

***B* decays and models for *CP* violation**

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The decay modes $B \rightarrow \pi\pi$, ψK_S , $K^- D$, πK , and ηK are promising channels to study the unitarity triangle of the CP -violating Cabibbo-Kobayashi-Maskawa (CKM) matrix. In this paper I study the consequences of these measurements in the Weinberg model. I show that using the same set of measurements, the following different mechanisms for CP violation can be distinguished: (1) CP is violated in the CKM sector only; (2) CP is violated spontaneously in the Higgs sector only; and (3) CP is violated in both the CKM and Higgs sectors. [S0556-2821(96)05711-6]

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I. INTRODUCTION

CP violation is one of the unresolved mysteries in particle physics. The explanation in the standard model (SM) based on the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] is still not established; although there is no conflict between the observation of CP violation in the neutral K system [2] and theory [3], intriguing hints of other plausible explanations emerge from consideration of the baryon asymmetry of the universe [4]. Models based on additional Higgs bosons [5,6] can equally well explain the existing laboratory data [7] and provide large CP violation required from baryon asymmetry [4]. It is important to carry out more experiments to find out the origin of CP violation. It is for this reason that exploration of CP violation in the B system is so crucial. The B system offers several final states that provide a rich source for the study of this phenomena [8]. Several methods using B decay modes have been proposed to measure the phase angles, $\alpha = \text{Arg}(-V_{td}V_{tb}^*/V_{ub}^*V_{ud})$, $\beta = \text{Arg}(-V_{cd}V_{cb}^*/V_{tb}^*V_{td})$ and $\gamma = \text{Arg}(-V_{ud}V_{ub}^*/V_{cb}^*V_{cd})$ in the unitarity triangle of the CKM matrix [9–14]. It has been shown that $\bar{B}^0(B^-) \rightarrow \pi^+\pi^-, \pi^0\pi^0(\pi^-\pi^0)$ [11], $\bar{B}^0 \rightarrow \psi K_S$ [12], and $B^- \rightarrow K^- D$ [13] decays can be used to determine α , β , and γ , respectively. Recently it has been shown that $B^- \rightarrow \pi^-\bar{K}^0, \pi^0 K^-, \eta K^-$, and $B^- \rightarrow \pi^-\pi^0$ can also be used to determine γ [14]. If the sum of these three angles is 180° , the SM is a good model for CP violation. Otherwise a new mechanism for CP violation is needed. In this paper I study the consequences of these measurements in the Weinberg model.

In the Weinberg model CP can be violated in the CKM sector and Higgs sector. If CP is violated spontaneously, it occurs in the Higgs sector only. I will refer to the model with CP violation in both the CKM and Higgs sectors as WM-I, and the model with CP violation only in the Higgs sector as WM-II. There are many ways to distinguish the SM and Weinberg model for CP violation. For example the neutron electric dipole moment in the Weinberg model can be several orders of magnitude larger than the SM prediction [15]. However, the neutron electric dipole moment measurement alone cannot distinguish the WM-I from the WM-II. I show that measurements of CP violation in B decays not only can

be used to distinguish the SM from the Weinberg model, but can also be used to determine whether CP is violated in the Higgs sector only or in both the CKM and Higgs sectors.

A. B decay amplitudes in the SM

CP violation in the SM is due to the phase in the CKM mixing matrix in the charged current interaction:

$$L = -\frac{g}{2\sqrt{2}} \bar{U} \gamma^\mu (1 - \gamma_5) V_{\text{CKM}} D W_\mu^+ + \text{H.c.}, \quad (1)$$

where $U = (u, c, t)$, and $D = (d, s, b)$. V_{CKM} is the CKM matrix. For three generations, it is a 3×3 unitary matrix. It has three rotation angles and one nonremovable phase which is the source of CP violation in the SM. I will use the Maiani, Wolfenstein, and Chau and Keung [16] convention for the CKM matrix, in which V_{ub}^* has the phase γ , and V_{td}^* has the phase β and other CKM elements have no or very small phases.

The effective Hamiltonian responsible for $\Delta C = 0$ hadronic $B \rightarrow \pi\pi, \pi K, \eta K, \psi K_S$ decays at the quark level to the one loop level in electroweak interaction can be parametrized as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{uq}^* (c_1 O_{1u}^q + c_2 O_{2u}^q) + V_{cb} V_{cq}^* (c_1 O_{1c}^q + c_2 O_{2c}^q) - \sum_{i=3}^{12} (V_{ub} V_{uq}^* c_i^u + V_{cb} V_{cq}^* c_i^c + V_{tb} V_{tq}^* c_i^t) O_i^q \right], \quad (2)$$

where c_i^f ($f = u, c, t$) are Wilson coefficients (WC's) of the corresponding quark and gluon operators O_i^q . The superscript f indicates the internal quarks. q can be d or s quark

depending on if the decay is a $\Delta S=0$ or $\Delta S=-1$ process. The operators O_i^q are defined as

