Relativistic description of exclusive heavy-to-light semileptonic decays $B \to \pi(\rho)e\nu$

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The method of calculating electroweak decay matrix elements between heavy-heavy and heavy-light meson states is developed in the framework of a relativistic quark model based on the quasipotential approach in quantum field theory. This method is applied to the study of exclusive semileptonic $B \to \pi(\rho)$ decays. It is shown that the large value of the final $\pi(\rho)$ meson recoil momentum allows for the expansion in inverse powers of the b-quark mass of the decay form factors at $q^2 = 0$, where q is a momentum carried by the lepton pair. This $1/m_b$ expansion considerably simplifies the analysis of these decays and is carried out up to the second order. The q^2 dependence of the form factors is investigated. It is found that the q^2 behavior of the axial form factor A_1 is different from the other form factors. It is argued that the ratios $\Gamma(B \to \rho e\nu)/\Gamma(B \to \pi e\nu)$ and Γ_L/Γ_T are sensitive probes of the A_1 q^2 dependence, and, thus, their experimental measurement may discriminate between different approaches. We find $\Gamma(B \to \pi e\nu) = (3.0 \pm 0.6) \times |V_{ub}|^2 \times 10^{12} \text{s}^{-1}$ and $\Gamma(B \to \rho e\nu) = (5.4 \pm 1.2) \times |V_{ub}|^2 \times 10^{12} \text{s}^{-1}$. The relation between semileptonic and rare radiative B decays is discussed.

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I. INTRODUCTION

The investigation of semileptonic decays of B mesons into light mesons is important for the determination of the Cabibbo-Kobayashi-Maskawa matrix element V_{ub}, which is the most poorly studied. At present the value of V_{ub} is mainly determined from the end point of the lepton spectrum in semileptonic B decays [1]. Unfortunately, the theoretical interpretation of the end point region of the lepton spectrum in inclusive $B \to X_u \ell \bar{\nu}$ decays is very complicated and suffers from large uncertainties [2]. The other way to determine V_{ub} is to consider exclusive semileptonic decays $B \to \pi(\rho)e\nu$. These are the heavy-to-light transitions with a wide kinematic range. In contrast with the heavy-to-heavy transitions, here we cannot expand matrix elements in the inverse powers of the final quark mass. It is also necessary to mention that the final meson has a large recoil momentum almost in the whole kinematical range. Thus the motion of final $\pi(\rho)$ meson should be treated relativistically. If we consider the point of maximum recoil of the final meson, we find that $\pi(\rho)$ bears the large relativistic recoil momentum $|\mathbf{\Delta}_{\max}|$ of order $m_b/2$ and the energy of the same order. Thus at this kinematical point it is possible to expand the matrix element of the weak current both in inverse powers of b-quark mass of the initial B meson and in inverse powers of the recoil momentum $|\mathbf{\Delta}_{\max}|$ of the final $\pi(\rho)$ meson. As a result the expansion in powers $1/m_b$ arises for the $B \to \pi(\rho)$ semileptonic form factors at $q^2 = 0$, where q is a momentum carried by the lepton pair. The aim of this paper is to realize such expansion in the framework of relativistic quark model. We show that this expansion considerably simplifies the analysis of exclusive $B \to \pi(\rho)e\nu$ semileptonic decays.

Our relativistic quark model is based on the quasipotential approach in quantum field theory with the specific choice of the $q\bar{q}$ potential. It provides a consistent scheme for calculation of all relativistic corrections at a given order of v^2/c^2

and allows for the heavy quark $1/m_Q$ expansion. This model has been applied for the calculations of meson mass spectra [3], radiative decay widths [4], pseudoscalar decay constants [5], heavy-to-heavy semileptonic [6], and nonleptonic [7] decay rates. The heavy quark $1/m_Q$ expansion in our model for the heavy-to-heavy semileptonic transitions has been developed in [8] up to $1/m_Q^2$ order. The results are in agreement with the model independent predictions of the heavy quark effective theory (HQET) [9]. The $1/m_b$ expansion of rare radiative decay form factors of B mesons has been carried out in [10] along the same lines as in the present paper. We have briefly presented the results for $B \to \pi e\nu$ and $B \to \rho e\nu$ decays in Ref. [11], where the expansion up to the first order in $1/m_b$ has been carried out. In the present paper we extend the analysis up to the second order and give a detailed discussion of the expansion method and results.

The paper is organized as follows. The relativistic quark model is described in Sec. II. In Sec. III we give the detailed description of the method of calculating decay matrix elements between heavy-heavy and heavy-light meson states, based on the quasipotential approach. We show that the heavy-to-heavy decay matrix elements can be expanded in inverse powers of the heavy quark masses at zero recoil of the final meson. On the other hand, the heavy-to-light decay matrix elements can be expanded in inverse powers of the initial heavy quark mass at the maximum recoil of the final light meson. These expansions permit the calculation of decay matrix elements with the account of relativistic effects. In Sec. IV the method is applied to the calculation of the semileptonic $B \to \pi(\rho)e\nu$ decay form factors. Our numerical results for the form factors and decay rates are presented in Sec. V. Therein we discuss the q^2 dependence of the form factors and the relations between semileptonic $B \to \rho e\nu$ and rare radiative $B \to \rho(K^*)\gamma$ decays. Section VI contains our conclusions. The formulas for the form factors at the point of maximum recoil of the final light meson are given in the Appendix.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach [12] meson with the mass M and relative momentum of quarks $\mathbf{p}$ is described by a single-time quasipotential wave function $\Psi_M(\mathbf{p})$, projected onto positive-energy states. This wave function satisfies the quasipotential equation

$$\left[M - (\mathbf{p}^2 + m_a^2)^{1/2} - (\mathbf{p}^2 + m_b^2)^{1/2}\right]\Psi_M(\mathbf{p})$$
$$= \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q}). \quad (1)$$

The quasipotential equation (1) can be transformed into a local Schrödinger-like equation [13]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M)\Psi_M(\mathbf{q}), \quad (2)$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_a^2 - m_b^2)^2}{4M^3}; \quad (3)$$

and the square of the relative momentum on the mass shell is

$$b^2(M) = \frac{[M^2 - (m_a + m_b)^2][M^2 - (m_a - m_b)^2]}{4M^2}; \quad (4)$$

$m_{a,b}$ are the quark masses. While constructing the kernel of this equation $V(\mathbf{p}, \mathbf{q}; M)$, the quasipotential of quark-antiquark interaction, we have assumed that effective interaction is the sum of the one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials. We have also assumed that at large distances the vector long-range potential contains the Pauli interaction. The quasipotential is defined by [3]

$$V(\mathbf{p}, \mathbf{q}, M) = \bar{u}_a(p)\bar{u}_b(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}, M)u_a(q)u_b(-q)$$
$$= \bar{u}_a(p)\bar{u}_b(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_a^\mu\gamma_b^\nu\right.$$
$$\left. + V_{\text{conf}}^V(\mathbf{k})\Gamma_a^\mu\Gamma_{b;\mu} + V_{\text{conf}}^S(\mathbf{k})\right\}u_a(q)u_b(-q),$$
$$(5)$$

where α_S is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator; γ_μ and $u(p)$ are the Dirac matrices and spinors; $\mathbf{k} = \mathbf{p} - \mathbf{q}$; the effective long-range vector vertex is

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu; \quad (6)$$

κ is the Pauli interaction constant. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\text{conf}}^V(r) = (1-\varepsilon)(Ar+B), \quad V_{\text{conf}}^S(r) = \varepsilon(Ar+B), \quad (7)$$

reproducing $V_{\text{nonrel}}^{\text{conf}}(r) = V_{\text{conf}}^S + V_{\text{conf}}^V = Ar + B$, where ε is the mixing coefficient. The explicit expression for the quasipotential with the account of the relativistic corrections of order v^2/c^2 can be found in Ref. [3]. All the parameters of our model: quark masses, parameters of linear confining potential A and B, mixing coefficient ε, and Pauli interaction constant κ were fixed from the analysis of meson masses [3] and radiative decays [4]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV, and the parameters of linear potential: $A = 0.18$ GeV^2; $B = -0.30$ GeV have standard values for quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -0.9$ has been primarily chosen from the consideration of meson radiative decays, which are very sensitive to the Lorentz structure of the confining potential: the resulting leading relativistic corrections coming from vector and scalar potentials have opposite signs for the radiative $M1$ decays [4]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J states [3].

Recently we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson states up to the second order in inverse powers of the heavy quark masses [8]. It has been found that the general structure of leading, subleading, and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential of our model. The analysis of the first order corrections [8] allowed to fix the value of Pauli interaction constant $\kappa = -1$, which coincides with the result, obtained from the mass spectra [3]. The mixing parameter of vector and scalar confining potentials has been found from the comparison of the second order corrections to be $\varepsilon = -1$. This value is very close to the previous one $\varepsilon = -0.9$ determined from radiative decays of mesons [4]. Therefore, we have got QCD and heavy quark symmetry motivation for the choice of the main parameters of our model. The found values of ε and κ imply that confining quark-antiquark potential has predominantly Lorentz-vector structure, while the scalar potential is anticonfining and helps to reproduce the initial nonrelativistic potential.

III. MATRIX ELEMENTS OF ELECTROWEAK CURRENT BETWEEN HEAVY-HEAVY AND HEAVY-LIGHT MESON STATES

The matrix element of the local current J between bound states in the quasipotential method has the form [14]

$$\langle M'|J_\mu(0)|M\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}_{M'}(\mathbf{p})\Gamma_\mu(\mathbf{p}, \mathbf{q})\Psi_M(\mathbf{q}), \quad (8)$$

where $M(M')$ is initial (final) meson, $\Gamma_\mu(\mathbf{p}, \mathbf{q})$ is the two-particle vertex function, and $\Psi_{M,M'}$ are the meson wave functions projected onto the positive energy states of quarks.

