Phenomenology of the $1/m_Q$ expansion in inclusive B and D meson decays

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We apply a recent theoretical analysis of hadronic observables in inclusive semileptonic heavy hadron decays to the phenomenology of B and D mesons. Correlated bounds on the nonperturbative parameters $\bar{\Lambda}$ and λ_1 are derived by considering data from B decays and, independently, data from D decays. The two sets of bounds are found to be consistent with each other. The data from B decays then are used to extract a lower limit on the CKM matrix element $|V_{cb}|$. We address the issue of the convergence of the perturbative expansions used in the analysis, and compare our bounds on $\bar{\Lambda}$ and λ_1 to lattice and QCD sum rule results. Finally, we argue that a comparison of the analyses of D and D_s decays provides evidence for the applicability of parton-hadron duality in the semileptonic decay of charmed hadrons. [S0556-2821(96)06311-4]

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I. INTRODUCTION

The heavy quark limit of QCD is of enormous practical use, because with it one may describe a wide variety of heavy hadron decay rates and matrix elements in terms of a small number of parameters. These parameters reflect nonperturbative QCD effects and cannot be computed directly. Instead, they must either be modeled or, preferably, be extracted from experimental data. One of the most important applications of the analysis of inclusive decays is the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cb}|$ from the process $B \to X_c \ell \nu$, which is complementary to the extraction from the exclusive decay $B \to D^* \ell \nu$. The computation involves an expansion in powers of $1/m_b$, and to $O(1/m_b^2)$ there appear three nonperturbative parameters: $\bar{\Lambda}$ (or equivalently, the quark mass m_b), λ_1, and λ_2. While λ_2 may be extracted directly from the B-B^* mass splitting, $\bar{\Lambda}$ and λ_1 are not directly measurable. Two approaches currently popular in the literature are to employ various QCD sum rules to estimate these parameters and to use an analysis of inclusive semileptonic D decay to fix one linear combination of them. However, each of these methods has a severe disadvantage: The QCD sum rule results are not model-independent consequences of QCD, and the expansions in $\alpha_s(m_c)$ and $1/m_c$ may or may not work well at the low scales relevant for D decay [1,2].

In a recent analysis [3], we calculated the leading perturbative and nonperturbative contributions to moments of the hadronic energy and invariant mass spectra in semileptonic heavy hadron decays. These predictions are particularly interesting, because experimental information on invariant masses of the hadrons produced in these decays may be derived from the reported branching ratios to exclusive final states. Furthermore, they rest on the same theoretical basis as earlier analyses of other semileptonic quantities such as the decay rate and the lepton energy spectrum [4].

In Ref. [3] we performed some preliminary phenomenology based on this theoretical analysis, deriving correlated bounds on the nonperturbative parameters $\bar{\Lambda}$ and λ_1. In this paper we will develop this phenomenology further, incorporating additional data and including in the discussion the semileptonic decays of charmed mesons. Our main conclusions are as follows.

(1) The perturbation series appearing in the analysis are under better control than had previously been thought. While the two-loop corrections relating $\bar{\Lambda}$ and λ_1 to the semileptonic decay rate and to the first moment of the invariant mass spectrum in $B \to X_c e \bar{\nu}$ are large, these corrections partially cancel in the relation between the semileptonic decay rate and the first moment of the invariant mass spectrum. The corresponding perturbation series relating the two physical quantities appears to be better behaved. Using the scale setting technique of Brodsky, Lepage, and Mackenzie (BLM) [5], we find a BLM scale $\mu_{\mathrm{BLM}} = 0.26 m_b$ for the relation between the first moment and the semileptonic width. This extends our previous result [3] to the case of finite charm quark mass.

(2) When combined with the measured semileptonic width of the B, the moments of the invariant mass spectrum and the measured branching fraction to excited states yield the constraint

$$|V_{cb}| > \left[0.040 - 2.9 \times 10^{-4} \left(\frac{\lambda_1}{0.1 \ \mathrm{GeV}^2} \right) \right] \left(\frac{\tau_B}{1.60 \ \mathrm{ps}} \right)^{-1/2}. \tag{1.1}$$

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While this is consistent with previous determinations of $|V_{cb}|$ from inclusive B decays [6], this result differs from previous extractions in that it does not depend on any assumptions about the size of $\bar{\Lambda}$ or on QCD sum rule estimates of the quark masses.

(3) The values of $\bar{\Lambda}$ and λ_1 extracted from the semileptonic decay width and first moment for D and D_s decays are consistent with those obtained from B decays. The B results are also consistent with recent lattice extractions of the modified minimal subtraction scheme ($\overline{\text{MS}}$) mass $\bar{m}_b(m_b)$ [7,8]. The combined results from B and D decays are inconsistent with the large negative value of λ_1 extracted from certain QCD sum rules [9].

(4) The theoretical prediction for the *difference* of the first moments of the invariant mass spectrum in D and D_s decays is well behaved and provides a test of parton-hadron duality. The comparison with experimental data is quite successful, providing additional evidence for the applicability of duality to the decays of charmed hadrons.

Since our conclusions on D decays disagree significantly with those presented in Ref. [2], it is worth commenting on the discrepancy. The authors of Ref. [2] used the extraction of m_b from Ref. [10] along with the QCD sum rules extraction of $\lambda_1 = -0.6 \pm 0.1$ GeV^2 [9] to conclude that $1.25 < m_c^{\text{pole}} < 1.40$. This results in a semileptonic decay width for the D meson which is at least a factor of 2 smaller than observed. However, it is difficult to relate this extraction of the pole mass to physical quantities. The radiative corrections in the relation between m_c^{pole} and the semileptonic charm width are so large that the perturbation series appears uncontrolled; whether or not this is the case for the relation between the moments of $\sigma(e^+e^- \to \bar{b}b)$ (from which m_b^{pole} and, hence, m_c^{pole} are extracted) and the charm quark semileptonic width requires a higher-order calculation. Given this uncertainty, we prefer to treat λ_1 and $\bar{\Lambda}$ as free parameters, to be fixed by relations between the decay widths, moments, and $\overline{\text{MS}}$ masses, in which case we find that all the data on charm and bottom are consistent with the smaller value $\lambda_1 \simeq -0.1$ GeV^2. This is also consistent with the observations of Ref. [11], where it is argued that the correct QCD sum rule should give a substantially smaller value of λ_1 than that found in Ref. [9].

Finally, we note that we will consider values of λ_1 which violate the constraint $\lambda_1 \lesssim -3\lambda_2 \approx -0.36$ GeV^2 which was proposed in Ref. [12]. The proof of this bound is criticized in Ref. [13], where it is demonstrated that the inclusion of radiative corrections precludes any rigorous constraint on λ_1. Hence we do not apply this proposed limit to our analysis.

