Radiative decays of heavy and light mesons in a quark triangle approach

N. R. Jones and Dongsheng Liu
Physics Department, University of Tasmania, GPO Box 252C, Hobart, Tasmania 7001, Australia

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The radiative meson decays $V \to P\gamma$ and $P \to \gamma\gamma$ are analyzed using the quark triangle diagram. Experimental data yield well determined estimates of the universal quark-antiquark-meson couplings $g_{Vq\overline{q'}}$ and $g_{Pq\overline{q'}}$ for the light meson sector. Also predictions for the ratios of neutral to charged heavy meson decay coupling constants are given and await experimental confirmation. [S0556-2821(96)06411-9]

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I. INTRODUCTION

In earlier work [1] we used a supermultiplet theory uniting the vector and pseudoscalar mesons to attempt to obtain a universal three-point coupling constant. The relativistic multispinor fields of the supermultiplet theory described pointlike mesons with correct spin, parity, flavor, and color degrees of freedom without necessarily invoking the notion of constituent quarks. Despite the apparent contradiction with the modern understanding of the quark nature of mesons it has been shown [2] that such a field is dynamically equivalent to a system of two quarks moving at equal velocity and on shell. This relatively simple scheme does nonetheless compare favorably with experimental results for strong vector and pseudoscalar interactions in the light and heavy meson sectors. When we included the radiative decays by incorporating vector meson dominance with the scheme we once again found reasonable agreement, but there were some unexpected and significant deviations from the theory, particularly in the $K^* \to K\gamma$ decays.

The supermultiplet theory and exact SU(3) predict the coupling ratio $|g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}|$ to equal 2. But experimental measurement currently estimates the ratio as 1.51 ± 0.13, a substantial difference. One possible reason why the supermultiplet scheme did not comply with the experimental measure is that the exact form of the vector meson dominance is not known in the $q^2 \to 0$ limit; it is only accurately known for $q^2 = m_V^2$ from $V \to l^+ l^-$ decays. Hence there is some uncertainty in extrapolating vector meson dominance to the off-shell case. However, the difference in theory and experiment is so large that it is unlikely this is the only contributing factor. Thus we also implemented some symmetry breaking in the supermultiplet scheme, but the K^* radiative decays seemed impervious to our attempts at matching theory with experiment as large discrepancies remained.

To understand these deviations further, we sought a method which easily allowed for off-shell propagation of the quarks so that we could evaluate the magnitude of this necessary correction. A convenient and apparently successful method for doing this was by use of a quark triangle diagram, which so far has given accurate predictions for $\pi^0 \to \gamma\gamma$ decay widths [3] and pion and kaon charge radii [3,4]. A form which used chiral and isospin symmetry has also been successfully applied to the K^* radiative decay problem and some radiative decays in the light meson sector [5]. The resulting loop integral accounted for the difference in quark masses and hence propagators in the loop, and correspondence between theory and experiment was achieved.

Nonetheless, a crucial assumption of the quark triangle diagram is that the meson-quark-antiquark vertex has the form $g_{Pq\overline{q'}}\gamma_5$ for the pseudoscalar meson and $g_{Vq\overline{q'}}\gamma^\mu$ for the vector meson. If we are to confidently use the quark triangle method, we wish to test the appropriateness of this assumption. We do this by extracting the couplings from experimental measurements and examine the extent to which they carry the spin and flavor symmetries. With this in mind, we first formulate the limit free form of the integral and derive a limiting case which uses chiral symmetry. This enables us to compare our result with others. We also determine a heavy quark expansion. Following this, the scheme is applied to $P \to \gamma\gamma$ decays, which yields estimates of $g_{Pq\overline{q'}}$, and then to $V \to P\gamma$ decays to obtain the product $g_{Vq\overline{q'}}g_{Pq\overline{q'}}$ for different quark flavors.

The results indicate that the meson-quark-antiquark couplings in the VVP sector determined from different channels which involve common constituent quarks are remarkably uniform, suggesting that the effective vertex in the quark triangle diagram is valid. The data also demonstrate that the triangle method should be highly predictive due to the stability of the couplings. Finally we use the method in the heavy meson sector, to predict coupling ratios of the form $g_{V^0P^0\gamma}/g_{V^+P^+\gamma}$ where the only free parameters required are the constituent quark masses. Our result for D^* decays falls within other theoretical estimates, while that of B^* is sensitive to b quark mass.

II. QUARK TRIANGLE

In the quark triangle formulation of Fig. 1, the decay from vector meson to pseudoscalar meson and photon state is mediated by a quark loop with flavors of constituent mass m and $\overline{m}$. (The choice of constituent mass rather than current mass is supported [3,4].) The quark triangle diagrams correspond to the Feynman amplitude for the decay

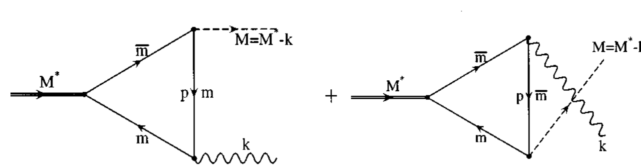

FIG. 1. Quark triangle diagrams contributing to $V \to P\gamma$.

$$A(V\to P\gamma)=-N_C g_{Vq\bar{q}'}g_{Pq\bar{q}'}eQ\kappa^{\mu}\epsilon^{\nu}\int\frac{d^4p}{(2\pi)^4}\times\mathrm{Tr}\left(\gamma_{\nu}\frac{1}{\not p-m}\gamma^5\frac{1}{\not p+\not M^*-\not k-\bar{m}}\gamma_{\mu}\times\frac{1}{\not p-\not k-m}\right)+(m\leftrightarrow\bar{m},Q\leftrightarrow\bar{Q}),\tag{1}$$

where $\kappa^{\mu}(\epsilon^{\nu})$ is the vector meson (photon) polarization vector, $M^*(M)$ is the vector (pseudoscalar) four-momenta, $g_{Vq\bar{q}'}(g_{Pq\bar{q}'})$ is the vector-quark (pseudoscalar-quark) coupling constant, and eQ is the electric charge of the quark of mass m in the loop.

The Feynman loop integral involved in the amplitude (1) may be solved with standard techniques. We maintain the notation of Bramon and Scadron (BS) [5] and call the integral J, a dimensionless quantity after multiplication by m. Subsequently,

$$J=m\int_0^1 du\int_0^{1-u}dv\times\frac{m+(\bar{m}-m)u}{m^2+(\bar{m}^2-m^2)u-M^{*2}uv-M^2u(1-u-v)}\tag{2}$$

$$=-\frac{m^2}{M^{*2}-M^2}\int_0^1 du(\delta+1/u)(\ln|j^*|-\ln|j|)$$

$$=\frac{m^2}{M^{*2}-M^2}[J_1^*-J_1+J_2^*-J_2],\tag{3}$$

where

$$\delta=\frac{m-\bar{m}}{m},$$

$$j=[m^2-(m^2-\bar{m}^2+M^2)u+M^2u^2]/m^2,$$

$$J_1=-\int_0^1 du\ln|j|/u,$$

$$J_2=-\delta\int_0^1 du\ln|j|,$$

and $j^*(J_1^*,J_2^*)$ corresponds to $j(J_1,J_2)$ with $M\leftrightarrow M^*$, respectively. Now $M^*(M)$ refers to the vector (pseudoscalar) meson mass.