This relation is valid for the general structure of the current $J = \bar{Q}'GQ$, where G can be an arbitrary combination of Dirac matrices. The contributions to Γ come from Figs. 1 and 2. Thus the vertex functions look like

$$\Gamma^{(1)}(\mathbf{p}, \mathbf{q}) = \bar{u}_{Q'}(p_1)Gu_Q(q_1)(2\pi)^3\delta(\mathbf{p}_2 - \mathbf{q}_2), \quad (9)$$

and

$$\Gamma^{(2)}(\mathbf{p},\mathbf{q})=\bar{u}_{Q'}(p_1)\bar{u}_q(p_2)\left\{ G\frac{\Lambda_Q^{(-)}(k_1)}{\varepsilon_Q(k_1)+\varepsilon_Q(p_1)}\gamma_1^0\mathscr{V}(\mathbf{p}_2-\mathbf{q}_2)+\mathscr{V}(\mathbf{p}_2-\mathbf{q}_2)\frac{\Lambda_{Q'}^{(-)}(k_1')}{\varepsilon_{Q'}(k_1')+\varepsilon_{Q'}(q_1)}\gamma_1^0 G\right\}u_Q(q_1)u_q(q_2),$$

$$(10)$$

where $\mathbf{k}_1=\mathbf{p}_1-\mathbf{\Delta}$, $\mathbf{k}_1'=\mathbf{q}_1+\mathbf{\Delta}$, $\mathbf{\Delta}=\mathbf{p}_M-\mathbf{p}_{M'}$, $\varepsilon(p)=(m^2+\mathbf{p}^2)^{1/2}$,

$$\Lambda^{(-)}(p)=\frac{\varepsilon(p)-[m\gamma^0+\gamma^0(\boldsymbol{\gamma}\mathbf{p})]}{2\varepsilon(p)}$$

and

$$p_{1,2}=\varepsilon_{1,2}(p)\frac{p_{M'}}{M'}\pm\sum_{i=1}^3 n^{(i)}(p_{M'})p^i,$$

$$q_{1,2}=\varepsilon_{1,2}(q)\frac{p_M}{M}\pm\sum_{i=1}^3 n^{(i)}(p_M)q^i;$$

here

$$n^{(i)\mu}(p)=L_{p_i}^\mu=\left\{\frac{p^i}{M},\delta_{ij}+\frac{p^ip^j}{M(E+M)}\right\},$$

Note that the contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. The form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of $q\bar{q}$ interaction.

The general structure of the current matrix element (8) is rather complicated, because it is necessary to integrate both with respect to d^3p and d^3q. The δ function in the expression (9) for the vertex function $\Gamma^{(1)}$ permits to perform one of these integrations. As a result the contribution of $\Gamma^{(1)}$ to the current matrix element has usual structure and can be calculated without any expansion, if the wave functions of initial and final meson are known. The situation with the contribution $\Gamma^{(2)}$ is different. Here instead of δ function we have a complicated structure, containing the potential of $q\bar{q}$ interaction in meson. Thus in general case we cannot get rid of one of the integrations in the contribution of $\Gamma^{(2)}$ to the matrix element (8). Therefore, it is necessary to use some additional considerations. The main idea is to expand the vertex function $\Gamma^{(2)}$, given by (10), in such a way that it will be possible to use the quasipotential equation (1) in order to perform one of the integrations in the current matrix element (8). The realization of such expansion differs for the cases of heavy-to-heavy and heavy-to-light transitions.
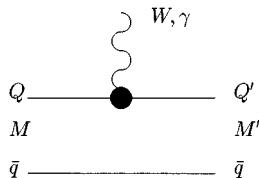
A. Heavy-to-heavy decay matrix elements

At first we consider the heavy-to-heavy meson decays, such as semileptonic $B\to De\nu$ decays, and radiative transitions in quarkonia and $B^*\to B\gamma$, $D^*\to D\gamma$. Here we have two natural expansion parameters, which are the heavy quark masses in initial and final meson. The most convenient point for the expansion of vertex function $\Gamma^{(2)}$ in inverse powers of the heavy quark masses for semileptonic decays is the point of zero recoil of final meson, where $\mathbf{\Delta}=0$. For radiative decays the momentum transfer is fixed $|\mathbf{\Delta}|=(M_M^2-M_{M'}^2)/2M_M$. The difference of initial and final meson masses is proportional to the fine or hyperfine splitting and thus $|\mathbf{\Delta}|/M=O(1/M^2)$, so zero recoil is a good approximation.

It is easy to see that $\Gamma^{(2)}$ contributes to the current matrix element at first order of $1/m_Q$ expansion for transitons between mesons consisting from heavy and light quarks (B, D mesons) [8] and at second order of v/c expansion for mesons consisting from two heavy quarks of the same flavour (quarkonia Υ, J/ψ) [4]. We limit our analysis to the consideration of the terms up to the second order in $1/m_Q$ or v/c expansions. We substitute the Dirac matrices G and spinors u in the vertex function $\Gamma^{(2)}$ and consider the cases of Lorentz-scalar and Lorentz-vector (with Pauli term) $q\bar{q}$-interaction potential. Then we expand $\Gamma^{(2)}$ to the desired order and see that it is possible to integrate either with respect to d^3p or d^3q in the current matrix element (8) using quasipotential equation (1). Performing these integrations and taking the sum of the contributions of $\Gamma^{(1)}$ and $\Gamma^{(2)}$ we get the expression for the current matrix element, which contains the ordinary mean values between meson wave functions. Thus this matrix element can be easily calculated numerically if the meson wave functions are known. The described method has been applied to the calculations of heavy-to-heavy semileptonic decays in [6,8] and radiative decays in [4].

B. Heavy-to-light decay matrix elements

Now we consider the heavy-to-light meson decays, such as semileptonic $B\to\pi(\rho)e\nu$ and rare radiative $B\to K^*\gamma$ decays. In these decays the final meson contains only light quarks (u, d, s); thus, in contrast to the heavy-to-heavy transitions, we cannot expand matrix elements in inverse

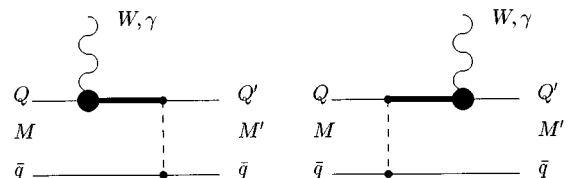FIG. 1. Lowest order vertex function.

FIG. 2. Vertex function with the account of the quark interaction. Dashed line corresponds to the effective potential (5). Bold line denotes the negative-energy part of the quark propagator.

powers of the final quark mass. The expansion of $\Gamma^{(2)}$ only in inverse powers of the initial heavy quark mass at $\boldsymbol{\Delta}=0$ does not allow to use the quasipotential equation for performing one of the integrations in corresponding current matrix element (8). However, as was already mentioned in the Introduction, the final light meson has the large recoil momentum almost in the whole kinematical range. At the point of maximum recoil of final light meson[1] the large value of recoil momentum $\boldsymbol{\Delta}_{\max}\sim m_Q/2$ allows for the expansion of decay matrix element in $1/m_Q$. The contributions to this expansion come both from the inverse powers of heavy m_Q from initial meson and from inverse powers of the recoil momentum $|\boldsymbol{\Delta}_{\max}|$ of the final light meson. The large value of recoil momentum $|\boldsymbol{\Delta}_{\max}|$ permits to neglect $\mathbf{p}^2$ in comparison with $\boldsymbol{\Delta}_{\max}^2$ in the light quark energy $\varepsilon_{q,Q'}(p+\Delta)$ in final meson in the expression for the matrix element originating from $\Gamma^{(2)}$. Such approximation corresponds to omitting terms of the third order in $1/m_Q$ expansion and is compatible

with our analysis, which is carried out up to the second order. It is easy to see that we can now perform one of the integrations in the current matrix element (8) using the quasipotential equation as in the case of heavy final meson. As a result we again get the expression for the current matrix element, which contains only the ordinary mean values between meson wave functions, but in this case at the point of maximum recoil of final light meson. This method has been applied to calculation of rare radiative decays of B mesons in Ref. [10] and in the next section we use it for consideration of $B\to\pi(\rho)e\nu$ semileptonic decays.

IV. $B\to\pi(\rho)e\nu$ DECAY FORM FACTORS

A. Decay form factors at $q^2=0$

The form factors of the semileptonic decays $B\to\pi e\nu$ and $B\to\rho e\nu$ are defined in the standard way as

$$\langle\pi(p_\pi)|\bar{q}\gamma_\mu b|B(p_B)\rangle=f_+(q^2)(p_B+p_\pi)_\mu+f_-(q^2)(p_B-p_\pi)_\mu,\tag{11}$$

$$\langle\rho(p_\rho,e)|\bar{q}\gamma_\mu(1-\gamma^5)b|B(p_B)\rangle=-(M_B+M_\rho)A_1(q^2)e_\mu^*+\frac{A_2(q^2)}{M_B+M_\rho}(e^*p_B)(p_B+p_\rho)_\mu+\frac{A_3(q^2)}{M_B+M_\rho}(e^*p_B)(p_B-p_\rho)_\mu$$

$$+\frac{2V(q^2)}{M_B+M_\rho}i\epsilon_{\mu\nu\tau\sigma}e^{*\nu}p_B^\tau p_\rho^\sigma,\tag{12}$$

where $q=p_B-p_{\pi(\rho)}$ and e is a polarization vector of ρ meson. In the limit of vanishing lepton mass, the form factors f_- and A_3 do not contribute to the decay rates and thus will not be considered.

It is convenient to consider the decay $B\to\pi(\rho)e\nu$ in the B meson rest frame. Then the wave function of the final $\pi(\rho)$ meson moving with the recoil momentum $\boldsymbol{\Delta}$ is connected with the wave function at rest by the transformation [14]

$$\Psi_{\pi(\rho)\boldsymbol{\Delta}}(\mathbf{p})=D_q^{1/2}(R_{L\boldsymbol{\Delta}}^W)D_q^{1/2}(R_{L\boldsymbol{\Delta}}^W)\Psi_{\pi(\rho)\mathbf{0}}(\mathbf{p}),\tag{13}$$

where $D^{1/2}(R)$ is the well-known rotation matrix and R^W is the Wigner rotation.

The meson wave functions in the rest frame have been calculated by numerical solution of the quasipotential equation (2) [15]. However, it is more convenient to use analytical expressions for meson wave functions. The examination of numerical results for the ground state wave functions of mesons containing at least one light quark has shown that they can be well approximated by the Gaussian functions

$$\Psi_M(\mathbf{p})\equiv\Psi_{M\mathbf{0}}(\mathbf{p})=\left(\frac{4\pi}{\beta_M^2}\right)^{3/4}\exp\left(-\frac{\mathbf{p}^2}{2\beta_M^2}\right),\tag{14}$$

with the deviation less than 5%.