II. CONSTRAINTS FROM B DECAYS

A. Theoretical expressions

We begin by discussing the constraints which may be obtained from inclusive semileptonic B decays. The theoretical treatment of these decays involves a double expansion in powers of $\alpha_s(m_b)$ and $1/m_b$, employing an operator product expansion (OPE) and heavy quark symmetry. From Ref. [3] we have the expressions for the first two moments of the hadronic invariant mass spectrum for the process $B \to X_c \ell \nu$:

$$\langle s_H - \bar{m}_D^2 \rangle = m_B^2 \left[0.051 \frac{\alpha_s}{\pi} + 0.23 \frac{\bar{\Lambda}}{m_B} \left(1 + 0.43 \frac{\alpha_s}{\pi} \right) \right.$$
$$\left. + 0.26 \frac{\bar{\Lambda}^2}{m_B^2} + 1.01 \frac{\lambda_1}{m_B^2} - 0.31 \frac{\lambda_2}{m_B^2} \right],$$

$$\langle (s_H^2 - \bar{m}_D^2)^2 \rangle = m_B^4 \left[0.0053 \frac{\alpha_s}{\pi} + 0.067 \frac{\bar{\Lambda}}{m_B} \frac{\alpha_s}{\pi} \right.$$
$$\left. + 0.065 \frac{\bar{\Lambda}^2}{m_B^2} - 0.14 \frac{\lambda_1}{m_B^2} \right], \tag{2.1}$$

where $m_\ell = 0$, and we have defined the spin-averaged D and B meson masses

$$\bar{m}_D \equiv \frac{m_D + 3m_{D^*}}{4} = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_D} + \cdots \simeq 1975 \text{ MeV},$$

$$\bar{m}_B \equiv \frac{m_B + 3m_{B^*}}{4} = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_B} + \cdots \simeq 5313 \text{ MeV}. \tag{2.2}$$

In deriving the expressions (2.1), we have eliminated the ratio of pole masses m_c/m_b by instead writing the heavy quark expansion in terms of $\bar{m}_D/\bar{m}_B$:

$$\frac{m_c}{m_b} = \frac{\bar{m}_D}{\bar{m}_B} - \frac{\bar{\Lambda}}{m_B} \left(1 - \frac{\bar{m}_D}{\bar{m}_B} \right) - \frac{\bar{\Lambda}^2}{m_B^2} \left(1 - \frac{\bar{m}_D}{\bar{m}_B} \right)$$
$$+ \frac{\lambda_1}{2m_B m_D} \left(1 - \frac{\bar{m}_D^2}{\bar{m}_B^2} \right)$$
$$= 0.372 - 0.628 \frac{\bar{\Lambda}}{m_B} - 0.628 \frac{\bar{\Lambda}^2}{m_B^2} + 1.16 \frac{\lambda_1}{m_B^2}. \tag{2.3}$$

Performing a similar substitution in the expression for the semileptonic decay rate, we find (for $m_\ell = 0$)

$$\Gamma_{\text{sl}}(B) = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192 \pi^3} 0.369 \left[1 - 1.54 \frac{\alpha_s}{\pi} - 1.65 \frac{\bar{\Lambda}}{m_B} \right.$$
$$\left. \times \left(1 - 0.87 \frac{\alpha_s}{\pi} \right) - 0.95 \frac{\bar{\Lambda}^2}{m_B^2} - 3.18 \frac{\lambda_1}{m_B^2} + 0.02 \frac{\lambda_2}{m_B^2} \right]. \tag{2.4}$$

The advantage of writing $\Gamma_{\text{sl}}(B)$ and the moments (2.1) in this way is that there is now no hidden dependence on the heavy quark masses; the coefficients arising at each order in the OPE are determined by measurable quantities.

The moments of the invariant mass spectrum depend only on the nonperturbative parameters $\bar{\Lambda}$, λ_1, and λ_2, and on the strong coupling constant $\alpha_s(m_b)$ at leading order. Since $\lambda_2(m_b) = 0.12$ GeV^2 is known from the B-B^* mass splitting,[1] and $\alpha_s(m_b)$ is measured in other processes, these moments provide direct information on the unknown had-

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[1] In this paper, we will neglect the small running of $\lambda_2(\mu)$ between m_b and m_c.

ronic matrix elements $\overline{\Lambda}$ and λ_1. This information may then be inserted into the expression for $\Gamma_{sl}(B)$ to determine $|V_{cb}|$ from the measured decay rate.

B. BLM scale setting for finite m_c

We begin by addressing the question of whether the perturbative corrections to the relation between the semileptonic decay width and the moments of the hadronic invariant mass spectrum are in fact well behaved. Earlier analyses [1] have indicated that the two-loop corrections to $\Gamma_{sl}(B)$ are uncomfortably large. In these analyses, one computes that part of the two-loop correction which is proportional to the first coefficient $\beta_0 = 11 - \frac{2}{3} n_f$ in the QCD β function, and from this derives a BLM scale [5] for the process. One finds the result

$$\Gamma(B \to X_c \ell \overline{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192 \pi^3} 0.369 \left[1 - 1.54 \frac{\alpha_s(m_b)}{\pi} \right.$$
$$\left. - 1.43 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 - 1.65 \frac{\overline{\Lambda}}{m_B} + \cdots \right],$$
$$(2.5)$$

which, since $\beta_0 \alpha_s(m_b)/\pi \sim 0.6$, leads to a perturbation series which is quite poorly behaved. Following the BLM prescription of absorbing this term into the $O(\alpha_s)$ correction by a change of scale, one finds[2] a low BLM scale $\mu_{BLM} = 0.16 m_b \approx 800$ MeV.

In Ref. [3] we discussed a similar situation in the analysis of the decay $b \to u \ell \nu$. There we considered two perturbation series, neither of which is particularly well behaved:

$$\Gamma(B \to X_u \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_B^5}{192 \pi^3} \left[1 - 2.41 \frac{\alpha_s(m_b)}{\pi} \right.$$
$$\left. - 2.98 \beta_0 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 - 5 \frac{\overline{\Lambda}}{m_B} + \cdots \right],$$

$$\langle s_H \rangle = m_B^2 \left[0.20 \frac{\alpha_s(m_b)}{\pi} + 0.35 \beta_0 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \right.$$
$$\left. + \frac{7}{10} \frac{\overline{\Lambda}}{m_B} + \cdots \right].$$
$$(2.6)$$

The BLM scale for $\Gamma(B \to X_u \ell \nu)$ is $\mu_{BLM} = 0.08 m_b$, while for $\langle s_H \rangle$ it is $\mu_{BLM} = 0.03 m_b$. However, both of the expressions (2.6) depend on the nonperturbative parameter $\overline{\Lambda}$, which is defined only up to certain arbitrary conventions [14]. If the poor convergence of the perturbation series can be absorbed into $\overline{\Lambda}$, then the large higher-order terms will be of no consequence, since ultimately $\overline{\Lambda}$ is eliminated from relations between physical observables.

