Cold plus hot dark matter cosmology in the light of solar and atmospheric neutrino oscillations

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We explore the implications of possible neutrino oscillations, as indicated by the solar and atmospheric neutrino experiments, for the cold plus hot dark matter scenario of large-scale structure formation. We find that there are essentially three distinct schemes that can accommodate the oscillation data and which also allow for dark matter neutrinos. These include (i) three nearly degenerate (in mass) neutrinos, (ii) nondegenerate masses with ν_τ in the eV range, and (iii) a nearly degenerate ν_μ-ν_τ pair (in the eV range), with the additional possibility that the electron neutrino is cosmologically significant. The last two schemes invoke a "sterile" neutrino which is light ($\lesssim$ eV). We discuss the implications of these schemes for $\bar{\nu}_\mu$-$\bar{\nu}_e$ and ν_μ-ν_τ oscillation, and find that scheme (ii), in particular, predicts them to be in the observable range. As far as structure formation is concerned we compare the one neutrino flavor case with a variety of other possibilities, including two and three degenerate neutrino flavors. We show, both analytically and numerically, the effects of these neutrino mass scenarios on the amplitude of cosmological density fluctuations. With a Hubble constant of 50 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$, a spectral index of unity, and $\Omega_{\mathrm{baryon}} = 0.05$, the two and three flavor scenarios fit the observational data marginally better than the single flavor scheme. However, taking account of the uncertainties in these parameters, we show that it is premature to pick a clear winner.

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I. INTRODUCTION

The idea that the dark matter in the universe may contain both a "cold" as well as a "hot" component [1] first arose as a serious possibility [2] within the framework of grand unified theories (which also inspired inflation, cosmic strings, baryogenesis, etc.). In the original models the cold component was the axion, although nowadays the lightest supersymmetric particle (LSP) often plays that role. Lightly massive neutrinos are the hot component, and it was usually assumed that the neutrinos would display the hierarchical mass spectrum characteristic of the quarks and charged leptons, with a single neutrino (e.g., ν_τ) dominating the mass density of the hot component.

However, it was noted from the beginning [1] that because of their special nature, the existence of nearly degenerate massive neutrinos is a logical, albeit not necessarily the simplest, possibility. The cosmological implications of neutrinos that are closely degenerate in mass have received much recent attention [3–5], inspired to some extent by the remarkable series of neutrino oscillation experiments (solar, atmospheric, and more recently accelerator) which suggest that two or more neutrino flavors may contribute to the hot dark matter in the universe.

Before discussing the impact of neutrino oscillation experiments on the cold plus hot dark matter (C+HDM) scenario, let us briefly recall why the latter has attracted so much recent attention. From the mid to the late 1980s evidence was mounting that the so-called "standard" cold dark matter (CDM) scenario had trouble providing a consistent explanation of small- ($\sim$ galactic) as well as large-scale structure. On the contrary, a C+HDM model (with $\Omega_\nu \sim 0.15$–0.35) provided a far more consistent fit to the data [6]. In order to provide additional tests, the quadrupole anisotropy of the microwave background expected in C+HDM models was estimated in 1989 [7,8] and compared with the CDM prediction. Normalized to the "small"-scale data, it was found that the C+HDM prediction exceeded the CDM value by a factor of about 2, and this was dramatically verified when the Cosmic Background Explorer (COBE) team [9] made its announcement in 1992 [10].

In the last two years, particularly after extensive numerical simulations of galaxy and cluster formation [11] and more detailed analytical work [12,13], additional support for the C+HDM model has emerged. It appears that this model provides the simplest (consistent) realization of an inflationary scenario for large-scale structure formation.

It has recently been pointed out [3,4] (and our results are in agreement) that with two or more neutrino flavors contributing to the hot dark matter, somewhat better fits to the present data may be possible. For fixed Ω_ν there is also some dependence both on the spectral index as well as the Hubble constant. It must be admitted, though, that within the framework of grand unified theories, it may not be easy to realize scenarios with two or three

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nearly degenerate (in mass) neutrinos. Furthermore, in some of the scenarios we consider, one needs to invoke a fourth (sterile) neutrino which is nearly degenerate in mass with the electron neutrino. Precisely why this singlet state can be so light is an important question for future research.

The layout of the paper is as follows. In Sec. II A we list three sets of observations related to solar, atmospheric, and dark matter neutrinos which provide hints for nonzero neutrino masses. For completeness, we also summarize the results of the Liquid Scintillation Neutrino Detector (LSND) experiment at Los Alamos, although we will not include them among the constraints to be satisfied. We then proceed to a discussion of three distinct schemes which are consistent with the observed solar and atmospheric ν oscillations, and which also allow for neutrino dark matter. We discuss the implications of these schemes for a variety of experiments, particularly the $\bar{\nu}_\mu$-$\bar{\nu}_e$ and ν_μ-ν_τ oscillation experiments at accelerators. In Sec. III we address the main issues raised by neutrino oscillations for cosmological structure formation within the C+HDM framework. We review the physics of the differences between having one and more than one massive neutrino flavor, giving some analytic formula to estimate the scale (Sec. III A) and amplitude (Sec. III B) of the effects on the growth of primordial density fluctuations. In Sec. III C we give results of some more detailed numerical computations of the cosmological models based on the solutions to the oscillation data, including some models, in Sec. III D, motivated by the LSND results. Finally, in Sec. III E we discuss the dependence on the cosmological parameters (Hubble constant, spectral index n, and baryonic density fraction) of conclusions drawn from comparing specific models to large-scale structure data.

II. NEUTRINO MASSES AND MIXINGS

A. Observational hints

One of the greatest challenges in particle physics today pertains to the issue of the neutrino masses. There are indications from a variety of experiments that one (or more) of the known neutrinos may possess nonzero mass(es). In this section we briefly summarize the relevant observations based on solar and atmospheric neutrinos, as well as present arguments for neutrino dark matter. (For completeness, we also summarize the recent findings of the LSND experiment at Los Alamos.) In Sec. II B, we discuss scenarios for neutrino masses and mixings which can simultaneously accommodate these observations. We find that there are essentially three viable schemes, and remarkably enough, all three will be tested in ongoing and planned neutrino oscillation experiments (particularly Los Alamos LSND, Rutherford KARMEN, the CERN CHORUS or NOMAD, and Fermilab E803 experiments).

