0$ inside the bubble wall:

$$B \propto 4\mu_{Q^3} + 2\mu_{t_R} + 4f\mu_{V_L} - (4\phi_{Q^3} + 4f\phi_{V_L}) = 0 \tag{2.34}$$

and

$$Y \propto 2\mu_{Q^3} + 4\mu_{t_R} + 2f\mu_{V_L}$$
$$+2\mu_H - (2\phi_{Q^3} + 2f\phi_{V_L}) = 0 \ , \quad (2.35)$$

where $\phi_{Q^3} \equiv \phi_q = \beta^2\dot{\theta}$ and $\phi_{V_L} = (\alpha^2/\beta^2)\phi_q$. f corresponds to the Boltzmann suppression factor (2.21) for the case of the V quark. Solving these for μ_{Q^3} then gives

$$\mu_{Q^3} = \frac{5}{(6+5f)}\phi_q\left(1+\frac{\alpha^2}{\beta^2}f\right) . \quad (2.36)$$

Thus from (2.15) we obtain for the rate of baryon generation within the wall in this case

$$\frac{dn_B}{dt} = -\frac{15}{(6+5f)}\phi_q\left(1+\frac{\alpha^2}{\beta^2}f\right)\frac{\Gamma_{\rm sp}}{T} . \quad (2.37)$$

We see that for $f \to 0$, corresponding to V quarks much heavier than T, we recover the rate of B generation for the case where the V quark Yukawa coupling is out of equilibrium. In general, the rates in the two cases are not much different. For simplicity, we will focus on the case with $|\beta| \lesssim 0.3$ when discussing the baryon asymmetry in the next section.

III. AN EXPLICIT EXAMPLE: SPONTANEOUS *CP* VIOLATION FROM A GAUGE SINGLET SCALAR SECTOR

We have shown in the previous section that it is essential that the mass mixing term in the case of heavy-light quark mixing has a spacetime-dependent phase. To be dependent on x within the bubble wall, the CP-violating phase must depend on the Higgs field. To give a simple example where this is realized, consider a complex scalar field S, with a tree-level scalar potential given by $V = V_1 + V_2 + V_3$, where

$$V_1 = -\mu_S^2 S^\dagger S + \lambda_{SH}S^\dagger SH^\dagger H + \lambda_{SS}(S^\dagger S)^2 \ , \quad (3.1a)$$

$$V_2 = (m_{S'}^2 S^2 + \text{H.c.}) + (\lambda_\beta S^4 + \text{H.c.}) \ , \quad (3.1b)$$

and

$$V_3 = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 \ . \quad (3.1c)$$

One can also consider other couplings (such as $S^2H^\dagger H$), but for now we will only consider the terms in (3.1), which are the important ones for electroweak baryogenesis. The scalar potential is assumed to have a discrete symmetry, which corresponds to the product of a CP transformation $Z_{CP} : S \leftrightarrow S^\dagger$ (this implies that all the couplings in (3.1) are real; we also take them to be positive in the following) and a Z_2 symmetry $Z_S : S \leftrightarrow -S$.

The coupling of the S scalar to the vector quarks and Q_L^i is given by

$$\lambda_V^i S\bar{V}_R Q_L^i + \text{H.c.} \quad (3.2)$$

The phase of the S field is then responsible for the CP-violating phase; thus the CP violation in this model is necessarily spontaneous in nature. We will concentrate on the third generation ($i = 3$) in the following, which is the most important for electroweak baryogenesis. One could also consider terms such as $\bar{u}_R\tilde{H}V_L$ (where $\tilde{H}^i = \epsilon^{ij}H^{j*}$) or $\bar{d}_R H V_L$. However, for simplicity we will not include these terms. If one extends the discrete symmetry Z_S to the V quarks, then the V quarks must be odd under Z_S, which will eliminate these additional terms.