In Ref. [3] we investigated whether this might be so by eliminating $\overline{\Lambda}$ from Eqs. (2.6), solving for $\Gamma(B \to X_u \ell \nu)$ in terms of $\langle s_H \rangle$. Doing so, we found

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[2] This scale arises from taking $m_c/m_b = 0.37$ [see Eq. (2.3)], and differs slightly from the result of Ref. [1], where m_c/m_b was taken to be 0.3.

$$\Gamma(B \to X_u \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_B^5}{192 \pi^3} \left[1 - 0.98 \frac{\alpha_s(m_b)}{\pi} \right.$$
$$\left. - 0.48 \beta_0 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 - 7.14 \frac{\langle s_H \rangle}{m_B^2} + \cdots \right], \quad (2.7)$$

leading to a much higher BLM scale $\mu_{BLM} = 0.38 m_b$. The apparent convergence of the perturbation series improves considerably under such a reorganization.

For finite charm quark mass we may perform a similar analysis. We use standard techniques [15] to extract a two-loop term of the form $\beta_0 (\alpha_s/\pi)^2$ which contributes to the first moment $\langle s_H - \overline{m}_D^2 \rangle$. The calculation is straightforward but tedious, with the final integrals performed numerically. We find

$$\langle s_H - \overline{m}_D^2 \rangle = m_B^2 \left[0.051 \frac{\alpha_s(m_b)}{\pi} + 0.096 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 \right.$$
$$\left. + 0.23 \frac{\overline{\Lambda}}{m_B} + \cdots \right], \quad (2.8)$$

which again leads to a perturbation series which appears to be badly behaved, with a very low BLM scale $\mu_{BLM} = 0.02 m_b$. However, if instead we use the expression (2.8) to eliminate $\overline{\Lambda}$ from the semileptonic width (2.5), we obtain

$$\Gamma(B \to X_c \ell \overline{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_B^5}{192 \pi^3} 0.369 \left[1 - 1.17 \frac{\alpha_s(m_b)}{\pi} \right.$$
$$\left. - 0.74 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 \right.$$
$$\left. - 7.17 \frac{\langle s_H - \overline{m}_D^2 \rangle}{m_B^2} + \cdots \right]. \quad (2.9)$$

The two-loop correction in Eq. (2.9) has been reduced by a factor of almost 2 compared with that in Eq. (2.5). The one-loop correction is reduced as well, and so the change in the BLM scale is less dramatic; the new BLM scale is $\mu_{BLM} = 0.28 m_b$. This reorganization of the perturbation series gives us hope that the expansion $\alpha_s(m_b)/\pi$ is now under control, although, of course, the full $O(\alpha_s^2)$ correction remains an important source of uncertainty.

For the second moment of the invariant mass spectrum there is, as in the massless case, no such cancellation. We find, for $\alpha_s(m_b) = 0.22$,

$$\langle (s_H - \overline{m}_D^2)^2 \rangle = m_B^4 \left[0.0053 \frac{\alpha_s(m_b)}{\pi} + 0.0078 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 \right.$$
$$\left. + 0.067 \frac{\overline{\Lambda}}{m_B} \frac{\alpha_s(m_b)}{\pi} + \cdots \right]$$
$$\simeq m_B^4 \left[3.7 \times 10^{-4} + 3.4 \times 10^{-4} \right.$$
$$\left. + 0.067 \frac{\overline{\Lambda}}{m_B} \frac{\alpha_s(m_b)}{\pi} + \cdots \right]. \quad (2.10)$$

Since the $O(\bar{\Lambda})$ term comes with an explicit factor of α_s, substituting a physical quantity for $\bar{\Lambda}$ will not introduce a term of $O(\alpha_s^2 \beta_0)$ to cancel the large two-loop correction in Eq. (2.10). Therefore, we expect that constraints from the second moment will be more sensitive to higher-order corrections than those from the first moment and hence less reliable. Fortunately, the most useful constraints in the $\bar{\Lambda}$-λ_1 plane will come from the first moment of $s_H - \bar{m}_D^2$.

C. Bounds on $\bar{\Lambda}$ and λ_1

Using the theoretical expressions (2.1) and experimental data, we now derive constraints on the nonperturbative parameters $\bar{\Lambda}$ and λ_1. These quantities are dependent on the scheme by which perturbation theory is defined; the bounds which we will derive are for $\bar{\Lambda}$ and λ_1 at one loop in QCD in the $\overline{\text{MS}}$ scheme, with the renormalization scale $\mu = m_b$. We make no claim that this is the ''natural'' definition of these quantities, and in any case the scheme dependence drops out of relations between physical observables. However, although they are unphysical, it is convenient to retain these parameters in intermediate stages of calculations, and in order to compare the values of $\bar{\Lambda}$ and λ_1 obtained from different observables we must specify some convention for their definition.

In Ref. [3], we used the known branching ratio of B mesons to excited charmed mesons to estimate experimental lower bounds on $\langle (s_H - \bar{m}_D^2)^n \rangle$. This estimate was based on the OPAL measurement [16] of $(34 \pm 7)\%$ for the fraction of semileptonic decays to the states D_1 and D_2^*. However, while the sum of the two branching fractions is consistent with the recent CLEO 90% C.L. upper limit [17] of 30%, there appears to be a discrepancy with the branching fractions to the individual D_1 and D_2^* final states. In Ref. [3] we took the average invariant mass of the produced $D_1(2420)$ and $D_2^*(2460)$ states to be 2450 GeV. Here we will assume that only the lower mass D_1 is produced, giving a more conservative lower limit on $\langle (s_H - \bar{m}_D^2)^n \rangle$. We will also take the 1σ OPAL lower limit on the fraction of semileptonic decays, 27%, so as to be consistent with the CLEO result. Doing so, and using the results of Ref. [3] for the contribution to the moments from the D and D^*, we find the experimental lower limits

$$\langle s_H - \bar{m}_D^2 \rangle_{\min} = 0.49 \text{ GeV}^2, \quad \langle (s_H - \bar{m}_D^2)^2 \rangle_{\min} = 1.1 \text{ GeV}^4. \tag{2.11}$$

Note that in obtaining these limits we have assumed that no other excited states are produced. It is more realistic to assume that there will also be production of the p-wave doublet D_0^* and D_1, which will raise the average invariant mass of the final hadronic state. However, since there is no experimental information on these states, we are conservative and do not include them in our estimates of $\langle (s_H - \bar{m}_D^2)^n \rangle_{\min}$.