The parameters are

$$\beta_B=0.41\,\text{GeV},\quad\beta_{\pi(\rho)}=0.31\,\text{GeV}.$$

Now we apply the method for calculation of decay matrix elements, described in the previous section. At the point of maximal recoil of final light meson

$$|\boldsymbol{\Delta}_{\max}|=\frac{M_B^2-M_{\pi(\rho)}^2}{2M_B},\tag{15}$$

we expand the vertex function $\Gamma^{(2)}$ for the Lorentz-scalar and Lorentz-vector (with Pauli term) $q\bar{q}$ interactions up to the second order in $1/m_b$. Then we substitute the vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ in the matrix element (8) and take into account the Lorentz transformation of the final meson wave function (13). Performing one of the integrations in the current matrix element (8) (using the δ-function in $\Gamma^{(1)}$ and the quasipotential equation in the contribution of $\Gamma^{(2)}$) we get for the form factors of $B\to\pi e\nu$ and $B\to\rho e\nu$ decays the following expressions at $q^2=0$ point:

$$f_+(0)=f_+^{(1)}(0)+\varepsilon f_+^{S(2)}(0)+(1-\varepsilon)f_+^{V(2)}(0),\tag{16}$$

$$A_1(0)=A_1^{(1)}(0)+\varepsilon A_1^{S(2)}(0)+(1-\varepsilon)A_1^{V(2)}(0),\tag{17}$$

$$A_2(0)=A_2^{(1)}(0)+\varepsilon A_2^{S(2)}(0)+(1-\varepsilon)A_2^{V(2)}(0),\tag{18}$$

$$V(0)=V^{(1)}(0)+\varepsilon V^{S(2)}(0)+(1-\varepsilon)V^{V(2)}(0),\tag{19}$$

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[1]In the case of rare radiative decays the recoil momentum of final light meson is fixed at the maximum value $\boldsymbol{\Delta}_{\max}$.

where $f_+^{(1)}$, $f_+^{S,V(2)}$, $A_{1,2}^{(1)}$, $A_{1,2}^{S,V(2)}$, $V^{(1)}$, and $V^{S,V(2)}$ are given in Appendix, the superscripts ''(1)'' and ''(2)'' correspond to Figs. 1 and 2, and S and V to the scalar and vector potentials of the $q\bar{q}$ interaction.

B. $1/m_b$ expansion for decay form factors

Let us proceed further and for the sake of consistensy carry out the complete expansion of form factors (16)–(19) in inverse powers of b-quark mass. For this expansion we will use some model independent results obtained in HQET [9].

In HQET the mass of B meson has the following expansion in $1/m_b$ [9]:

$$M_B = m_b + \bar{\Lambda} + \frac{\Delta m_B^2}{2m_b} + O\left(\frac{1}{m_b^2}\right), \tag{20}$$

where parameter $\bar{\Lambda}$ is the difference between the meson and quark masses in the limit of infinitely heavy quark mass. In our model $\bar{\Lambda}$ is equal to the mean value of light quark energy inside the heavy meson $\bar{\Lambda} = \langle \varepsilon_q \rangle_B \approx 0.54$ GeV [8]. Δm_B^2 arises from the first-order power corrections to the HQET Lagrangian and has the form [9]

$$\Delta m_B^2 = -\lambda_1 - 3\lambda_2. \tag{21}$$

The parameter λ_1 results from the mass shift due to the kinetic operator, while λ_2 parameterizes the chromomagnetic interaction [9]. The value of spin-symmetry breaking parameter λ_2 is related to the vector-pseudoscalar mass splitting:

$$\lambda_2 \approx \frac{1}{4}(M_{B*}^2 - M_B^2) = 0.12 \pm 0.01 \text{ GeV}^2.$$

The parameter λ_1 is not directly connected with observable quantities. Theoretical predictions for it vary in a wide range: $\lambda_1 = -0.30 \pm 0.30$ GeV^2 [9,16].

The Gaussian parameter β_B in the wave function (14) has the following expansion [8] in our model:

$$\beta_B = \beta - \frac{\Delta \beta^2}{m_b} + O\left(\frac{1}{m_b^2}\right), \quad \beta \approx 0.42 \text{ GeV}, \tag{22}$$

where $\Delta \beta^2 \approx 0.045$ GeV^2 [8].

Substituting (20) in (15) and (70) we get the $1/m_b$ expansion of the recoil momentum and the energy of final vector meson:

$$|\mathbf{\Delta}_{\max}| = \frac{m_b}{2}\left[1 + \frac{1}{m_b}\bar{\Lambda} + \frac{1}{m_b^2}\left(\frac{\Delta m_B^2}{2} - M_{\pi(\rho)}^2\right)\right] + O\left(\frac{1}{m_b}\right),$$

$$E_{\pi(\rho)} = \frac{m_b}{2}\left[1 + \frac{1}{m_b}\bar{\Lambda} + \frac{1}{m_b^2}\left(\frac{\Delta m_B^2}{2} + M_{\pi(\rho)}^2\right)\right] + O\left(\frac{1}{m_b}\right). \tag{23}$$

Now we use the Gaussian approximation for the wave functions (14). Then shifting the integration variable $\mathbf{p}$ in (58)–(69) by $-[\varepsilon_q/(E_{\pi(\rho)} + M_{\pi(\rho)})]\mathbf{\Delta}_{\max}$, we can factor

out the $\mathbf{\Delta}_{\max}$ dependence of the meson wave function overlap in form factors f_+, $A_{1,2}$, V. The result can be written in the form

$$f_+(0) = \mathcal{F}_+(\mathbf{\Delta}_{\max}^2)\exp(-\zeta\mathbf{\Delta}_{\max}^2), \tag{24}$$

$$A_{1,2}(0) = \mathcal{A}_{1,2}(\mathbf{\Delta}_{\max}^2)\exp(-\zeta\mathbf{\Delta}_{\max}^2), \tag{25}$$

$$V(0) = \mathcal{V}(\mathbf{\Delta}_{\max}^2)\exp(-\zeta\mathbf{\Delta}_{\max}^2), \tag{26}$$

where $|\mathbf{\Delta}_{\max}|$ is given by (15) and

$$\zeta\mathbf{\Delta}_{\max}^2 = \frac{2\tilde{\Lambda}^2\mathbf{\Delta}_{\max}^2}{(\beta_B^2 + \beta_{\pi(\rho)}^2)(E_{\pi(\rho)} + M_{\pi(\rho)})^2}$$

$$= \frac{\tilde{\Lambda}^2}{\beta_B^2}\eta\left(\frac{M_B - M_{\pi(\rho)}}{M_B + M_{\pi(\rho)}}\right)^2; \tag{27}$$

here $\eta = 2\beta_B^2/(\beta_B^2 + \beta_{\pi(\rho)}^2)$ and $\tilde{\Lambda}$ is equal to the mean value of light quark energy between B and $\pi(\rho)$ meson states:

$$\tilde{\Lambda} = \langle \varepsilon_q \rangle \approx 0.53 \text{ GeV}. \tag{28}$$

Expanding (27) in powers of $1/m_b$ we get

$$\zeta\mathbf{\Delta}_{\max}^2 = \frac{\tilde{\Lambda}^2}{\beta^2}\eta\left(1 - 4\frac{M_{\pi(\rho)}}{m_b}\right) + O\left(\frac{1}{m_b^2}\right). \tag{29}$$

We see that the first term in this expansion for the decay into ρ meson is large. Really, $4M_\rho/m_b \approx 0.63$. The value of this correction is also increased by the exponentiating in (24)–(26). Therefore, we conclude that the first order correction in $1/m_b$ expansion for the form factors of $B \to \pi(\rho)e\nu$ decay, arising from the meson wave function overlap, is large.[2] Thus, taking into account that our method of calculating decay matrix elements does not require the expansion of the meson wave function overlap, we use unexpanded expression (27) in the exponential of the form factors (24)–(26).

In contrast with the meson wave function overlap the factors $\mathcal{F}_1(\mathbf{\Delta}_{\max}^2)$, $\mathcal{A}_{1,2}(\mathbf{\Delta}_{\max}^2)$, and $\mathcal{V}(\mathbf{\Delta}_{\max}^2)$, which originate from the vertex functions $\Gamma^{(1),(2)}$ and Lorentz transformation (13) of the final meson wave function, have a well defined $1/m_b$ expansion. The first and second order corrections are small. Substituting the Gaussian wave functions (13) in the expressions for the form factor (16)–(19) and (A1)–(A12), with the value of Pauli interaction constant $\kappa = -1$, and using (24)–(26) and the expansions (20)–(23), we get up to the second order in $1/m_b$ expansion:

(a) $B \to \pi e \nu$ decay

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[2] The same situation occurs for the rare radiative B decays [10].

$$\mathscr{F}_+(\boldsymbol{\Delta}_{\max}^2) = \mathscr{F}_+^{(1)}(\boldsymbol{\Delta}_{\max}^2) + \varepsilon \mathscr{F}_+^{S(2)}(\boldsymbol{\Delta}_{\max}^2) + (1-\varepsilon)\mathscr{F}_+^{V(2)}(\boldsymbol{\Delta}_{\max}^2); \tag{30}$$

$$\mathscr{F}_1^{(1)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ 1 + \frac{1}{m_b}X_1 + \frac{1}{m_b^2}\left[Y_+ - 2\langle\mathbf{p}^2\rangle - \frac{3}{4}\widetilde{\Lambda}^2\eta^2 + \overline{\Lambda}\eta(2m_q+M_\pi) + \frac{2}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle\left(2\widetilde{\Lambda}\eta - m_q - 2M_\pi - \frac{1}{2}\overline{\Lambda}\right) \right] \right\}; \tag{31}$$

$$\mathscr{F}_+^{S(2)}(\boldsymbol{\Delta}_{\max}^2) = N\frac{1}{m_b^2}\left\{ -2m_q(M_\pi - 2\langle\overline{\varepsilon}_q\rangle) - \frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle[M_\pi(1-R)+2m_q(1-2R)-2\overline{\Lambda}R] + \frac{2}{3}\langle\mathbf{p}^2\rangle(1-2R) \right.$$
$$\left. + \frac{1}{2}\widetilde{\Lambda}\eta\left(2\langle\overline{\varepsilon}_q\rangle(1-2R) - M_\pi(1-R) + 2\overline{\Lambda}R - \frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q}\right\rangle \right) \right\}; \tag{32}$$

$$\mathscr{F}_+^{V(2)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ -\frac{1}{m_b}[2RZ_1 + (1+2R)Z_2] + \frac{1}{m_b^2}\left[-\frac{2}{3}m_q RZ_3 + 4\langle\mathbf{p}^2\rangle\left(1-\frac{3}{4}R\right) - \left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle\left(2M_\pi\left(1-\frac{1}{3}R\right)\right.\right.\right.$$
$$\left.\left.\left. + 4m_q\left(1-\frac{5}{4}R\right) + \frac{5}{3}\overline{\Lambda}Rb\right) + \left(\left(2M_\pi + \eta\frac{\beta_\pi^2}{\beta^2}\frac{\Delta\beta^2}{\beta}\right)(1+2R) - 2m_q(1-2R)\right)Z_2 \right] \right\}; \tag{33}$$