We should note that similar models have been suggested in the past as a way of accounting for low-energy CP-violating phenomenology. In particular, essentially the same model as we are considering here was studied with respect to low-energy CP violation in Ref. [24]. [In addition, flavor mixing and CP violation due to vector pairs of charge $-\frac{1}{3}$ quarks and gauge singlet scalars were also features of E(6) based "superstring inspired" models [25], while the possible significance of spontaneous CP violation and vector quark pairs as a solution to the strong CP problem was earlier pointed out by Nelson and Barr [26].] As we have noted in the previous section, there is no $T = 0$ CP violation due to a constant phase in the mass term (2.1). In the case of the above model, however, this phase comes from the expectation value of the S scalar. Although at $T = 0$ we can rotate away the constant phase from the mass term mixing the light and heavy quarks, this rotation will have the effect of introducing a phase on the Yukawa coupling coming from (3.2) between the physical scalar associated with S (S', where $S = \langle S\rangle + S'$) and the light and heavy quark doublets. According to the discussion of Ref. [24], this can result in a CP-violating phase in the Kobayashi-Maskawa matrix as well as giving significant contributions to the neutron electric dipole moment. However, this $T = 0$ CP violation, which is due to the constant phase at $T = 0$, is not directly related to the CP violation responsible for the baryon asymmetry, which is due to the spatial dependence of this phase in the vicinity of the electroweak bubble wall.

On introducing the S expectation value $\langle S\rangle = (\rho/\sqrt{2})e^{i\theta}$, we find that $\tilde{m}(x)e^{i\theta(x)} = (\lambda_v/\sqrt{2})\rho e^{i\theta}$. ρ and θ are dependent on the Higgs field within the bubble wall via the coupling λ_{SH}. On introducing the Higgs expectation value $\langle H\rangle = h/\sqrt{2}$ and minimizing with respect to ρ and θ for a given h, we obtain

$$\rho^2 = \frac{1}{\lambda_{SS}}\left(\mu_S^2 - \frac{\lambda_{SH}}{2}h^2\right) \quad (3.3)$$

and

$$\cos 2\theta = -\frac{m_{S'}^2}{2\lambda_\beta \rho^2} \ , \quad (3.4)$$

where (3.4) is true so long as on the right-hand side $|m_{S'}^2/2\lambda_\beta\rho^2| < 1$; otherwise $\cos 2\theta = -1$. We see that ρ^2 is $h(x)$ dependent, and so θ is x dependent also, provided that $|\cos 2\theta| < 1$.

The baryon asymmetry generated during the electroweak phase transition (in the absence of diffusion effects) then follows from the now standard spontaneous baryogenesis analysis [5,6]. From the discussion of the last section, for the case where $|\beta| \lesssim 0.3$, the rate of B generation in the wall is given by

$$\frac{dn_B}{dt} = -\frac{5}{2}\frac{\Gamma_{sp}}{T}|\beta|^2\dot{\theta} \ . \tag{3.5}$$

We can roughly approximate the rate of sphaleron transitions by [5,6,8] (we use the term "sphaleron transition" to denote any transition between gauge vacua of different topology, even when the sphaleron barrier disappears at high T)

$$\Gamma_{sp} = K(\alpha_W T)^4, \quad m_W(h) \lesssim \sigma\alpha_W T \ , \tag{3.6a}$$

$$\Gamma_{sp} = 0, \quad m_W(h) \gtrsim \sigma\alpha_W T \ , \tag{3.6b}$$

where $\alpha_W \equiv g^2/4\pi = 3.4\times10^{-2}$, and σ and K are numerical factors reflecting the uncertainty in the estimates of the transition rates between vacua of different $B+L$; K has been estimated numerically to be between 0.1 and 1 [27], while σ is estimated to be between 2 and 7 [6,7,28]. (This estimate of K has been recently questioned [7], and K has even been estimated to be as large as 20 or more [4,8].) $m_W(h)$ is the W mass as a function of the Higgs field, $m_W = gh/2$. The form of (3.6) originates from the fact that for $m_W \lesssim \alpha_W T$ the weak-coupling calculation of the sphaleron transition rate breaks down and thermal fluctuations between vacua of different $B+L$ become dominated by field configurations other than the sphaleron (the temperature being greater than or of the order of the energy of the sphaleron barrier $E_{sp} \approx 3m_W/\alpha_W$). As $m_W(h)$ increases the sphaleron barrier becomes important and the transition rate is exponentially suppressed and contributes negligibly to the baryon asymmetry for h greater than a critical value $h_c \approx 2\sigma\alpha_W T/g$, with the uncertainty in the value of h_c, due to the lack of an analytic weak-coupling expression for the rate at small h [4], being parametrized by σ. We then integrate (3.5) over the time when sphaleron processes are rapid, which gives