A. Determination of J_1

In attempting to find an expression for J_1, we rewrite the argument of the natural logarithm in a form similar to that of the dilogarithm. To do this, we factorize j as

$$j=1+[\delta^2+2\delta-(M/m)^2]u+M^2u^2/m^2$$
$$=(1-v_1u)(1-v_2u),$$

where

$$v_{1,2}=-(\delta^2+2\delta-(M/m)^2\mp\{[\delta^2+2\delta-(M/m)^2]^2-(2M/m)^2\}^{1/2})/2\tag{4}$$

$$=\{m^2-\bar{m}^2+M^2\pm\lambda^{1/2}(m^2,\bar{m}^2,M^2)\}/2m^2\tag{5}$$

and

$$\lambda(m^2,\bar{m}^2,M^2)=[M^2-(m+\bar{m})^2][M^2-(m-\bar{m})^2].\tag{6}$$

We obtain a similar expression for $v_{1,2}^*$ upon substitution of M by M^*. The factorization we have performed does not necessarily lead to real v_k^* or v_k and we must consider the case for both real and complex arguments.

1. Real v_k^* or v_k

We first consider the simplest case, that when either v_k^* or v_k is real. For the moment we simply deal with real v_k and extend our findings to real v_k^* by substitution of M by M^*. The factor v_k is only real when $\lambda(m^2,\bar{m}^2,M^2)\geqslant 0$ which from (6) implies

$$M\geqslant m+\bar{m}\ \text{ or }\ M\leqslant|m-\bar{m}|.\tag{7}$$

When we are assured of real v_k the solution of J_1 is related to the standard dilogarithm function:

$$J_1=-\int_0^1 du\sum_{k=1}^{2}\frac{\ln|1-v_ku|}{u}\equiv\sum_{k=1}^{2}\mathrm{Li}_2(v_k,0)$$

$$=\begin{cases}\sum_{k=i}^{2}\mathrm{Li}_2(v_k) & \text{for } v_k\leqslant 1,\\ \sum_{k=1}^{2}\mathrm{Li}_2(v_k)+i\pi\ln|v_k| & \text{for } v_k>1.\end{cases}\tag{8}$$

2. Complex v_k^* or v_k

The v_k are complex if $\lambda(m^2,\bar{m}^2,M^2)<0$ and we need an appropriate method for handling this situation. Fortunately the dilogarithm of a complex argument does exist, and so we may proceed.

We express J_1 as

$$J_1=-\int_0^1 du\sum_{k=1}^{2}\frac{\ln|1-v_ku|}{u}$$

$$=-\sum_{k=1}^{2}\int_0^{\rho e^{i\phi_k}}\frac{\ln(1-z)}{z}dz$$

$$=-\sum_{k=1}^{2}\frac{1}{2}\int_0^{\rho}\frac{\ln(1-2x\cos\phi_k+x^2)}{x}dx+i\int_0^{\rho}\arctan\left[\frac{y\sin\phi_k}{1-y\cos\phi_k}\right]\frac{dy}{y},$$

$$J_1=-\int_0^{\rho}\frac{\ln(1-2x\cos\phi+x^2)}{x}dx\equiv 2\mathrm{Li}_2(\rho,\phi),\tag{9}$$

where

$$\rho = M/m, \quad \phi = \phi_1 = -\phi_2, \quad \text{and } \cos\phi = \frac{m^2 - \overline{m}^2 + M^2}{2Mm}, \tag{10}$$

and J_1^* corresponds to J_1 with $M \leftrightarrow M^*$.

B. Determination of J_2

J_2 is a simpler integral to evaluate as it does not contain the $1/u$ dependence. Recall

$$J_2 = -\delta \int_0^1 du \ln[1 - (1 - \overline{m}^2/m^2 + M^2/m^2)u + M^2u^2/m^2],$$

which, by standard techniques, reduces to

$$J_2 = \frac{m - \overline{m}}{mM^2}\left[(\overline{m}^2 - m^2 + M^2)\ln\frac{\overline{m}}{m} - \lambda^{1/2}(m^2,\overline{m}^2,M^2) \times \operatorname{arctanh}\left(\frac{\lambda^{1/2}(m^2,\overline{m}^2,M^2)}{\overline{m}^2 + m^2 - M^2}\right)\right],$$

and is valid for all $m, \overline{m}, M$. We obtain a similar expression J_2^* when we substitute $M \to M^*$.

It is also useful to express J_2 in terms of real v_k. Following a similar method to that used for deriving J_1, we find

$$J_2 = -\delta \sum_{k=1}^{2} (1 - 1/v_k)\ln|1 - v_k| \quad \text{for } \lambda(m^2,\overline{m}^2,M^2) \geqslant 0. \tag{11}$$

III. COMPARISON WITH COVARIANT AMPLITUDE

Our final form for the loop integral is

$$J = \frac{m^2}{M^{*2} - M^2}[J_1^* - J_1 + J_2^* - J_2]. \tag{12}$$

Here we have not considered the imaginary part in J_1 which is irrelevant to the decay process:

$$J_1 = \begin{cases} \sum_{k=1}^{2} \mathrm{Li}_2(v_k,0) & \text{if } \lambda(m^2,\overline{m}^2,M^2) \geqslant 0 \quad \text{where } v_{1,2} = [m^2 - \overline{m}^2 + M^2 \pm \lambda^{1/2}(m^2,\overline{m}^2,M^2)]/2m^2, \\ 2\mathrm{Li}_2(\rho,\phi) & \text{if } \lambda(m^2,\overline{m}^2,M^2) < 0 \quad \text{where } \rho = M/m, \ \cos\phi = (m^2 - \overline{m}^2 + M^2)/2Mm \end{cases} \tag{13}$$

and

$$J_2 = \frac{m - \overline{m}}{mM^2}\left[(\overline{m}^2 - m^2 + M^2)\ln\frac{\overline{m}}{m} - \lambda^{1/2}(m^2,\overline{m}^2,M^2) \times \operatorname{arctanh}\left(\frac{\lambda^{1/2}(m^2,\overline{m}^2,M^2)}{\overline{m}^2 + m^2 - M^2}\right)\right].$$

Thus, our Feynman amplitude for the decay is

$$A(V \to P\gamma) = iN_C e g_{Vq\overline{q}'} g_{Pq\overline{q}'} \epsilon_{\mu\nu\rho\sigma} \kappa^\mu \epsilon^\nu P^\rho k^\sigma \times [QJ/m + \overline{QJ/m}]/4\pi^2, \tag{14}$$

where $J \to \overline{J}$ when $m \leftrightarrow \overline{m}$ (from the momentum crossed Feynman diagram) and $\overline{Q}$ is the charge of the quark with mass $\overline{m}$. We compare this with the general covariant amplitude for the process $V \to P\gamma$,

$$A(V \to P\gamma) = i g_{VP\gamma} \epsilon_{\alpha\beta\mu\nu} P^\alpha k^\beta \kappa^\mu \epsilon^\nu,$$

so that our quark triangle approach resolves the $g_{VP\gamma}$ covariant coupling constant as

$$g_{VP\gamma} = N_C e g_{Vq\overline{q}'} g_{Pq\overline{q}'} [QJ/m + \overline{QJ/m}]/4\pi^2. \tag{15}$$