(b) $B\rightarrow\rho e\nu$ decay

$$\mathscr{A}_1(\boldsymbol{\Delta}_{\max}^2) = \mathscr{A}_1^{(1)}(\boldsymbol{\Delta}_{\max}^2) + \varepsilon\mathscr{A}_1^{S(2)}(\boldsymbol{\Delta}_{\max}^2) + (1-\varepsilon)\mathscr{A}_1^{V(2)}(\boldsymbol{\Delta}_{\max}^2); \tag{34}$$

$$\mathscr{A}_1^{(1)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ 1 + \frac{1}{m_b}X_- + \frac{1}{m_b^2}\left(Y_- + M_\rho(M_\rho - m_q) + \overline{\Lambda}(M_\rho + m_q) - \frac{1}{2}\widetilde{\Lambda}\eta(5m_q + 3M_\rho) \right.\right.$$
$$\left.\left. + \frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle(2\widetilde{\Lambda}\eta - m_q - 3M_\rho) \right) \right\}; \tag{35}$$

$$\mathscr{A}_1^{S(2)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ \frac{2}{m_b}(M_\rho - 2\langle\overline{\varepsilon}_q\rangle) + \frac{1}{m_b^2}\left[-2(M_\rho + \overline{\Lambda} + 3m_q)(M_\rho - 2\langle\overline{\varepsilon}_q\rangle) - \frac{2}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle[M_\rho R + m_q(1+3R) + \overline{\Lambda}(1+R)] \right.\right.$$
$$\left.\left. + \frac{2}{3}\langle\mathbf{p}^2\rangle(1+3R) - \widetilde{\Lambda}\eta\left(M_\rho(2+R) - \langle\overline{\varepsilon}_q\rangle(7+3R) + \overline{\Lambda}(3+R) + \frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q}\right\rangle(1+R) \right) \right] \right\}; \tag{36}$$

$$\mathscr{A}_1^{V(2)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ \frac{1}{m_b}(4RZ_1 + 2RZ_2) + \frac{1}{m_b^2}\left[\frac{4}{3}(m_q - 3M_\rho)RZ_3 - 10\langle\mathbf{p}^2\rangle\left(1-\frac{2}{3}R\right) + \left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle\left(2M_\rho\left(2+\frac{13}{3}R\right)\right.\right.\right.$$
$$\left.\left.\left. + 8m_q\left(1-\frac{4}{3}R\right)\right) + 2\left(2m_q(1-R) - 2M_\rho R + \eta\frac{\beta_\rho^2}{\beta^2}\frac{\Delta\beta^2}{\beta}R\right)Z_2 + 2\widetilde{\Lambda}\eta\left[\left\langle\frac{1}{\overline{\varepsilon}_q+m_q}\right\rangle\widetilde{\Lambda}\eta\left(M_\rho\left(1-\frac{5}{3}R\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left. - \overline{\Lambda} + m_q\left(1-\frac{10}{3}R\right)\right) + 2(3\overline{\Lambda} - M_\rho - \langle\overline{\varepsilon}_q\rangle) - \widetilde{\Lambda}\eta\left(1-\frac{10}{3}R\right) - \frac{10}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q}\right\rangle \right] \right] \right\}; \tag{37}$$

$$\mathscr{A}_2(\boldsymbol{\Delta}_{\max}^2) = \mathscr{A}_2^{(1)}(\boldsymbol{\Delta}_{\max}^2) + \varepsilon\mathscr{A}_2^{S(2)}(\boldsymbol{\Delta}_{\max}^2) + (1-\varepsilon)\mathscr{A}_2^{V(2)}(\boldsymbol{\Delta}_{\max}^2); \tag{38}$$

$$\mathscr{A}_2^{(1)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ 1 + \frac{1}{m_b}X_- + \frac{1}{m_b^2}\left(Y_- - M_\rho(3M_\rho - m_q) + \overline{\Lambda}(M_\rho - m_q) - \frac{1}{2}\widetilde{\Lambda}\eta(M_\rho - 7m_q) \right.\right.$$
$$\left.\left. - 2\widetilde{\Lambda}^2\eta^2 + \frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q+m_q}\right\rangle(2\widetilde{\Lambda}\eta - m_q + 5M_\rho) \right) \right\}; \tag{39}$$

$$\mathscr{A}_2^{S(2)}(\boldsymbol{\Delta}_{\max}^2) = \mathscr{A}_1^{S(2)}(\boldsymbol{\Delta}_{\max}^2) + N\left\{ \frac{1}{m_b^2}4M_\rho(M_\rho - 2\langle\overline{\varepsilon}_q\rangle) \right\}; \tag{40}$$

$$\mathscr{A}_2^{V(2)}(\boldsymbol{\Delta}_{\max}^2) = N \left\{ \frac{1}{m_b}(4RZ_1 + 2RZ_2) + \frac{1}{m_b^2}b\left[\frac{4}{3}(M_\rho - m_q)RZ_3 - 2\langle\mathbf{p}^2\rangle\left(1 - \frac{10}{3}R\right) + \left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle\left(\frac{2}{3}M_\rho(1 - 11R)\right. \right.$$

$$+ \frac{2}{3}\overline{\Lambda} + 2m_q(1+R)\Big) + 2\left(m_q(1+3R) - 3M_\rho R + \eta\frac{\beta_\rho^2}{\beta^2}\frac{\Delta\beta^2}{\beta}R \right)Z_2 + 2\widetilde{\Lambda}\eta\left[-\left\langle \frac{1}{\overline{\varepsilon}_q + m_q} \right\rangle\widetilde{\Lambda}\eta\left(M_\rho\left(1 + \frac{5}{3}R\right)\right.$$

$$\left. + \overline{\Lambda} + m_q\left(3 + \frac{10}{3}R\right)\right) + 3\overline{\Lambda} - M_\rho - \langle\overline{\varepsilon}_q\rangle + \widetilde{\Lambda}\eta\left(3 + \frac{10}{3}R\right) - \frac{1}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q}\right\rangle\Big]\Big]\Big\}; \tag{41}$$

$$\mathscr{V}(\boldsymbol{\Delta}_{\max}^2) = \mathscr{V}^{(1)}(\boldsymbol{\Delta}_{\max}^2) + \varepsilon\mathscr{V}^{S(2)}(\boldsymbol{\Delta}_{\max}^2) + (1-\varepsilon)\mathscr{V}^{A(2)}(\boldsymbol{\Delta}_{\max}^2); \tag{42}$$

$$\mathscr{V}^{(1)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ 1 + \frac{1}{m_b}X_+ + \frac{1}{m_b^2}\left(Y_- + M_\rho(M_\rho - m_q) + \frac{1}{2}m_q^2 + \overline{\Lambda}(2m_q - M_\rho) + \frac{7}{2}\widetilde{\Lambda}\eta M_\rho - 4\frac{1}{3}\langle\mathbf{p}^2\rangle \right.$$

$$\left. - \frac{1}{3}\left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle(M_\rho - m_q)\right)\right\}; \tag{43}$$

$$\mathscr{V}^{S(2)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ -\frac{2}{m_b}(M_\rho - 2\langle\overline{\varepsilon}_q\rangle) + \frac{1}{m_b^2}\left[-2(M_\rho - \overline{\Lambda} - m_q)(M_\rho - 2\langle\overline{\varepsilon}_q\rangle) + \frac{2}{3}\left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle[M_\rho(1-R) + m_q(2-3R)] \right.$$

$$\left. - \overline{\Lambda}R] + \frac{2}{3}\langle\mathbf{p}^2\rangle(2-3R) + \widetilde{\Lambda}\eta\left(-M_\rho(1+2R) + 6\langle\overline{\varepsilon}_q\rangle R + 2\overline{\Lambda}(1-R) - \frac{2}{3}\left\langle\frac{\mathbf{p}^2}{\overline{\varepsilon}_q}\right\rangle(2-R) \right)\right]\right\}; \tag{44}$$

$$\mathscr{V}^{A(2)}(\boldsymbol{\Delta}_{\max}^2) = N\left\{ \frac{1}{m_b}(8RZ_1 - 2(1-R)Z_2) + \frac{1}{m_b^2}\left[-\frac{8}{3}(M_\rho - m_q)RZ_3 + 20\langle\mathbf{p}^2\rangle - \left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle\left(2M_\rho\left(\frac{11}{3} - 4R\right)\right. \right. \right.$$

$$\left. + \frac{22}{3}\overline{\Lambda} + 4m_q(5+2R)\right) + 2\left(\overline{\Lambda} + \left(M_\rho R + \eta\frac{\beta_\rho^2}{\beta^2}\frac{\Delta\beta^2}{\beta} \right)(1-R) + m_q(1+R) \right)\mathbf{Z_2}$$

$$\left. + \widetilde{\Lambda}^2\eta^2\left(17 - \left\langle \frac{1}{\overline{\varepsilon}_q + m_q} \right\rangle\left(\frac{16}{3}M_\rho + \frac{19}{3}\overline{\Lambda} + 17m_q \right) \right)\right]\right\}, \tag{45}$$

where $N = (2\beta_B\beta_{\pi(\rho)}/[\beta_B^2 + \beta_{\pi(\rho)}^2])^{3/2} = ([\beta_{\pi(\rho)}/\beta_B]\eta)^{3/2}$ is due to the normalization of Gaussian wave functions in (14); $\overline{\varepsilon}_q = \sqrt{\mathbf{p}^2 + m_q^2 + \widetilde{\Lambda}^2\eta^2}$, i.e., the energies of light quarks in final light meson acquire additional contribution from the recoil momentum. The averaging $\langle\cdots\rangle$ is taken over the Gaussian wave functions of B and $\pi(\rho)$ mesons, so it can be carried out analytically. For example,

$$\langle\overline{\varepsilon}_q\rangle = N^{-1}\int \frac{d^3p}{(2\pi)^3}\overline{\Psi}_{\pi(\rho)}(\mathbf{p})\overline{\varepsilon}_q(p)\Psi_B(\mathbf{p})$$

$$= \frac{1}{\sqrt{\pi}}\frac{\overline{m}_q^2}{\beta_{\pi(\rho)}\sqrt{\eta}}e^z K_1(z), \tag{46}$$

where $\overline{m}_q^2 = m_q^2 + \widetilde{\Lambda}^2\eta^2$ and $K_1(z)$ is the modified Bessel function; $z = \overline{m}_q^2/(2\eta\beta_{\pi(\rho)}^2)$. Analogous expressions can be obtained for the other matrix elements in (30)–(45).

