Renormalization group evolution of R-parity-violating Yukawa couplings

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We study the evolution of R-parity-violating (RPV) couplings in the minimum supersymmetric standard model, between the electroweak and grand unification scales, assuming a family hierarchy for these coupling strengths. Particular attention is given to solutions where both the R-conserving and R-violating top-quark Yukawa couplings simultaneously approach infrared fixed points; these we analyze both algebraically and with numerical solutions of the evolution equations at the one-loop level. We identify constraints on these couplings at the GUT scale, arising from lower limits on the top-quark mass. We show that fixed points offer a new source of bounds on RPV couplings at the electroweak scale. We derive evolution equations for the CKM matrix, and show that RPV couplings affect the scaling of the unitarity triangle. The fixed-point behavior is compatible with all present experimental constraints. However, fixed-point values of RPV top-quark couplings would require the corresponding sleptons or squarks to have a mass $\gtrsim m_t$ to suppress strong new top-quark decays to sparticles. [S0556-2821(96)00511-5]

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I. INTRODUCTION

Supersymmetry is a very attractive extension of the standard model (SM), with low-energy implications that are being actively pursued, both theoretically and experimentally [1,2]. In the minimal supersymmetric extension of the standard model (MSSM), with minimum new particle content, a discrete symmetry (R parity) is assumed to forbid rapid proton decay. In terms of baryon number B, lepton number L, and spin S, the R parity of a particle is $R\equiv(-1)^{3B+L+2S}$, with value $R=+1$ for particles and $R=-1$ for sparticles. An important consequence of R conservation is that the lightest sparticle is stable and is thus a candidate for cold dark matter. However, since R conservation is not theoretically motivated by any known principle, the possibility of R nonconservation deserves equally serious consideration. In addition to the Yukawa superpotential in the MSSM,

$$\mathscr{W}=(\mathbf{U})_{ab}H_2Q_L^a\overline{U_R^b}+(\mathbf{D})_{ab}H_1Q_L^a\overline{D_R^b}+(\mathbf{E})_{ab}H_1L_L^a\overline{E_R^b}, \quad (1)$$

there are two classes of R-violating couplings in the MSSM superpotential, allowed by supersymmetry and renormalizability [3]. The superpotential terms for the first class violate lepton number L,

$$\mathscr{W}=\frac{1}{2}\lambda_{abc}L_L^aL_L^b\overline{E_R^c}+\lambda'_{abc}L_L^aQ_L^b\overline{D_R^c}+\mu_iH_2L_i, \quad (2)$$

while those of the second class violate baryon number B,

$$\mathscr{W}=\frac{1}{2}\lambda''_{abc}\overline{D_R^a}\,\overline{D_R^b}\,\overline{U_R^c}. \quad (3)$$

Here, $L,Q,\overline{E},\overline{D},\overline{U}$ stand for the doublet lepton, doublet quark, singlet antilepton, singlet d-type antiquark, singlet u-type antiquark superfields, respectively, and a,b,c are generation indices. The $(\mathbf{U})_{ab}$, $(\mathbf{D})_{ab}$, and $(\mathbf{E})_{ab}$ in Eq. (1) are the Yukawa coupling matrices. In our notation, the superfields above are the weak interaction eigenstates, which might be expected as the natural choice at the grand unified scale, rather than the mass eigenstates. The term $\mu_iL_iH_2$ in the superpotential can be rotated away into the R-parity conserving term μH_1H_2 via an SU(4) rotation between the superfields H_1 and L_i. However, this operation must be performed at some energy scale, and the mixing is regenerated at other scales through the renormalization group equations. The Yukawa couplings λ_{abc} and λ''_{abc} are antisymmetric in their first two indices because of superfield antisymmetry. These superpotential terms lead to the interaction Lagrangians

$$\mathscr{L}=\frac{1}{2}\lambda_{abc}\{\tilde{\nu}_{aL}\overline{e_{cR}}e_{bL}+\tilde{e}_{bL}\overline{e_{cR}}\nu_{aL}+(\tilde{e}_{cR})^*(\overline{\nu_{aL}})^ce_{bL}-(a\leftrightarrow b)\}+\text{H.c.} \quad (4)$$

for the λ terms, whereas the λ' terms yield

$$\mathscr{L}=\lambda'_{abc}\{\tilde{\nu}_{aL}\overline{d_{cR}}d_{bL}+\tilde{d}_{bL}\overline{d_{cR}}\nu_{aL}+(\tilde{d}_{cR})^*(\overline{\nu_{aL}})^cd_{bL}-\tilde{e}_{aL}\overline{d_{cR}}u_{bL}-\tilde{u}_{bL}\overline{d_{cR}}e_{aL}-(\tilde{d}_{cR})^*(\overline{e_{aL}})^cu_{bL}\}+\text{H.c.}, \quad (5)$$

with corresponding terms for each of these generations. In the case of a B-violating superpotential, the Lagrangian reads

$$\mathscr{L}=\frac{1}{2}\lambda''_{abc}\{u_c^cd_a^c\tilde{d}_b^*+u_c^c\tilde{d}_a^*d_b^c+\tilde{u}_c^*d_a^cd_b^c\}+\text{H.c.} \quad (6)$$

To escape the proton-lifetime constraints, it is sufficient that only one of these classes be absent or very highly suppressed. Phenomenological studies of the consequences of

TABLE I. $16\pi^2\gamma_{\phi_j}^{\phi_i}$ in the MSSM plus additional terms for lepton or baryon number-violating couplings, where i and j are flavor indices.

$\phi_{i,j}$	MSSM	Lepton no. violation	Baryon no. violation
L_i, H_1	—	$\lambda^{iab}\mathbf{E}_{ab}+3\lambda'^{iab}\mathbf{D}_{ab}$	—
$L_{i,j}$	$\mathbf{E}\mathbf{E}^\dagger-\frac{3}{2}g_2^2-\frac{3}{10}g_1^2$	$\lambda_{iab}\lambda^{jab}+3\lambda'_{iab}\lambda'^{jab}$	—
$E_{i,j}$	$2\mathbf{E}^\dagger\mathbf{E}-\frac{6}{5}g_1^2$	$\lambda^{abi}\lambda_{abj}$	—
$D_{i,j}$	$2\mathbf{D}^\dagger\mathbf{D}-\frac{8}{3}g_3^2-\frac{2}{15}g_1^2$	$2\lambda'^{abi}\lambda'_{abj}$	$2\lambda''^{iab}\lambda''_{jab}$
$U_{i,j}$	$2\mathbf{U}^\dagger\mathbf{U}-\frac{8}{3}g_3^2-\frac{8}{15}g_1^2$	—	$\lambda''^{abi}\lambda''_{abj}$
$Q_{i,j}$	$\mathbf{U}\mathbf{U}^\dagger+\mathbf{D}\mathbf{D}^\dagger-\frac{8}{3}g_3^2-\frac{3}{2}g_2^2-\frac{1}{30}g_1^2$	$\lambda'_{aib}\lambda'^{ajb}$	—
H_1	$\mathrm{Tr}(\mathbf{E}\mathbf{E}^\dagger)+3\mathrm{Tr}(\mathbf{D}\mathbf{D}^\dagger)-\frac{3}{2}g_2^2-\frac{3}{10}g_1^2$	—	—
H_2	$3\mathrm{Tr}(\mathbf{U}\mathbf{U}^\dagger)-\frac{3}{2}g_2^2-\frac{3}{10}g_1^2$	—	—