$$\begin{aligned}
O_{1f}^q &= \bar{q}_\alpha \gamma_\mu L f \bar{f}_\beta \gamma^\mu L b_\alpha, \\
O_{2f}^q &= \bar{q} \gamma_\mu L f \bar{f} \gamma^\mu L b, \\
O_{3(5)} &= \bar{q} \gamma_\mu L b \Sigma \bar{q}' \gamma^\mu L(R) q', \\
O_{4(6)} &= \bar{q}_\alpha \gamma_\mu L b_\beta \Sigma \bar{q}'_\beta \gamma^\mu L(R) q'_\alpha, \\
O_{7(9)} &= \frac{3}{2} \bar{q} \gamma_\mu L b \Sigma e_{q'} \bar{q}' \gamma^\mu R(L) q', \\
O_{8(10)} &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu L b_\beta \Sigma e_{q'} \bar{q}'_\beta \gamma^\mu R(L) q'_\alpha, \\
O_{11} &= \frac{g_s}{32\pi^2} m_b \bar{q} \sigma_{\mu\nu} R T_{ab} G_a^{\mu\nu}, \\
Q_{12} &= \frac{e}{32\pi^2} m_b \bar{q} \sigma_{\mu\nu} R b F^{\mu\nu},
\end{aligned} \tag{3}$$

where $L(R) = (1 \mp \gamma_5)$, and q' is summed over u, d, s , and c quarks. The subscripts α and β are the color indices. T^a is the color SU(3) generator with the normalization $\text{Tr}(T^a T^b) = \delta^{ab}/2$. $G_a^{\mu\nu}$ and $F_{\mu\nu}$ are the gluon and photon field strengths, respectively. O_1, O_2 are the tree level and QCD corrected operators. O_{3-6} are the gluon induced strong penguin operators. O_{7-10} are the electroweak penguin operators due to γ and Z exchange, and ‘‘box’’ diagrams at loop level. The WC's c_{1-10} have been evaluated at the next-to-leading-log QCD corrections [17]. The operators $O_{11,12}$ are the dipole penguin operators. Their WC's have been evaluated at the leading order in QCD correction [18], and their phenomenological implications in B decays have also been studied [19].

One can generically parametrize the decay amplitude of B as

$$\bar{A}_{\text{SM}} = \langle \text{final state} | H_{\text{eff}}^q | B \rangle = V_{fb} V_{fq}^* T(q)^f + V_{tb} V_{tq}^* P(q), \tag{4}$$

where $T(q)$ contains the *tree* and *penguin* diagrams due to internal u and c quark contributions, while $P(q)$ contains *penguin* contributions from internal t and c or u quarks. I use \bar{A} for the decay amplitude of B meson containing a b quark, and A for a B meson containing a \bar{b} quark. The WC's involved in T are much larger than the ones in P . One expects the hadronic matrix elements arising from quark operators to be the same order of magnitudes. The relative strength of the amplitudes T and P are predominantly determined by their corresponding WC's in the effective Hamiltonian. In general $|P|$, if not zero, is about or less than 10% of $|T|$.

For $\bar{B}^0 \rightarrow \psi K_S$, the decay amplitude can be written as

$$\begin{aligned}
\bar{A}_{\text{SM}}(\psi K) &= V_{cb} V_{cs}^* T_{\psi K} + V_{tb} V_{ts}^* P_{\psi K} \\
&= |V_{cb} V_{cs}| (T_{\psi K} - P_{\psi K}) + |V_{ub} V_{us}^*| e^{-i\gamma} P_{\psi K}.
\end{aligned} \tag{5}$$

The second term is about 10^3 times smaller than the first term and can be safely neglected. To this level, the decay amplitude for $\bar{B}^0 \rightarrow \psi K_S$ does not contain a weak

CP -violating phase. This decay mode provides a clean way to measure the phase angle β in the SM [12].

One can parametrize the decay amplitudes for $B \rightarrow \pi\pi, K\pi, \eta K$ in a similar way. Further if flavor SU(3) symmetry is a good symmetry there are certain relations among the decay amplitudes [20]. I will assume the validity of the SU(3) symmetry in my later analysis. The operators $Q_{1,2}^u, O_{1,2}^c, O_{3-6,11,12}$, and O_{7-10} transform under SU(3) symmetry as $\bar{\mathbf{3}}_a + \bar{\mathbf{3}}_b + \mathbf{6} + \mathbf{15}, \bar{\mathbf{3}}, \bar{\mathbf{3}}$, and $\bar{\mathbf{3}}_a + \bar{\mathbf{3}}_b + \mathbf{6} + \mathbf{15}$, respectively. Flavor SU(3) symmetry predicts

$$\sqrt{2}\bar{A}(\pi^0\pi^0) + \sqrt{2}\bar{A}(\pi^-\pi^0) = \bar{A}(\pi^+\pi^-), \tag{6}$$

$$\sqrt{2}\bar{A}(\pi^0K^-) - 2\bar{A}(\pi^-\bar{K}^0) = \sqrt{6}\bar{A}(\eta_8K^-). \tag{7}$$

Isospin symmetry also implies Eq. (6). These relations form two triangles in the complex plan which provide important information for obtaining phase angles α and γ [11,14].

I parametrize the decay amplitudes in the SM as

$$\begin{aligned}
\bar{A}_{\text{SM}}(\pi^-\pi^0) &= |V_{ub} V_{ud}^*| e^{-i\gamma} T_{\pi^-\pi^0} + |V_{tb} V_{td}^*| e^{i\beta} P_{\pi^-\pi^0}, \\
\bar{A}_{\text{SM}}(\pi^+\pi^-) &= |V_{ub} V_{ud}^*| e^{-i\gamma} T_{\pi^+\pi^-} + |V_{tb} V_{td}^*| e^{i\beta} P_{\pi^+\pi^-}, \\
\bar{A}_{\text{SM}}(K^-\pi^0) &= |V_{ub} V_{us}^*| e^{-i\gamma} T_{K^-\pi^0} + |V_{tb} V_{ts}^*| P_{K^-\pi^0}, \\
\bar{A}_{\text{SM}}(\bar{K}^0\pi^-) &= |V_{ub} V_{us}^*| e^{-i\gamma} T_{\bar{K}^0\pi^-} + |V_{tb} V_{ts}^*| P_{\bar{K}^0\pi^-}.
\end{aligned} \tag{8}$$

The decay amplitudes $\bar{A}_{\text{SM}}(\pi^0\pi^0)$ and $\bar{A}_{\text{SM}}(K^-\eta)$ are obtained by the SU(3) relations in Eqs. (6) and (7).