We have introduced the notation

$$X_1 = \frac{2}{3}\left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle - \frac{1}{2}\widetilde{\Lambda}\eta,$$

$$X_\pm = \frac{1}{3}\left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle + \frac{1}{2}\widetilde{\Lambda}\eta \pm (M_\rho - m_q),$$

$$Y_\pm = -\frac{11}{24}\langle\mathbf{p}^2\rangle + \frac{1}{2}(M_{\pi(\rho)}^2 - m_q^2)$$

$$\pm \frac{1}{2}\widetilde{\Lambda}\eta^2\left(\frac{1}{4}\widetilde{\Lambda} + \frac{\beta_{\pi(\rho)}^2}{\beta^2}\frac{\Delta\beta^2}{\beta} \right),$$

$$Z_1 = \frac{1}{3}\left\langle \frac{\mathbf{p}^2}{(\overline{\varepsilon}_q + m_q)^2} \right\rangle(\overline{\Lambda} + M_{\pi(\rho)} + 3m_q) - \left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q + m_q} \right\rangle,$$

$$Z_2 = \widetilde{\Lambda}\eta\left(\frac{1}{3}\left\langle \frac{\mathbf{p}^2}{\overline{\varepsilon}_q(\overline{\varepsilon}_q + m_q)} \right\rangle \right.$$

$$\left. + \left\langle \frac{1}{\overline{\varepsilon}_q + m_q} \right\rangle(\overline{\Lambda} + M_{\pi(\rho)} + 3m_q) - 3 \right),$$

TABLE I. Semileptonic $B \to \pi$ and $B \to \rho$ decay form factors.

Ref.	$f_+^{B \to \pi}(0)$	$A_1^{B \to \rho}(0)$	$A_2^{B \to \rho}(0)$	$V^{B \to \rho}(0)$
Our results	0.20 ± 0.02	0.26 ± 0.03	0.31 ± 0.03	0.29 ± 0.03
[17] [a]	0.33	0.28	0.28	0.33
[18] [a]	0.09	0.05	0.02	0.27
[19] [b]	0.26 ± 0.02	0.5 ± 0.1	0.4 ± 0.2	0.6 ± 0.2
[20] [b]	0.23 ± 0.02	0.38 ± 0.04	0.45 ± 0.05	0.45 ± 0.05
[21] [b]		0.24 ± 0.04		0.28 ± 0.06
[22] [c]	0.35 ± 0.08	0.24 ± 0.12	0.27 ± 0.80	0.53 ± 0.31
[23] [c]	$0.30 \pm 0.14 \pm 0.05$	0.22 ± 0.05	$0.49 \pm 0.21 \pm 0.05$	0.37 ± 0.11

[a]Quark models.
[b]QCD sum rules.
[c]Lattice.

$$Z_3 = \left\langle \frac{\mathbf{p}^2}{(\bar{\varepsilon}_q + m_q)^2} \right\rangle (\bar{\Lambda} + M_{\pi(\rho)} + 3 m_q)$$

and

$$R = \frac{m_b}{2[\varepsilon_b(\Delta_{max}) + m_b]} = \frac{1}{\sqrt{5} + 2}.$$

V. RESULTS AND DISCUSSION

Using the parameters of Gaussian wave functions (14) and the value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ [8] in the expressions (24), (30)–(33) for the $B \to \pi$ transition form factor $f_+(0)$ and the Eqs. (25), (26), (34)–(45) for the $B \to \rho$ transition form factors $A_1(0)$, $A_2(0)$, and $V(0)$ we get

$$f_+^{B \to \pi}(0) = 0.20 \pm 0.02, \quad V^{B \to \rho}(0) = 0.29 \pm 0.03,$$

$$A_1^{B \to \rho}(0) = 0.26 \pm 0.03, \quad A_2^{B \to \rho}(0) = 0.31 \pm 0.03. \quad (47)$$

The presented theoretical uncertainty is estimated within our model. It results mostly from the approximation of the wave functions by Gaussians (14). The contribution of higher order terms in $1/m_b$ is almost negligible. In Ref. [11] we have presented the results for $B \to \pi(\rho) e \nu$ decay form factors up to the first order in $1/m_b$ expansion. The values of form factors found in [11] are close to (47), this indicates that the second order correctons in $1/m_Q$ are small (less than 5% of form factor values). Certainly additional theoretical uncertainties due to our model approach exist.

We compare our results (47) for the form factors of $B \to \pi(\rho) e \nu$ decays with the predictions of quark models [17,18], QCD sum rules [19–21] and lattice calculations [22,23] in Table I. There is an agreement between our value of $f_+^{B \to \pi}(0)$ and QCD sum rule and lattice predictions. Our $B \to \rho e \nu$ form factors agree with lattice and QCD sum rule ones [21], while they are approximately 1.5 times less than QCD sum rule results of Refs. [19,20].

To calculate the $B \to \pi(\rho)$ semileptonic decay rates it is necessary to determine the q^2 dependence of the form factors. Analyzing the Δ_{max}^2 dependence of the expressions

(A1)–(A12), (24)–(27) for the form factors f_+, A_1, A_2, and V, we find that the q^2 dependence of these form factors near $q^2 = 0$ is given by

$$f_+(q^2) = \frac{M_B + M_\pi}{2 \sqrt{M_B M_\pi}} \tilde{\xi}(w) . \mathscr{F}_+(\Delta_{max}^2), \quad (48)$$

$$A_1(q^2) = \frac{2 \sqrt{M_B M_\rho}}{M_B + M_\rho} \frac{1}{2} (1 + w) \tilde{\xi}(w) . \mathscr{A}_1(\Delta_{max}^2), \quad (49)$$

$$A_2(q^2) = \frac{M_B + M_\rho}{2 \sqrt{M_B M_\rho}} \tilde{\xi}(w) . \mathscr{A}_2(\Delta_{max}^2), \quad (50)$$

$$V(q^2) = \frac{M_B + M_\rho}{2 \sqrt{M_B M_\rho}} \tilde{\xi}(w) \mathscr{V}(\Delta_{max}^2), \quad (51)$$

where $w = (M_B^2 + M_{\pi(\rho)}^2 - q^2)/2 M_B M_{\pi(\rho)}$; $\mathscr{F}_+(\Delta_{max}^2)$ and $\mathscr{A}_{1,2}(\Delta_{max}^2)$, $\mathscr{V}(\Delta_{max}^2)$ are defined by (24)–(26), (30)–(45). We have introduced the function

$$\tilde{\xi}(w) = \left(\frac{2}{w + 1} \right)^{1/2} \exp\left(- \eta \frac{\tilde{\Lambda}^2}{\beta_B^2} \frac{w - 1}{w + 1} \right), \quad (52)$$

which in the limit of infinitely heavy quarks in the initial and final mesons coincides with the Isgur-Wise function of our model [8]. In this limit Eqs. (48)–(51) reproduce the leading order prediction of HQET [9].

It is important to note that the form factor A_1 in (49) has a different q^2 dependence than the other form factors (48), (50), (51). In the quark models it is usually assumed the pole [17] or exponential [18] q^2 behavior for all form factors. However, the recent QCD sum rule analysis indicates that the form factor A_1 has q^2 dependence different from other form factors [19–21]. In [20] it even decreases with the increasing q^2 as

$$A_1(q^2) \simeq \left(1 - \frac{q^2}{M_b^2} \right) A_1(0) \simeq \frac{2 M_B M_\rho}{(M_B + M_\rho)^2} (1 + w) A_1(0). \quad (53)$$

Such behavior corresponds to replacing $\tilde{\xi}(w)$ in (49) by $\tilde{\xi}(w_{max})$.

TABLE II. Semileptonic decay rates $\Gamma(B \to \pi e \nu)$, $\Gamma(B \to \rho e \nu)$ ($\times |V_{ub}|^2 \times 10^{12}$ s^{-1}), the ratio of the rates for longitudinally (L) and transversely (T) polarized ρ meson, and the ratio of ρ and π rates.

Ref.	$\Gamma(B \to \pi e \nu)$	$\Gamma(B \to \rho e \nu)$	Γ_L/Γ_T	$\Gamma^{B \to \rho}/\Gamma^{B \to \pi}$
Our results				
Model A	3.0 ± 0.6	5.4 ± 1.2	0.5 ± 0.3	1.8 ± 0.6
Model B	2.9 ± 0.6	5.0 ± 1.2	0.5 ± 0.3	1.7 ± 0.6
[17] [a]	7.4	26	1.34	3.5
[18] [a]	2.1	8.3	0.75	4.0
[19] [b]	5.1 ± 1.1	12 ± 4	0.06 ± 0.02	2.4 ± 0.9
[20] [b]	3.6 ± 0.6	5.1 ± 1.0	0.13 ± 0.08	1.4 ± 0.2
[22] [c]	8 ± 4			

[a]Quark models.
[b]QCD sum rules.
[c]Lattice.