(1) *Solar neutrino puzzle.* The apparent deficit in the flux of solar ν_e's [14] that has persisted in the chlorine experiment for over two decades [15] has been confirmed in

the last several years by three independent experiments: the Kamiokande water Cherenkov detector [16], and the SAGE [17] and GALLEX [18] radiochemical experiments. These four observations, which probe different energy regimes in the solar neutrino spectrum, can all be simultaneously explained in terms of two flavor neutrino oscillations. The matter-enhanced Mikheyev-Smirnov-Wolfenstein (MSW) [19] oscillation for a two-flavor (ν_e-ν_x) system admits two branches [20]: (i) the small angle nonadiabatic solution, which requires $\sin^2 2\theta = (3.5 \times 10^{-3}$–$1.5 \times 10^{-2})$ and $\Delta m^2 \equiv (m_2^2 - m_1^2) = (3.4 \times 10^{-6}$–$1.2 \times 10^{-5})$ eV^2; (ii) the large angle MSW solution requiring $\sin^2 2\theta = (0.6$–$0.9)$ and $\Delta m^2 = (7 \times 10^{-6}$–$5 \times 10^{-5})$ eV^2. Here ν_x could be ν_μ, ν_τ, or a sterile neutrino ν_s for the small angle MSW solution, while $\nu_x = (\nu_\mu$ or $\nu_\tau)$ in the large angle MSW solution. There is also the (ν_e-ν_x) vacuum oscillation solution [21] for $\nu_x = (\nu_\mu$ or $\nu_\tau)$, with $\sin^2 2\theta = (0.6$–$1.0)$ and $\Delta m^2 \sim 10^{-10}$ eV^2.

(2) *Atmospheric neutrino anomaly.* The ratio of muon to electron neutrinos from atmospheric cascades measured by the Kamiokande [22] and the IMB [23] experiments appears to suggest a deficit of about a factor of 2 when compared to Monte Carlo simulations [24]. For sub-GeV neutrinos (i.e., neutrinos with energy less than ~ 1 GeV), Kamiokande finds for this ratio of ratios (which is expected to be 1 in the absence of neutrino oscillations) $R = 0.60^{+0.06}_{-0.05} \pm 0.05$ which is in good agreement with the IMB value of $R = 0.54 \pm 0.05 \pm 0.12$. Recently Kamiokande has also reported results for the multi-GeV neutrinos [25] where the ratio is $R = 0.57^{+0.08}_{-0.07} \pm 0.07$, in agreement with the sub-GeV data. This apparent deficit of ν_μ's or the excess of ν_e's can be attributed to either (ν_μ-ν_τ) oscillations with $\sin^2 2\theta = (0.65$–$1.0)$ and $\Delta m^2 = (5 \times 10^{-3}$–$3 \times 10^{-2})$ eV^2, or to (ν_e-ν_μ) oscillations with $\sin^2 2\theta = (0.55$–$1.0)$ and $\Delta m^2 = (7 \times 10^{-3}$–$7 \times 10^{-2})$ eV^2. The resolution of the anomaly in terms of (ν_μ-ν_s) oscillations, where ν_s is a sterile neutrino, will run into difficulty with primordial nucleosynthesis calculations which require that [26] $\Delta m^2 \sin^2 2\theta \leq 1.6 \times 10^{-6}$ eV^2. [Note that the solar (ν_e-ν_s) oscillation parameters satisfy this constraint.] Among the other atmospheric neutrino experiments, only Soudan II [27] sees an anomaly with a preliminary value of $R = 0.69 \pm 0.19 \pm 0.09$. The Frejus [28] results are marginally in conflict with the combined Kamiokande and IMB data. Considering the low statistics of Frejus experiment compared with the high statistical significance of Kamiokande and IMB data, we shall ignore this marginal discrepancy in our theoretical discussions.

(3) *Hot component of dark matter.* As already noted in the Introduction, and further explained in the next section, a combined fit to the COBE data as well as the data on the distribution of galaxies on large and small angular scales is difficult to achieve within the cold dark matter scenario. The simplest consistent scenario requires a significant hot component (15–30)% in the dark matter [13,29,30], leading to the C+HDM scheme, with neutrinos being the natural candidates for the hot component. The mass of the neutrino comprising the hot dark matter should be in the few eV range.

(4) *$\bar{\nu}_\mu$-$\bar{\nu}_e$ oscillation at accelerators.* Recently the

LSND experiment at the LAMPF facility in Los Alamos has reported positive evidence for $\bar{\nu}_\mu$-$\bar{\nu}_e$ oscillations [31]. If these results survive further scrutiny and there is independent verification (e.g., at the KARMEN experiment [32]), it certainly will have a strong impact on particle and nuclear physics as well as cosmology and astrophysics. The initial LSND data, if interpreted in terms of two-neutrino oscillations, suggest a mixing probability of $P_{\bar{\nu}_\mu \to \bar{\nu}_e} = (0.34^{+0.20}_{-0.18} \pm 0.07)\%$, when oscillation constraints from KARMEN and BNL-E776 [33] accelerator searches as well as reactor neutrino constraints from Bugey facility [34] are folded in. In the discussion that follows, we shall take into consideration the sensitivity of the LSND and other accelerator and reactor experiments in searching for $\bar{\nu}_\mu$-$\bar{\nu}_e$ oscillations, but we shall not demand that the theoretical scenarios discussed correctly reproduce the precise numerical values reported by the LSND collaboration. We feel that this position is justified at this time, especially in view of the fact that a different analysis of the LSND data has yielded considerably weaker limits on the mixing parameters [35]. We find it remarkable, however, that independent of the LSND data, two of the three scenarios which can accommodate points (1)–(3) above imply an observable signal at accelerator ($\bar{\nu}_\mu$-$\bar{\nu}_e$) oscillation experiments.

B. Theoretical schemes

Let us focus on the solar, atmospheric, and hot dark matter neutrinos, leaving out the LSND results which will not play a role in determining the viable scenarios. It is clear that the mass splittings required for explaining these three observations do not overlap ($\Delta m^2 \sim 10^{-5}$ eV^2 for solar neutrinos, $\Delta m^2 \sim 10^{-2}$ eV^2 for atmospheric neutrinos, and $m_\nu \sim$ (1 to few) eV for hot dark matter). Assuming that there are no additional light neutrinos, one concludes that the three neutrinos (ν_e, ν_μ, ν_τ) should be nearly degenerate in mass. (Note that we often identify a mass eigenstate through its dominant flavor. The respective masses are m_1, m_2, and m_3.) This leads us to our first scenario.