$$n_B = -\tfrac{5}{2}K\alpha_W^4 T^3|\beta|^2\delta\theta_c \ , \tag{3.7}$$

where $\delta\theta_c$ is the change in θ as h goes from 0 to h_c. With the entropy density given by $s = 2\pi^2 g(T)/45T^3$ [where $g(T)$ is the number of relativistic degrees of freedom in thermal equilibrium [23]], we obtain for the baryon number to entropy ratio

$$\frac{n_B}{s} = -\frac{225}{4\pi^2 g(T)}K\alpha_W^4|\beta|^2\delta\theta_c \ . \tag{3.8}$$

For example, with $g(T) \approx 100$ for the standard model at the electroweak phase transition, in order to account for the observed baryon asymmetry $(n_B/s)_{obs} = (0.4\text{–}1)\times 10^{-10}$ we require

$$K|\beta|^2|\delta\theta_c| \gtrsim 5.3\times10^{-4} \ . \tag{3.9}$$

For K in the range 0.1 to 1, this can be satisfied so long as $|\beta|$ and $|\delta\theta_c|$ are not much smaller than 0.1. We note that the largest possible change in the right-hand side of (3.4) for which $|\cos 2\theta| < 1$ is still valid corresponds to $|\delta\theta_c| = \pi/4 \approx 0.8$. Thus, in general, we require that $|\beta| \gtrsim 0.026/K^{1/2}$ if we are to account for the observed baryon asymmetry.

For the case of the gauge singlet scalar model, for $m_{S'}^2/2\lambda_\beta\rho^2$ small compared with 1, we can write $\cos 2\theta \approx -2\theta' \sin 2\theta_0$, where $\theta = \theta_0 + \theta'$, $2\theta_0 = n\pi/2$ (n odd), and θ' is small compared with 1. Thus from (3.4) we obtain

$$\delta\theta_c \approx \frac{1}{4}A_0\frac{s_n\lambda_{SH}h_c^2}{\mu_S^2} \ , \tag{3.10}$$

where $s_n = \sin n\pi/2$, and we have assumed that μ_S^2 is large compared with $(\lambda_{SH}/2)h_c^2$, so that the fractional charge in ρ due to the Higgs field of the bubble wall is small, as one would expect for the small Higgs expectation value within the wall. We have written $\delta\theta_c$ in terms of $A_0 = m_{S'^2}/2\lambda_\beta\rho_0^2$ (where $\rho_0^2 = \mu_S^2/\lambda_{SS}$), which has been assumed to be small compared to 1 in order that (3.4) is valid, so allowing θ to vary within the bubble wall. $|\beta|^2$ for the gauge singlet scalar model is given by

$$|\beta|^2 = \frac{\lambda_V^2}{2\lambda_{SS}}\frac{\mu_S^2}{m_V^2} \ . \tag{3.11}$$

Thus from (3.8) we obtain for the baryon asymmetry resulting from the passage of the wall past a point in space

$$\frac{n_B}{s} = -\frac{45}{16\pi^3 g(T)}\frac{5}{2}s_n K\sigma^2\alpha_W A_0\frac{\lambda_V^2\lambda_{SH}}{\lambda_{SS}}\frac{T^2}{m_V^2} \ . \tag{3.12}$$

In general, in the absence of an explicit CP-violating phase in the S potential, there will be as many regions in the observed Universe with a negative B asymmetry as with a positive asymmetry, resulting in a net asymmetry far below that which could account for the observed asymmetry. (Since at the electroweak phase transition there are around 10^{38} horizon volumes in the volume corresponding to the presently observable Universe, the largest possible net baryon asymmetry that could be generated would only be $\sim 10^{-19}$ times the baryon asymmetry generated in a single domain.) However, it can be shown that an extremely small explicit phase can eliminate the domains of negative baryon number [29]. For example, for S scalars of mass scale 100 GeV to 1 TeV, an explicit phase on the $m_{S'}^2$ term as small as 10^{-16} can eliminate all negative baryon number domains prior to the electroweak phase transition [29].