We have calculated the decay rates of $B \to \pi(\rho)e\nu$ using our form factor values at $q^2 = 0$ and the q^2 dependence (48)–(52) in the whole kinematical region (model A). We have also used the pole dependence for form factors $f_+(q^2)$, $A_2(q^2)$, $V(q^2)$, and

$$A_1(q^2) = \frac{2 M_B M_\rho}{(M_B + M_\rho)^2} (1+w) \frac{A_1(0)}{(1 - q^2/m_P^2)} \quad \text{(modelB)},$$

which corresponds to replacing the function $\widetilde{\xi}(w)$ (52) by the pole form factor. The results are presented in Table II in comparison with the quark model [17,18], QCD sum rule [19,20], and lattice (for $B \to \pi e \nu$) [22] predictions. Lattice accuracy, at present, is not enough to estimate $B \to \rho e \nu$ rates [24]. We see that our results for the above mentioned models A and B of form factor q^2 dependence coincide within errors. The ratio of the rates $\Gamma(B \to \rho e \nu)/\Gamma(B \to \pi e \nu)$ is considerably reduced in our model compared to the Bauer-Stech-Wirbel (BSW) [17] and Isgur-Scora-Grinstein-Wise (ISGW) [18] models with the simple pole or exponential q^2 behavior of all form factors. Meanwhile our prediction for this ratio is in agreement with QCD sum rule results [19,20]. The absolute values of the rates $\Gamma(B \to \pi e \nu)$ and $\Gamma(B \to \rho e \nu)$ in our model are close to those from QCD sum rules [20]. The predictions for the rates with longitudinally and transversely polarized ρ meson differ considerably in these approaches. This is mainly due to different q^2 behavior of A_1 [see (49), (53) or pole dominance model [17]]. Thus the measurement of the ratios $\Gamma(B \to \rho e \nu)/\Gamma(B \to \pi e \nu)$ and Γ_L/Γ_T should provide the test of q^2 dependence of A_1 and may discriminate between these approaches.

Recently CLEO presented [25] the first experimental measurement of B semileptonic decays to π and ρ:

$$\mathcal{B}(B^0 \to \pi^- l^+ \nu) = (1.34 \pm 0.35 \pm 0.28) \times 10^{-4},$$

$$\mathcal{B}(B^0 \to \rho^- l^+ \nu) = (2.28 \pm 0.36 \pm 0.59^{+0.00}_{-0.46}) \times 10^{-4},$$

$$\frac{\Gamma(B^0 \to \rho^- l^+ \nu)}{\Gamma(B^0 \to \pi^- l^+ \nu)} = 1.70^{+0.80}_{-0.50} \pm 0.58^{+0.00}_{-0.34}.$$

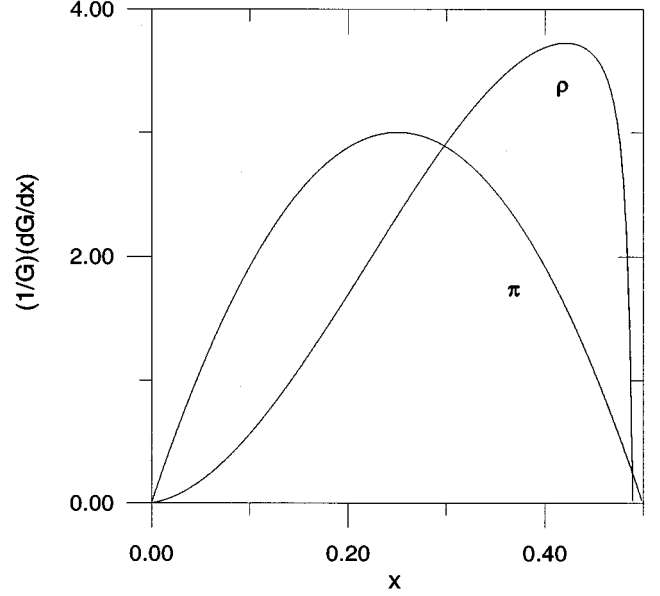
FIG. 3. The differential decay spectra (model A) $(1/\Gamma)(d\Gamma/dx)$ for $B \to \pi(\rho)$ semileptonic transitions, where $x = E_l/M_B$ and E_l is lepton energy. Absolute rates $d\Gamma/dx$ can be obtained using $\Gamma[B \to \pi(\rho)e\nu]$ from Table II.

The present experimental accuracy is not enough to make definite conclusions. However, we see from the Table II that the experimental ratio of the ρ and π rates favors the models with a specific q^2 behavior of A_1 form factor. For the CKM matrix element V_{ub} we find, in our model,

$$|V_{ub}| = (5.2 \pm 1.3 \pm 0.5) \times 10^{-3} \quad (B \to \pi l \nu),$$

$$|V_{ub}| = (5.2^{+1.2}_{-1.9} \pm 0.6) \times 10^{-3} \quad (B \to \rho l \nu),$$

where the first error is experimental and the second one is theoretical. The experimental errors are rather large at present, but we find the agreement between V_{ub} values extracted from $B \to \pi e \nu$ and $B \to \rho e \nu$ decays. There is also no contradiction with the value obtained from the analysis of inclusive decay rates.

The differential decay spectra $(1/\Gamma)(d\Gamma/dx)$ for $B \to \pi(\rho)$ semileptonic transitions, where $x = E_l/M_B$ and E_l is the lepton energy, are presented in Fig. 3 (see also the second paper of Ref. [6]).

We can use our results for V and A_1 to test the HQET relation [26] between the form factors of the semileptonic and rare radiative decays of B mesons. Isgur and Wise [26] have shown that in the limit of infinitely heavy b-quark mass an exact relation connects the form factors V and A_1 with the rare radiative decay $B \to \rho \gamma$ form factor F_1 defined by

$$\langle \rho(p_\rho, e) | \bar{u} i \sigma_{\mu\nu} q^\nu P_R b | B(p_B) \rangle$$
$$= i \epsilon_{\mu\nu\tau\sigma} e^{*\nu} p_B^\tau p_\rho^\sigma F_1(q^2) + [e_\mu^* (M_B^2 - M_\rho^2)$$
$$- (e^* q)(p_B + p_\rho)_\mu] G_2(q^2). \quad (54)$$

This relation is valid for q^2 values sufficiently close to $q_{max}^2 = (M_B - M_\rho)^2$ and reads

$$F_1(q^2) = \frac{q^2 + M_B^2 - M_\rho^2}{2 M_B} \frac{V(q^2)}{M_B + M_\rho} + \frac{M_B + M_\rho}{2 M_B} A_1(q^2). \quad (55)$$

It has been argued in [27,28,20] that in these processes the soft contributions dominate over the hard perturbative ones, and thus the Isgur-Wise relations (55) could be extended to the whole range of q^2. In [10] we developed $1/m_b$ expansion for the rare radiative decay form factor $F_1(0)$ using the same ideas as in the present discussion of semileptonic decays. It was shown that Isgur-Wise relation (55) is satisfied in our model at leading order of $1/m_b$ expansion. The found value of the form factor of rare radiative decay $B \to \rho \gamma$ up to the second order in $1/m_b$ expansion is [10]

$$F_1^{B \to \rho}(0) = 0.26 \pm 0.03. \tag{56}$$

Using (55) and the values of form factors (47) we find

$$F_1^{B \to \rho}(0) = 0.27 \pm 0.03, \tag{57}$$

which is in accord with (56). Thus we conclude that $1/m_b$ and $1/m_b^2$ corrections do not break the Isgur-Wise relation (55) in our model.

VI. CONCLUSIONS

We have presented in detail the method of the calculation of electroweak decay matrix elements for transitions between different meson states, based on the quasipotential approach in quantum field theory. It has been shown that the heavy-to-heavy decay matrix element can be expanded in inverse powers of the initial and final heavy quark masses at the point of zero recoil of the final meson. On the other hand, the heavy-to-light decay matrix element can be expanded in inverse powers of the initial heavy quark mass and large recoil momentum of final light meson at the point of maximum recoil of final meson. As a result the expansion of the heavy-to-light decay matrix element in inverse powers of initial heavy quark mass arises. This method permits the calculation

of various radiative and weak decays of heavy mesons with the complete account of relativistic effects.

This method has been applied to the investigation of the semileptonic decays of B mesons into light mesons. The recoil momentum of final $\pi(\rho)$ meson is large compared to the $\pi(\rho)$ mass almost in the whole kinematical range. This requires the completely relativistic treatment of these decays. On the other hand, the presence of large recoil momentum, which for $q^2 = 0$ is of order $m_b/2$, allows for the $1/m_b$ expansion of weak decay matrix element at this point. The contributions to this expansion come both from the heavy b-quark mass and large recoil momentum of the light final meson.

We have performed the $1/m_b$ expansion of the semileptonic decay form factors at $q^2 = 0$ up to the second order. The q^2 dependence of the form factors near $q^2 = 0$ has been determined. It has been found that the axial form factor A_1 has a q^2 behavior different from other form factors [see (48)–(51)]. This is in agreement with recent QCD sum rule results [19–21]. The ratios $\Gamma(B \to \rho e \nu)/\Gamma(B \to \pi e \nu)$ and Γ_L/Γ_T are very sensitive to the q^2 dependence of A_1, and thus their experimental measurement may discriminate between different approaches.

We have considered the relation between semileptonic decay form factors and the rare radiative decay form factor [26], obtained in the limit of the infinitely heavy b quark. It has been found that in our model $1/m_b$ corrections do not violate this relation.

APPENDIX: HEAVY-TO-LIGHT SEMILEPTONIC DECAY FORM FACTORS AT $q^2 = 0$

(a) $B \to \pi e \nu$ decay form factor

$$
\begin{aligned}
f_+^{(1)}(0) = {} & \sqrt{\frac{E_\pi}{M_B}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\pi \left(\mathbf{p} + \frac{2\varepsilon_q}{E_\pi + M_\pi} \mathbf{\Delta}_{\max} \right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max}) + m_q}{2\varepsilon_q(p+\Delta_{\max})}} \sqrt{\frac{\varepsilon_b(p) + m_b}{2m_b}} \\
& \times \Bigg\{ 1 + \frac{M_B - E_\pi}{\varepsilon_q(p+\Delta_{\max}) + m_q} + \frac{(\mathbf{p}\mathbf{\Delta}_{\max})}{\mathbf{\Delta}_{\max}^2} \left[\frac{\varepsilon_q(p+\Delta_{\max}) - m_q}{2m_b} + (M_B - E_\pi)\left(\frac{1}{\varepsilon_q(p+\Delta_{\max}) + m_q} + \frac{1}{2m_b} \right) \right] \\
& + (p_x^2 + p_y^2)\Bigg[\frac{E_\pi - M_\pi}{2m_b(\varepsilon_q(p+\Delta_{\max}) + m_q)} \left(\frac{1}{\varepsilon_q(p) + m_q} - \frac{1}{\varepsilon_q(p+\Delta_{\max}) + m_q} \right) \\
& + \frac{M_B - E_\pi}{E_\pi + M_\pi} \left(\frac{1}{\varepsilon_q(p+\Delta_{\max}) + m_q} - \frac{1}{2m_b} \right) \left(\frac{1}{\varepsilon_q(p) + m_q} - \frac{1}{\varepsilon_q(p+\Delta_{\max}) + m_q} \right) \Bigg] \Bigg\} \Psi_B(\mathbf{p}),
\end{aligned} \tag{A1}
$$