R-parity violation (RPV) have placed constraints on the various couplings $\lambda_{abc},\lambda'_{abc},\lambda''_{abc}$ [4–8], but considerable latitude remains for RPV.

Studies of the renormalization group evolution equations (RGE's), relating couplings at the electroweak scale to their values at the grand unified theory (GUT) scale, have led to new insights and constraints on the observable low-energy parameters in the R-conserving scenario. It therefore seems worthwhile to see what can be learned from similar studies of RPV scenarios. An initial study of this type addressed the evolution of λ''_{133} and λ''_{233} couplings [8]. This was subsequently extended to all the baryon-violating couplings λ''_{ijk} [9]. In the present work, we undertake a somewhat more general study of the RGE's for RPV interactions, paying particular attention to solutions for which both the R-conserving and R-violating top-quark Yukawa couplings simultaneously approach infrared fixed points. Such fixed-point behavior requires a coupling λ,λ', or λ'' to be of order unity at the electroweak scale. After our study was completed, a related work on RGE's for RPV couplings appeared [10], which, however, has a different focus and is largely complementary to the present paper.

In the context of grand unified theories, one is led to consider the possible unification of RPV parameters. If, for example, the RPV interactions arose from an SU(5)-invariant term, then in fact the L-violating RPV couplings would be related to the B-violating ones [11] at the GUT scale. We could then no longer set one or the other arbitrarily to zero and the proton lifetime (which places very strong constraints on products of L-violating and B-violating RPV couplings, typically requiring products $\lambda'\lambda''$ to be smaller than 5×10^{-17} [11]) would strongly constrain all types of RPV couplings. It can be argued that some products of B-violating and L-violating couplings, containing several high-generation indices, would not contribute directly to proton decay [12]; however, proton decay would still be induced at the one-loop level by flavor mixing [11], so in fact all RPV couplings would have to be very small. In such scenarios, the fixed-point solutions for RPV couplings would be excluded; our present studies therefore implicitly assume that this kind of RPV unification does not occur. Furthermore, since RPV unification is analogous to the popular hypothesis of $\lambda_b=\lambda_\tau$ Yukawa unification, it would appear somewhat inconsistent (though not completely unthinkable) to assume one without the other. Accordingly, in our present work, we do not try to impose the additional constraint of $\lambda_b=\lambda_\tau$ unification.

II. RENORMALIZATION GROUP EQUATIONS AND FIXED POINTS

For any trilinear term in the superpotential $d_{abc}\Phi^a\Phi^b\Phi^c$ involving superfields Φ^a,Φ^b,Φ^c, the evolution of the couplings d_{abc} with the scale μ is given by the RGE's

$$\mu\frac{\partial}{\partial\mu}d_{abc}=\gamma_a^e d_{ebc}+\gamma_b^e d_{aec}+\gamma_c^e d_{abe}, \tag{7}$$

where the γ_a^e are elements of the anomalous dimension matrix. Table I gives the anomalous dimensions for the superfields. The first column of the table gives the results for the MSSM in matrix form; here **U, D**, and **E** are the matrices of Yukawa couplings to the up quarks, down quarks, and charged leptons, respectively, and a unit matrix is understood in front of the terms involving SU(3), SU(2), and U(1) gauge couplings g_3, g_2, and g_1 and the terms with traces. The second column of Table I gives the additions to the anomalous dimension matrix due to L-violating terms λ_{abc} and λ'_{abc}, while the third column gives the corresponding additions due to B-violating λ''_{abc} terms. In our notation, an RPV coupling with upper indices is the complex conjugate of the same coupling with lower indices, e.g., $\lambda^{abc}=\lambda^*_{abc}$.

The evolution equations for the R-conserving Yukawa matrices $\mathbf{U},\mathbf{D},\mathbf{E}$ of Eq. (1) are obtained from Eq. (7) with the index c belonging to a Higgs field. The general forms of the RGE's are

$$\mu\frac{\partial}{\partial\mu}(\mathbf{U})_{ab}=(\mathbf{U})_{ib}\gamma_{Q_a}^{Q_i}+(\mathbf{U})_{ai}\gamma_{\overline{U}_b}^{\overline{U}_i}+(\mathbf{U})_{ab}\gamma_{H_2}^{H_2}, \tag{8}$$

$$\mu\frac{\partial}{\partial\mu}(\mathbf{D})_{ab}=(\mathbf{D})_{ib}\gamma_{Q_a}^{Q_i}+(\mathbf{D})_{ai}\gamma_{\overline{D}_b}^{\overline{D}_i}+(\mathbf{D})_{ab}\gamma_{H_1}^{H_1}+\lambda'_{iab}\gamma_{H_1}^{L_i}, \tag{9}$$

$$\mu\frac{\partial}{\partial\mu}(\mathbf{E})_{ab}=(\mathbf{E})_{ib}\gamma_{L_a}^{L_i}+(\mathbf{E})_{ai}\gamma_{\overline{E}_b}^{\overline{E}_i}+(\mathbf{E})_{ab}\gamma_{H_1}^{H_1}+\lambda_{iab}\gamma_{H_1}^{L_i}. \tag{10}$$

When we solve Eqs. (8)–(10) for the general R-parity-violating case, we get additional contributions from