In order to estimate the magnitude of the baryon asymmetry which can be generated in this model, we will consider a particular example. Consider the case where $m_V = 2T$ (and where $s_n = -1$). Then with $g(T) \approx 100$ for the SM, we obtain

$$\frac{n_B}{s} = 5.4\times10^{-11}(K\sigma^2 A_0)\frac{\lambda_V^2\lambda_{SH}}{\lambda_{SS}} \ . \tag{3.13}$$

For the estimated range of possible values for K (0.1–1) and σ (2–7), $K\sigma^2$ can have values in the range 0.4–50. With the relatively conservative assumptions $K = 0.3$ and $\sigma \approx 2$, we obtain

$$\frac{n_B}{s} \approx 6.4\times10^{-11}A_0\frac{\lambda_V^2\lambda_{SH}}{\lambda_{SS}} \ . \tag{3.14}$$

Comparing with the observed baryon to entropy ratio, $(n_B/s)_{\rm obs} = (0.4–1) \times 10^{-10}$, we see that λ_V and A_0 cannot be much smaller than 1 and $\lambda_{SH}/\lambda_{SS}$ should not be too small if we are to account for the observed baryon asymmetry. The V quark mass should also not be too much heavier than T, which is typically around 100 GeV for Higgs masses less than or of the order of 100 GeV [10]. It is interesting to note from (3.10) and (3.11) that the value of $|\beta|^2$ can be decreased and the value of $|\delta\theta_c|$ can be increased by decreasing the value μ_S^2, without altering the value of n_B/s.

Of course, there are, at present, considerable uncertainties in the calculation, particularly in the estimates of K and σ; for example, if K was equal to 20 or more [4,7], then we could consider λ_V or order 0.1 rather than order 1. However, given the relatively conservative nature of our assumptions regarding the uncertain parameters of the calculation, it seems reasonable to conclude that the observed baryon asymmetry probably can be accounted for by adding to the standard model vector quark pairs of mass not much heavier than a few hundred GeV.

The calculation we have given is for the case of the "old" spontaneous baryogenesis scenario, which assumes that the baryon number inside the wall is initially zero and increases towards the equilibrium density only via sphaleron processes [5,6]. It has been subsequently pointed out that the B asymmetry inside the bubble wall could also move towards the local equilibrium density by diffusion of quarks into the wall region, thus requiring a new analysis in order to obtain the B asymmetry generated [8,9]. For the case of a fermionic hypercharge charge potential, the inclusion of diffusion effects was found to result in a large enhancement of the B asymmetry [8]. However, the fermionic hypercharge case is a special case, in that one has suppression factors associated with strong sphalerons [22] and with the fact that, in the case where it is the phase of an isodoublet Higgs field which is responsible for the CP violation in the wall region, the B asymmetry depends on the magnitude of the hypercharge breaking Higgs field within the wall (Dine-Thomas suppression [7]). These suppression factors are reduced once diffusion is taken into account, resulting in a large enhancement of the B asymmetry in the fermionic hypercharge case [8]. In the case of the charge potential (2.8), there are no such suppression effects in the "old" spontaneous baryogenesis scenario. By a straightforward chemical potential analysis similar to that in the last section, one can show that the effect of strong sphalerons (which bring the b_R quarks into thermal equilibrium within the bubble wall region via chirality changing transitions of the form $u_L d_L \leftrightarrow u_R d_R$ [22]) is to reduce the B asymmetry by a factor of only $\frac{3}{5}$, while the CP-violating phase in this case is from the S expectation value, and so no Dine-Thomas suppression occurs. In this case, the only change in the B asymmetry will be due directly to diffusion and not due to the elimination of suppression factors. We have recently estimated the direct effect of including diffusion, and find that typically the order of magnitude of the resulting B asymmetry is the same as in the "old" spontaneous baryogenesis scenario, with little enhancement due to diffusion [30]. In general, however, the "old" spontaneous baryogenesis calculation will at least give a useful lower bound on the baryon asymmetry generated.