Scenario (i): Three nearly degenerate neutrinos. Assume that ν_e, ν_μ, and ν_τ have a nearly common mass of about (1–3) eV. Their masses are split by small amounts, such that $\Delta m^2_{12} \equiv (m_2^2 - m_1^2) \sim 10^{-5}$ eV^2 and $|\Delta m^2_{23}| \equiv |(m_3^2 - m_2^2)| \sim 10^{-2}$ eV^2. Phenomenological neutrino mass matrices that are consistent with these assumptions are easily constructed [36]. This scheme will account for the solar neutrino data via (ν_e-ν_μ) MSW oscillations and the atmospheric neutrino anomaly in terms of (ν_μ-ν_τ) oscillations. The required mixing angles are free parameters and can be adjusted to the suggested values. The (ν_e-ν_τ) mixing angle can be arbitrarily small, but it can also be as large as about 0.05. According to this scenario, neutrinoless double-β decay ($\beta\beta_{0\nu}$) is at (or even above?) the present experimental limit (modulo nuclear matrix element uncertainties), and so that it can be ruled out if $\beta\beta_{0\nu}$ is not observed in the near future. Also according to this scheme the ν_μ-ν_τ oscillation

experiments at accelerators (CHORUS/NOMAD, E803 [37,38]), as well as ν_e-ν_μ oscillation searches at reactors, should fail to find anything interesting. In both cases the relevant $\Delta m^2 \lesssim 10^{-2}$ eV^2, which is not in the range accessible in these experiments. However, this statement is strictly true only to the extent that the leptonic mixing matrix is assumed to be unitary. Deviation from unitarity can produce oscillation signals, although there should be no spatial dependence. This situation can arise if the ordinary neutrinos mix with a neutral heavy lepton with mass on the order of the electroweak scale. Let us consider this in more detail.

For definiteness, let us assume that the three light neutrinos have an admixture of a heavy fourth-generation neutrino with mass greater than M_Z. Writing

$$\nu_e = \sum_{i=1}^{4} U_{ei}\nu_i \tag{1}$$

and similarly for the weak eigenstates ν_μ, ν_τ, one sees that muon neutrinos produced in π decays will be the weak eigenstates with the heavy ν_4 component removed. The ν_e and ν_μ states are no longer orthogonal, leading to an apparent oscillation probability given by [39]

$$P_{\nu_e-\nu_\mu} = |(U_{e1}^* U_{\mu1} + U_{e2}^* U_{\mu2} + U_{e3}^* U_{\mu3})|^2$$
$$= |-(U_{e4}^* U_{\mu4})|^2 . \tag{2}$$

Here we have set the Δm^2s to zero since they are all $\lesssim 10^{-2}$ eV^2 and thus negligible for accelerator neutrino experiments. If the mixing parameters $U_{e4}, U_{\mu4} \sim (\frac{1}{4}-\frac{1}{5})$, then the oscillation probability at the LSND experiment will be in the range of $(2-3)\times10^{-3}$ with no position dependence. The data presented in Ref. [31] is consistent with such an interpretation, as can be inferred from the large Δm^2 region of their Fig. 3. Various other constraints on such leakage to a heavy fourth-generation neutrino have been studied in [40], where it is shown that the best existing limit is from neutrino oscillation experiments. A ν_μ-ν_τ oscillation probability of $(2-3)\times10^{-3}$ can also be achieved by a similar mechanism, but this may be more difficult to measure.

If the heavy neutrino is a standard model singlet, there are stringent limits from Z-boson decays which make the nonunitary oscillations unobservable at accelerators. To see this, consider the mixing of ordinary neutrinos with a heavy isospin singlet neutral lepton N of mass m. In the presence of such mixings, the invisible decay width of Z will be modified so that the number of effective neutrino species coupling to Z is given by

$$N_\nu = 3 + (1 - |\alpha|^2)[-(1 + |\alpha|^2)$$
$$+ 2F|\alpha|^2 + F'(1 - |\alpha|^2)], \tag{3}$$

where F and F' are phase space factors given by

$$F = 1 - \frac{3}{2}x^2 + \frac{1}{2}x^3,$$
$$F' = \sqrt{1 - 4x^2}(1 - x^2), \tag{4}$$

with $x \equiv m/m_Z$. It is to be understood here that for $m \leq m_Z/2$, all terms in Eq. (3) will contribute to N_ν, while for $m \geq m_Z$, the F and F' terms do not contribute. If $m_Z/2 \leq m \leq m_Z$, the term proportional to F' should be set to zero, while F is nonzero. The mixing parameter $|\alpha|$ is equal to the (4,4) element of the unitary matrix U_ν, which diagonalizes the neutrino mass matrix. It is related to the leptonic mixing matrix U appearing in the charged current via

$$|\alpha|^2 = 1 - |U_{e4}|^2 - |U_{\mu4}|^2 - |U_{\tau4}|^2 . \qquad (5)$$

Although U, now a 3×4 matrix, is not unitary, the ν_e-ν_μ transition probability is still given by Eq. (2), since each row of U is normalized to 1. It becomes apparent from Eqs. (2) and (5) that $P_{\nu_e\text{-}\nu_\mu} \leq (1 - |\alpha|^2)^2/4$. If the mass of N is greater than m_Z, then $F = F' = 0$, so that $P_{\nu_e\text{-}\nu_\mu} \simeq [(3 - N_\nu)/(1 + |\alpha|^2)]^2/4 \equiv [\Delta N_\nu/(1 + |\alpha|^2)]^2/4$. ΔN_ν can be as large as 0.042 at 1σ [41], which when combined with $|\alpha|^2 \simeq 1$ yields $P_{\nu_e\text{-}\nu_\mu} \leq 1.1 \times 10^{-4}$, too small to be observable. (Even with $\Delta N_\nu = 0.067$, the 2σ value, $P_{\nu_e\text{-}\nu_\mu} \leq 2.8 \times 10^{-4}$.) If the mass of N is less than $m_Z/2$, then both the F and F' terms in Eq. (3) will be relevant, making N_ν more consistent with 3. (As $F, F' \to 1$, $N_\nu \to 3$.) However, such a scenario is ruled out by direct search limits for a heavy neutral lepton decaying into the usual charged leptons (e, μ), as should happen here. (The limit on the mass of such a neutral lepton is $m \geq 46$ GeV.) If the mass of N obeys $m_Z/2 \leq m \leq m_Z$, then $F \neq 0$, but $F' = 0$ in Eq. (3). In this case, noting that F is at most 11/16 (for $x = 1/2$), we see that $P_{\nu_e\text{-}\nu_\mu}$ can be as large as $(4\Delta N_\nu/5)^2 \simeq 10^{-3}$, which is close to present experimental sensitivity. However, since the branching ratio $B[Z \to (N\bar{\nu} + \overline{N}\nu)] \sim 2 \times 10^{-4}$, a few hundred such events should have been observed at LEP with the N decaying subsequently into $l^+l^-\nu$. This possibility is also excluded based on the nonobservation of such events.