IV. $P \to \gamma\gamma$ IN THE QUARK TRIANGLE SCHEME

We are interested in understanding the behavior and obtaining actual values for the coupling constants $g_{Pq\overline{q}'}$ and $g_{Vq\overline{q}'}$ in the light quark sector so that we can use appropriate estimates for these couplings in the heavy quark sector. To this end we can use the well-documented decay data for $V \to P\gamma$ in the light vector meson sector, as well as the decays $P \to \gamma\gamma$. These latter processes are particularly useful as they only involve the coupling $g_{Pq\overline{q}'}$, and not the product $g_{Vq\overline{q}'} g_{Pq\overline{q}'}$ as does the first case. Consequently, we must derive the amplitude for the decay of a pseudoscalar meson into two photons. This we may do by following a similar derivation as above, but it is much simpler to make the following substitutions in the $V \to P\gamma$ amplitude (14):

$$M^* \to M, \quad M \to 0, \quad g_{Vq\overline{q}'} \to g_{Pq\overline{q}'}, \quad \text{and } g_{Pq\overline{q}'} \to eQ,$$

and since all pseudoscalar mesons involved in $P \to \gamma\gamma$ decays must be quark flavor singlets, $m = \overline{m}, Q = \overline{Q}, J = \overline{J}$. Subsequently (15) is reduced to

$$g_{P\gamma\gamma} = 2N_C g_{Pq\overline{q}'} e^2 [Q^2 J/m]/4\pi^2, \tag{16}$$

where

$$J = \begin{cases} \frac{m^2}{M^2}\sum_{k=1}^{2} \mathrm{Li}_2(v_k,0) & \text{if } M \geqslant 2m \quad \text{where } v_{1,2} = M[M/m \pm (M^2/m^2 - 4)^{1/2}]/2m, \\ 2\frac{m^2}{M^2}\mathrm{Li}_2(\rho,\phi) & \text{if } 0 \leqslant M \leqslant 2m \quad \text{where } \rho = M/m, \quad \cos\phi = M/2m. \end{cases}$$

V. CHIRAL LIMIT

The chiral symmetry limit is useful in the light meson sector, and gives our work direct comparison with that of Bramon and Scadron [5]. The limit corresponds to a small pseudoscalar mass when compared to the vector mass, that is, $M^{*2} \gg M^2$. Such a limit is entirely appropriate for the study of the radiative decays of K^* mesons, and it is reassuring to know that our J reduces to the J_M of [5] in the chiral limit.

The chiral limit, corresponding to $M \to 0$ in J, enables us to use the real form for J_1 in (8) as $M \leqslant |m - \overline{m}|$,

$$v_k(M \to 0) = \begin{cases} 0 & \text{for } k=1, \\ -(\delta^2 + 2\delta) & \text{for } k=2, \end{cases}$$

from (4) so that

$$\begin{aligned} J_1 &= \sum_{k=1}^{2} \mathrm{Li}_2(v_k,0) \\ &= \mathrm{Li}_2(0,0) + \mathrm{Li}_2(-\delta^2 - 2\delta,0) \\ &= \mathrm{Li}_2(-\delta^2 - 2\delta,0) \end{aligned}$$

and

$$\begin{aligned} J_2 &= -\delta \sum_{k=1}^{2} (1 - 1/v_k)\ln|1 - v_k| \\ &= -\delta - \delta[1 + 1/(\delta^2 + 2\delta)]\ln|1 + 2\delta + \delta^2| \\ &= -\delta - \frac{2(\delta+1)^2}{\delta+2}\ln|1+\delta|. \end{aligned}$$

Thus J in the chiral limit, which we denote as J^{CL}, becomes

$$J^{\mathrm{CL}} = \frac{m^2}{M^{*2}} \left\{ \delta + \sum_{k=1}^{2} [\mathrm{Li}_2(v_k^*,0) - \delta(1 - 1/v_k^*)\ln|1 - v_k^*|] \cdots - \mathrm{Li}_2(-\delta^2 - 2\delta,0) + \frac{2(1+\delta)^2}{2+\delta}\ln|1+\delta| \right\}, \tag{17}$$

where we have assumed v_k^* is real. This form may be simplified even further near the isospin symmetry limit whereby $m \approx \overline{m}$. In this instance we ignore δ terms of order 2 and higher:

$$\mathrm{Li}_2(-\delta^2 - 2\delta,0) \to \mathrm{Li}_2(-2\delta,0),$$

$$\frac{2(1+\delta)^2}{2+\delta}\ln|1+\delta| \to (\delta + \tfrac{1}{2})\ln|1+2\delta|,$$

$$\begin{aligned} v_{1,2}{}^* &\to -(2\delta - (M^*/m)^2 \mp \{[2\delta + (M^*/m)^2]^2 - (2M^*/m)^2\}^{1/2})/2 \\ &= \frac{M^{*2}}{2m^2} - \delta \pm \left[\left(\frac{M^{*2}}{2m^2} - \delta \right)^2 - \frac{M^{*2}}{m^2} \right]^{1/2}. \end{aligned} \tag{18}$$

Thus we find J^{CL} incorporating isospin symmetry between quark flavors reduces to

$$J^{\mathrm{CL}} = \frac{m^2}{M^{*2}} \left\{ \delta + \sum_{k=1}^{2} [\mathrm{Li}_2(v_k^*,0) - \delta(1 - 1/v_k^*)\ln|1 - v_k^*|] - \mathrm{Li}_2(-2\delta,0) + (\delta + \tfrac{1}{2})\ln|1+2\delta| \right\},$$

with v_k^* defined in Eq. (18). This form is very similar to the J_M of Bramon and Scadron [5]. There is a subtle difference in their use of the dilogarithm function $\mathrm{Li}_2(z)$ versus our function $\mathrm{Li}_2(z,0)$ which is similar to the dilogarithm, but which only allows real solutions [the term $i\pi\ln|v_k|$ in Eq. (8) ensures this]. In the chiral limit with $M^* > m + \overline{m}$ these two functions are equivalent. In addition they have the term $(\delta - 1/2)\ln|1+2\delta|$ whereas we have $(\delta + 1/2)\ln|1+2\delta|$. We believe that this difference is due to a typographical mistake as the argument of the logarithmic function is linked to the multiplier outside, so that there should be no difference between them (the missing multiplication factor of 2 is easily accounted for, but not the sign change).

A. Chiral limit of $P \to \gamma\gamma$

There is a well-known chiral limit of the $P \to \gamma\gamma$ case, namely, $\pi^0 \to \gamma\gamma$ [6,3], when $g_{\pi^0\gamma\gamma} = e^2 N_C g_{\pi^0} Q^2 / 4\pi^2 m$. This implies that $J = 1/2$ for the pion. We can establish this from our full formulas. The chiral limit implies that $M_{\pi^0} \to 0$ so that the appropriate form of J is

$$J = 2m^2 \mathrm{Li}_2(\rho, \phi)/M^2,$$

and we define $M/m = \epsilon$ with $\epsilon \to 0$ as $M \to 0$. We subsequently find $\rho = \epsilon$, $\cos\phi = \epsilon/2$, and, therefore,

$$\begin{aligned} J &= \frac{2}{\epsilon^2}\left[-\frac{1}{2}\int_0^{\epsilon} \frac{\ln(1 - \epsilon x + x^2)}{x}\,dx \right] \\ &\approx -\frac{1}{\epsilon^2}\int_0^{\epsilon} dx[-\epsilon + (1 - \epsilon^2/2)x + \epsilon(\epsilon^2+3)x/3 + \cdots] \\ &= \frac{1}{2} - \epsilon^2/12 + \epsilon^4/9 + \cdots \quad . \end{aligned}$$

Therefore J reproduces the $\pi^0 \to \gamma\gamma$ result in the chiral limit.