Hermitian matrices involving the RPV couplings that are analogous to combinations like $\mathbf{D}^\dagger\mathbf{D}$ for the usual Yukawa matrices. For example, the matrix equation for the Yukawa matrices $\mathbf{U}$ and $\mathbf{D}$ become

$$\frac{d\mathbf{U}}{dt}=\frac{1}{16\pi^2}\left\{\left[-\frac{16}{3}\alpha_3-3\alpha_2-\frac{13}{15}\alpha_1+3\mathbf{U}\mathbf{U}^\dagger+\mathbf{D}\mathbf{D}^\dagger+\mathbf{Tr}[3\mathbf{U}\mathbf{U}^\dagger]+\mathbf{M}'^{(Q)}\right]\mathbf{U}+\mathbf{U}\mathbf{M}''^{(U)}\right\}, \tag{11}$$

$$\frac{d\mathbf{D}}{dt}=\frac{1}{16\pi^2}\left\{\left[-\frac{16}{3}\alpha_3-3\alpha_2-\frac{7}{15}\alpha_1+3\mathbf{D}\mathbf{D}^\dagger+\mathbf{U}\mathbf{U}^\dagger+\mathbf{Tr}[3\mathbf{D}\mathbf{D}^\dagger+\mathbf{E}\mathbf{E}^\dagger]+\mathbf{M}'^{(Q)}\right]\mathbf{D}+2\mathbf{D}\mathbf{M}''^{(D)}+2\mathbf{D}\mathbf{M}'^{(D)}\right\}+\lambda'_{iab}(\lambda^{icd}\mathbf{E}_{cd}+3\lambda'^{icd}\mathbf{D}_{cd}), \tag{12}$$

where $\mathbf{M}'^{(Q)}_{ij}\equiv\lambda'_{aib}\lambda'^{ajb}$, $\mathbf{M}'^{(D)}_{ij}\equiv\lambda'^{abi}\lambda'_{abj}$, $\mathbf{M}''^{(U)}_{ij}\equiv\lambda''^{abi}\lambda''_{abj}$, and $\mathbf{M}''^{(D)}_{ij}\equiv\lambda''^{iab}\lambda''_{jab}$ are the combinations of RPV couplings appearing in Table I. The variable is

$$t=\ln(\mu/M_G), \tag{13}$$

where μ is the running mass scale and M_G is the GUT unification mass.

The gauge couplings are not affected by the presence of R-violating couplings at the one-loop level.

The third generation Yukawa couplings are dominant, so if we retain in the anomalous dimensions only the (3,3) elements $\lambda_t,\lambda_b,\lambda_\tau$ in $\mathbf{U},\mathbf{D},\mathbf{E}$, setting all other elements to zero, Eqs. (8)–(10) read

$$\mu\frac{\partial}{\partial\mu}\lambda_t=\lambda_t[\gamma^{Q_3}_{Q_3}+\gamma^{\overline{U}_3}_{U_3}+\gamma^{H_2}_{H_2}], \tag{14}$$

$$\mu\frac{\partial}{\partial\mu}\lambda_b=\lambda_b[\gamma^{Q_3}_{Q_3}+\gamma^{\overline{D}_3}_{D_3}+\gamma^{H_1}_{H_1}]+\lambda'_{i33}\gamma^{L_i}_{H_1}, \tag{15}$$

$$\mu\frac{\partial}{\partial\mu}\lambda_\tau=\lambda_\tau[\gamma^{L_3}_{L_3}+\gamma^{\overline{E}_3}_{E_3}+\gamma^{H_1}_{H_1}]+\lambda_{i33}\gamma^{L_i}_{H_1}. \tag{16}$$

Since there are 36 independent RPV couplings $\lambda_{abc},\lambda'_{abc}$ in the L-violating sector (9 independent couplings λ''_{abc} in the B-violating sector) to be added to the three dominant R-conserving Higgs couplings $\lambda_t,\lambda_b,\lambda_\tau$, we would have to consider 39 (12) coupled nonlinear evolution equations, in general. Some further radical simplifications in the RPV sector are clearly needed to make the system of equations tractable.

It is plausible that there may exist a generational hierarchy among the RPV couplings, analogous to that of the conventional Higgs couplings; indeed, the RPV couplings to higher generations evolve more strongly because of larger Higgs couplings in their RGE's, and hence have the potential to take larger values than RPV couplings to lower generations. Thus, we consider retaining only the couplings λ_{233} and λ'_{333}, or λ''_{233}, neglecting all others. This restriction is also motivated by the fact that the experimental upper limits are stronger for the couplings with lower indices.

To simplify the form of the RGE's, we adopt the notation

$$Y_i=\frac{1}{4\pi}\lambda_i^2(i=t,b,\tau),\quad Y''=\frac{1}{4\pi}\lambda''^2_{233},$$

$$Y'=\frac{1}{4\pi}\lambda'^2_{333},\quad Y=\frac{1}{4\pi}\lambda^2_{233}.$$

The one-loop RGE's then take the following forms, where $\alpha_i=1/4\pi g_i^2$:

$$\frac{d\alpha_i}{dt}=\frac{1}{2\pi}b_i\alpha_i^2,\quad b_i=\{33/5,1,-3\}, \tag{17}$$

$$\frac{dY_t}{dt}=\frac{1}{2\pi}Y_t\left(6Y_t+Y_b+Y'+2Y''-\frac{16}{3}\alpha_3-3\alpha_2-\frac{13}{15}\alpha_1\right), \tag{18}$$

$$\frac{dY_b}{dt}=\frac{1}{2\pi}Y_b\left(Y_t+6Y_b+Y_\tau+6Y'+2Y''-\frac{16}{3}\alpha_3-3\alpha_2-\frac{7}{15}\alpha_1\right), \tag{19}$$

$$\frac{dY_\tau}{dt}=\frac{1}{2\pi}Y_\tau\left(3Y_b+4Y_\tau+4Y+3Y'-3\alpha_2-\frac{9}{5}\alpha_1\right), \tag{20}$$

$$\frac{dY}{dt}=\frac{1}{2\pi}Y\left(4Y_\tau+4Y+3Y'-3\alpha_2-\frac{9}{5}\alpha_1\right), \tag{21}$$

$$\frac{dY'}{dt}=\frac{1}{2\pi}Y'\left(Y_t+6Y_b+Y_\tau+Y+6Y'-\frac{16}{3}\alpha_3-3\alpha_2-\frac{7}{15}\alpha_1\right), \tag{22}$$

$$\frac{dY''}{dt}=\frac{1}{2\pi}Y''\left(2Y_t+2Y_b+6Y''-8\alpha_3-\frac{4}{5}\alpha_1\right). \tag{23}$$

Here, it is understood that one takes *either* $Y=Y'=0$ *or* $Y''=0$.