I would like to point out that $\bar{A}_{\text{SM}}(\pi^-\pi^0)$ and $\sqrt{2}\bar{A}(K^-\pi^0) - \bar{A}(\bar{K}^0\pi^-)$ only receive contributions from the effective operators which transform as $\bar{\mathbf{15}}$ [14,22]:

$$\bar{A}_{\text{SM}}(\pi^-\pi^0) = V_{ub} V_{ud}^* C_{\bar{\mathbf{15}}}^T + V_{tb} V_{td}^* C_{\bar{\mathbf{15}}}^P, \tag{9}$$

$$\bar{A}_{\text{SM}}(K^-\pi^0) - \bar{A}_{\text{SM}}(\bar{K}^0\pi^-) / \sqrt{2} = V_{ub} V_{us}^* C_{\bar{\mathbf{15}}}^T + V_{tb} V_{ts}^* C_{\bar{\mathbf{15}}}^P,$$

where $C_{\bar{\mathbf{15}}}$ is the invariant amplitude due to operators that transform as $\bar{\mathbf{15}}$ under SU(3) symmetry. This is an important property useful for my later discussions. The second term in $\bar{A}_{\text{SM}}(\pi^-\pi^0)$ is less than 3% of the first term [21]. For all practical purposes it can be neglected. However, the second term on the right-hand side of the second equation in Eq. (9) cannot be neglected because there is an enhancement factor $|V_{tb} V_{ts}^*| / |V_{ub} V_{us}^*|$ which is about 50 [23].

The effective Hamiltonian responsible for $B \rightarrow DK$ decay is given by

$$\begin{aligned}
H_{\text{eff}} &= \frac{G_F}{\sqrt{2}} [V_{ub} V_{cs}^* (c_1 \bar{u}^\alpha \gamma_\mu L b_\beta \bar{s}^\beta \gamma^\mu L c_\alpha + c_2 \bar{u} \gamma_\mu L b \bar{s} \gamma^\mu L c) \\
&\quad + V_{cb} V_{us}^* (c_1 \bar{c}^\alpha \gamma_\mu L b_\beta \bar{s}^\beta \gamma^\mu L u_\alpha + c_2 \bar{c} \gamma_\mu L b \bar{s} \gamma^\mu L u)].
\end{aligned} \tag{10}$$

The decay amplitudes for $B^- \rightarrow K^- D^0$ and $B^- \rightarrow K^- \bar{D}^0$, respectively, are given by

$$\begin{aligned}\bar{A}_{\text{SM}}(K^- D^0) &= |V_{ub} V_{cs}^*| a_{KD} e^{-i\gamma}, \\ \bar{A}_{\text{SM}}(K^- \bar{D}^0) &= |V_{cb} V_{us}^*| b_{KD}.\end{aligned}\quad (11)$$

From the above, one easily obtains the decay amplitude for $B^- \rightarrow K^- D_{CP}$ with $D_{CP} = (D^0 - \bar{D}^0)/\sqrt{2}$ being the CP even eigenstate:

$$\bar{A}_{\text{SM}}(K^- D_{CP}) = \frac{1}{\sqrt{2}} [\bar{A}_{\text{SM}}(K^- D^0) - \bar{A}_{\text{SM}}(K^- \bar{D}^0)]. \quad (12)$$

This relation forms a triangle in the complex plane which is useful in determining the phase angle γ in the SM [13].

B. B decay amplitudes in the Weinberg model

In the Weinberg model, besides the CP -violating phase in the CKM matrix, CP violation for hadronic B decays can also arise from the exchange of a charged Higgs boson at the tree and loop levels, and also a neutral Higgs boson at loop levels. In this model, there are two physical charged Higgs particles and three neutral Higgs particles. The neutral Higgs couplings to fermions are flavor conserving and proportional to the fermion masses. The flavor-changing decay amplitude can only be generated at the loop level. For the cases in consideration, all involving light fermions, the CP -violating amplitude generated by neutral Higgs exchange is very small and can be neglected. The exchange of charged Higgs bosons may generate sizable CP -violating decay amplitudes, however. The charged Higgs couplings to fermions are given by [25]

$$\begin{aligned}L &= 2^{7/4} G_F^{1/2} \bar{U} [V_{\text{KM}} M_D (\alpha_1 H_1^+ + \alpha_2 H_2^+) R \\ &+ M_U V_{\text{KM}} (\beta_1 H_1^+ + \beta_2 H_2^+) L] D + \text{H.c.},\end{aligned}\quad (13)$$

where $M_{U,D}$ are the diagonal up and down quark mass matrices. The parameters α_i and β_i are obtained from diagonalizing charged Higgs boson masses and can be written as