VI. HEAVY QUARK EXPANSION

Since we are particularly interested in the heavy meson decays $D^* \to D\gamma$ and $B^* \to B\gamma$, we feel it is of interest to examine the heavy quark expansion of our loop integral J. To derive this we consider an expansion in terms of the light to heavy quark mass ratio in each of the loop integrals. We make the arbitrary choice of $m = m_q$ and $\overline{m} = m_Q$, where m_q is the light quark mass and m_Q the heavy quark mass. These lead to the definitions

$$\frac{m}{\overline{m}} = \epsilon, \tag{19}$$

$$M = \overline{m} + \Lambda, \tag{20}$$

$$M^* = \overline{m} + \Lambda^*, \tag{21}$$

where $\epsilon \to 0$ in the heavy quark limit, and Λ, Λ^* is the combined binding energy and light quark mass.

A. Heavy quark expansion of J

When dealing with J, Eqs. (20) and (21) lead to

$$M/m = r + 1/\epsilon,$$

$$M^*/m = r^* + 1/\epsilon,$$

where $r = \Lambda/m$ and $r^* = \Lambda^*/m$ which are independent of the heavy quark mass. We express J of Eq. (3) using these relations to find

$$J \approx \int_0^1 du \int_0^{1-u} dv \times \frac{u/\epsilon}{u/\epsilon^2 - 2(r^*-r)uv/\epsilon - (2r+1/\epsilon)u(1-u)/\epsilon},$$

ignoring the constant term in ϵ. This reduces to

$$J^{\mathrm{HQL}} = -\frac{\epsilon(r^*\ln|2r^*\epsilon| - r\ln|2r\epsilon|)}{(r^*-r)[1+2\epsilon(r^*+r)]}$$

as the highest order terms in the expansion. Note that this term is of the form $\epsilon\ln\epsilon$ and we can thus expect slow convergence of the heavy quark expansion.

B. Heavy quark expansion of $\overline{J}$

We maintain our definition of ϵ and r; however, since $\overline{J}$ corresponds to J with $m \leftrightarrow \overline{m}$, we shall need the relations

$$M/\overline{m} = 1 + \Lambda/\overline{m} = 1 + r\epsilon,$$

$$M^*/\overline{m} = 1 + \Lambda^*/\overline{m} = 1 + r^*\epsilon,$$

from which

$$\overline{J} \approx \int_0^1 du \int_0^{1-u} dv \frac{1+(\epsilon-1)u}{1-u-2(r^*-r)\epsilon uv - (1+2r\epsilon)u(1-u)},$$

ignoring the ϵ^2 contribution. Thus our heavy quark expansion of $\overline{J}$ reduces to

$$\overline{J}^{\mathrm{HQL}} = \mathrm{Li}_2(1+2r^*\epsilon,0) - \mathrm{Li}_2(1+2r\epsilon,0) + \frac{2\epsilon(1-\epsilon)}{1+2\epsilon(r^*+r)}(r^*\ln|2r^*\epsilon| - r\ln|2r\epsilon|).$$

To simplify this form further, we consider an expansion of the dilogarithm. Since

$$\begin{aligned}
\mathrm{Li}_2(1+2r^*\epsilon,0) - \mathrm{Li}_2(1+2r\epsilon,0) &= -\int_{1+2r\epsilon}^{1+2r^*\epsilon} du \frac{\ln|1-u|}{u} \\
&\approx -\int_{2r\epsilon}^{2r^*\epsilon} dz \ln z(1 - z + z^2 - z^3 + \cdots) \\
&= -2\epsilon[r^*\ln|2r^*\epsilon| - r\ln|2r\epsilon| - (r^*-r) + O(\epsilon)],
\end{aligned}$$

we then arrive at our final form

$$\overline{J}^{\mathrm{HQL}} = 1 - \frac{\epsilon}{r^*-r}(r^*\ln|2r^*\epsilon| - r\ln|2r\epsilon|).$$

Once again observe the $\epsilon\ln\epsilon$ dependence, indicative of slow convergence.

It appears that both expansions J^{HQL} and $\overline{J}^{\mathrm{HQL}}$ will only converge slowly to their true counterparts J and $\overline{J}$. Thus, unfortunately, they are not so useful approximations for either the c or b quark cases.

There are other possible expansions we could consider, namely, that of the $P \to \gamma\gamma$ and $V \to P\gamma$ loop integrals where there is only one quark flavor in the loop (such as $\eta_c \to \gamma\gamma$, $\eta_b \to \gamma\gamma$, $J/\psi \to \eta_c\gamma$, or $\Upsilon \to \eta_b\gamma$) and consider some expansion as the quark mass becomes large. Unfortunately, such an expansion fails to be a good approximation, simply due to the assumption one has to make about the pseudoscalar and/or vector mass. For example, in the $P \to \gamma\gamma$ case one would assume the pseudoscalar mass M would consist of the sum of the quark masses along with some binding energy so that $M = 2m + \Delta$. Then the loop integral could be expressed as

$$J = \overline{J} = \int_0^1 du \int_0^{1-u} \frac{dv}{1-\rho^2 uv},$$

where $\rho = M/m = 2 + \Delta/m$, and one would attempt to do some sort of expansion near $\rho = 2$. Unfortunately such an expansion is impractical as the integral contains a pole at $\rho = 2, u = 1/2$.

VII. RESULTS

A. $K^* \to K\gamma$ and the coupling ratio

The observed K^* branching fraction of

$$\Gamma_{K^{*0} \to K^0\gamma}/\Gamma_{K^{*+} \to K^+\gamma} = 2.31 \pm 0.29,$$

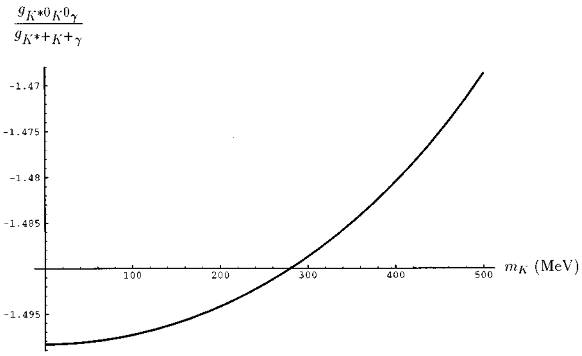

FIG. 2. Appropriateness of chiral limit, shown in the lack of sensitivity of $g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}$ to K meson mass.