An extremely interesting possibility in the RGE's is that Y_t is large at the GUT scale and consequently, is driven toward a fixed point at the electroweak scale [13,14]. In particular, in the MSSM $\lambda_t\to 1.1$ as $\mu\to m_t$; since $\lambda_t(m_t)=\sqrt{2}m_t(m_t)/(v\sin\beta)$, this leads to the relation, for low $\tan\beta$ [14]

$$m_t(\text{pole})=(200\ \text{GeV})\sin\beta, \tag{24}$$

where $\tan\beta=v_2/v_1$ is the ratio of the Higgs vacuum expectation values (VEV's) and m_t(pole) is the mass at the t-propagator pole. It is interesting to examine the impact of RPV couplings on this fixed-point result [8].

A. λ_t fixed point in the MSSM

We first review the λ_t fixed-point behavior in the MSSM limit, where RPV couplings are neglected. Setting $dY_t/dt\simeq 0$ at $\mu\simeq m_t$ gives the fixed-point condition

$$6Y_t+Y_b=\frac{16}{3}\ \alpha_3+3\alpha_2+\frac{13}{15}\ \alpha_1. \quad (25)$$

The λ_t and λ_b couplings at $\mu=m_t$ are related to the running masses

$$\lambda_t(m_t)=\frac{\sqrt{2}m_t(m_t)}{v\sin\beta}, \quad \lambda_b(m_t)=\frac{\sqrt{2}m_b(m_b)}{\eta_b v\cos\beta}, \quad (26)$$

with $v=(\sqrt{2}G_F)^{-1/2}=246$ GeV. Here, η_b gives the QCD or QED running of $m_b(\mu)$ between $\mu=m_b$ and $\mu=m_t$; $\eta_b\simeq 1.5$ for $\alpha_s(m_t)\simeq 0.10$ [14]. Thus, we can express $\lambda_b(m_t)$ in terms of $\lambda_t(m_t)$, $\tan\beta$, and the known running masses:

$$\lambda_b(m_t)=\frac{m_b(m_b)}{m_t(m_t)}\frac{\tan\beta}{\eta_b}\lambda_t(m_t)\simeq 0.017\ \tan\beta\lambda_t(m_t), \quad (27)$$

taking $m_b(m_b)=4.25$ GeV, $m_t(m_t)=167$ GeV, and hence

$$Y_b(m_t)\simeq 3\times 10^{-4}\tan^2\beta Y_t(m_t). \quad (28)$$

For small or moderate values of $\tan\beta\lesssim 20$, we obtain $Y_b/(6Y_t)<0.02$ so we can safely neglect the Y_b contribution. In this case, taking the approximate values

$$\alpha_3=1/10, \quad \alpha_2=1/30, \quad \alpha_1=1/58 \quad \text{at} \quad \mu=m_t, \quad (29)$$

we find the numerical value

$$Y_t(m_t)=0.108, \quad \lambda_t(m_t)=1.16. \quad (30)$$

For large $\tan\beta\sim m_t/m_b$, we can express the λ_t fixed-point relation as

$$\begin{aligned}Y_t(m_t)&=\frac{\lambda_t^2(m_t)}{4\pi}\\ &=\left(\frac{8}{9}\alpha_3+\frac{1}{2}\alpha_2+\frac{13}{90}\alpha_1\right)\Big/(1+5\times 10^{-5}\tan^2\beta).\end{aligned} \quad (31)$$

B. λ'', λ_t simultaneous fixed points

Next, we consider the B-violating scenario with $Y=Y'=0$ and Y'' nonzero, investigating the possibility that fixed-point limits are approached for both Y_t and Y'' couplings, as found numerically in Ref. [8] (note that these authors use a different definition of λ''_{abc}). This requires $dY_t/dt\simeq 0$ and $dY''/dt\simeq 0$ at $\mu\simeq m_t$, giving the conditions

$$6Y_t+Y_b+2Y''-\frac{16}{3}\ \alpha_3-3\alpha_2-\frac{13}{15}\alpha_1\simeq 0, \quad (32)$$

$$2Y_t+2Y_b+6Y''-8\alpha_3-\frac{4}{5}\ \alpha_1\simeq 0. \quad (33)$$

Taking linear combinations to solve for Y_t and Y'', we obtain (with $Y_b\ll Y_t$)

$$Y_t\simeq\frac{1}{16}\left(8\alpha_3+9\alpha_2+\frac{9}{5}\ \alpha_1\right)\simeq 0.071, \quad \lambda_t\simeq 0.94, \quad (34)$$

$$Y''\simeq\frac{1}{16}\left(\frac{56}{3}\ \alpha_3-3\alpha_2+\frac{23}{15}\ \alpha_1\right)\simeq 0.112, \quad \lambda''_{233}\simeq 1.18, \quad (35)$$

showing a considerable downward displacement in λ_t due to λ''_{233}. Such a large value of λ''_{233} would imply substantial $t\to b\tilde{s}, s\tilde{b}$ decay, if kinematically allowed.

If both λ_t and λ''_{233} fixed points are realized as above, then the predicted physical top-quark mass is

$$m_t(\text{pole})\simeq(150\ \ \text{GeV})\sin\beta. \quad (36)$$

Even for moderate values of $\tan\beta$ ($\tan\beta>5$) one has $\sin\beta\simeq 1$ ($\sin\beta>0.98$). This prediction is at the lower end of the present data [15,16]:

$$\begin{aligned}&m_t=176\pm 8\pm 10\ \ \text{GeV}\\ &\qquad[\text{Collider Detector at Fermilab (CDF)}], \quad (37)\\ &m_t=199^{+10}_{-21}\pm 22\ \ \text{GeV} \quad (\text{D0}).\end{aligned}$$

When the data become more precise, the fixed-point possibility for λ''_{233} could be excluded, if the measured central value of m_t is unchanged.