Another observable which depends on $\bar{\Lambda}$ and λ_1 is the ratio of partial widths,

$$R_\tau = \frac{\Gamma(B \to X_c \tau \bar{\nu})}{\Gamma(B \to X_c e \bar{\nu})}. \tag{2.12}$$
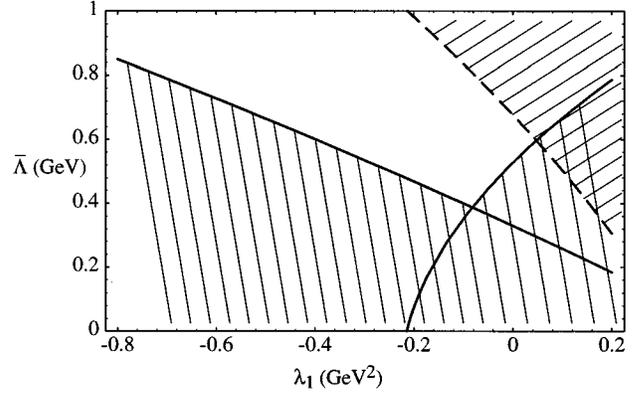

FIG. 1. The limits on $\bar{\Lambda}$ and λ_1 from data on semileptonic B decays. The solid curves are lower limits on $\bar{\Lambda}$ from the first two moments of the hadronic invariant mass spectrum, while the dashed curve is a 1σ upper limit from the ratio R_τ. The solid curve on the left corresponds to the bound from $\langle s_H - \bar{m}_D^2 \rangle$, the one on the right to the bound from $\langle (s_H - \bar{m}_D^2)^2 \rangle$.

The theoretical expression for R_τ also depends on the ratio of masses, m_τ/m_b, both at the tree level and in the nonperturbative [18,19] and perturbative [20,21] corrections. As before, m_τ/m_b may be reexpanded in terms of the observable m_τ/m_B, yielding the result

$$R_\tau = 0.224 \left[1 + 0.24 \frac{\alpha_s}{\pi} - 0.29 \frac{\bar{\Lambda}}{m_B} \left(1 - 1.33 \frac{\alpha_s}{\pi} \right) - 0.68 \frac{\bar{\Lambda}^2}{m_B^2} \right.$$
$$\left. - 3.85 \frac{\lambda_1}{m_B^2} - 7.54 \frac{\lambda_2}{m_B^2} \right]. \tag{2.13}$$

At present, there are only data on the average b hadron semitauonic branching fraction, obtained at the CERN $e^+ e^-$ collider LEP, where the identity of the bottom hadron is not determined. The experimental result is

$$B(b \to X_c \tau \bar{\nu}) = 2.75 \pm 0.48\%. \tag{2.14}$$

This differs from $B(B \to X_c \tau \bar{\nu})$ by contamination from the B_s and Λ_b. However, this difference is small compared with the experimental uncertainty. The ratio of the theoretical expressions for R_τ in the B and Λ_b sectors is

$$\frac{R_\tau^B}{R_\tau^{\Lambda_b}} \simeq 1 + O\left(\frac{\Lambda_{\text{QCD}}^2}{m_B^2} \right). \tag{2.15}$$

Since only about 10% of b hadrons at LEP are Λ_b's, the effect on R_τ should be much less than 1%. Hence we use the measurement (2.14), along with $B(B \to X_c e \nu) = 10.7 \pm 0.5$, [22] to obtain

$$R_\tau = 0.26 \pm 0.05. \tag{2.16}$$

The comparison of the theoretical predictions (2.1) and (2.13) with experiment leads to limits on $\bar{\Lambda}$ and λ_1. The experimental central value for R_τ yields a curve which is entirely inconsistent with the other data, giving a negative value for $\bar{\Lambda}$. Therefore in Fig. 1 we show the curve corresponding to the 1σ lower limit on R_τ, along with the con-

straints from the moments of the invariant mass spectrum, where we have taken $\alpha_s(m_b) = 0.22$.

Since the experimental error on R_τ is relatively large, the 2σ constraint is uninteresting, allowing all values of $\overline{\Lambda}$ and λ_1 in the displayed region of Fig. 1. Therefore, at present we can only conclude that R_τ favors a negative value of λ_1. However, if the experimental uncertainty in R_τ is reduced in the future, it may become an important quantity for constraining $\overline{\Lambda}$ and λ_1. For now, the most interesting constraints in the $\overline{\Lambda}$-λ_1 plane come from $\langle s_H - \overline{m}_D^2 \rangle$.

We note that a very similar discussion of the limits on $\overline{\Lambda}$ and λ_1 which may be obtained from R_τ has been given by Ligeti and Nir [23]. Our analysis is organized somewhat differently from theirs in its treatment of experimental masses and errors, leading to results of a superficially different form, but the physics and the uncertainties are largely the same.

D. Constraints on $|V_{cb}|$

Leaving aside the weak constraints from R_τ, we now take the information on $\overline{\Lambda}$ and λ_1 obtained from the analysis of the moments of s_H and apply it to the extraction of $|V_{cb}|$ from the semileptonic width. We use the theoretical expression (2.4), the experimental semileptonic branching ratio of 10.7%, [22] the central value $\tau_B = 1.60$ ps for the B lifetime,

and the strong coupling constant $\alpha_s(m_b) = 0.22$. The experimental lower limit on $\langle s_H - \overline{m}_D^2 \rangle$ gives a restriction on the one-loop value of $\overline{\Lambda}$:

$$\overline{\Lambda} > \left[0.40 - 1.15\frac{\alpha_s(m_b)}{\pi} - 0.07\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\ \mathrm{GeV}$$

$$> \left[0.32 - 0.07\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\ \mathrm{GeV}. \qquad (2.17)$$

Including the $O(\alpha_s^2 \beta_0)$ term and assuming that this dominates the two-loop result, we obtain for the two-loop value of $\overline{\Lambda}$ the constraint,

$$\overline{\Lambda}^{(2\ \mathrm{loop})} > \left[0.40 - 1.15\frac{\alpha_s(m_b)}{\pi} - 1.88\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \right.$$

$$\left. - 0.07\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\ \mathrm{GeV}$$

$$> \left[0.25 - 0.07\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\ \mathrm{GeV}. \qquad (2.18)$$