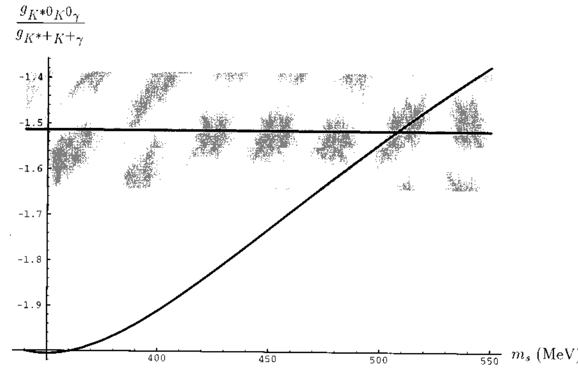

FIG. 3. Breaking of SU(3) by s quark mass. The experimental measurement is included.

and corresponding coupling constant ratio of

$$|g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}|=1.514\pm0.125 \tag{22}$$

is far from its SU(3) predicted value of 2, but is simply understood in the quark loop formalism as shown by Bramon and Scadron [5]. We quickly reiterate this point. Using Eq. (15) and assuming $g_{Vus}=g_{Vds}$ and $g_{Pus}=g_{Pds}$, then

$$\frac{g_{K^{*0}K^0\gamma}}{g_{K^{*+}K^+\gamma}}=-\frac{J_{d,s}[K^{*0},K^0]/m_d+J_{s,d}[K^{*0},K^0]/m_s}{2J_{u,s}[K^{*+},K^+]/m_u-J_{s,u}[K^{*+},K^+]/m_s}$$
$$=-1.47, \tag{23}$$

where we have used a more complete notation $J_{q,\overline{q'}}[V,P]$ to denote J for quarks $q,\overline{q'}$, vector meson V, and pseudoscalar meson P of masses m, $\overline{m}$, M^*, and M, respectively. We used quark masses $m_u=m_d=340$ MeV and $m_s=510$ MeV. The experimental uncertainty in the coupling ratio permits the s quark mass range $475<m_s<545$ MeV, when $m_u=340$ MeV, and a fixed s quark mass of $m_s=510$ MeV permits a u quark range of $225<m_u<385$ MeV. Since an s quark mass of $m_s=m_\phi/2$ gives such a good comparison between the quark triangle diagram and the experimental measurement, we will choose such a mass throughout this work, along with $m_u=m_d=340$ MeV. The result of Eq. (23) compares well with that of [5], indicating that their chiral limit formulas are appropriate. In fact we can observe the variation from the chiral limit to $M=M_K$ using our J; as Fig. 2 shows there is very little change in the result.

We can also dramatically show how the s quark mass breaks the SU(3) symmetry. Figure 3 displays the behavior of $g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}$ from $m_s=m_u=m_d=340$ MeV [the SU(3) limit] to $m_s=550$ MeV, clearly indicating that it is the violation of constituent quark masses from SU(3) symmetry that is responsible for the large deviation in K^* mesons from the expected symmetry. The K^* radiative decays are particularly sensitive to SU(3) violations as they involve the constituent masses of strange and nonstrange quarks in the loop of the corresponding triangle diagram. Subsequently, the heavier meson cases are best computed by the triangle diagram which can allow for different constituent masses in the loop, rather than an SU(4) or SU(5) symmetry.

B. Measurements of $g_{Pq\overline{q'}}$

We may obtain estimates for the $g_{Pq\overline{q'}}$ coupling constants using experimental measurements of the $P\rightarrow\gamma\gamma$ decay widths. In particular the widths for $\pi^0\rightarrow\gamma\gamma$, $\eta\rightarrow\gamma\gamma$, and $\eta'\rightarrow\gamma\gamma$ processes are well known and we should be able to determine $g_{Pu\overline{u}}$ and $g_{Ps\overline{s}}$ to reasonable accuracy (we make the isosymmetric approximation $g_{Pu\overline{u}}=g_{Pd\overline{d}}$, $m_u=m_d$).

Beginning with the π^0 meson which is the antisymmetric mixture $(u\overline{u}-d\overline{d})/\sqrt{2}$, we find

$$g_{\pi^0\gamma\gamma}=\frac{6e^2}{4\pi^2}\left\{\left(\frac{2}{3}\right)^2 g_{\pi^0u\overline{u}}\frac{J_u[\pi^0]}{m_u}+\left(-\frac{1}{3}\right)^2 g_{\pi^0d\overline{d}}\frac{J_d[\pi^0]}{m_d}\right\}$$
$$=\frac{e^2}{6\pi^2}\left\{4\frac{g_{Pu\overline{u}}}{\sqrt{2}}-\frac{g_{Pd\overline{d}}}{\sqrt{2}}\right\}J_u[\pi^0]/m_u$$
$$=\frac{e^2}{2\sqrt{2}\pi^2}g_{Pu\overline{u}}J_u[\pi^0]/m_u. \tag{24}$$

In a similar fashion, we use the standard octet-singlet pseudoscalar mixing angle θ_P to ascribe the η-η' mixing by

$$\eta=\frac{1}{\sqrt{6}}[(\cos\theta_P-\sqrt{2}\sin\theta_P)(u\overline{u}+d\overline{d})$$
$$-\sqrt{2}(\sin\theta_P+\sqrt{2}\cos\theta_P)s\overline{s}],$$

$$\eta'=\frac{1}{\sqrt{6}}[(\sin\theta_P+\sqrt{2}\cos\theta_P)(u\overline{u}+d\overline{d})$$
$$+\sqrt{2}(\cos\theta_P-\sqrt{2}\sin\theta_P)s\overline{s}].$$

Following a methodology such as Eq. (24) we obtain relations between the covariant amplitudes and meson-quark couplings which are given in Table I. We have used $m_u=340$ MeV, $m_s=510$ MeV, and two different mixing angles $\theta_P=-10.5°$ (in accordance with the quadratic Gell-Mann–Okubo relation) and $\theta_P=-20°$ (which is partly favored by pseudoscalar decay processes) along with the covariant couplings from the measured decay rates [7]:

$$\Gamma_{P\rightarrow\gamma\gamma}=m_P^3g_{P\gamma\gamma}^2/64\pi,$$

TABLE I. Radiative decays of ground state mesons and relations between covariant couplings $g_{P\gamma\gamma}$, $g_{VP\gamma}$, or $g_{PV\gamma}$ and meson-quark-antiquark couplings $g_{Vq\bar{q}'}, g_{Pq\bar{q}'}$.