One can also consider the case of large $\tan\beta$ where the coupling Y_b is non-negligible, and, in fact, may be near its own fixed point. In that case, we add another equation, $dY_b/dt\simeq 0$, to those above. This gives

$$Y_t+6Y_b+Y_\tau+2Y''-\frac{16}{3}\ \alpha_3-3\alpha_2-\frac{7}{15}\ \alpha_1\simeq 0. \quad (38)$$

A new coupling Y_τ enters here, but it can be related to Y_b since

$$\lambda_\tau(m_t)=\frac{\sqrt{2}m_\tau(m_t)}{\eta_\tau v\cos\beta}, \quad (39)$$

and hence

$$\begin{aligned}\lambda_\tau(m_t)&=\frac{m_\tau(m_\tau)}{m_b(m_b)}\frac{\eta_b}{\eta_\tau}\lambda_b(m_t)=0.6\lambda_b(m_t),\\ Y_\tau(m_t)&=0.4Y_b(m_t),\end{aligned} \quad (40)$$

by arguments similar to those above relating $\lambda_b(m_t)$ to $\lambda_t(m_t)$. Then we have three simultaneous equations in three unknowns, that give the solutions

$$Y_t\simeq 0.067, \quad \lambda_t\simeq 0.92, \quad (41)$$

$$Y_b\simeq 0.061, \quad \lambda_b\simeq 0.88, \quad (42)$$

$$Y''\simeq 0.092, \quad \lambda''_{233}\simeq 1.08. \quad (43)$$

C. λ, λ', λ_t simultaneous fixed points

If, instead, fixed points should occur simultaneously for Y_t and Y' (with $Y''=0$), the conditions at $\mu\simeq m_t$, found from $dY_t/dt\simeq 0$ and $dY'/dt\simeq 0$, are

$$Y_t = \frac{1}{35}\left[\frac{80}{3}\alpha_3 + 15\alpha_2 + \frac{71}{15}\alpha_1 + Y_\tau + Y\right], \qquad (44)$$

$$Y' = \frac{1}{35}\left[\frac{80}{3}\alpha_3 + 15\alpha_2 + \frac{29}{15}\alpha_1 - 35Y_b - 6Y_\tau - 6Y\right]. \qquad (45)$$

If Y is small and we also neglect Y_b and Y_τ (e.g., assuming small $\tan\beta$), then Y_t and Y' approach almost the same fixed-point value

$$\lambda_t(m_t) \simeq \lambda'_{333} \simeq 1.07. \qquad (46)$$

In this case, $\lambda_t(m_t)$ is only slightly displaced below the MSSM value, while λ'_{333} has quite a large value. The latter would imply substantial $t \to b\tilde{\bar{\tau}}, \widetilde{\tau b}$ decays, if kinematically allowed; the $t \to b\tilde{\bar{\tau}}$ mode is more likely, since $\tilde{\bar{\tau}}$ is usually expected to be lighter than $\tilde{b}$, and we discuss its implications later.

If Y' is negligible, Y_t and Y can approach fixed points simultaneously; in this case, the two conditions essentially decouple, giving the MSSM result for Y_t. If Y_b and Y_τ are negligible, the solution is

$$\lambda_t(m_t) \simeq 1.16, \quad \lambda_{233} \simeq 0.64, \qquad (47)$$

but if Y_b too is large and approaches its fixed point, the three corresponding conditions give

$$\lambda_t(m_t) \simeq 1.09, \quad \lambda_b \simeq 1.04, \qquad (48)$$

and the λ_{233} fixed point is very small and never truly reached in numerical studies. It is also not possible for Y, Y', and Y_t to have simultaneous fixed points; the conditions $dY/dt = dY'/dt = dY_t/dt = 0$ cannot be satisfied with all three couplings positive.

D. CKM evolution

The presence of nonzero RPV couplings can also change the evolution of Cabibbo-Kobayashi-Maskawa (CKM) mixing angles. This has interesting implications for the prediction of fermion mixings at the electroweak scale from an ansatz for Yukawa matrices at the GUT scale. In a model such as the MSSM (or the SM) with no RPV terms, the evolution of the CKM angles at the one-loop level comes entirely from the Yukawa matrix terms in the anomalous dimension $\gamma_{Q_j}^{Q_i}$. The Yukawa matrices $\mathbf{U}$ and $\mathbf{D}$ can be diagonalized by biunitary transformations

$$\mathbf{U}^{\mathrm{diag}} = V_U^L \mathbf{U} V_U^{R\dagger}, \qquad (49)$$

$$\mathbf{D}^{\mathrm{diag}} = V_D^L \mathbf{D} V_D^{R\dagger}. \qquad (50)$$

The CKM matrix is then given by

$$V \equiv V_U^L V_D^{L\dagger}. \qquad (51)$$

In the presence of RPV, there are additional contributions to the anomalous dimensions and hence to the CKM RGE's. Consider, for example, the case in which only the λ'' couplings are nonzero, for which there are new contributions $\mathbf{M}''^{(U)}_{ij}$ and $\mathbf{M}''^{(D)}_{ij}$ to the RGE's as defined following Eq. (12). The RPV contributions to the RGE's can be diagonalized by

$$\mathbf{M}''^{(U),\mathrm{diag}} = V_{(U)}^R \mathbf{M}''^{(U)} V_{(U)}^{R\dagger} \equiv \{\lambda''^2_u, \lambda''^2_c, \lambda''^2_t\}, \qquad (52)$$

$$\mathbf{M}''^{(D),\mathrm{diag}} = V_{(D)}^R \mathbf{M}''^{(D)} V_{(D)}^{R\dagger} \equiv \{\lambda''^2_d, \lambda''^2_s, \lambda''^2_b\}, \qquad (53)$$

for which new matrices

$$V^{(U)} \equiv V_U^R V_{(U)}^{R\dagger}, \qquad (54)$$

$$V^{(D)} \equiv V_D^R V_{(D)}^{R\dagger}, \qquad (55)$$

can be defined. We find the RGE's take the form

$$\begin{aligned}\frac{dV_{i\alpha}}{dt} = \frac{1}{16\pi^2}\Bigg[&\sum_{\beta, j\neq i} \frac{\lambda_i^2 + \lambda_j^2}{\lambda_i^2 - \lambda_j^2}\lambda_\beta^2 V_{i\beta}V^*_{j\beta}V_{j\alpha} \\ &+ \sum_{j,\beta\neq\alpha} \frac{\lambda_\alpha^2 + \lambda_\beta^2}{\lambda_\alpha^2 - \lambda_\beta^2}\lambda_j^2 V^*_{j\beta}V_{j\alpha}V_{i\beta} \\ &+ \sum_{k,j\neq i} \frac{\lambda_i\lambda_j}{\lambda_i^2 - \lambda_j^2}\lambda''^2_k V^{(U)}_{ik}V^{(U)*}_{jk}V_{j\alpha} \\ &+ \sum_{\gamma,\beta\neq\alpha} \frac{2\lambda_\alpha\lambda_\beta}{\lambda_\alpha^2 - \lambda_\beta^2}\lambda''^2_\gamma V^{(D)*}_{\gamma\beta}V^{(D)}_{\gamma\alpha}V_{i\beta}\Bigg], \end{aligned} \qquad (56)$$

where $i,j,k = u,c,t$ and $\alpha,\beta,\gamma = d,s,b$. One observes that generally there is a contribution to the evolution of the CKM matrix from the RPV sector.