Incorporating the latter bound into our expression for the semileptonic width and solving for $|V_{cb}|$, we find

$$|V_{cb}| > \left[0.038 + 0.023\frac{\alpha_s(m_b)}{\pi} + 0.018\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 - 2.9\times10^{-4}\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\left(\frac{\tau_B}{1.60\ \mathrm{ps}}\right)^{-1/2}$$

$$> \left[0.038 + 0.0016 + 0.0006 - 2.9\times10^{-4}\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\left(\frac{\tau_B}{1.60\ \mathrm{ps}}\right)^{-1/2}$$

$$> \left[0.040 - 2.9\times10^{-4}\left(\frac{\lambda_1}{0.1\ \mathrm{GeV}^2}\right) \right]\left(\frac{\tau_B}{1.60\ \mathrm{ps}}\right)^{-1/2}. \qquad (2.19)$$

In the second line above we display the tree-level, one-loop, and (partial) two-loop contributions to the bound. As we showed earlier [see Eq. (2.9)], the perturbation series appears to be well behaved. We also note that this lower limit on $|V_{cb}|$ is relatively insensitive to the experimental error on $\langle s_H - \overline{m}_D^2 \rangle$. If the semileptonic production of D_1 and D_2^* mesons is reduced to its 2σ OPAL lower limit of 20%, then the bound on $\overline{\Lambda}$ is weakened by 100 MeV. However, we still obtain $|V_{cb}| > [0.038 - 2.8\times10^{-4}(\lambda_1/0.1\ \mathrm{GeV}^2)](\tau_B/1.60\ \mathrm{ps})^{-1/2}$.

E. Constraints on $\overline{m}_b(m_b)$

Our bounds on $\overline{\Lambda}$ and λ_1 may be translated into bounds on the $\overline{\mathrm{MS}}$ quark mass $\overline{m}_b(m_b)$. However, these results should be treated with some caution because the two-loop corrections between $\overline{m}_b(m_b)$ and the first two moments of $s_H - \overline{m}_D^2$ are quite large, indicating a poorly behaved perturbation series.

We define the dimensionless parameter

$$x_B = \frac{m_B - \overline{m}_b(m_b)}{m_B}$$

$$= \frac{\overline{\Lambda}}{m_B} + \frac{4}{3}\frac{\alpha_s(m_b)}{\pi} + 1.56\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \cdots, \qquad (2.20)$$

where we keep only the large $O(\alpha_s^2 \beta_0)$ contribution to the two-loop term. We then find

$$\frac{\langle s_H - \overline{m}_D^2 \rangle}{m_B^2} = 0.23 x_B \left(1 - 0.52\frac{\alpha_s(m_b)}{\pi} + 1.13 x_B \right)$$

$$- 0.26\frac{\alpha_s(m_b)}{\pi} - 0.26\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2$$

$$+ 1.13\frac{\lambda_1}{m_B^2} + 0.03\frac{\lambda_2}{m_B^2} + \cdots \qquad (2.21)$$
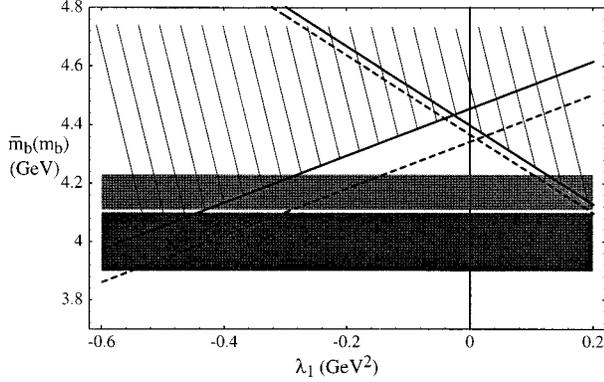
for the first moment and

FIG. 2. The two-loop constraints on $\bar{m}_b(m_b)$ and λ_1 from the first two moments of $s_H - \bar{m}_D^2$, evaluated at $\mu = m_b$ (solid lines) and $\mu = m_b/2$ (dashed lines). The hatched region is excluded. Note that the constraints become more stringent as μ is lowered. The two solid bands are the lattice results from Refs. [7,8] with the quoted uncertainties. The two-loop calculation includes only terms of order $\alpha_s^2 \beta_0$.

$$
\frac{\langle (s_H - \bar{m}_D)^2 \rangle}{m_B^4} = x_B \left(-0.11 \frac{\alpha_s(m_b)}{\pi} + 0.065 x_B \right)
$$
$$
+ 0.0053 \frac{\alpha_s(m_b)}{\pi} + 0.0078 \beta_0 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2
$$
$$
- 0.14 \frac{\lambda_1}{m_B^2} + \cdots \tag{2.22}
$$

for the second. The $O(\alpha_s^2 \beta_0)$ corrections are clearly substantial. The corresponding BLM scales in Eqs. (2.21) and (2.22) are $\mu_{\mathrm{BLM}} = 0.14 m_b$ and $0.05 m_b$, respectively, corresponding to $O(\alpha_s^2 \beta_0)$ terms which are roughly 60% and 80% of the one-loop term. Therefore there are likely to be much larger uncalculated radiative corrections in the relations between the first moment and $\bar{m}_b(m_b)$ than between the first moment and the semileptonic B width.

With these caveats in mind, we combine these results with the experimental limits (2.11) to yield the constraints on $\bar{m}_b(m_b)$ and λ_1 shown in Fig. 2. To illustrate some of the remaining dependence on the renormalization scale μ, the constraints are plotted for both $\mu = m_b$ and $\mu = m_b/2$. We also display, with the quoted uncertainties, the results of two recent lattice extractions of $\bar{m}_b(m_b)$:

$$
\bar{m}_b(m_b) = 4.17 \pm 0.06 \text{ GeV} [7] \tag{2.23}
$$

and

$$
\bar{m}_b(m_b) = 4.0 \pm 0.1 \text{ GeV} [8]. \tag{2.24}
$$

We see that our bounds are consistent with these lattice results.