$$\begin{aligned}\alpha_1 &= s_1 c_3 / c_1, \alpha_2 = s_1 s_3 / c_1, \\ \beta_1 &= (c_1 c_2 c_3 + s_2 s_3 e^{i\delta_H}) / s_1 c_2, \\ \beta_2 &= (c_1 c_2 s_3 - s_2 c_3 e^{i\delta_H}) / s_1 c_2,\end{aligned}\quad (14)$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$ with θ_i being the rotation angles, and δ_H is a CP -violating phase. The decay amplitudes due to exchange of a charged Higgs boson at the tree level will be proportional to $V_{fb} V_{f'q}^* (m_b m_{f'} / m_{H_i}^2) \alpha_i \beta_i^*$. Therefore if a decay involves light quark, the amplitude will be suppressed. However, at the one loop level if the internal quark masses are large, sizable CP -violating decay amplitude may be generated. The leading term is from the strong dipole penguin interaction with the top quark in the loop [26]:

$$L_{\text{DP}} = \tilde{f} O_{11},$$

$$\begin{aligned}\tilde{f} &= \frac{G_F}{16\sqrt{2}} \sum_i^2 \alpha_i^* \beta_i V_{tb} V_{tq}^* \frac{m_t^2}{m_{H_i}^2 - m_t^2} \\ &\times \left(\frac{m_{H_i}^4}{(m_{H_i}^2 - m_t^2)^2} \ln \frac{m_{H_i}^2}{m_t^2} - \frac{m_{H_i}^2}{m_{H_i}^2 - m_t^2} - \frac{1}{2} \right).\end{aligned}\quad (15)$$

This is not suppressed compared with the penguin contributions in the SM. There is also a similar contribution from the operator O_{12} . However the contribution from this operator is suppressed by a factor of $\alpha_{\text{em}}/\alpha_s$ and can be neglected. I write the O_{11} contribution to B decays as

$$\bar{A}_{\text{final}} = V_{tb} V_{tq}^* a_{\text{final}} e^{i\alpha_H}, \quad (16)$$

where α_H is the phase in \tilde{f} which is decay mode independent, and $a_{\text{final}} = |\tilde{f}| \langle \text{final state} | O_{11} | B \rangle$ which is decay mode dependent. Note that L_{DP} transforms as $\bar{3}$ under $SU(3)$ symmetry. It does not contribute to $\bar{A}(\pi^- \pi^0)$ and $\sqrt{2} \bar{A}(K^- \pi^0) - \bar{A}(\bar{K}^0 \pi^-)$.

The decay amplitudes in the Weinberg model can be written as

$$\bar{A}_W(\pi^+ \pi^-) = \bar{A}_{\text{SM}}(\pi^+ \pi^-) + V_{tb} V_{td}^* e^{i\alpha_H} a_{\pi\pi},$$

$$\bar{A}_W(\pi^- \pi^0) = \bar{A}_{\text{SM}}(\pi^- \pi^0),$$

$$\bar{A}_W(K^- \pi^0) = \bar{A}_{\text{SM}}(K^- \pi^0) + \frac{1}{\sqrt{2}} V_{tb} V_{ts}^* e^{i\alpha_H} a_{K\pi},$$

$$\bar{A}_W(\bar{K}^0 \pi^-) = \bar{A}_{\text{SM}}(\bar{K}^0 \pi^-) + V_{tb} V_{ts}^* e^{i\alpha_H} a_{K\pi},$$

$$\bar{A}_W(\psi K_S) = \bar{A}_{\text{SM}}(\psi K_S) + V_{tb} V_{ts}^* e^{i\alpha_H} a_{\psi K}. \quad (17)$$

In the $SU(3)$ limit $a_{\pi\pi} = a_{K\pi}$.

The decay amplitudes for $B \rightarrow KD$ only have contributions from tree operators. Because the CP -violating amplitude from the tree level charged Higgs boson exchange is negligibly small, to a good approximation:

$$\bar{A}_W(KD) = \bar{A}_{\text{SM}}(KD). \quad (18)$$

The decay amplitudes for both the WM-I and WM-II have the same form given in Eqs. (17) and (18). In the WM-I CP is violated in both the CKM and Higgs sectors with $\alpha\beta\gamma\alpha_H \neq 0$. In the WM-II CP is violated only in the Higgs sector with $\alpha = \beta = \gamma = 0$, but $\alpha_H \neq 0$. I will drop the asterisk of the CKM matrix elements in the WM-II.

II. CP VIOLATION IN B DECAYS

A. $B \rightarrow \pi\pi$ decays

In the time evolution of the rate asymmetry for $\bar{B}^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow \pi^- \pi^+$, there are two terms varying with time, one varies as a cosine function and the other as a

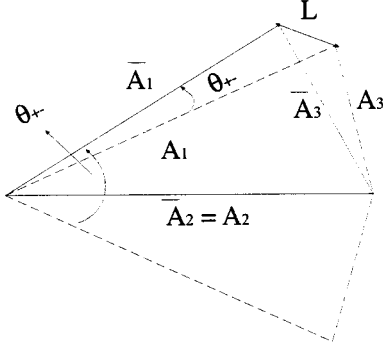


FIG. 1. The isospin triangle for $B \rightarrow \pi\pi$. In the SM and WM-I, $\bar{A}_1 = e^{i\gamma} \bar{A}_{SM,W}(\pi^+ \pi^-)$, $\bar{A}_2 = \sqrt{2} e^{i\gamma} \bar{A}_{SM,W}(\pi^- \pi^0)$, $\bar{A}_3 = \sqrt{2} e^{i\gamma} \bar{A}_{SM,W}(\pi^0 \pi^0)$, and $A_1 = e^{-i\gamma} A_{SM,W}(\pi^- \pi^+)$, $A_2 = \sqrt{2} e^{-i\gamma} A_{SM,W}(\pi^+ \pi^0)$, $A_3 = \sqrt{2} e^{-i\gamma} A_{SM,W}(\pi^0 \pi^0)$. In the WM-II, the same as for the WM-I except that all weak phases α , β , and γ are zero.

sine function. The coefficients of these two terms can be measured experimentally. The coefficient of the sine term is given by [12]

$$\text{Im}\lambda = \text{Im}\left(\frac{q}{p} \frac{\bar{A}(\pi^+ \pi^-)}{A(\pi^- \pi^+)}\right), \quad (19)$$

where p and q are the mixing parameters defined by

$$|B_H\rangle = p|\bar{B}^0\rangle + q|B^0\rangle, \quad |B_L\rangle = q|\bar{B}^0\rangle - p|B^0\rangle, \quad (20)$$

where $|B_{H,L}\rangle$ are the heavy and light mass eigenstates, respectively.