Process	Relation between covariant couplings and mesonquark couplings
$\pi^0\to\gamma\gamma$	$g_{\pi^0\gamma\gamma}=\frac{e^2}{2\sqrt{2}\pi^2}g_{Pu\bar{u}}J_u[\pi^0]/m_u$
$\eta\to\gamma\gamma$	$g_{\eta\gamma\gamma}=\frac{e^2}{6\sqrt{6}\pi^2}\{5(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Pu\bar{u}}J_u[\eta]/m_u$ $-\sqrt{2}(\sin\theta_P+\sqrt{2}\cos\theta_P)g_{Ps\bar{s}}J_s[\eta]/m_s\}$
$\eta'\to\gamma\gamma$	$g_{\eta'\gamma\gamma}=\frac{e^2}{6\sqrt{6}\pi^2}\{5(\sin\theta_P+\sqrt{2}\cos\theta_P)g_{Pu\bar{u}}J_u[\eta']/m_u$ $+\sqrt{2}(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Ps\bar{s}}J_s[\eta']/m_s\}$
$\eta_c\to\gamma\gamma$	$g_{\eta_c\to\gamma\gamma}=\frac{2e^2}{3\pi^2}g_{Pc\bar{c}}J_c[\eta_c]/m_c$
$\rho^0\to\pi^0\gamma$	$g_{\rho^0\to\pi^0\gamma}=\frac{e}{4\pi^2}g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\rho^0,\pi^0]/m_u$
$\rho^+\to\pi^+\gamma$	$g_{\rho^+\to\pi^+\gamma}=\frac{e}{4\pi^2}g_{Vu\bar{d}}g_{Pu\bar{d}}J_{u,d}[\rho^+,\pi^+]/m_u$
$\rho^0\to\eta\gamma$	$g_{\rho^0\to\eta\gamma}=\frac{\sqrt{3}e}{4\pi^2}(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\rho^0,\eta]/m_u$
$\omega\to\pi^0\gamma$	$g_{\omega\to\pi^0\gamma}=\frac{\sqrt{3}e}{4\pi^2}(\sin\theta_V+\sqrt{2}\cos\theta_V)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\omega,\pi^0]/m_u$
$\omega\to\eta\gamma$	$g_{\omega\to\eta\gamma}=\frac{e}{12\pi^2}\{(\sin\theta_V+\sqrt{2}\cos\theta_V)(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\omega,\eta]/m_u$ $+2(\cos\theta_V-\sqrt{2}\sin\theta_V)(\sin\theta_P+\sqrt{2}\cos\theta_P)g_{Vs\bar{s}}g_{Ps\bar{s}}J_{s,s}[\omega,\eta]/m_s\}$
$\eta'\to\rho^0\gamma$	$g_{\eta'\to\rho^0\gamma}=\frac{\sqrt{3}e}{4\pi^2}(\sin\theta_P+\sqrt{2}\cos\theta_P)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\eta',\rho^0]/m_u$
$\eta'\to\omega\gamma$	$g_{\eta'\to\omega\gamma}=\frac{e}{12\pi^2}\{(\sin\theta_V+\sqrt{2}\cos\theta_V)(\sin\theta_P+\sqrt{2}\cos\theta_P)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\eta',\omega]/m_u$ $-2(\cos\theta_V-\sqrt{2}\sin\theta_V)(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Vs\bar{s}}g_{Ps\bar{s}}J_{s,s}[\eta',\omega]/m_s\}$
$\phi\to\pi^0\gamma$	$g_{\phi\to\pi^0\gamma}=\frac{\sqrt{3}e}{4\pi^2}(\cos\theta_V-\sqrt{2}\sin\theta_V)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\phi,\pi^0]/m_u$
$\phi\to\eta\gamma$	$g_{\phi\to\eta\gamma}=\frac{e}{12\pi^2}\{(\cos\theta_V-\sqrt{2}\sin\theta_V)(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\phi,\eta]/m_u$ $-2(\sin\theta_V+\sqrt{2}\cos\theta_V)(\sin\theta_P+\sqrt{2}\cos\theta_P)g_{Vs\bar{s}}g_{Ps\bar{s}}J_{s,s}[\phi,\eta]/m_s\}$
$K^{*0}\to K^0\gamma$	$g_{K^{*0}\to K^0\gamma}=\frac{e}{4\pi^2}g_{Vd\bar{s}}g_{Pd\bar{s}}(J_{d,s}[K^{*0},K^0]/m_d+J_{s,d}[K^{*0},K^0]/m_s)$
$K^{*+}\to K^+\gamma$	$g_{K^{*+}\to K^+\gamma}=\frac{e}{4\pi^2}g_{Vu\bar{s}}g_{Pu\bar{s}}(2J_{u,s}[K^{*+},K^+]/m_u-J_{s,u}[K^{*+},K^+]/m_s)$
$J/\psi\to\eta_c\gamma$	$g_{J/\psi\to\eta_c\gamma}=\frac{e}{\pi^2}g_{Vc\bar{c}}g_{Pc\bar{c}}J_{c,c}[J/\psi,\eta_c]/m_c$

to obtain several estimates of the pseudoscalar-quark couplings as given in Table II. We used the $\eta\to\gamma\gamma$ and $\eta'\to\gamma\gamma$ to determine simultaneously the values.

From the results, it appears $g_{Pu\bar{u}}$ differs as determined from π^0, η, and η' processes. Considering that the η meson is about 4 times as massive as the pion, it may be appropriate to allow for such a mass dependence in the coupling constant. Suppose we label the first coupling constant from $\pi^0\to\gamma\gamma$ as $g_{Pu\bar{u}}(m_{\pi^0}^2)$, while the second from $\eta\to\gamma\gamma$ and $\eta'\to\gamma\gamma$ as a coupling constant somewhere between m_η and $m_{\eta'}$. Numerically we took the appropriate mass as the equal weight average $(m_\eta^2+m_{\eta'}^2)/2$. By linearly interpolating between these two couplings, we estimated a value of $g_{Pu\bar{u}}(m_\eta^2)=4.61\pm0.19$ for $\theta_P=-10.5°$ and $g_{Pu\bar{u}}(m_\eta^2)=4.39\pm0.17$ for $\theta_P=-20°$.

The Goldberger-Treiman (GT) relation at the quark level, gives us a good check of our results. For the pion, the relation reads

$$f_\pi g_{Pu\bar{u}}(m_{\pi^0}^2)/\sqrt{2}=m_u\,.$$

Using our coupling value in Table II along with $m_u=340$ MeV we predict $f_\pi=93.5\pm3.46$ MeV which compares well with the experimental result $f_\pi=92.4\pm0.26$ MeV [7].

TABLE II. Determination of meson-quark-antiquark couplings.

Experimental result ($\times 10^{-4}$ MeV^{-1}) [7]	Meson-quark-antiquark coupling [a] $\theta_P=-10.5°$	$\theta_P=-20°$
$\lvert g_{\pi^0\gamma\gamma}\rvert = 0.2516\pm0.0091$	$g_{Pu\bar{u}}=5.14\pm0.19$	
$\lvert g_{\eta\gamma\gamma}\rvert = 0.239\pm0.011$	$g_{Pu\bar{u}}=4.03\pm0.14$,	$g_{Pu\bar{u}}=3.56\pm0.12$,
$\lvert g_{\eta'\gamma\gamma}\rvert = 0.312\pm0.016$	$g_{Ps\bar{s}}=6.42\pm0.67$	$g_{Ps\bar{s}}=8.19\pm0.65$
$\lvert g_{\eta_c\gamma\gamma}\rvert = 0.07297\pm0.01366$	$g_{Pc\bar{c}}=2.03\pm0.38$	
$\lvert g_{\rho^0\to\pi^0\gamma}\rvert = 2.96\pm0.38$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=14.82\pm1.90$	
$\lvert g_{\rho^+\to\pi^+\gamma}\rvert = 2.24\pm0.13$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=11.32\pm0.64$	
$\lvert g_{\rho^0\to\eta\gamma}\rvert = 5.67\pm0.53$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=10.96\pm1.02$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=9.56\pm0.89$
$\lvert g_{\omega\to\pi^0\gamma}\rvert = 7.04\pm0.21$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=12.53\pm0.38$	
$\lvert g_{\omega\to\eta\gamma}\rvert = 1.83\pm0.23$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=12.3\pm1.5$,	$g_{Vu\bar{u}}g_{Pu\bar{u}}=10.7\pm1.3$,
$\lvert g_{\phi\to\eta\gamma}\rvert = 2.117\pm0.052$	$g_{Vs\bar{s}}g_{Ps\bar{s}}=7.08\pm0.17$	$g_{Vs\bar{s}}g_{Ps\bar{s}}=8.66\pm0.21$
$\lvert g_{\phi\to\pi^0\gamma}\rvert = 0.417\pm0.021$	$g_{Vu\bar{u}}g_{Pu\bar{u}}=25.1\pm1.3$ [b]	
$\lvert g_{K^{*0}\to K^0\gamma}\rvert = 3.84\pm0.17$	$g_{Vd\bar{s}}g_{Pd\bar{s}}=8.43\pm0.37$	
$\lvert g_{K^{*+}\to K^+\gamma}\rvert = 2.534\pm0.115$	$g_{Vu\bar{s}}g_{Pu\bar{s}}=8.21\pm0.37$	
$\lvert g_{J/\psi\to\eta_c\gamma}\rvert = 1.67\pm0.26$	$g_{Vc\bar{c}}g_{Pc\bar{c}}=1.87\pm0.30$	

[a] $m_u=m_d=340$ MeV, $m_s=510$ MeV, $m_c=1550$ MeV, $\theta_V=219.4°$.
[b] $g_{Vu\bar{u}}g_{Pu\bar{u}}=11.9\pm0.6$ for $\theta_V=224°$.