However, it is much more difficult to compare these limits to the extraction of the b quark pole mass from high moments of $\sigma(e^+ e^- \to \bar{b}b)$ [10]:

$$
m_b = 4.827 \pm 0.007 \text{ GeV}. \tag{2.25}
$$

This is simply because the ambiguity in the renormalization scale introduces a large uncertainty, of order $\beta_0 (\alpha_s/\pi)^2 m_b \sim 200$ MeV, in the relation of the "one-loop" pole mass m_b^{pole} to $\bar{m}_b(m_b)$. For example, if we use $\alpha_s(m_b)$ in the one-loop relation,

$$
m_b^{\mathrm{pole}} \equiv \bar{m}_b(m_b) \left(1 + \frac{4}{3} \frac{\alpha_s}{\pi} \right), \tag{2.26}
$$

then Eq. (2.25) yields the value $\bar{m}_b(m_b) = 4.42$ GeV, somewhat higher than the lattice results and barely consistent with our analysis of the moments. Furthermore, such a value certainly would be inconsistent with the combination of the moments analysis and the QCD sum rules extraction of $\lambda_1 = -0.6$ GeV^2. On the other hand, in Ref. [10] it was argued that the natural scale for matching onto nonrelativistic theory is $\mu \sim 0.63 m_b$. Taking this lower scale, we find $\bar{m}_b(m_b) = 4.34$ GeV, still somewhat higher than the lattice calculations and still inconsistent with a large negative λ_1. Of course, using an even lower scale in the one loop relation between m_b^{pole} and $\bar{m}_b(m_b)$ would lower the extracted value of $\bar{m}_b(m_b)$ even further. While this scale ambiguity is formally of higher order in α_s than our calculation, we see that numerically it is quite significant. Without a higher-loop calculation of m_b^{pole} from QCD sum rules, it is impossible to determine whether or not this extraction of m_b^{pole} is consistent with the other constraints.

III. D MESON DECAYS

In Ref. [2] it was argued that the semileptonic decays of charmed mesons are not well described in the heavy quark expansion, since the value of m_c which is required to fit the observed semileptonic D decay rate lies significantly above 1.4 GeV. This is the upper limit suggested by combining the value of of m_b extracted from the Υ spectrum [10] and the large negative value of λ_1 found from certain QCD sum rules [9]. However, we believe that this argument should be reconsidered in light of the uncertainty inherent in relating the pole mass derived in Ref. [10] with physical quantities. Indeed, we will find that the values of λ_1 and $\bar{\Lambda}$ implied by the semileptonic decay rate and the first moments of the D and D_s invariant mass spectra are in reasonable agreement with the limits from the corresponding observables in the bottom sector.

The theoretical analysis in the D sector proceeds as before. Since m_s is of order Λ_{QCD}, for consistency we keep only terms of order m_s^2/m_c^2 in the theoretical expressions for $\langle s_H^n \rangle$ and $\Gamma(D \to X_s e \bar{\nu})$. As we are neglecting terms of order $\alpha_s \lambda_1$ and $\alpha_s \lambda_2$, we also omit terms of order $\alpha_s m_s^2$ but keep logarithmically enhanced terms of order $\alpha_s m_s^2 \ln(m_s^2/m_c^2)$. Thus we find, for the Cabibbo allowed semileptonic width,

$$\Gamma(D \to X_s e \bar{\nu}) = \frac{G_F^2 m_c^5 |V_{cs}|^2}{192\pi^3}\left[1 + \left(\frac{25}{6} - \frac{2}{3}\pi^2\right)\frac{\alpha_s}{\pi}\right.$$
$$\left. - 8\frac{m_s^2}{m_D^2}\left(1 - 2\frac{\alpha_s}{\pi}\ln\frac{m_s^2}{m_c^2}\right) + \frac{\lambda_1 - 9\lambda_2}{2m_D^2}\right]$$
$$= \frac{G_F^2 m_c^5 |V_{cs}|^2}{192\pi^3}\left[1 + \left(\frac{25}{6} - \frac{2}{3}\pi^2\right)\frac{\alpha_s}{\pi}\right.$$
$$\left. - 8\frac{\bar{m}_s^2(m_c)}{m_D^2} + \frac{\lambda_1 - 9\lambda_2}{2m_D^2}\right] \qquad (3.1)$$

and, for the first moment,

$$\langle s_H \rangle_D = m_s^2\left(1 - 2\frac{\alpha_s}{\pi}\ln\frac{m_s^2}{m_c^2}\right) + m_D^2\left[\frac{91}{450}\frac{\alpha_s}{\pi} + \frac{7\bar{\Lambda}}{10m_D}\right.$$
$$\times\left(1 - \frac{227}{630}\frac{\alpha_s}{\pi}\right) + \frac{3}{10m_D^2}(\bar{\Lambda}^2 + \lambda_1 - \lambda_2)\right]$$
$$= \bar{m}_s^2(m_c) + m_D^2\left[\frac{91}{450}\frac{\alpha_s}{\pi} + \frac{7\bar{\Lambda}}{10m_D}\left(1 - \frac{227}{630}\frac{\alpha_s}{\pi}\right)\right.$$
$$\left. + \frac{3}{10m_D^2}(\bar{\Lambda}^2 + \lambda_1 - \lambda_2)\right]. \qquad (3.2)$$

Note that the large infrared logarithms of the pole mass m_s may be absorbed naturally into the $\overline{\text{MS}}$ mass renormalized at m_c, $\bar{m}_s(m_c)$. As one would expect, the individual terms in the perturbative expansion, which arises from an operator product expansion performed at the scale $\mu = m_c$, remain insensitive to physics below m_c.

In addition to the usual nonperturbative parameters, the theoretical expressions for the decay rate and moments also depend on the strange quark mass $\bar{m}_s(m_c)$. We use the range

$$100 \text{ MeV} < \bar{m}_s(1 \text{ GeV}) < 300 \text{ MeV}, \qquad (3.3)$$

given by the Particle Data Group [25] (which changes only slightly when evolved from $\mu = 1$ GeV to $\mu = m_c$). Finally, note that we have not reexpanded the m_c^5 term appearing in the semileptonic width (3.1) in terms of the meson mass $\bar{m}_D$. This is because here we have no analogue of the expansion (2.3), since the strange quark is not heavy. As a result, the parameter $\bar{\Lambda}$ does not appear explicitly in Eq. (3.1). Instead, we will solve Eq. (3.1) directly for m_c, linearize in λ_1, and then use the heavy quark expansion (2.2) to relate m_c to $\bar{\Lambda}$ and λ_1.