We conclude that the case of an isosinglet neutrino does not lead to any observable deviation from unitarity in the accelerator neutrino oscillation experiments. Note that there is no such constraint on a sequential fourth-generation neutrino (or neutrinos in vectorlike families).

Scenario (ii): No degenerate neutrinos. If there is no degeneracy in the neutrino masses and we assume some kind of a mass hierarchy, accounting for the solar, atmospheric, and hot dark matter neutrinos would require the introduction of a light (sterile) neutrino ν_s. There is then just one consistent scheme in this case. As far as we are aware, this possibility has not been discussed in the literature, and so we shall elaborate on it.

By assumption, in this scenario the known neutrinos are nondegenerate, and indeed have hierarchical masses. The solar neutrino puzzle is resolved via $(\nu_e$-$\nu_s)$ MSW oscillations such that $m_{\nu_s}^2 - m_{\nu_e}^2 \sim 10^{-5}$ eV^2. The atmospheric neutrino anomaly is explained via $(\nu_\mu$-$\nu_e)$ oscillations with $m_{\nu_\mu} \sim 10^{-1}$ eV. The τ neutrino with a few eV mass constitutes the "hot" component of dark matter.

As far as $(\bar{\nu}_\mu$-$\bar{\nu}_e)$ oscillations at accelerators are concerned, the "direct" transition is not possible because

$\Delta m^2 \sim 10^{-2}$ eV^2. However, it has recently been pointed out [42] that an "indirect" $(\bar{\nu}_\mu$-$\bar{\nu}_e)$ transition via a virtual $\bar{\nu}_\tau$ in the few eV mass range can still occur in such a scheme. For this to be within experimental reach, both $(\nu_e$-$\nu_\tau)$ oscillations (at reactors) and $(\nu_\mu$-$\nu_\tau)$ oscillations (at accelerators) should be in the observable range. As shown in [42], the present limits from the Bugey reactor [34] on ν_e-ν_τ oscillations, and the CHARM-II, Fermilab E531, and CDHS limits [43] on $(\nu_\mu$-$\nu_\tau)$ oscillations translate into an observable "indirect" $(\nu_e$-$\nu_\mu)$ oscillations for a ν_τ mass in the few eV range, which can account for the LSND results.

Let us point out that in this second scenario, the allowed MSW parameter region will be somewhat shifted because of the rapid $(\nu_e$-$\nu_\mu)$ oscillations from the Sun to Earth, leading to a further depletion of ν_e's by a factor of $\frac{1}{2}\sin^2 2\theta_{e\mu} \sim (\frac{1}{4}$-$\frac{1}{2})$. This prediction will be tested in the forthcoming solar neutrino experiments.

This scheme may be on a somewhat better theoretical footing than the first one because no mass degeneracy is assumed. A light "sterile" neutrino state, following [44], may arise from the hidden sector of some fundamental theory. If the hidden sector contains SU(3)×SU(2)×U(1), one can define an unbroken "parity" at the Lagrangian level. The ν_R's needed for the seesaw mechanism, having no gauge quantum numbers, can freely mix from the observable and hidden sectors. This would lead to mixing between the light neutrinos and their mirror partners (which are also light). Gravitationally induced interactions could also mix the neutrinos from the two sectors.

Scenario (iii): Nearly degenerate $(\nu_\mu$-$\nu_\tau)$ pair. This scheme also requires a light sterile neutrino ν_s. In addition, an approximate lepton number symmetry such as $L_e + L_\mu - L_\tau$ is necessary to make ν_μ and ν_τ nearly degenerate in mass [36,44,45]. When this symmetry breaks by a small amount, maximal $(\nu_\mu$-$\nu_\tau)$ mixing occurs, facilitating a resolution of the atmospheric neutrino problem. The solar neutrino puzzle is explained via $(\nu_e$-$\nu_s)$ oscillations. The common mass of ν_μ and ν_τ is assumed to be $\sim$ few eV, so that they constitute a "two-flavor" hot dark matter. For accelerator neutrinos, there are direct $(\nu_e$-$\nu_\mu)$ oscillations at an observable level. In particular, the LSND data can be accommodated. However, $(\nu_\mu$-$\nu_\tau)$ oscillations will be beyond the reach of CHORUS/NOMAD and E803.

Aside from the two-flavor hot dark matter component, this scenario has other special cases relevant for cosmology. The solar neutrino data require the mass splitting Δm^2 between ν_e and ν_s to be $\sim 10^{-5}$ eV^2, but the masses themselves may be in the cosmologically interesting range of a few eV. Since this pair behaves like Dirac neutrinos, there is no conflict with $\beta\beta_{0\nu}$ constraints. For cosmology, however, the hot dark matter may be a linear combination of ν_e and $(\nu_\mu + \nu_\tau)$. If the total mass of the (ν_μ, ν_τ) pair exceeds that of ν_e, we call it a (2+1) C+HDM scheme, and if the ν_e mass dominates over (ν_μ, ν_τ), it is a (1+2) scheme. Of course, one could also recover the one-flavor case by making the mass of $(\nu_\mu$-$\nu_\tau)$ pair much smaller than 1 eV. (This case will correspond to an inverted mass hierarchy [46].)