IV. CONCLUSIONS

We have considered the possibility that the additional CP violation required in order to account for the baryon asymmetry of the Universe originates in the mixing of standard model quarks with heavy vector quarks. In order that the CP-violating phase in the mass matrix of the standard model quarks and heavy vector quarks results in a charge potential term leading to electroweak baryogenesis, it is necessary that the phase be space dependent within the bubble wall. This suggests that the CP violation driving electroweak baryogenesis is spontaneous in nature, as is true in the case of the simplest models of light-heavy quark mixing, where the mixing is due to the expectation value of a complex scalar. We have considered the B asymmetry generated in the "old" spontaneous baryogenesis scenario (without diffusion effects). For the case of a model based on having a complex gauge singlet scalar as the source of spontaneous CP violation, we find that it is possible to generate a sufficiently large B asymmetry in order to be able to account for the observed B asymmetry, even with quite conservative assumptions regarding the uncertain parameters in the calculation. In order to do so, our calculation suggests that the couplings of the third-generation standard model quarks and vector quarks to the S scalar responsible for light-heavy quark mixing must be of order 1, and that the mass of the heavy quarks should not be larger than a few hundred GeV [say (2–3)T, where typically $T \approx 100$ GeV at the electroweak phase transition]. In general, the mixing angle of the SM quarks and vector quarks should not be much smaller than order 10^{-1}. Since there is no significant strong sphaleron or other such suppression of the B asymmetry in this case (unlike the case of a fermionic hypercharge potential), including the effect of diffusion will probably not alter the B asymmetry much from the old spontaneous baryogenesis value, which in any case gives at least a useful lower bound on the baryon asymmetry. A more complete analysis including the effects of diffusion is beyond the scope of the present paper; we hope to return to this question in the future.

We have only addressed the CP problem of electroweak baryogenesis in this paper. In order to have a complete and phenomenologically consistent model, we also need to consider the Higgs mass problem, as well as the $T = 0$ phenomenology of the model, in particular light-heavy quark mixing. Depending on the details of the electroweak phase transition and the evolution of $\langle S \rangle$ with T, it is possible the Higgs mass problem could be solved by the modification of the phase transition in the presence of a nonzero $\langle S \rangle$. (A similar possibility in the context of singlet Majoron models has previously been analyzed [31].) It is also possible that one could have $\langle S \rangle \to 0$ as $T \to 0$, which would eliminate or suppress any phenomenological effects due to light-heavy quark mixing. All of these issues require a detailed analysis of the finite-temperature effective potential and electroweak phase transition in the model, as well as a study of its low-energy phenomenology.

Although we have focused on the case where the heavy vector quarks transform as weak isodoublets, we expect that essentially the same analysis and results will hold for the case of isosinglet vector quarks, transforming as either u_R or d_R with respect to the standard model gauge group.

In conclusion, the possibility that the additional CP violation needed for electroweak baryogenesis could be accounted for by the mixing of standard model quarks with heavy vector quark pairs provides us with an interesting motivation for "beyond-the-standard-model" physics, suggesting spontaneous CP violation and new scalars, and fermions not much heavier than a few hundred GeV. Such models should be testable at the Large Hadron Collider (LHC). Further study of the details of the electroweak phase transition and of the low-energy phenomenology of the model is necessary in order to establish whether such a scheme could provide a complete and phenomenologically consistent basis for explaining the origin of the baryon asymmetry of the Universe.

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APPENDIX: LOCAL EQUILIBRIUM WITHIN THE BUBBLE WALL

In this Appendix we estimate the conditions under which the interactions and mass terms involving light and heavy quarks can be considered to be locally in equilibrium within the bubble wall.