The inclusive semielectronic D branching fraction recently has been measured to be [24]

$$B(D^0 \to Xe^+\nu) = [6.64 \pm 0.18(\text{stat}) \pm 0.29(\text{syst})]\% . \qquad (3.4)$$

The same CLEO analysis indicates that the Cabibbo-allowed inclusive rate is saturated, within errors, by the exclusive modes $D^0 \to (K^-, K^{*-})e^+\nu$, with stringent upper limits reported on the channels $D^0 \to (K_1^-(1270), K^{*-}(1430))e^+\nu$. The data on these decays yield for the first moment of the $D \to X_s \ell^+\nu$ invariant mass spectrum the value
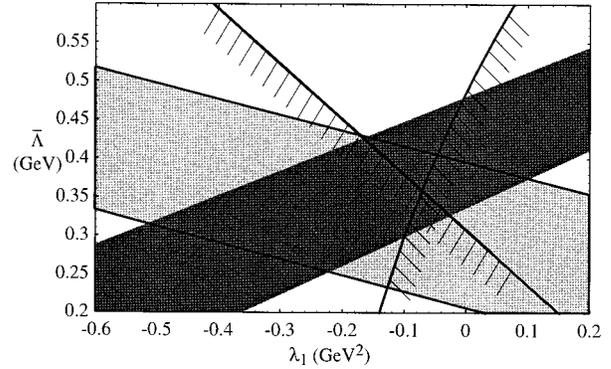

FIG. 3. The restrictions on $\bar{\Lambda}$ and λ_1 from the semileptonic D decay rate (darker shaded region) and from $\langle s_H \rangle$ in D decay (lighter shaded region). We also show the lower bounds on $\bar{\Lambda}$ from the moments of the B decay spectrum (crosshatched black lines).

$$\langle s_H \rangle_D = (0.49 \pm 0.03) \text{ GeV}^2, \qquad (3.5)$$

where we have added errors in quadrature and neglected the widths of the excited K mesons.

The result of this analysis is the set of constraints displayed in Fig. 3, in which we also show the limits obtained earlier from B decays. Both theoretical and experimental uncertainties are included in the displayed bands. In fact, the dominant uncertainties are theoretical, and have two distinct sources. First, there is the uncertainty in the strange quark mass (3.3). Second, there is the effect of uncomputed terms in the mass expansion of order $1/m_c^3$, which will be more substantial than in bottom decays. The theoretical analysis at order $1/m_c^3$ is quite complex and involves a number of new nonperturbative parameters, and so we do not attempt to include these terms systematically.[3] Instead, we obtain a minimal estimate of the size of the uncertainty arising from these effects by extracting the bounds from $\Gamma(D \to X_s \ell^+\nu)$ in two ways: on the one hand, by solving directly for the width in terms of the charm quark pole mass and, on the other, by proceeding through the intermediate step of calculating the ''decay mass'' m_c^Γ [3]. These two procedures, which are formally the same only up to order $1/m_c^2$, yield bounds on $\bar{\Lambda}$ which differ by approximately 70 MeV. It would be hard to argue convincingly that $1/m_c^3$ effects were intrinsically smaller than this. For the analysis of $\langle s_H \rangle_D$, we employ the simpler procedure of including a term $n(0.5 \text{ GeV}/m_D)^3$ in the theoretical expression (3.2) and varying n between -1 and 1.

Note that the constraints from the charm sector are compatible with those derived from bottom decays, although they do not appear to agree with the lattice extractions of m_b shown in Fig. 2. However, Fig. 3 must be interpreted with caution, because of the possible presence of large higher-loop effects. As discussed in Sec. II C, each curve in Fig. 3 may be interpreted as giving a constraint on $\bar{\Lambda}$ derived from a particular process. A reliable comparison between the con-

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[3]In Ref. [2] the $1/m_c^3$ corrections to the semileptonic decay rate were estimated using the factorization hypothesis and other arguments, and found to be small.

straints derived from two different physical quantities requires that when one is expressed in terms of the other, the perturbative expansion in α_s be well behaved. For example, we demonstrated earlier that although there are large two-loop corrections to the theoretical expressions for Γ_{sl} and $\langle s_H \rangle$ for B or D decays alone, these corrections partially cancel out when $\bar{\Lambda}$ is eliminated and one of these physical quantities is expressed in terms of the other. Unfortunately, this requirement may no longer be satisfied when expressions from the bottom and charm sectors are compared with each other. For example, writing $\langle s_H - \bar{m}_D^2 \rangle$ for B decays in terms of $\langle s_H \rangle$ for D decays, we find

$$\langle s_H - \bar{m}_D^2 \rangle = m_B^2 \left[0.028 \frac{\alpha_s(m_c)}{\pi} + 0.029 \beta_0 \left(\frac{\alpha_s(m_c)}{\pi} \right)^2 \right.$$
$$\left. + 0.12 \frac{\langle s_H \rangle - \bar{m}_s^2(m_c)}{m_D^2} \right], \quad (3.6)$$

resulting in a pertubative expansion which does not appear to converge well. Thus while the rough consistency of the constraints from B and D decays is encouraging, it may not be particularly significant. Still, it is amusing to note that if we were to take the combined constraints as legitimate, then we would conclude that λ_1 is small and negative, of order -0.1 GeV^2. Hence it would make a negligible contribution to most observables, including $|V_{cb}|$.

We do not compare the results from the charm sector with $\bar{m}_c(m_c)$, which may be extracted from the lattice measurement of $\bar{m}_b(m_b)$ using the heavy quark mass relations. This is because the radiative corrections between these quantities are so large that perturbation theory appears uncontrolled, making it difficult to conclude whether or not the regions in the $\bar{\Lambda}$-λ_1 plane indicated by the different observables are consistent with each other.

A better test of duality in charm decays comes from comparing D and D_s decays. There is only an upper bound on the semileptonic branching ratio of the D_s, [25]

$$Br(D_s \rightarrow X e^+ \nu) < 20\%, \quad (3.7)$$

which is not strong enough to provide an interesting constraint from the semileptonic decay rate. The observed exclusive semileptonic modes are $D_s \rightarrow (\eta, \eta', \phi) \ell^+ \nu$, for which CLEO has recently reported relative branching ratios [26]. Adding the reported systematic and statistical errors in quadrature, we estimate a value for the first moment of the invariant mass spectrum:

$$\langle s_H \rangle_{D_s} = (0.68 \pm 0.03) \text{ GeV}^2. \quad (3.8)$$

We have assumed that decays to η, η', and ϕ saturate the inclusive semileptonic rate, an approximation for which we do not assign an error. We merely note that since semileptonic D^0 decays are known to be saturated by relatively few exclusive modes, the error due to this approximation for D_s might well be small. In light of this uncertainty, the upper limit on $\langle s_H \rangle_{D_s}$ is less firm than the lower limit.