III. COSMOLOGICAL IMPLICATIONS

In studies of large-scale structure, it has become clear that a critical density cold dark matter (CDM) universe with density perturbations that have roughly a Harrison-Zeldovich spectrum cannot simultaneously fit observations of structure on large and small scales. Normalized on large scales to fit the COBE observations, CDM produces too many clusters of galaxies and galactic pairwise velocities far in excess of observations [47]. This basic problem is neatly resolved in a cold plus hot dark matter scenario because the growth of the density perturbations on small scales is damped by the presence of the hot dark matter component [1,7,11]. Since the neutrino oscillation experiments may be telling us that the neutrino component is spread among several flavors, we would like to understand, in a quantitative way, how this changes the usual (one-flavor) cold plus hot dark matter model. We will begin with a general description of the effects of a hot component on the growth of density fluctuations. First we will give some analytic formulas for estimating the size of the effects, and then we will show some results of more detailed calculations. Finally we conclude with some remarks about the uncertainties in cosmological parameters which currently prevent one from using cosmological data to pin down the values of the neutrino masses.

A. Free streaming or Jeans masses

Neutrinos retain large velocities from the time before primordial nucleosynthesis when they were in thermal equilibrium. This means that if one initially sets up density perturbations, the neutrinos will rearrange themselves to a different pattern at some later time, erasing neutrino density fluctuations on length scales over which the neutrinos can have traveled. This "free-streaming" or "Jeans" length scale λ_J is given at time t by

$$\lambda_J(t) = a(t)\tau(t)v(t), \tag{6}$$

where $a(t)$ is the scale factor which describes the expansion of the universe, $\tau(t)$ is the conformal time $[d\tau = dt/a(t)]$ and so $a(t)\tau(t)$ is the physical horizon size, and $v(t)$ is the average neutrino velocity. An analytic fit for v is given by [48]

$$v(t) = \left[1 + \left(\frac{a}{a_{\rm nr}}\right)^2\right]^{-1/2}, \tag{7}$$

where $a_{\rm nr}$ is the scale factor when the neutrinos become nonrelativistic, and we use units where the velocity of light is unity. Normalizing the present value of the scale factor to unity, we have

$$a_{\rm nr} = 2.25 T_\gamma^0/m_\nu, \tag{8}$$

where $T_\gamma^0 = 2.35 \times 10^{-4}$ eV is the present cosmic background photon temperature. The value of the Jeans length at the current time t_0 is

$$\lambda_J(t_0) = 3t_0 a_{\rm nr} = 3.2 \left(\frac{1\ {\rm eV}}{m_\nu}\right) h^{-1}\ {\rm Mpc}. \tag{9}$$

h is the parametrization of our ignorance of the true value of the Hubble constant $H_0 \equiv 100\ h$ km s^{-1} Mpc^{-1}. To relate this length to the masses of known astrophysical structures we convert to the total mass contained in the volume $4\pi(\lambda_J/2)^3/3$ (the Jeans mass). Currently, the Jeans mass $M_{J_\nu}(t_0)$ is

$$M_{J_\nu}(t_0) = M_{H_0}(a_{\rm nr})^3 = 4.6 \times 10^{12} \left(\frac{1\ {\rm eV}}{m_\nu}\right)^3 h^{-1}\ M_\odot,$$
$$\tag{10}$$

where $M_{H_0} = 3.14 \times 10^{22}\ h^{-1}\ M_\odot$ is the mass contained in the current Hubble volume. Interestingly enough, the mass in (10) is comparable to a galactic mass for neutrinos in the range $m_\nu \simeq 1$–4 eV. Note that in the formulas presented here we assume a critical density of matter, and only a fraction is in the form of hot dark matter; the remainder is comprised of cold dark matter and baryons.

During the matter-dominated epoch, $\rho \propto a^{-3}$, $v \propto a^{-1}$ (while the neutrinos are nonrelativistic), and $a\tau \propto a^{3/2}$; so the Jeans mass $M_{J\nu} \propto a^{-3/2}$ and decreases with time. When the neutrinos are relativistic, $v \sim 1$, and the Jeans mass increases with time. The maximum in the Jeans mass occurs when the neutrinos are just becoming nonrelativistic, i.e., when $a(t) = a_{\rm nr}$:

$$M_{J\nu}({\rm max}) \simeq M_{H_0}(a_{\rm nr})^{3/2}$$

$$= 4 \times 10^{17} \left(\frac{1\ {\rm eV}}{m_\nu}\right)^{3/2} h^{-1}\ M_\odot. \tag{11}$$

This relation is approximate, because the time when neutrinos become relativistic is quite close to the time when matter begins to dominate over radiation in the universe, and so the horizon size does not scale exactly as $a^{-3/2}$. The exact formula must be determined numerically for each value of the neutrino mass. The formula in (11) is accurate to within an order of magnitude, which is good enough for estimating the size of the effects under study here. In the older models of structure formation in which the dark matter is totally composed of neutrinos of mass ~ 30 eV, this maximum Jeans mass is somewhat larger than a typical mass of a cluster of galaxies.

In studies of single massive flavor C+HDM models, a more typical value of the neutrino mass, ~ 6 eV (for $h = 0.5$), was preferred, yielding a maximum Jeans mass of $M_{J_\nu}({\rm max}) = 3 \times 10^{16}\ h^{-1}\ M_\odot$, which is slightly smaller than the so-called "great attractor" [49], the largest concentration of mass which is pulling galaxies gravitationally on scales of order $40\ h^{-1}$ Mpc. If we assume, as in scenario (i), that the same Ω_ν is now composed of three nearly degenerate neutrinos of mass ~ 2 eV, $M_{J_\nu}({\rm max}) = 1 \times 10^{17}\ h^{-1}\ M_\odot$, which is somewhat larger than the "great attractor" mass.

We will proceed to examine how these two mass scales [$M_{J_\nu}(t_0)$ and $M_{J_\nu}({\rm max})$] are incorporated in predictions of structure formation.