The width of the electroweak bubble wall d_w has been estimated by various groups, with values ranging from $10/T$ to $40/T$ [8,12]. We will make the assumption of a thick wall of width $40/T$ in our discussion, which is the most appropriate for the spontaneous baryogenesis scenario. The mean free path of the quarks, l_q, due to elastic scattering via gluon exchange, has been estimated to be $3/T$ to $15/T$ [8,9,12]. We will use the value $5/T$ in our discussion. The rate of chirality changing processes, due to scattering with Higgs scalars or Higgs exchange, is estimated to be [9]

$$\Gamma_q \approx 0.2\alpha_S \lambda_q^2 T \tag{A1}$$

where $\alpha_S = g_S^2/4\pi$ is the QCD fine-structure constant and λ_q is the quark Yukawa coupling. We first estimate the condition for the $t_L \leftrightarrow t_R + h$ Yukawa interaction to be in equilibrium within the bubble wall. Due to elastic scattering within the bubble wall, the quarks will perform a random walk. The average time for the quarks to cross a distance d_w via a random walk is

$$t_w = \frac{d_w^2}{2D_q} \tag{A2}$$

where D_q is the diffusion constant for the quarks, given by $D_q = l_q/3$ for relativistic quarks. Thus the average total path length of the quarks within the bubble wall will be

$$l_w \approx \frac{3}{2}\frac{d_w^2}{l_q} \; . \tag{A3}$$

With our assumed values for d_w and l_q, this gives $l_w \approx 480/T$. If the mean free path for the Yukawa interaction is smaller than this, then we can consider the Yukawa interaction to be in equilibrium within the wall. This will be true if $\Gamma_q^{-1} < 480/T$, which implies that

$$\lambda_q > \left(\frac{1}{480(0.2\alpha_S)}\right)^{1/2} \; . \tag{A4}$$

With $\alpha_S \approx 0.1$, this gives $\lambda_q > 0.32$. With a top quark mass of 170 GeV and a single Higgs doublet, we have $\lambda_t \approx 1.0$, so this is well satisfied for the t quark. If we consider the case of a heavy V quark, with a Yukawa coupling from mixing with the t quark given by $\beta\lambda_t$, then this will be out of equilibrium within the bubble wall if $|\beta| \lesssim 0.3$. For the case of a massive V quark with a Dirac mass term, the $V_L \leftrightarrow V_R$ transitions are primarily due to the one-loop color magnetic moment of the quark. We can roughly compare the rate of these chirality changing scatterings with the rate of the usual gluon exchange quark scatterings, which determine the quark mean free path l_q, by using the ratio of the gluon exchange cross section for $e^+e^- \to \overline{q_L}q_L$ and $e^+e^- \to \overline{q_R}q_L g$ (where the additional gluon is required by angular momentum conservation). This has been given, for example, in Ref. [32],

$$\frac{\sigma_{q\bar{q}g}}{\sigma_{q\bar{q}}} \approx \frac{\alpha_s}{\pi}\frac{\kappa^2 s}{18m_q^2} \; , \tag{A5}$$

where $\kappa \approx \alpha_s/\pi$ is the anomalous color magnetic moment of the quark, and s is the center-of-mass energy squared. All other scattering and annihilation processes will involve roughly the same ratio of quark-gluon vertex to the color magnetic moment vertex, up to factors of order 1. In our case we expect $s \approx (3T + 3T)^2$, where we have used the average thermal energy of the fermions in the plasma, $\approx 3T$. Thus we expect that the ratio of the mean free path for $V_L \leftrightarrow V_R$ transitions to the mean free path of the V quarks will be roughly given by $\sigma_{q\bar{q}}/\sigma_{q\bar{q}g} \approx (\pi/\alpha_s)^3 m_V^2/36T^2 \approx 860 m_V^2/T^2$, where we have used $\alpha_s \approx 0.1$. Multiplying by the quark mean free path $5/T$, and noting that we are interested in the case where $m_V \gtrsim T$, we see that the mean free path for $V_L \leftrightarrow V_R$ transitions will be greater than about $4000/T$, and so will be out of equilibrium within the bubble wall.

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