In the SM, the mixing is dominated by the top quark loop in the box diagram, and

$$\text{Im}\lambda = \text{Im}\left(e^{-2i(\beta+\gamma+\beta_H)} \frac{\bar{A}_{SM}(\pi^+ \pi^-) + V_{tb} V_{td}^* e^{i(\alpha_H+\gamma)} a_{\pi\pi}}{A_{SM}(\pi^+ \pi^-) + V_{tb} V_{td}^* e^{-i(\alpha_H+\gamma)} a_{\pi\pi}}\right) = \frac{|\bar{A}_W(\pi^+ \pi^-)|}{|A_W(\pi^- \pi^+)|} \sin(2\alpha - 2\beta_H + \theta_{+-}^H). \quad (24)$$

This equation has the same form as Eq. (22) for the SM. The determination of $\alpha - \beta_H$ is exactly the same as α in the SM except that in this case $\bar{A}_1 = e^{i\gamma} \bar{A}_W(\pi^+ \pi^-)$ and $A_1 = e^{-i\gamma} A_W(\pi^- \pi^+)$. The phase β_H can be neglected because it is suppressed by a factor of $m_b m_t / m_H^2$. The measurement proposed here still measures α even though there is an additional CP -violating phase in $\bar{B}^0 \rightarrow \pi^+ \pi^- (\pi^0 \pi^0)$ decay amplitudes.

If CP is violated spontaneously, the result will be dramatically different. Here the CP -violating weak phases in \bar{A}_{SM} are all zero. The amplitude $\bar{A}_2 = \sqrt{2} \bar{A}_W(\pi^- \pi^0)$ is equal to $A_2 = \sqrt{2} A_W(\pi^+ \pi^0)$, and can be normalized to be real. Now using the isospin triangle in Fig. 1, one easily obtains the phase in $\bar{A}_W(\pi^+ \pi^-)/A_W(\pi^- \pi^+)$, and therefore determines the phase angle β_H . One would obtain a very small value. This will be a test for spontaneous CP -violation model WM-II.

Using the isospin triangle in Fig. 1, the CP -violating am-

$$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = e^{-2i\beta}. \quad (21)$$

One obtains

$$\begin{aligned} \text{Im}\lambda &= \text{Im}\left(e^{-2i(\beta+\gamma)} \frac{e^{i\gamma} \bar{A}_{SM}(\pi^+ \pi^-)}{e^{-i\gamma} A_{SM}(\pi^- \pi^+)}\right) \\ &= \text{Im}\left(e^{2i\alpha} \frac{|V_{ub} V_{ud}^*| T_{\pi^+ \pi^-} + |V_{tb} V_{td}^*| P_{\pi^+ \pi^-} e^{i(\beta+\gamma)}}{|V_{ub} V_{ud}^*| T_{\pi^+ \pi^-} + |V_{tb} V_{td}^*| P_{\pi^+ \pi^-} e^{-i(\beta+\gamma)}}\right) \\ &= \frac{|\bar{A}_{SM}(\pi^+ \pi^-)|}{|A_{SM}(\pi^- \pi^+)|} \sin(2\alpha + \theta_{+-}). \end{aligned} \quad (22)$$

The ratio $|\bar{A}_{SM}|/|A_{SM}|$ can be determined from time integrated rate asymmetry at symmetric [24] and asymmetric colliders [8]. If θ_{+-} can be determined, the phase angle α can be determined. To determine θ_{+-} , Gronau and London [11] proposed to use the isospin relation in Eq. (6),

$$\sqrt{2} \bar{A}(\pi^0 \pi^0) + \sqrt{2} \bar{A}(\pi^- \pi^0) = \bar{A}(\pi^+ \pi^-), \quad (23)$$

and normalize the amplitudes $\bar{A}_2 = \sqrt{2} e^{i\gamma} A_{SM}(\pi^- \pi^0)$ and $A_2 = \sqrt{2} e^{-i\gamma} A_{SM}(\pi^+ \pi^0)$ on the real axis. The triangle is shown in Fig. 1. It is easy to see from Eq. (22) that the angle θ_{+-} is given by phase angle difference between $\bar{A}_1 = e^{i\gamma} \bar{A}_{SM}(\pi^+ \pi^-)$ and $A_1 = e^{-i\gamma} A_{SM}(\pi^- \pi^+)$. It can be easily read off from Fig. 1.

In the Weinberg model, a similar measurement will obtain a different result. In the WM-I, in addition to the phase β , there is also a phase β_H in q/p due to charged Higgs exchange in the box diagram. One obtains $q/p = e^{-2i(\beta+\beta_H)}$, and

plitude $|a_{\pi\pi}|^2 \sin^2 \alpha_H$ in the WM-II can also be determined. It is given by

$$|a_H|^2 \sin^2 \alpha_H = \frac{L^2}{4|V_{tb} V_{td}|^2}. \quad (25)$$

This measurement will also serve as a test for the WM-II. I will come back to this later.