B. Growth of density perturbations

In the early universe, when matter begins to dominate the energy density of the universe, the CDM and C+HDM mass fluctuation spectra are identical. The only feature imprinted on the initial spectrum at this time is a decrease of amplitude on length scales smaller than the horizon at matter domination. These scales experience a smaller amount of growth during the radiation-dominated era due to the effects of radiation pressure. After matter domination the fluctuation amplitudes on these scales can grow, driven by their own self-gravitation. If a volume of space contains a mass $\tilde{M}$ which is slightly different than the average M for that size, and the dark matter is strictly CDM, the fluctuation amplitude will grow proportional to the scale factor a:

$$\frac{\delta M}{M}(M) \equiv \frac{\tilde{M} - M}{M} \propto a. \qquad (12)$$

Growth in the C+HDM model is more interesting. When a mass fluctuation contains a mass $M < M_{J_\nu}(t)$ at time t, the neutrinos will stream out of the fluctuation and change the local gravitational potential. This smaller potential also reduces the growth rate of the mass fluctuations to [1,50]

$$\frac{\delta M}{M}(M) \propto a^p, \quad p = \frac{1}{4}\left(-1 + \sqrt{25 - 24\Omega_\nu}\right). \qquad (13)$$

The fact that the Jeans mass decreases with time means that if $M < M_{J_\nu}$ at some early time, sometime in the future $M > M_{J_\nu}$ will be true. Once $M > M_{J_\nu}$, the growth rate returns to the CDM growth rate, as the neutrinos will then effectively be "cold" on this mass scale. This leads to scale-dependent changes in the mass fluctuation spectrum. The equations for the Jeans mass, Eqs. (11) and (10), and the growth rate, Eq. (13), also illustrate the effect of having multiple flavors of massive neutrinos with comparable masses. The Jeans mass depends on the individual neutrino mass, while the growth rate depends only on the *sum* of the neutrino masses, as

$$\Omega_\nu = h^{-2} \sum_{i=e,\mu,\tau} \frac{m_{\nu_i}}{94 \text{ eV}}. \qquad (14)$$

So increasing the number of degenerate flavors means that the Jeans mass will be increased for a given value of Ω_ν.

The growth of fluctuations on a mass scale M in the intermediate region $M_{J_\nu}(\text{max}) < M < M_{J_\nu}(\text{now})$ will be "damped" relative to their value in a universe dominated only by CDM. The amount of damping between the time when the neutrinos become nonrelativistic and when the Jeans mass becomes smaller than scale M is given by

$$\text{damping factor} = \left[\frac{M_{J_\nu}(\text{max})}{M}\right]^{(2p-2)/3}. \qquad (15)$$

The maximum amount of damping occurs for mass scales $M \ll M_{J_\nu}(t_0)$ which is a constant given by

$$\text{max damping factor} = \left[\frac{M_{J_\nu}(\text{max})}{M_{J_\nu}(t_0)}\right]^{(2p-2)/3} = (a_{\text{nr}})^{1-p}. \qquad (16)$$

The difference, then, between having the mass equally spread among $\mathcal{N}_\nu$ flavors as opposed to 1 flavor results in an additional small scale damping of $\mathcal{N}_\nu^{1-p}$, which is 10%–20% for $0.20 < \Omega_\nu < 0.30$.

The combination of hot and cold components of dark matter thus has the following effects, when compared to the a model with only a cold component. First of all, the growth of density perturbations with masses in the intermediate range $M_{J_\nu}(\text{now}) < M < M_{J_\nu}(\text{max})$ is reduced by the factor given in Eq. (15) which depends mainly on Ω_ν. Below $M_{J_\nu}(\text{now})$ the mass fluctuation spectrum follows the scale dependence of a CDM model but with an amplitude reduced by the factor in Eq. (16). Splitting the neutrino density among $\mathcal{N}_\nu$ flavors of approximately degenerate neutrinos produces an additional damping of $\mathcal{N}_\nu^{p-1}$ below a somewhat higher value of $M_{J_\nu}(\text{max})$. This additional damping may be important for reconciling observations of clusters of galaxies with a COBE normalized spectrum of density fluctuations.

C. Detailed comparisons to structure formation

In order to do a more careful comparison of these models to observed large-scale structure, a more accurate evaluation of the effects expressed above is required. We achieve this by integrating the linearized general relativistic evolution equations for the photons, neutrinos (massive and effectively massless flavors), baryons, and CDM particles. The procedure we use is described in detail in Ref. [51]. The models have a baryon mass fraction $\Omega_{\text{baryon}} = 0.0125/h^2$, which is consistent with nucleosynthesis [52]. The CDM fraction is then given by $\Omega_{\text{CDM}} = 1 - \Omega_\nu - \Omega_{\text{baryon}}$ (since here we are only considering critical density universes). In Figs. 1, 2, and 3, we use a Harrison-Zeldovich (scale-free) spectrum normalized to COBE [53] with $h = 0.5$. After smoothing the density field with a low pass Gaussian filter, we arrive at the curves shown in the figures. The masses on the x axis are $(2\pi)^{3/2}r_f^3\rho$, as appropriate for a Gaussian filter radius of r_f. In the three panels of Figs. 1(a), 1(b), and 1(c), we show curves for $\Omega_\nu = 0.20$, 0.25, and 0.30, respectively. Within each panel we show curves for one, two, and three degenerate neutrino flavors. In panel (a), we also display the analogous CDM model curve for comparison.

The models under consideration are now known to fit a large amount of data on large-scale structure [6,12,13,29], but here we will concentrate on the two toughest constraints for C+HDM models to satisfy. The first constraint is on the amplitude of mass fluctuations which form clusters. It has been argued [54] that the number density of rich clusters requires that on a scale $6 \times 10^{15} \ h^{-1}M_\odot$, $\delta M/M \sim 0.52$–0.62. As stated in Ref. [54] the errors are hard to estimate because of possible systematic uncertainties in the masses assigned to clusters. Indeed, the mass scales for two different deter-

minations of the cluster mass function in Refs. [55] and [56] differ by almost a factor of 2. Recently, a new estimate of the amplitude of mass fluctuations has been given which avoids the necessity of finding the absolute mass by comparing the x-ray temperatures between simulated and observed clusters. Reference [57] gives the amplitude (with 95% confidence errors) $\delta M/M = 0.62^{+0.19}_{-0.14}$ at $M = 6 \times 10^{14} \, h^{-1} M_\odot$. This amplitude has been derived using the "top-hat" filtering of the density field. To convert this constraint to the Gaussian filtering at the cluster scale in our plots, one must multiply by a factor 1.50/1.68 (see Ref. [58]).