Assuming, as we do, that only the RPV couplings λ_{233}, λ'_{333}, or λ''_{233} are nonzero, the off-diagonal elements of the matrices defined in Eqs. (54) and (55) vanish. Then the one-loop RGE's for mixing angles and the CP-violation parameter $J = \mathrm{Im}(V_{ud}V_{cs}V^*_{us}V^*_{cd})$ have the same forms as in the MSSM, namely [17]

$$\frac{dW}{dt} = -\frac{W}{8\pi^2}(\lambda_t^2 + \lambda_b^2), \qquad (57)$$

where $W = |V_{ub}|^2$, $|V_{cb}|^2$, $|V_{td}|^2$, $|V_{ts}|^2$, or J. Nevertheless, the evolution of CKM angles differs from the MSSM because the evolution of the Yukawa couplings on the right-hand side (RHS) is altered by the RPV couplings.

III. NUMERICAL RGE STUDIES

In the previous section, we identified the quasi-infrared fixed points that can be determined through the algebraic solutions to the RGE equations. The one-loop RGE's form a set of coupled first-order differential equations that must be solved numerically.

Figure 1 shows the fixed-point behavior of each of the three RPV couplings considered in this paper $(\lambda''_{233}, \lambda'_{333}, \lambda_{233})$ along with the corresponding fixed-point behavior for λ_t, assuming that $\tan\beta$ is small and hence λ_b and λ_τ are negligible. It can be seen that for all $\lambda \gtrsim 1$ at the GUT scale, the respective Yukawa coupling approaches its fixed point at the electroweak scale. These infrared fixed points provide the theoretical upper limits for the RPV-Yukawa couplings at the electroweak scale summarized in

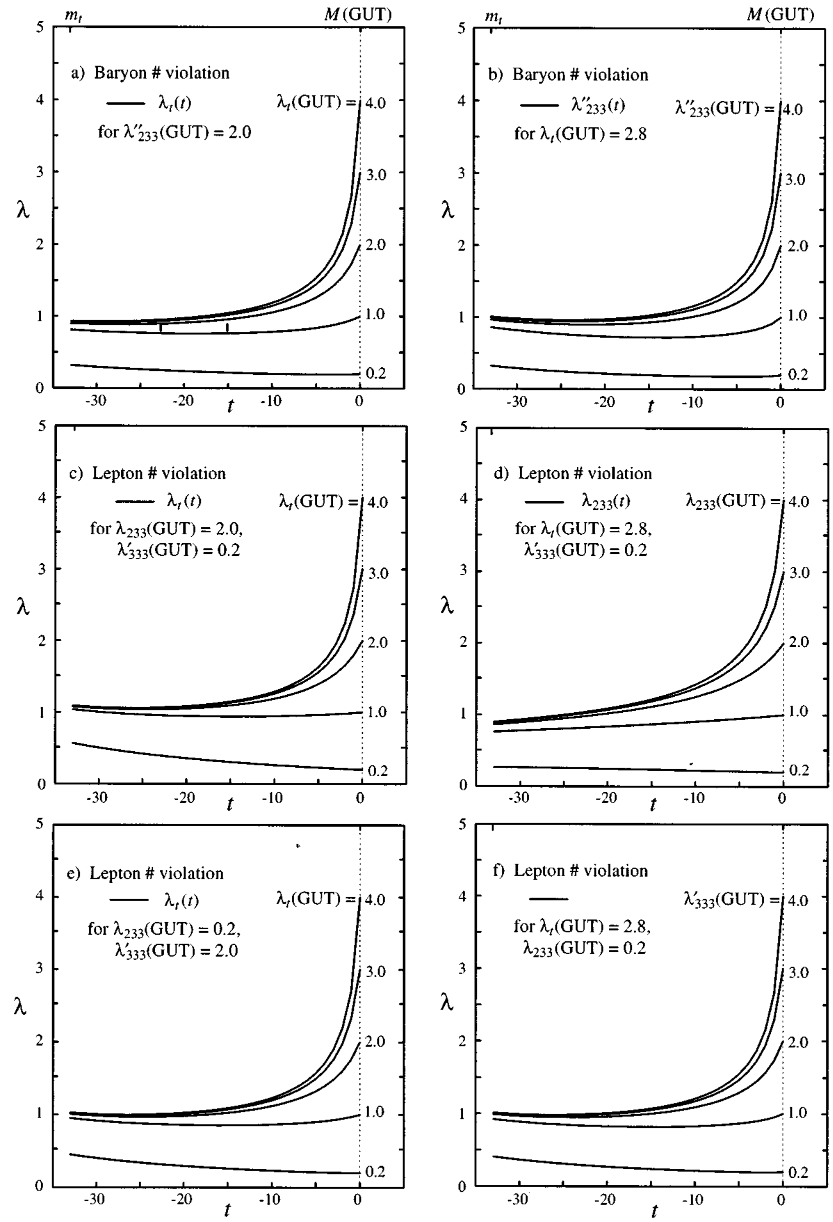

FIG. 1. Couplings λ as a function of the energy scale t for λ_t in (a) baryon-number RPV, (c) lepton-number RPV with $\lambda_{233} \gg \lambda'_{333}$ and (e) lepton-number RPV with $\lambda'_{333} \gg \lambda_{233}$ for different starting points at the GUT scale ($t=0$). Panels (b), (d), and (f) show the same for λ''_{233}, λ_{233} ($\lambda_{233} \gg \lambda'_{333}$) and λ'_{333} ($\lambda'_{333} \gg \lambda_{233}$), respectively. Here $t \simeq -33$ represents the electroweak scale, where these couplings reach their fixed points.

Table II. The numerical evolution of the fixed points approaches but does not exactly reproduce the approximate analytical values of Eqs. (34), (35), (46), and (47).

We obtain additional restrictions on the RPV couplings from the experimental lower bound on m_t (that we take to be $m_t > 150$ GeV [15,16]). These additional limits are shown in Fig. 2; the dark shaded region is excluded in all types of models only by assuming this lower bound on the top mass.

One might hope that RPV interactions could help to explain the measured value of $R_b = \Gamma(Z \rightarrow b\bar{b})/$

TABLE II. Fixed points for the different Yukawa couplings λ in different models for (i) $\tan\beta \lesssim 30$ and (ii) $\tan\beta \sim m_t/m_b$. In the case of large $\tan\beta$, λ_b also reaches a fixed point.