B. $B \rightarrow \psi K_S$ decay

In the SM, the cleanest way to measure β is to measure the parameter $\text{Im}\lambda$ for $\bar{B}^0 \rightarrow \psi K_S$ decay [12]. In this case,

$$\text{Im}\lambda_{\psi K} = \text{Im}\left(\frac{q}{p} \frac{\bar{A}_{SM}(\psi K_S)}{A_{SM}(\psi K_S)}\right). \quad (26)$$

Neglecting the small term proportional to $V_{cb} V_{cs}^*$, one obtains

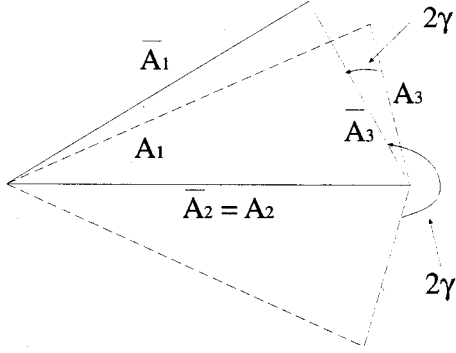


FIG. 2. The triangle relation for $B \rightarrow KD$. $\bar{A}_1 = \sqrt{2}\bar{A}_{SM,W}(K^- D_{CP})$, $\bar{A}_2 = \bar{A}_{SM,W}(K^- \bar{D}^0)$, $\bar{A}_3 = \bar{A}_{SM,W}(K^- D^0)$, and $A_1 \sqrt{2}A_{SM,W}(K^+ D_{CP})$, $A_2 = A_{SM,W}(K^+ D^0)$, $A_3 = A_{SM,W}(K^+ \bar{D}^0)$.

$$\text{Im}\lambda_{\psi K} = \text{Im}\left(\frac{q}{p} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) = -\sin(2\beta). \quad (27)$$

This is a very clean way to measure the phase angle β in the SM.

In the Weinberg model, the same measurement will give a different result. In the WM-I, one has

$$\text{Im}\lambda_{\psi K} = \text{Im}\left(e^{-2i(\beta+\beta_H)} \frac{\bar{A}_{SM}(\psi K_S) + V_{tb} V_{ts}^* e^{i\alpha_H} a_{\psi K}}{A_{SM}(\psi K_S) + V_{tb} V_{ts}^* e^{-i\alpha_H} a_{\psi K}}\right). \quad (28)$$

The amplitude from the new contribution proportional to $a_{\psi K}$ is expected to be about 10% of the SM contribution. Even though β_H is small, $\text{Im}\lambda_{\psi K}$ in the WM-I will be different from $-\sin(2\beta)$. This measurement alone will not be able to distinguish the SM and WM-I. However, combining the result from this measurement and knowledge about α determined from the previous section and γ to be determined in the next section, one can distinguish the SM from the WM-I. If the SM is the correct model, the phase angles α , γ , and β will add up to 180° . However, if the WM-I is the right model and one naively interprets $\text{Im}\lambda_{\psi K_S}$ to be $-\sin(2\beta)$, the sum of the three phase angles will not be 180° .

In the WM-II, the measurement $\text{Im}\lambda_{\psi K_S}$ will only measure the phase difference between $\bar{A}(\psi K_S)$ and $A(\psi K_S)$ which is smaller than the value for the SM and WM-I. A small experimental value for $\text{Im}\lambda_{\psi K}$ is an indication for the WM-II.

C. $B^- \rightarrow K^- D$ decays

In the SM and WM-I, the triangle relation

$$A(K^- D^0) - A(K^- \bar{D}^0) = \sqrt{2}A(K^- D_{CP}), \quad (29)$$

provides a measurement for the phase angle γ . The phase γ is given as shown in Fig. 2 [13].

In the WM-II, there are no CP -violating weak phase angles in these decay amplitudes. The triangles for the particle decays and antiparticle decays will be identical. One should be aware that in the SM and WM-I if the strong rescattering phases are all zero the triangles for the particle and antiparticle decays will also be identical, one must put

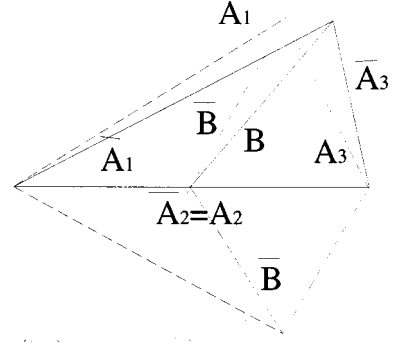


FIG. 3. The triangle relation for $B \rightarrow K\pi, K\eta$ in the SM. $\bar{A}_1 = \sqrt{2}\bar{A}_{SM}(K^- \pi^0)$, $\bar{A}_2 = 2\bar{A}_{SM}(\bar{K}^0 \pi^-)$, $\bar{A}_3 = \sqrt{6}\bar{A}_{SM}(K^- \eta_8)$, and $A_1 = \sqrt{2}A_{SM}(K^+ \pi^0)$, $A_2 = 2A_{SM}(K^0 \pi^+)$, $A_3 = \sqrt{6}A_{SM}(K^+ \eta_8)$.

the two triangles on the opposite side as shown in Fig. 2 to determine the value for γ . However, if the two triangles for the particle and antiparticle decays are not identical, the WM-II is ruled out.