$$
f_+^{S(2)}(0) = \sqrt{\frac{E_\pi}{M_B}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\pi\left(\mathbf{p} + \frac{2\varepsilon_q}{E_\pi + M_\pi}\boldsymbol{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}} \left\{ \left(\frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q} - \frac{M_B - E_\pi}{\varepsilon_q(\Delta_{\max})+m_q}\right) \right.
$$

$$
\times \frac{1}{\varepsilon_q(\Delta_{\max})}\left[M_\pi - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right] + \frac{(\mathbf{p}\boldsymbol{\Delta}_{\max})}{2\boldsymbol{\Delta}_{\max}^2}\left[\left(\frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})(\varepsilon_q(\Delta_{\max})+m_q)}\right.\right.
$$

$$
- (M_B - E_\pi)\left(\frac{1}{\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]} + \frac{1}{m_b[\varepsilon_b(\Delta_{\max})+m_b]}\right)\right)\left(M_B + M_\pi - \varepsilon_b(p) - \varepsilon_q(p)\right.
$$

$$
\left. - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right) - \frac{\varepsilon_q(\Delta_{\max})-m_q}{2m_b[\varepsilon_b(\Delta_{\max})+m_b]}[M_B - \varepsilon_b(p) - \varepsilon_q(p)]
$$

$$
+ \frac{M_B - E_\pi}{2m_b\varepsilon_q(\Delta_{\max})}\frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q}\left(M_\pi - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right)\right]
$$

$$
+ \frac{p_x^2 + p_y^2}{2[\varepsilon_q(p)+m_q]}\left[\left(\frac{E_\pi - M_\pi}{\varepsilon_q(\Delta_{\max})[\varepsilon_q(p+\Delta_{\max})+m_q]^2}\right.\right.
$$

$$
- \frac{M_B - E_\pi}{E_\pi + M_\pi}\left(\frac{1}{\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]} - \frac{1}{m_b[\varepsilon_b(\Delta_{\max})+m_b]}\right)\right)\left(M_B + M_\pi - \varepsilon_b(p) - \varepsilon_q(p)\right.
$$

$$
\left. - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right) + \frac{(E_\pi - M_\pi)[M_B - \varepsilon_b(p) - \varepsilon_q(p)]}{m_b[\varepsilon_b(\Delta_{\max})+m_b][\varepsilon_q(\Delta_{\max})+m_q]} - \frac{M_B - E_\pi}{(E_\pi + M_\pi)m_b\varepsilon_q(\Delta_{\max})}
$$

$$
\left.\left. \times \frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q}\left(M_\pi - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right)\right]\right\} \Psi_B(\mathbf{p}), \tag{A2}
$$

$$
f_+^{V(2)}(0) = \sqrt{\frac{E_\pi}{M_B}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\pi\left(\mathbf{p} + \frac{2\varepsilon_q}{E_\pi + M_\pi}\boldsymbol{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}} \left\{ -\frac{p_x^2 + p_y^2}{[\varepsilon_q(p)+m_q]}\left(\frac{1}{\varepsilon_q(p)+m_q} - \frac{1}{\varepsilon_q(\Delta_{\max})+m_q}\right) \right.
$$

$$
\times \left[\frac{E_\pi - M_\pi}{\varepsilon_q(p+\Delta_{\max})+m_q}\left(\frac{1}{\varepsilon_b(\Delta_{\max})+m_b} + \frac{1}{\varepsilon_q(\Delta_{\max})+m_q}\right) - \frac{M_B - E_\pi}{E_\pi + M_\pi}\left(\frac{1}{\varepsilon_q(\Delta_{\max})+m_q} - \frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\right)\right]
$$

$$
\times \left[M_B + M_\pi - \varepsilon_b(p) - \varepsilon_q(p) - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right] + \frac{(\mathbf{p}\boldsymbol{\Delta}_{\max})}{\boldsymbol{\Delta}_{\max}^2}\frac{1}{\varepsilon_q(p)+m_q}\left[[\varepsilon_q(\Delta_{\max})-m_q]\left(\frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\right.\right.
$$

$$
\left. - \frac{m_q}{\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\right) + (M_B - E_\pi)\left(\frac{1}{\varepsilon_q(\Delta_{\max})+m_q} + \frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\right)\right]\left[M_B + M_\pi - \varepsilon_b(p) - \varepsilon_q(p)\right.
$$

$$
\left. - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)\right] + \frac{\mathbf{p}^2}{\varepsilon_q(p)+m_q}\left[\frac{M_B - M_\pi - \varepsilon_b(p) - \varepsilon_q(p) + 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)}{2\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\right.
$$

$$
- \frac{M_B - E_\pi}{\varepsilon_q(\Delta_{\max})+m_q}\frac{M_B + M_\pi - \varepsilon_b(p) - \varepsilon_q(p) - 2\varepsilon_q\left(p + \frac{2\varepsilon_q}{E_\pi + M_\pi}\Delta_{\max}\right)}{2\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}
$$

$$
\left.\left. + \left(\frac{1}{\varepsilon_q(\Delta_{\max})+m_q} - \frac{1}{2m_b}\left(1 - \frac{M_B - E_\pi}{\varepsilon_q(\Delta_{\max})+m_q}\right)\right)\frac{M_B - \varepsilon_b(p) - \varepsilon_q(p)}{\varepsilon_b(\Delta_{\max})+m_b}\right]\right\} \Psi_B(\mathbf{p}), \tag{A3}
$$

(b) $B \to \rho e \nu$ decay form factors

$$
A_1^{(1)}(0) = \frac{2\sqrt{M_B M_\rho}}{M_B + M_\rho}\sqrt{\frac{E_\rho}{M_\rho}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p} + \frac{2\varepsilon_q}{E_\rho + M_\rho}\boldsymbol{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}}\sqrt{\frac{\varepsilon_b(p)+m_b}{2m_b}}
$$

$$
\times \left\{1 + \frac{1}{2m_b[\varepsilon_q(p+\Delta_{\max})+m_q]}\left((E_\rho - M_\rho)\frac{p_x^2 + p_y^2}{\varepsilon_q(p)+m_q} - \frac{1}{3}\mathbf{p}^2 - (\mathbf{p}\boldsymbol{\Delta}_{\max})\right)\right\} \Psi_B(\mathbf{p}), \tag{A4}
$$

$$A_1^{S(2)}(0) = \frac{2\sqrt{M_B M_\rho}}{M_B+M_\rho} \sqrt{\frac{E_\rho}{M_\rho}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\boldsymbol{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}} \left\{ \frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q} \frac{1}{\varepsilon_q(\Delta_{\max})} \right.$$

$$\times\left[M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right] - \frac{(p_x^2+p_y^2)(E_\rho-M_\rho)}{2m_b[\varepsilon_q(p)+m_q][\varepsilon_q(\Delta_{\max})+m_q]}$$

$$\times\left[\frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)+\frac{1}{\varepsilon_q(\Delta_{\max})}[M_B-\varepsilon_b(p)-\varepsilon_q(p)]\right]$$

$$+\frac{(\mathbf{p}\boldsymbol{\Delta}_{\max})}{\boldsymbol{\Delta}_{\max}^2}\frac{\varepsilon_q(\Delta_{\max})-m_q}{2}\left[\left(\frac{1}{m_b[\varepsilon_b(\Delta_{\max})+m_b]}+\frac{1}{\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\right)\left(M_B+M_\rho-\varepsilon_b(p)\right.\right.$$

$$\left.\left.\left.-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)+\frac{1}{m_b\varepsilon_q(\Delta_{\max})}[M_B-\varepsilon_b(p)-\varepsilon_q(p)]\right]\right\}\Psi_B(\mathbf{p}), \tag{A5}$$

$$A_1^{V(2)}(0) = \frac{2\sqrt{M_B M_\rho}}{M_B+M_\rho} \sqrt{\frac{E_\rho}{M_\rho}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\boldsymbol{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}}$$

$$\times\left\{\frac{(p_x^2+p_y^2)(E_\rho-M_\rho)}{[\varepsilon_q(p)+m_q][\varepsilon_q(\Delta_{\max})+m_q]}\left(\frac{1}{[\varepsilon_q(p)+m_q][\varepsilon_b(\Delta_{\max})+m_b]}+\frac{1}{[\varepsilon_q(\Delta_{\max})+m_q]^2}\right)\left[M_B+M_\rho-\varepsilon_b(p)\right.\right.$$

$$\left.-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right]+\frac{\mathbf{p}^2}{\varepsilon_q(p)+m_q}\left[\frac{1}{2\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\left[\frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q}\right.\right.$$

$$\times\left(M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)-[M_B-\varepsilon_b(p)-\varepsilon_q(p)]\right]-\frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\left(\frac{1}{3[\varepsilon_q(\Delta_{\max})+m_q]}+\frac{1}{m_b}\right)$$

$$\times\left(M_\rho-2\varepsilon\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right]-\frac{(\mathbf{p}\boldsymbol{\Delta}_{\max})}{\boldsymbol{\Delta}_{\max}^2}[\varepsilon_q(\Delta_{\max})-m_q]\left[\frac{1}{2\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\left[2[M_B-\varepsilon_b(p)\right.\right.$$

$$\left.\left.-\varepsilon_q(p)]+\frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q}\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right]\right.$$

$$+\frac{1}{\varepsilon_q(p)+m_q}\left(\frac{1}{\varepsilon_b(\Delta_{\max})+m_b}+\frac{m_q}{\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\right)\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)\right.$$

$$\left.\left.\left.-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right]\right\}\Psi_B(\mathbf{p}), \tag{A6}$$

$$A_2^{(1)}(0) = \frac{M_B+M_\rho}{2\sqrt{M_B M_\rho}} \frac{2\sqrt{E_\rho M_\rho}}{E_\rho+M_\rho} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\boldsymbol{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}} \sqrt{\frac{\varepsilon_b(p)+m_b}{2m_b}}$$