	Model	λ_t	λ_b	λ_{233}	λ'_{333}	λ''_{233}
(i)	MSSM	1.06	–	–	–	–
	Lepton No. violation ($\lambda \gg \lambda'$)	1.06	–	0.90	–	–
	Lepton No. violation ($\lambda' \gg \lambda$)	0.99	–	–	1.01	–
	Baryon No. violation	0.90	–	–	–	1.02
(ii)	MSSM	1.00	0.92	–	–	–
	Lepton No. violation ($\lambda' \gg \lambda$)	1.01	0.72	–	0.71	–
	Baryon No. violation	0.87	0.85	–	–	0.92

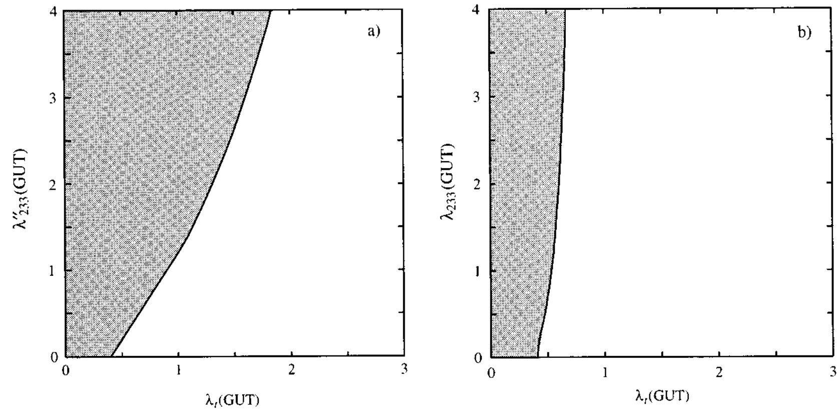

FIG. 2. Excluded regions in the (a) λ_t(GUT), λ''_{233}(GUT) plane and (b) λ_t(GUT), λ_{233}(GUT) [$\lambda_{233}(\mathrm{GUT})=\lambda'_{333}(\mathrm{GUT})$] plane obtained from $m_t>150$ GeV.

$\Gamma(Z\rightarrow \mathrm{hadrons})$, which differs from the SM prediction by over three standard deviations. However, while their contributions can have either sign, the RPV couplings must be significantly above their fixed-point values to explain the full discrepancy [5]. In the case of lepton RPV, the bounds on the leptonic partial widths are always strong enough to prevent RPV couplings from taking such large values.

Next, we address the question whether RPV couplings will significantly change the relation between electroweak scale and GUT scale values of the off-diagonal terms of the CKM matrix. When the masses and mixings of the CKM matrix satisfy a hierarchy, these relations are given by

$$W(\mathrm{GUT})=W(\mu)S(\mu),$$

where W is a CKM matrix element connecting the third generation to one of the lighter generations, and S is a scaling factor [17]. The other CKM elements do not change with scale to leading order in the hierarchy. The scaling factor $S(\mu)$ is determined by integrating Eq. (57) together with the other RGE's. In Fig. 3 we show the dependence of the scaling factor S on the GUT-scale RPV couplings λ_{233}, λ'_{333}, and λ''_{233}, respectively.

IV. RPV DECAYS OF THE TOP QUARK

The RPV couplings λ''_{233} and λ'_{333} give rise to new decay modes of the top-quark [18], if the necessary squark or slepton masses are small enough.

The L-violating coupling λ'_{333} leads to $t_R\rightarrow b_R\bar{\tilde{\tau}}_R$, $\tilde{b}_R\bar{\tau}_R$ decays, with partial widths [18]

$$\Gamma(t\rightarrow b\bar{\tilde{\tau}})=\frac{(\lambda'_{333})^2}{32\pi}m_t(1-m^2_{\tilde{\tau}}/m_t^2)^2, \tag{58}$$

$$\Gamma(t\rightarrow \tilde{b}\bar{\tau})=\frac{(\lambda'_{333})^2}{32\pi}m_t(1-m^2_{\tilde{b}}/m_t^2)^2, \tag{59}$$

neglecting m_b and m_τ. The former mode is more likely to be accessible, since sleptons are expected to be lighter than squarks. Since the SM top decay has partial width

$$\Gamma(t\rightarrow bW)=\frac{G_F m_t^3|V_{tb}|^2}{8\pi\sqrt{2}}(1-M_W^2/m_t^2)^2(1+2M_W^2/m_t^2), \tag{60}$$

the ratio of RPV to SM decays would be typically

$$\Gamma(t\rightarrow b\tilde{\tau}^+)/\Gamma(t\rightarrow bW^+)\simeq 0.70(\lambda'_{333})^2 \quad (\text{for } m_{\tilde{\tau}}\simeq M_W). \tag{61}$$

It is natural to assume that $\tilde{\tau}$ would decay mostly to τ plus the lightest neutralino χ_1^0 (which is also probably the lightest sparticle), followed by the RPV decay $\chi_1^0\rightarrow b\bar{b}\nu_\tau(\bar{\nu}_\tau)$, with a short lifetime [19]

$$\tau(\chi_1^0\rightarrow b\bar{b}\nu_\tau, b\bar{b}\bar{\nu}_\tau) \sim 3\times 10^{-21}\mathrm{sec}(m_{\tilde{b}}/m_\chi)^4(100\ \mathrm{GeV}/m_\chi)/(\lambda'_{333})^2, \tag{62}$$

giving altogether

$$t\rightarrow b\tilde{\tau}^+\rightarrow b\tau\chi_1^0\rightarrow bb\bar{b}\tau^+\nu_\tau(\bar{\nu}_\tau). \tag{63}$$

This mode could, in principle, be identified experimentally, e.g., by exploiting the large number of potentially taggable b jets and the presence of a τ. However, it would not be readily confused with the SM decay modes $t\rightarrow bW^+\rightarrow bq\bar{q}', b\ell\nu$, ($\ell=e,\mu$), that form the basis of the presently detected $p\bar{p}\rightarrow t\bar{t}X$ signals in the $(W\rightarrow\ell\nu)+4$ jet and dilepton channels (neglecting leptons from $\tau\rightarrow\ell\nu\nu$ that suffer from a small branching fraction and a soft spectrum). On the contrary, the RPV mode would deplete the SM signals by competition. With $m_{\tilde{\tau}}\sim M_W$, fixed-point values $\lambda'_{333}\simeq 0.9$ (Fig. 1) would suppress the SM signal rate by a factor $[1+0.70(\lambda'_{333})^2]^{-2}\simeq 0.4$, in contradiction to experiment where $p\bar{p}\rightarrow t\bar{t}X\rightarrow b\bar{b}WWX$ signals tend, if anything, to exceed SM expectations [15,16]. We conclude that either the fixed-point value is not approached or the $\tilde{\tau}$ mass is higher and reduces the RPV effect (e.g., $m_{\tilde{\tau}}=150$ GeV with $\lambda'_{333}=0.9$ would suppress the SM signal rate by 0.88 instead). Note that our discussion hinges on the fact that the RPV decays of present interest would *not* contribute to SM top signals; it is quite different from the approach of Ref. [7], which considers RPV couplings that would give hard electrons or muons and contribute in conventional top searches.