At the far left end of the figures we have also drawn a line which corresponds to the lower limit of the mass fluctuation amplitude implied by the measurements of the
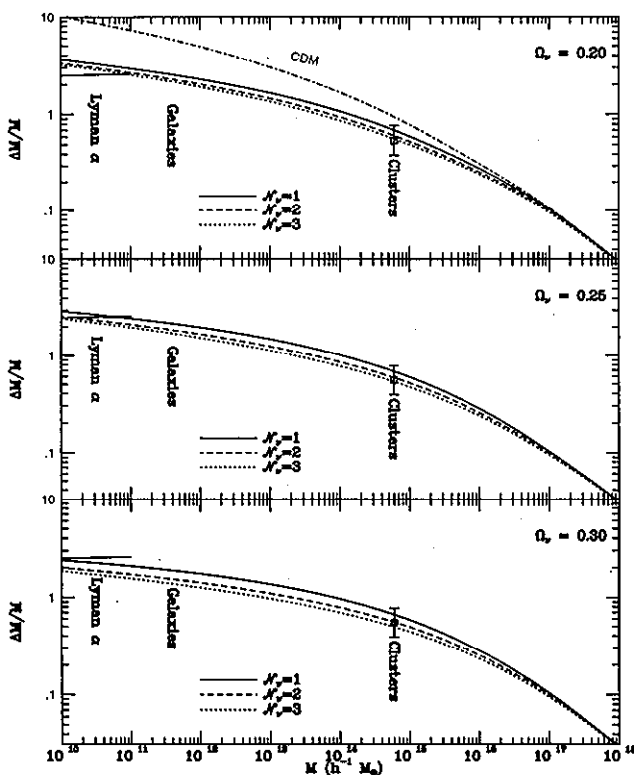

FIG. 1. We show the difference in the rms mass fluctuation amplitude ($\Delta M/M$) between having the hot dark matter (neutrino) component distributed among one, two, or three flavors of degenerate neutrinos which correspond to our scenarios (ii), (iii), and (i), respectively, for fitting the neutrino oscillation data. Here $h = 0.5$, $\Omega_{\text{baryon}} = 0.05$, and $n = 1$, with the spectrum normalized to COBE. The data points are (1) the 95% confidence level mass fluctuation estimate based on cluster formation [57] and (2) the lower limit on the mass fluctuations consistent with the formation of Lyman alpha cloud abundances at high redshift [59]. (The mass at which this lower limit is to be imposed should be somewhere along the constraint line.) Panels (a), (b), and (c) correspond to $\Omega_\nu = 0.20$, 0.25, and 0.30, respectively. It can be seen that increasing the number of neutrino flavors marginally improves the fit to the cluster data. In panel (a) we also show the curve for a pure cold dark matter model.

abundance of "damped Lyman α systems" as indicated by the recent survey of Ref. [59]; see also Ref. [60]. While early work [61] indicated that C+HDM models (with $\Omega_\nu = 0.3$) predictions were far below the observations, it now seems as though the assumptions used may have been too restrictive. There is little information about the exact nature of these damped Lyman α systems, other than their neutral hydrogen column densities and spectral linewidths. A more general analysis [58] shows how to properly estimate number abundances of objects on these scales [62]. We have used this latter procedure to estimate the amplitude of the constraint line in Figs. 1 and 2. It is still uncertain what mass is to be identified with these damped Lyman α systems. It seems reasonable that we should associate dark matter halo masses of $\sim (10^{10}\text{--}10^{11})h^{-1} \, M_\odot$ and we have placed our constraint line there in our figures. (The curves should pass above the constraint line somewhere in the range.) The mixed dark matter models are more compatible with the lower end of this mass range. However, with $\Omega_\nu = 0.30$ and $h = 0.5$ one has to take very small halo masses even to make this model work marginally. From Fig. 1 then it is apparent that $\Omega_\nu < 0.30$ if $h = 0.5$ with a pure Harrison-Zeldovich spectrum.

Figure 1 also points out an interesting aspect of spreading the neutrino mass density among a number of massive neutrino flavors, $\mathcal{N}_\nu$. With $\mathcal{N}_\nu = 1$, the amplitude of the density fluctuations on cluster scales seems to be slightly larger than what is required; increasing $\mathcal{N}_\nu$ improves the fit. This result depends only weakly on Ω_ν on this mass scale. [Equation (15) predicts less than 10% difference in the cluster scale $\delta M/M$ amplitude over the range $\Omega_\nu \sim 0.2\text{--}0.3$.] However, because of the Ω_ν dependence in the $\mathcal{N}_\nu^{p-1}$ damping factor (via p), $\mathcal{N}_\nu = 2$ works best for $\Omega_\nu = 0.30$ and $\mathcal{N}_\nu = 3$ works best for $\Omega_\nu = 0.20$. On the other hand increasing the number of flavors makes the disagreement with Lyman α systems datum worse for $\Omega_\nu = 0.30$, and so $\mathcal{N}_\nu > 1$ seems to work best for models with $\Omega_\nu < 0.30$.

We should point out here that the conclusions we draw from Fig. 1 hold only for the spectrum, Hubble constant, and baryon fraction used in the models, and altering any of these parameters changes the conclusions. We will explore this further later. The purpose of Fig. 1 is mainly to illustrate the effects of modifying the hot dark matter composition.

D. Additional models which can accommodate the LSND results

In the above we have considered the consequences of the three neutrino scenarios in which we have $\mathcal{N}_\nu$ flavors with roughly equal mass. However, if we take the LSND results as a serious constraint, then scenario (iii) admits the possibility that all three known neutrino flavors can have masses in the eV range, although the electron neutrino mass can still be significantly different from the μ and τ neutrino masses (which form a degenerate pair). Since only the square of the mass difference is measured, we have two cases to consider: (a) $m_{\nu_\mu} > m_{\nu_e}$ ("2+1")

and (b) $m_{\nu_e} > m_{\nu_\mu}$ ("1+2").