$$\times\left\{1+\frac{M_\rho}{M_B}\left(1-\frac{E_\rho+M_\rho}{\varepsilon_q(p+\Delta_{\max})+m_q}\right)-\frac{p_z^2}{2m_b[\varepsilon_q(p+\Delta_{\max})+m_q]}+\frac{p_x^2+p_y^2}{[\varepsilon_q(p+\Delta_{\max})+m_q][\varepsilon_q(p)+m_q]}\right.$$

$$\left.\times\left(\frac{E_\rho+M_\rho}{2m_b}+\frac{M_\rho}{M_B}\right)-\frac{(\mathbf{p}\boldsymbol{\Delta}_{\max})}{\boldsymbol{\Delta}_{\max}^2}\left[\frac{\varepsilon_q(p+\Delta_{\max})-m_q}{2m_b}+\frac{M_\rho}{M_B}\left(1+\frac{E_\rho}{m_b}\right)\right]\right\}\Psi_B(\mathbf{p}), \tag{A7}$$

$$A_2^{S(2)}(0) = \frac{M_B+M_\rho}{2\sqrt{M_BM_\rho}} \frac{2\sqrt{E_\rho M_\rho}}{E_\rho+M_\rho} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\mathbf{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}}\Biggl\{\Biggl(\frac{\varepsilon_q(\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})+m_q}$$

$$+2\frac{M_\rho}{M_B}\Biggr)\frac{1}{\varepsilon_q(\Delta_{\max})}\left[M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right]-\frac{p_x^2+p_y^2}{2m_b[\varepsilon_q(p)+m_q]}\left[\frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\Bigl(M_B+M_\rho-\varepsilon_b(p)\right.$$

$$\left.-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)+\frac{1}{\varepsilon_q(\Delta_{\max})}[M_B-\varepsilon_b(p)-\varepsilon_q(p)]\Bigr]+\frac{(\mathbf{p}\mathbf{\Delta}_{\max})}{\mathbf{\Delta}_{\max}^2}\frac{1}{2}\Biggl[\left(\frac{\varepsilon_q(\Delta_{\max})}{m_b[\varepsilon_b(\Delta_{\max})+m_b]}\right.$$

$$\left.+\frac{1}{\varepsilon_q(\Delta_{\max})+m_q}\right)\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)$$

$$+\frac{1}{m_b}[M_B-\varepsilon_b(p)-\varepsilon_q(p)]\Biggr]\Biggr\}\Psi_B(\mathbf{p}), \tag{A8}$$

$$A_2^{V(2)}(0) = \frac{M_B+M_\rho}{2\sqrt{M_BM_\rho}} \frac{2\sqrt{E_\rho M_\rho}}{E_\rho+M_\rho} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\mathbf{\Delta}_{\max}\right) \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}}$$

$$\times\Biggl\{\frac{p_x^2+p_y^2}{[\varepsilon_q(p)+m_q][\varepsilon_q(\Delta_{\max})+m_q]}\left(\frac{E_\rho+M_\rho}{[\varepsilon_q(p)+m_q][\varepsilon_b(\Delta_{\max})+m_b]}+\frac{1}{\varepsilon_q(\Delta_{\max})+m_q}\right)\left[M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)\right.$$

$$\left.-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right]-\frac{\mathbf{p}^2}{\varepsilon_q(p)+m_q}\left[\frac{1}{2\varepsilon_q(\Delta_{\max})[\varepsilon_q(\Delta_{\max})+m_q]}\Bigl(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)\right.$$

$$\left.-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)+\frac{1}{\varepsilon_b(\Delta_{\max})+m_b}\left(\frac{1}{3[\varepsilon_q(\Delta_{\max})+m_q]}+\frac{1}{m_b}\right)\left(M_\rho-2\varepsilon\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\Bigr]$$

$$-\frac{(\mathbf{p}\mathbf{\Delta}_{\max})}{\mathbf{\Delta}_{\max}^2}\left[\frac{1}{\varepsilon_q(p)+m_q}\left(\frac{\varepsilon_q(\Delta_{\max})+m_q}{\varepsilon_b(\Delta_{\max})+m_b}+\frac{m_q}{2\varepsilon_q(\Delta_{\max})}\right)\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right.$$

$$\left.+\frac{1}{\varepsilon_q(\Delta_{\max})}\left[3[M_B-\varepsilon_b(p)-\varepsilon_q(p)]-\left(M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right]\right]\Biggr\}\Psi_B(\mathbf{p}), \tag{A9}$$

$$V^{(1)}(0) = \frac{M_B+M_\rho}{2\sqrt{M_BM_\rho}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\mathbf{\Delta}_{\max}\right) \frac{2\sqrt{E_\rho M_\rho}}{\varepsilon_q(p+\Delta_{\max})+m_q} \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}}\sqrt{\frac{\varepsilon_b(p)+m_b}{2m_b}}$$

$$\times\Biggl\{1+\frac{p_x^2+p_y^2}{E_\rho+M_\rho}\left(\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2m_b[\varepsilon_q(p)+m_q]}-\frac{1}{\varepsilon_q(p+\Delta_{\max})+m_q}\right)+\frac{(\mathbf{p}\mathbf{\Delta}_{\max})}{\mathbf{\Delta}_{\max}^2}\left(1-\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2m_b}\right)\Biggr\}\Psi_B(\mathbf{p}), \tag{A10}$$

$$V^{S(2)}(0) = \frac{M_B+M_\rho}{2\sqrt{M_BM_\rho}} \int \frac{d^3p}{(2\pi)^3} \bar{\Psi}_\rho\left(\mathbf{p}+\frac{2\varepsilon_q}{E_\rho+M_\rho}\mathbf{\Delta}_{\max}\right) \frac{2\sqrt{E_\rho M_\rho}}{\varepsilon_q(p+\Delta_{\max})+m_q} \sqrt{\frac{\varepsilon_q(p+\Delta_{\max})+m_q}{2\varepsilon_q(p+\Delta_{\max})}}$$

$$\times\Biggl\{-\frac{1}{\varepsilon_q(\Delta_{\max})}\left[M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right]+\frac{p_x^2+p_y^2}{2m_b(E_\rho+M_\rho)[\varepsilon_q(p)+m_q]}\left[\frac{\varepsilon_q(p+\Delta_{\max})-m_q}{\varepsilon_q(\Delta_{\max})}\right.$$

$$\left.\times\left(M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)-\frac{\varepsilon_q(\Delta_{\max})+m_q}{\varepsilon_b(\Delta_{\max})+m_b}\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right]$$

$$-\frac{(\mathbf{p}\mathbf{\Delta}_{\max})}{\mathbf{\Delta}_{\max}^2}\left[\frac{1}{2}\left(\frac{1}{\varepsilon_q(\Delta_{\max})}-\frac{\varepsilon_q(\Delta_{\max})+m_q}{m_b[\varepsilon_b(\Delta_{\max})+m_b]}\right)\left(M_B+M_\rho-\varepsilon_b(p)-\varepsilon_q(p)-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right.$$

$$\left.-\frac{\varepsilon_q(\Delta_{\max})-m_q}{2m_b\varepsilon_q(\Delta_{\max})}\left(M_\rho-2\varepsilon_q\left(p+\frac{2\varepsilon_q}{E_\rho+M_\rho}\Delta_{\max}\right)\right)\right]\Biggr\}\Psi_B(\mathbf{p}), \tag{A11}$$

$$
V^{V(2)}(0) = \frac{M_B + M_\rho}{2\sqrt{M_B M_\rho}} \int \frac{d^3 p}{(2\pi)^3} \bar{\Psi}_\rho \left(\mathbf{p} + \frac{2\varepsilon_q}{E_\rho + M_\rho} \mathbf{\Delta}_{\max} \right) \frac{2\sqrt{E_\rho M_\rho}}{\varepsilon_q(p + \Delta_{\max}) + m_q} \sqrt{\frac{\varepsilon_q(p + \Delta_{\max}) + m_q}{2\varepsilon_q(p + \Delta_{\max})}}
$$

$$
\times \left\{ \frac{2(p_x^2 + p_y^2)}{(E_\rho + M_\rho)[\varepsilon_q(p) + m_q]} \left(\frac{\varepsilon_q(\Delta_{\max}) + m_q}{[\varepsilon_q(p) + m_q][\varepsilon_b(\Delta_{\max}) + m_b]} - \frac{1}{\varepsilon_q(\Delta_{\max}) + m_q} \right) \right.
$$

$$
\times \left[M_B + M_\rho - \varepsilon_b(p) - \varepsilon_q(p) - 2\varepsilon_q \left(p + \frac{2\varepsilon_q}{E_\rho + M_\rho} \Delta_{\max} \right) \right] - \frac{\mathbf{p}^2}{2m_b[\varepsilon_q(p) + m_q]} \left[\frac{1}{\varepsilon_b(\Delta_{\max}) + m_b} \right.
$$

$$
\times [M_B - \varepsilon_b(p) - \varepsilon_q(p)] + \frac{1}{\varepsilon_q(\Delta_{\max}) + m_q} \left(M_B + M_\rho - \varepsilon_b(p) - \varepsilon_q(p) - 2\varepsilon \left(p + \frac{2\varepsilon_q}{E_\rho + M_\rho} \Delta_{\max} \right) \right) \right]
$$

$$
+ \frac{(\mathbf{p\Delta}_{\max})}{\mathbf{\Delta}_{\max}^2} \frac{1}{\varepsilon_q(p) + m_q} \left(1 - \frac{\varepsilon_q(\Delta_{\max}) + m_q}{\varepsilon_b(\Delta_{\max}) + m_b} \right) \left[M_B + M_\rho - \varepsilon_b(p) - \varepsilon_q(p) - 2\varepsilon \left(p + \frac{2\varepsilon_q}{E_\rho + M_\rho} \Delta_{\max} \right) \right] \left. \right\} \Psi_B(\mathbf{p}),
$$

$$
\text{(A12)}
$$

where the superscripts ''(1)'' and ''(2)'' correspond to Figs. 1 and 2, S and V to the scalar and vector potentials of $q\bar{q}$ interaction:

$$
|\mathbf{\Delta}_{\max}| = \frac{M_B^2 - M_{\pi(\rho)}^2}{2M_B}; \quad E_{\pi(\rho)} = \sqrt{M_{\pi(\rho)}^2 + \mathbf{\Delta}_{\max}^2} = \frac{M_B^2 + M_{\pi(\rho)}^2}{2M_B}; \tag{A13}
$$

and the z axis is chosen in the direction of $\mathbf{\Delta}_{\max}$.

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