Also included in Table II is the estimate of $g_{Pc\bar{c}}$ using a charm quark mass of $m_c=1550$ MeV along with the experimentally determined width [7] of $\Gamma_{\eta_c\to\gamma\gamma}=7.0\pm2.6$ keV.

C. Measurements of $g_{Vq\bar{q}'}$

There exist many useful decay channels $V\to P\gamma$ and corresponding data from which we can determine the product $g_{Vq\bar{q}'}g_{Pq\bar{q}'}$. To this end we proceed in two steps. First, we interpret individual meson-meson-photon couplings in terms of meson-quark-antiquark couplings, deriving relations between them as shown in Table I. Assuming isospin symmetry there are only two unknown products of couplings involved in the light meson sector; one is $g_{Vu\bar{u}}g_{Pu\bar{u}}$ for nonstrange quarks while the other is $g_{Vs\bar{s}}g_{Ps\bar{s}}$ for strange quarks.

Following this we extract individual meson-meson-photon couplings from the most recently measured decay widths $\Gamma_{V\to P\gamma}=(m_V^2-m_P^2)^3 g_{VP\gamma}^2/12\pi m_V^3$ by simply removing the kinematic factors. The results are listed in the first column of Table II. As one can see, they scatter over a relatively wide range.

We are able to determine $g_{Vu\bar{u}}g_{Pu\bar{u}}$ solely from any one of the processes $\rho^0\to\pi^0\gamma$, $\rho^+\to\pi^+\gamma$, $\rho^0\to\eta\gamma$, $\omega\to\pi^0\gamma$, and $\omega\to\eta\gamma$. In addition, the decays $\omega\to\eta\gamma$ and $\phi\to\eta\gamma$ can be used to simultaneously solve for $g_{Vu\bar{u}}g_{Pu\bar{u}}$ and $g_{Vs\bar{s}}g_{Ps\bar{s}}$. Our numerical results are shown in the second column of Table II where we use the same quark masses as previously along with standard mixing angles.

The values of the product $g_{Vu\bar{u}}g_{Pu\bar{u}}$ turn out to lie in a quite small range, except for that from the $\phi\to\pi^0\gamma$. However, it would fall into this range had we chosen a mixing angle of about $\theta_V=224°$, a change of 4.6°. Such a high sensitivity of $\phi\to\pi^0\gamma$ to change in mixing angle suggests it is reasonable to exclude this channel from our analysis. Recalling the couplings of a pseudoscalar meson with quark-antiquark pairs discussed previously, we now obtain $g_{Vu\bar{u}}$.

As the light vector meson masses vary by less than 30%, we shall not attempt to distinguish between the slightly different couplings and thus on average we find $g_{Vu\bar{u}}=2.40\pm0.08$ (weighted average) for $\theta_P=-10.5°$ and $g_{Vu\bar{u}}=2.35\pm0.08$ (weighted average) for $\theta_P=-20°$. It differs from $g_{Pu\bar{u}}$, revealing a substantial violation of the spin symmetry in the triangle scheme.

We repeat this procedure in the analysis of $g_{Vs\bar{s}}$, but with fewer channels to determine a result. Consequently we have $g_{Vs\bar{s}}=1.10$ for $\theta_P=-10.5°$ and $g_{Vs\bar{s}}=1.06$ for $\theta_P=-20°$, indicating a large $SU(3)_V$ symmetry breaking once again. Estimates for $g_{Vc\bar{c}}$ using the $J/\psi\to\eta_c\gamma$ channel yield $g_{Vc\bar{c}}=0.92\pm0.23$. Note that $g_{Vc\bar{c}}$ and $g_{Pc\bar{c}}$ are not substantially different, perhaps indicative of a limit $g_{Vq\bar{q}}=g_{Pq\bar{q}}$ as m_q gets large.

For completeness, we wish to obtain a measure of $g_{Vu\bar{s}}$ using the product $g_{Vu\bar{s}}g_{Pu\bar{s}}$. However, we have no means of getting $g_{Pd\bar{s}}$ for the kaon in the triangle scheme. This is because, unlike π^0, the $K^0\to\gamma\gamma$ decay is not mediated by pure electromagnetic interactions. However assuming the Goldberger-Treiman relation at the quark level,

$$f_K g_{Pd\bar{s}}(m_K^2)=(m_u+m_s)/2,$$

we find $g_{Pd\bar{s}}=3.77\pm0.03$ where we have used $f_K=113.0\pm1.0$ MeV [7]. Subsequently, $g_{Vd\bar{s}}=2.21\pm0.10$ (averaged over the charged and neutral processes).

We ought to point out that the triangle scheme has also been applied to radiative decays of η' into ρ^0 or ω, but failed to yield coupling constants near the above range. This suggests to us that we should treat η' in a different way which would most likely incorporate the U(1) anomaly.

VIII. PREDICTIONS

A. $\phi\to\eta'\gamma$ coupling constant and branching fraction

We can use our best fit estimates of the meson-quark coupling constants to predict the decay width for the decay $\phi\to\eta'\gamma$,

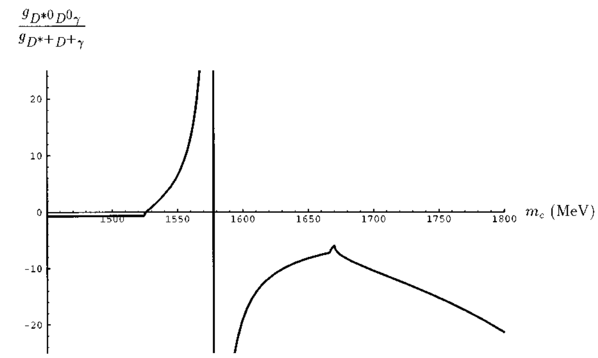

FIG. 4. Variation of coupling ratio with c quark mass.