D. $B^- \rightarrow \pi K, \eta K$ decays

Another method to measure the phase angle γ is to use the following B decays: $B^- \rightarrow \pi^- \bar{K}^0$, $\pi^0 K^-$, ηK^- , and $B^- \rightarrow \pi^- \pi^0$ [14]. This method requires the construction of the triangle mentioned in Eq. (7):

$$\sqrt{2}\bar{A}(K^- \pi^0) - 2\bar{A}(\bar{K}^0 \pi^-) = \sqrt{6}\bar{A}(K^- \eta_8). \quad (30)$$

In the spectator model, the contributions to $\bar{A}(\bar{K}^0 \pi^-)$ from the tree operators vanish [27]. To a good approximation, one has

$$\bar{A}_{SM}(\bar{K}^0 \pi^-) = A_{SM}(K^0 \pi^+). \quad (31)$$

These amplitudes do not have weak phases. They can be normalized to be real. From Fig. 3, one can determine the two amplitudes B and \bar{B} which are given by

$$\begin{aligned} B &= \sqrt{2}\bar{A}_{SM}(K^- \pi^0) - \bar{A}_{SM}(\bar{K}^0 \pi^-), \\ \bar{B} &= \sqrt{2}A_{SM}(K^+ \pi^0) - A_{SM}(K^0 \pi^+). \end{aligned} \quad (32)$$

Using the SU(3) relation in Eq. (9), one obtains

$$B - \bar{B} = -i2\sqrt{2}e^{i\delta^T} \frac{|V_{us}|}{|V_{ud}|} |\bar{A}_{SM}(\pi^- \pi^0)| \sin\gamma. \quad (33)$$

The angle δ^T denotes the strong final state rescattering phase of the tree amplitude of B (or \bar{B}). It is clear that $\sin\gamma$ can be determined [14].

In the WM-I, the result will be different. In this case even in the spectator model, $\bar{A}_W(\bar{K}^0 \pi^-)$ is not equal to $A_W(K^0 \pi^+)$ because the new contribution is proportional to $V_{tb} V_{ts}^* a_{K\pi} e^{i\alpha_H}$. There is no common side for the triangles for the particle and antiparticle decay amplitudes. No useful information about the phase γ can be obtained. However, if experiments will find $\bar{A}(\bar{K}^0 \pi^-) \neq A(K^0 \pi^+)$, it indicates that the SM may not be correct.

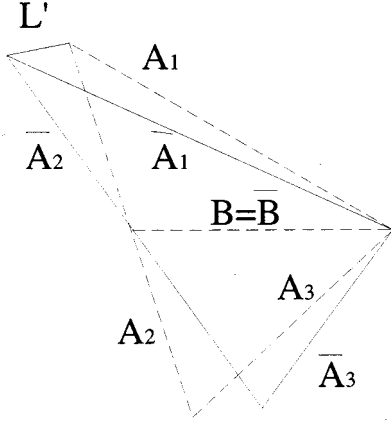


FIG. 4. The triangle relation for $B \rightarrow K\pi, K\eta$ in the WM-II. $\bar{A}_1 = \sqrt{2}A_W(K^0\pi^-)$, $\bar{A}_2 = 2\bar{A}_W(K^-\pi^0)$, $\bar{A}_3 = \sqrt{6}A_W(K^-\eta_8)$, and $A_1 = \sqrt{2}A_W(K^0\pi^+)$, $A_2 = 2A_W(K^+\pi^0)$, $A_3 = \sqrt{6}A_{SM}(K^+\eta_8)$.

In the WM-II, the analysis is again very different. In the analysis for the SM, the decay amplitudes for $\bar{A}(K^0\pi^-)$ and $A_{SM}(K^0\pi^+)$ are normalized to be real. In the WM-II because of the additional term $V_{tb}V_{ts}e^{i\alpha_H}a_{K\pi}$, one can no longer use this normalization. One needs to find amplitudes which can serve as the orientation axis. To this end I note that the amplitude B and \bar{B} in the above only receive contributions from operators in the effective Hamiltonian transforming as $\bar{\mathbf{15}}$ under SU(3). The strong dipole penguin, which transforms as $\bar{\mathbf{3}}$, does not contribute, and therefore in the WM-II, $B = \bar{B}$. One can normalize the triangles by putting B and \bar{B} on the real axis as shown in Fig. 4. The phases of the rest decay amplitudes can be easily read off from the figure. One particularly interesting amplitude is $|a_{K\pi}|^2 \sin^2 \alpha_H$. From Fig. 4, one obtains

$$|a_{K\pi}|^2 \sin^2 \alpha_H = \frac{L'^2}{4|V_{tb}V_{ts}^*|^2}. \quad (34)$$

Several comments on these measurements are in order. (1) In the SM, $\bar{A}(K^0\pi^-) = A(K^0\pi^+)$. This is not generally true in the Weinberg model. This can be used to test the SM. (2) In the WM-II, $B = \bar{B}$. This is a test for WM-II. In the SM and WM-I this happens only when the strong rescattering phases are zero which is, however, unlikely. (3) If SU(3) is a good symmetry, the quantity $|a_{K\pi}|^2 \sin^2 \alpha_H$ is equal to $|a_{\pi\pi}|^2 \sin^2 \alpha_H$ obtained in Eq. (25). This will serve as a test for the WM-II.

E. Rate differences in $\bar{B}^0 \rightarrow \pi^+\pi^-$ and $\bar{B}^0 \rightarrow K^-\pi^+$

In this section I comment on rate differences in $\bar{B}^0 \rightarrow \pi^+\pi^-$ and $\bar{B}^0 \rightarrow K^-\pi^+$.