Similarly, the B-violating coupling λ''_{233} leads to $t_R\rightarrow\bar{b}_R\bar{\tilde{s}}_R, \bar{\tilde{b}}_R\bar{s}_R$ decays, with partial widths

$$\Gamma(t\rightarrow\bar{b}\bar{\tilde{s}})=\Gamma(t\rightarrow\bar{\tilde{b}}\bar{s})=\frac{(\lambda''_{233})^2}{32\pi}m_t(1-m_{\tilde{q}}^2/m_t^2)^2, \tag{64}$$

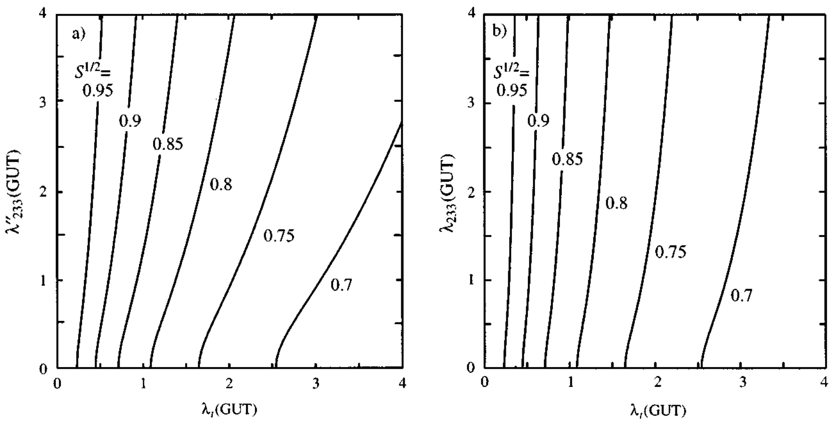

FIG. 3. Contours of constant $S^{1/2}$ for different values of (a) λ''_{233}(GUT) and λ_t(GUT) (baryon-number violation) and (b) λ_{233}(GUT) $=\lambda'_{333}$(GUT) and λ_t(GUT) (lepton-number violation).

neglecting m_b and m_s and assuming a common squark mass $m_{\tilde b}=m_{\tilde s}=m_{\tilde q}$. If the squarks were no heavier than 150 GeV, say, the ratio of RPV to SM decays would be

$$\Gamma(t\to\overline{b\tilde{s}},\overline{\tilde{b}s})/\Gamma(t\to bW^+)\simeq 0.16(\lambda''_{233})^2$$

$$(\text{for } m_{\tilde q}=150 \text{ GeV}). \qquad (65)$$

These RPV decays would plausibly be followed by $\tilde q\to q\chi_1^0$ and $\chi_1^0\to cbs,\overline{cbs}$ [via the same λ''_{233} coupling with a short lifetime analogous to Eq. (62)], giving altogether

$$t\to(b\tilde s,s\tilde b)\to bs\chi_1^0\to(cbbbs,\overline{cbbbs}). \qquad (66)$$

This all-hadronic mode could, in principle, be identified experimentally, through the multiple b jets plus the $t\to 5$-jet and $\chi_1^0\to 3$-jet invariant mass constraints. However, it would not be readily mistaken for the SM hadronic mode $t\to bW\to 3$ jet, and would simply reduce all the SM top signal rates. If the coupling approached the fixed-point value $\lambda''_{233}\simeq 1.0$, while $m_{\tilde q}\simeq 150$ GeV as assumed in Eq. (65), the SM top signals would be suppressed by a factor $[1+0.16(\lambda''_{233})^2]^{-2}\simeq 0.75$, which is strongly disfavored by the present data [15,16] but perhaps not yet firmly excluded.

If indeed the s and b squarks were lighter than t to allow the B-violating modes above, it is quite likely that the R-conserving decay $t\to\tilde t\chi_1^0$ would also be allowed, followed by $\tilde t\to c\chi_1^0$ (via a loop) and B-violating decays for both neutralinos, with net effect

$$t\to\tilde t\chi_1^0\to c\chi_1^0\chi_1^0\to(cccbbbb,ccbb\overline{cbb},c\overline{ccbbbb}). \qquad (67)$$

This seven-quark mode would look quite unlike the usual SM modes and would further suppress the SM signal rates. Depending on details of the sparticle spectrum, however, other decays such as $\tilde t\to bW\chi_1^0$ might take part too, leading to different final states; no general statement can be made except that they too would dilute the SM signals and therefore cannot be very important.

V. CONCLUSIONS

The renormalization group evolution of the standard Yukawa couplings can be affected by the presence of RPV couplings. In this paper we have done the following.

We have identified the fixed points that occur in the RPV couplings, under the usual assumption that only B-violating or only L-violating RPV interactions exist.

These fixed points provide process-independent upper bounds on RPV couplings at the electroweak scale; we confirm previously obtained bounds in the B-violating case and provide new results for the L-violating case (Fig. 1).

We have also addressed scenarios with large $\tan\beta$ where λ_b too can reach a fixed point.

The fixed point values are summarized in Table II. It is interesting that they are compatible with all present experimental constraints.

However, fixed-point values of the L-violating coupling λ'_{333} or the B-violating coupling λ''_{233} would require the corresponding sparticles to have mass $\gtrsim m_t$ to prevent unacceptably large fractions of top-quark decay to sleptons or squarks.

The fixed points lead to constraints, correlating the RPV couplings with the top-quark Yukawa coupling at the GUT scale, from lower bounds on the top mass (Fig. 2).

We have derived evolution equations for the CKM matrix and examined the evolution of the CKM mixing angles in the presence of RPV couplings (Fig. 3). In the most general case, new CKM-like angles occur in the RPV coupling sector and influence the scaling of the CKM unitarity triangle.

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