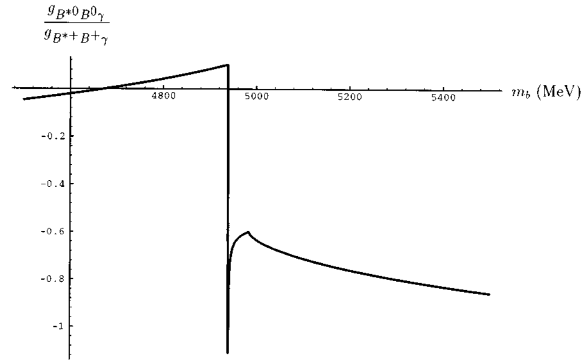

FIG. 5. Variation of coupling ratio with b quark mass.

$$g_{\phi\to\eta'\gamma}=\frac{e}{12\pi^2}\{(\cos\theta_V-\sqrt{2}\sin\theta_V)(\sin\theta_P+\sqrt{2}\cos\theta_P)\times g_{Vu\bar{u}}g_{Pu\bar{u}}J_{u,u}[\phi,\eta']/m_u+2(\sin\theta_V+\sqrt{2}\cos\theta_V)(\cos\theta_P-\sqrt{2}\sin\theta_P)g_{Vs\bar{s}}g_{Ps\bar{s}}J_{s,s}[\phi,\eta']/m_s\},$$

and using $m_u=340$ MeV, $m_s=510$ MeV, and mixing angles $\theta_P=-10.5°,\theta_V=219.4°$ along with our couplings from Table II, we compute the coupling to be $g_{\phi\to\eta'\gamma}=-5.69\times10^{-4}$. This gives a branching ratio of

$$\mathcal{B}(\phi\to\eta'\gamma)=4.16\times10^{-4},$$

which is slightly above the experimental upper limit of $\mathcal{B}(\phi\to\eta'\gamma)<4.1\times10^{-4}$ at 90% confidence level [7]. However, we note that a change to an s quark mass of $m_s=500$ MeV produces a branching fraction of $\mathcal{B}(\phi\to\eta'\gamma)=3.24\times10^{-4}$, so that the result displays very sensitive dependence on the choice of s quark mass and probably vector mixing angle. Also we remain cautious of predictions involving the η' meson due to its association with the U(1) anomaly.

B. $D^*\to D\gamma$ and $B^*\to B\gamma$ coupling ratios

Since our J is approximation free (i.e., no chiral limit assumptions), we can safely use it in the D^* and B^* meson cases. We do assume $g_{VuQ}=g_{VdQ}$ and $g_{PuQ}=g_{PdQ}$ where Q is either the c or b quark (much like we did in the $K^*\to K\gamma$ case) to obtain

$$\frac{g_{D^{*0}D^0\gamma}}{g_{D^{*+}D^+\gamma}}=\frac{2(J_{u,c}[D^{*0},D^0]/m_u+J_{c,u}[D^{*0},D^0]/m_c)}{-J_{d,c}[D^{*+},D^+]/m_d+2J_{c,d}[D^{*+},D^+]/m_c}\tag{25}$$

and

$$\frac{g_{B^{*0}B^0\gamma}}{g_{B^{*+}B^+\gamma}}=\frac{J_{d,b}[B^{*0},B^0]/m_d+J_{b,d}[B^{*0},B^0]/m_b}{J_{u,b}[B^{*+},B^+]/m_u-2J_{b,u}[B^{*+},B^+]/m_b}.\tag{26}$$

Relations (25) and (26) allow us to examine the coupling constant ratios as a function of the c and b quark mass, respectively. The results appear in Figs. 4 and 5. In order to give actual values we use a c quark mass of 1550 MeV (approximately half the J/ψ mass), yielding $g_{D^{*0}D^0\gamma}/g_{D^{*+}D^+\gamma}=6.47$ and a b quark mass of 4730 MeV (approximately half the Υ mass) which gives $g_{B^{*0}B^0\gamma}/g_{B^{*+}B^+\gamma}=0.018$. We can compare our results with those of other workers. These are presented in Table III. We hoped that our study of $g_{Vq\bar{q}'},g_{Pq\bar{q}'}$ measurements would enable us to make some reasonable guesses of $g_{Vu\bar{c}}g_{Pu\bar{c}}$ and

TABLE III. Summary of theoretical estimates.

Reference	$\lvert g_{D^{*0}D^0\gamma}/g_{D^{*+}D^+\gamma}\rvert$	$\lvert g_{B^{*0}B^0\gamma}/g_{B^{*+}B^+\gamma}\rvert$
[7]	-	-
This paper	6.47	0.018
[10]	3.05 ± 0.63	0.49 ± 0.38
[11]	2.98 ± 0.62	-
[12]	6.32 ± 2.97	0.64 ± 0.51
[13] [a]	11.0 ± 2.3	-
[13] [b]	12.9 ± 2.7	-
[14]	3.28 ± 0.83	-
[15]	5.4 ± 1.1	-
[16]	-	0.58 ± 0.44
[17] [c]	6.95 ± 1.44	-
[18]	6.61 ± 1.37	0.62 ± 0.48
[19]	5.54 ± 3.00	0.59 ± 0.48
[1]	60	0.8 [d]
[20]	3.50 ± 0.73	-
[21]	-	0.678 ± 0.523
[22]	3.84 ± 0.80	-
[23] [e]	1.66 ± 0.34	-
[24] [f]	3.93 ± 0.84	-
[24] [g]	4.49 ± 0.96	-
[24] [h]	3.92 ± 0.84	-
[25]	2.00 ± 0.41	-
[26]	3.78 ± 0.78	-

[a] Gaussian wave function with m_u=300 MeV, m_c = 1500 MeV.
[b] BS wave function with m_u=350.MeV, m_c = 1500 MeV.
[c] $m_u=m_d=300$ MeV, $m_s=500$ MeV.
[d] The original paper contained an error for this calculation. This is the corrected value.
[e] Zero anomalous magnetic moment of the charm quark.
[f] SU(4) symmetry.
[g] Broken SU(4) by $M1$ transition.
[h] Broken SU(4) by $1/M_V^2$.

$g_{Vu\bar{b}}g_{Pu\bar{b}}$, but the data do not allow this. Thus we cannot make predictions about actual decay widths.

IX. CONCLUSIONS

We have successfully evaluated $V \rightarrow P\gamma$ and $P \rightarrow \gamma\gamma$ processes in a quark triangle diagram scheme which is valid for arbitrary vector or pseudoscalar masses. By comparison with available experimental data, we found that this scheme works well for all radiative processes involving the light mesons (no charm or bottom quarks), except for $\phi \rightarrow \pi^0\gamma$ (due to the sensitivity of this channel to the mixing angle) and $\eta' \rightarrow \rho^0(\omega)\gamma$.

The scheme produces well-determined estimates of the meson-quark-antiquark couplings for the light mesons. The large difference between $g_{Vq\bar{q}'}$ and $g_{Pq\bar{q}'}$ indicates a substantial violation of spin symmetry in the quark triangle formalism. We also observed a relatively weak SU(3) chiral symmetry breaking due to the finite masses of the Goldstone-type pseudoscalar mesons, along with a more apparent $\mathrm{SU}(3)_V$ symmetry breakdown arising from the difference in light constituent quark masses. We note that these coupling constants are relatively insensitive to the pseudoscalar mixing angle.

A number of predictions have been made based on the scheme. First we note that our theoretical result for the $\phi \rightarrow \eta'\gamma$ decay width is around the present experimental upper limit and awaits comparison with further measurement. Second our prediction for $g_{D^{*0}D^0\gamma}/g_{D^{*\pm}D^\pm\gamma}=6.47$ with $m_c \approx m_{J/\psi}/2$ is within range of other theoretical estimates, while $g_{B^{*0}B^0\gamma}/g_{B^{*\pm}B^\pm\gamma}=0.018$ for $m_b \approx m_\Upsilon/2$ is small compared with the few results in the literature. We expect that future measurements of these radiative decays will distinguish between these predictions.

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