The decay amplitude for $\bar{B}^0 \rightarrow K^-\pi^+$ can be written as

$$A(K^-\pi^+) = V_{ub}V_{us}^*T_{K^-\pi^+} + V_{tb}V_{ts}^* \times \left(P_{K^-\pi^+} + \frac{1}{\sqrt{2}}e^{i\alpha_H}a_{K\pi} \right). \quad (35)$$

It has been shown that in the SU(3) limit, $T(P)_{\pi^+\pi^-} = T(P)_{K^-\pi^+}$, and $a_{\pi\pi} = a_{K\pi}$ [22]. Here $T(P)_{\pi^+\pi^-}$ and $a_{\pi\pi}$ are the corresponding amplitudes in $\bar{A}_{SM}(\pi^+\pi^-)$ given in Eq. (8). In the SM,

$$\begin{aligned} \Delta_{\pi\pi} &= \Gamma(\pi^+\pi^-) - \bar{\Gamma}(\pi^-\pi^+) \\ &= -\text{Im}(V_{ub}V_{ud}^*V_{tb}V_{td}^*)\text{Im}(T_{\pi^+\pi^-}P_{\pi^+\pi^-}^*) \frac{m_B \Lambda_{\pi\pi}}{4\pi}, \\ \Delta_{K\pi} &= \Gamma(K^-\pi^+) - \bar{\Gamma}(K^+\pi^-) \\ &= -\text{Im}(V_{ub}V_{ud}^*V_{tb}V_{td}^*)\text{Im}(T_{\pi^+K^-}P_{\pi^+K^-}^*) \frac{m_B \Lambda_{\pi K}}{4\pi}, \end{aligned} \quad (36)$$

where $\Lambda_{ab} = \sqrt{1 - 2(m_a^2 + m_b^2)/m_B^2 + (m_a^2 - m_b^2)^2/m_B^4}$. One obtains [22]

$$\frac{\Delta(\pi^+\pi^-)}{\Delta(\pi^+K^-)} = -1. \quad (37)$$

When SU(3) breaking effects are included,

$$\frac{\Delta(\pi^+\pi^-)}{\Delta(\pi^+K^-)} \approx -\frac{f_\pi^2}{f_K^2}. \quad (38)$$

The ratio is negative and of order one. In the Weinberg model, the prediction is very different. In the WM-I, the situation is complicated. It is difficult to obtain useful information about CP-violating parameters. In the simpler case, the WM-II,

$$\begin{aligned} \frac{\Delta(\pi^+\pi^-)}{\Delta(\pi^+K^-)} &= \frac{V_{td}V_{ub}V_{ud}\text{Im}(T_{\pi^+\pi^-}a_H^*) + V_{tb}V_{td}\text{Im}(P_{\pi^+\pi^-}a_H^*)}{V_{ts}V_{ub}V_{us}\text{Im}(T_{\pi^+K^-}a_H^*) + V_{tb}V_{ts}\text{Im}(P_{\pi^+K^-}a_H^*)}. \end{aligned} \quad (39)$$

This is very different from the SM prediction. The ratio can vary a large range. If the first term dominates, $\Delta(\pi^+\pi^-)/\Delta(\pi^+K^-) = V_{td}V_{ud}/V_{ts}V_{us}$ which is of order one. If the second term dominates, the ratio is given by V_{td}^2/V_{ts}^2 which is positive, but very small.

III. CONCLUSION

I have analyzed the consequences of several methods for measuring CP-violating observables. These measurements can be used to distinguish different models for CP violation.

The isospin triangle relation among $\bar{B}^0 \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ and $B^- \rightarrow \pi^-\pi^0$ provides critical information to correct strong penguin contamination in the determination of the phase angle α . The same measurement also determine the

phase angle α in the WM-I where CP is violated both in the CKM and Higgs sectors. In the WM-II where CP is violated spontaneously in the Higgs sector, one would obtain a very different value. The WM-II will be tested.

The asymmetry in time evolution for $\bar{B}^0 \rightarrow \psi K_S$ is an idea place to determine β in the SM. This is, however, not true for the WM-I because of the contamination from the Higgs boson induced strong dipole penguin operator. This measurement does not measure the true value of β in this case. In the WM-II the resulting $\text{Im}\lambda_{\psi K}$ is very small. This measurement again provides a test for the WM-II.

The triangle relation among $B^- \rightarrow K^- \bar{D}^0, K^0 D^0, K^- D_{CP}^0$ provides a clean way to measure the phase angle γ in both the SM and WM-I. Combining these measurements and the previous measurements for α and β , it is possible to distinguish the SM and WM-I because in the SM the true β is measured in $\bar{B}^0 \rightarrow \psi K_S$, whereas in the WM-I the measurement is contaminated. If the sum of the three phase angles is 180° , the SM is the correct one. If the WM-II is the correct model, experiments will find the triangles for particle and

antiparticle decays to be identical.

The method to measure γ using $B \rightarrow K\pi, K\eta, \pi\pi$ will provide different results for the three models. In principle the three models of CP violation can be distinguished. This analysis is based on a triangle relation obtained in $SU(3)$ symmetry. The validity of $SU(3)$ flavor symmetry has not been established. One should be careful when carrying out this analysis. $SU(3)$ breaking effects may change the results. Detailed study is needed.

Another way to distinguish models for CP violation is to use the rate asymmetry $\Delta(\pi^+ \pi^-)$ and $\Delta(\pi^+ K^-)$. The SM and Weinberg model have very different predictions.

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