The theoretical prediction for $\langle s_H \rangle_{D_s}$ is given by the expression (3.2) for $\langle s_H \rangle_D$, with the replacements of m_D by

m_{D_s} and $\bar{\Lambda}$ and λ_i by their strange counterparts $\bar{\Lambda}_s$ and λ_{is}. They are related to $\bar{\Lambda}$ and λ_i by

$$\bar{\Lambda}_s - \bar{\Lambda} = \frac{\bar{m}_B(\bar{m}_{B_s} - \bar{m}_B) - \bar{m}_D(\bar{m}_{D_s} - \bar{m}_D)}{\bar{m}_B - \bar{m}_D} \approx 95 \text{ MeV},$$

$$\lambda_{1s} - \lambda_1 = \frac{2\bar{m}_B \bar{m}_D}{\bar{m}_B - \bar{m}_D} [(\bar{m}_{B_s} - \bar{m}_B) - (\bar{m}_{D_s} - \bar{m}_D)]$$
$$\approx -0.02 \text{ GeV}^2,$$

$$\lambda_{2s} - \lambda_2 = \tfrac{1}{4} [(m_{D_s^*}^2 - m_{D_s}^2) - (m_{D^*}^2 - m_D^2)]$$
$$\approx -0.01 \text{ GeV}^2, \quad (3.9)$$

up to corrections to the mass expansions of relative order $1/m_{b,c}^3$.

The theoretical expression for $\Delta \langle s_H \rangle = \langle s_H \rangle_{D_s} - \langle s_H \rangle_D$ is particularly well behaved, since it vanishes up to SU(3)-violating effects. Expanding in powers of $1/m_D$, we find

$$\Delta \langle s_H \rangle = m_D(\bar{\Lambda}_s - \bar{\Lambda}) \left[\frac{7}{10} + \frac{137}{900} \frac{\alpha_s(m_c)}{\pi} \right] + \frac{13}{10} \bar{\Lambda}(\bar{\Lambda}_s - \bar{\Lambda})$$
$$+ (\bar{\Lambda}_s - \bar{\Lambda})^2 + \frac{3}{10}(\lambda_{1s} - \lambda_1) - \frac{3}{10}(\lambda_{2s} - \lambda_2). \quad (3.10)$$

The large radiative corrections to $\langle s_H \rangle$ almost precisely cancel in the difference.

The leading corrections to this result likely come from the SU(3)-violating Cabibbo-allowed annihilation channel in D_s decays. Although this channel typically has small s_H, it contributes to $\langle s_H \rangle_{D_s}$ primarily through its effect on the total semileptonic rate. For small lepton mass, helicity conservation suppresses the purely leptonic decay $D_s \rightarrow \ell \nu$ in favor of processes in which at least one gluon is emitted. Using vacuum saturation to guess the relevant hadronic matrix elements, one obtains a naive estimate of $4\pi \alpha_s(m_c) f_D^2 / m_D^2$ for the fractional correction to the D_s semileptonic decay width.[4] With $f_D \approx 200$ MeV and $\alpha_s(m_c) \approx 0.32$, this corresponds to an increase of order 5% in the semileptonic width of the D_s or a decrease of $\langle s_H \rangle$ by roughly the same percentage. Since the error in this estimate due to the use of vacuum saturation is unknown, we will take the effects of weak annihilation into account by decreasing the predicting for $\langle s_H \rangle_{D_s}$ by 5%, and assigning an additional error of $\pm 5\%$, or 100% of the naive estimate of the $1/m_D^3$ correction, to the prediction of the differences in the moments.

With the known values (3.9) of $\bar{\Lambda}_s - \bar{\Lambda}$ and $\lambda_{is} - \lambda_i$, the expression (3.10) reduces to $\Delta \langle s_H \rangle = [0.136 + 0.121(\bar{\Lambda}/1 \text{ GeV})]$GeV^2. For an analysis entirely within the charm sys-

---

[4] The authors of Ref. [2] have calculated a related quantity of the same order, namely, the contribution of "penguin-type" annihilation processes to the semileptonic decay of the D. They obtain the naive estimate above, multiplied by a coefficient $\frac{8}{9}[\ln(m_c/\mu) + 1/3] \approx \frac{1}{3}$ for $\mu \approx 500$ MeV.

tem, it is appropriate to estimate $\bar{\Lambda}$ from the overlap of the shaded bands in Fig. 3, from which we obtain $0.25 \text{ GeV} < \bar{\Lambda} < 0.45$ GeV. Including the estimated contribution and uncertainty from the annihilation channel in D_s decay, this range leads to $\Delta \langle s_H \rangle = (0.14 \pm 0.04) \text{GeV}^2$, which we may combine with the measured value of $\langle s_H \rangle_D$ to obtain the theoretical prediction

$$\langle s_H \rangle_{D_s} = (0.64 \pm 0.04) \text{GeV}^2. \tag{3.11}$$

This value agrees quite well with the experimental result $(0.68 \pm 0.03) \text{GeV}^2$.

Indeed, we view the successful prediction of $\langle s_H \rangle_{D_s}$ as evidence for the applicability of parton-hadron duality to inclusive semileptonic charm decays. The prediction of this moment is on a firmer theoretical footing than other quantities in the charm system, since the large radiative and power corrections up to $O(1/m_c^2)$ cancel out of the difference of the moments. This feature distinguishes this test of duality from those in which a comparison is made to the bottom system, while its relative insensitivity to the value of $\bar{\Lambda}$ makes it more stringent than the comparison of the first moment to the total D decay width. At the same time, duality is satisfied in a nontrivial way, as the exclusive final states in D and D_s decays are entirely different. We are encouraged by this success of the theoretical analysis.

IV. SUMMARY

In this paper we have explored the constraints on the nonperturbative parameters $\bar{\Lambda}$ and λ_1 which are obtained from semileptonic B and D decays. We have found that independent analyses of the bottom and charm systems yield limits which are consistent with one another. Taken together, they imply values of the order $\bar{\Lambda} \sim 450$ MeV (at one loop) and $\lambda_1 \sim -0.1 \text{ GeV}^2$. Whether or not one chooses to trust the numerical results of the charm analysis, we see no evidence that parton-hadron duality fails in these decays. On the contrary, our discussion of the difference of the first moments in D and D_s decays leads us to quite the opposite conclusion: In at least one nontrivial case, duality works well for charm.

A primary motivation for investigating inclusive decays is to extract the CKM matrix element $|V_{cb}|$ with high precision. Our analysis yields the lower limit $|V_{cb}| > [0.040 - 2.9 \times 10^{-4}(\lambda_1/0.1 \text{GeV}^2)](\tau_B/1.60 \text{ ps})^{-1/2}$ using the current measurement of the branching fraction to excited D meson states in B decays. This is consistent with the value of $|V_{cb}|$ obtained from exclusive B decays [6]. We have bolstered our theoretical analysis with a partial treatment of two-loop corrections to this bound, performing a BLM scale setting analysis which indicates that the relevant perturbation series is reasonably well behaved.

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