In a "2 + 1" model, where ν_μ and ν_τ are each heavier than ν_e, the lightest neutrino (ν_e) will have a very large maximum Jeans mass $[M_{J_\nu}^l(\text{max})]$ compared to the other two heavier neutrino Jeans masses $[M_{J_\nu}^h(\text{max})]$. In the "1 + 2" model, where ν_μ and ν_τ are lighter than ν_e, the lightest neutrinos will have very large $M_{J_\nu}^l(\text{max})$, compared to the remaining heavier ν_e neutrino $M_{J_\nu}^h(\text{max})$. For $M_{J_\nu}^l(\text{max}) > M > M_{J_\nu}^h(\text{max})$, there will be an amount of damping which would be there if only the lightest neutrino comprised Ω_ν. For $M_{J_\nu}^h(\text{max}) > M$ the damping will be intermediate between the one- and three-flavor massive neutrino case.

To illustrate these two cases, we will use values of $\Delta m^2 = 8$ and 20 eV^2, for the "2 + 1" and "1 + 2" models, respectively, in the context of an $\Omega_\nu = 0.25$, $h = 0.5$, $n = 1$ cosmology. In Fig. 2 we plot the curves for the 2+1 model ($m_{\nu_e} = 0.22$ eV and $m_{\nu_\tau}, m_{\nu_\mu} = 2.84$ eV) and the 1 + 2 model ($m_{\nu_e} = 4.524$ eV and m_{ν_τ}, $m_{\nu_\mu} = 0.685$ eV). For comparison we also plot the curves for $\mathcal{N}_\nu = 1$, 2, and 3 flavors for the same Ω_ν ($= 0.25$). The curve for 2 + 1 (1 + 2) lies in between the curves for the 2 and 3 (1 and 2) degenerate flavors.

We can estimate the difference between having three flavors with equal and unequal masses as follows. In the "1+2" and "2+1" models the lightest flavor mass $m_{\nu l}$ is lighter than the three-equal-flavor mass $m_{\nu 3}$. The heavier flavor mass $m_{\nu h}$ is greater than $m_{\nu 3}$. Consider a mass scale $M < M_{J_\nu}^h(\text{max})$. After some time, when $M_{J_\nu}^h(t) = M$, the amount of damping is the same in both the equal and unequal mass cases. After that time that scale will experience an additional damping of $[M_{J_\nu}^l(\text{max})/M_{J_\nu}^h(\text{max})]^{(2p_l-2)/3}$ [Eq. (15)] in the unequal mass case and $[M_{J_\nu}^{(3\ \text{flavors})}(\text{max})/M_{J_\nu}^h(\text{max})]^{(2p-2)/3}$ in the three-flavor case. Here p_l is p in Eq. (13) calculated assuming Ω_ν is only made from the lightest neutrino(s). The ratio of the damping in the 1+2 (and 2+1) cases to the three-equal-flavor mass case is [for scales $< M_{J_\nu}^h(\text{max})$]

$$\text{damping ratio} = \frac{(m_{\nu l}/m_{\nu h})^{(1-p_l)}}{(m_{\nu 3}/m_{\nu h})^{(1-p)}}. \qquad (17)$$

The ratio in (17) is always larger than 1, indicating that the damping is greatest in the three-equal-flavor case. However, if the ratio of the masses of heavy to light flavors is $\lesssim 4$ the difference between damping in the various cases differs from the three-flavor case by < 2 %. This is the reason for choosing the particular Δm^2 for plotting. Had we plotted, for example, the 2+1 and 1+2 models using $\Delta m^2 = 6$ eV^2 (the same value used in Ref. [4], but for $\Omega_\nu = 0.20$), both of the curves would be nearly indistinguishable from the three-degenerate-neutrino case for masses $< M_{J_\nu}^h(\text{max})$. When the light and heavy masses differ by an order of magnitude or more, as in Fig. 2, the 2 + 1 model goes to the two-degenerate-flavor model and the 1 + 2 goes to the single-flavor model. We note that because the lightest neutrino weighs less than the three-degenerate-flavor neutrino, on the very largest scales where $M_{J_\nu}^l(\text{max}) < M < M_{J_\nu}^{(3\ \text{flavors})}(\text{max})$, there will be some damping which is absent in the three-degenerate-flavor case.

We see that these additional models offer now a continuum of $\Delta M/M$ values between the one- and three-degenerate-flavor mass neutrino cases. The sole motivation for considering a case where the neutrinos have masses *and* mass differences of order a few eV in scenario (iii) is the possibility of detecting oscillations with Δm^2 in the few eV^2 range in the LSND (and also KARMEN) experiments. In the framework of grand unified theories (GUT's), however, such a mass spectrum will not be easy to understand.
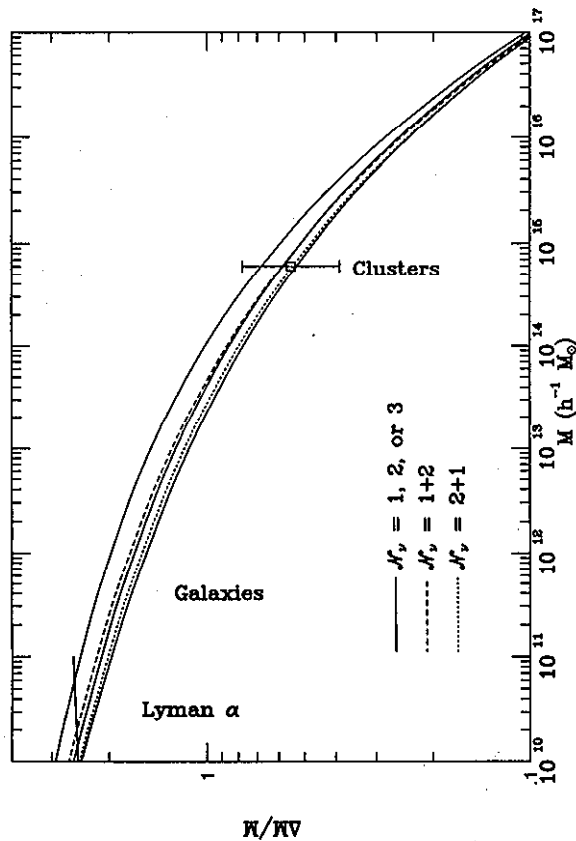

FIG. 2. The rms mass fluctuation amplitude in a special case of neutrino scenario (iii), which has been made consistent with an oscillation signal detectable at the LSND experiment in a model with $\Omega_\nu = 0.25$, $h = 0.5$, and $n = 1$. For comparison we also show (from top to bottom) the same Ω_ν with one, two, or three degenerate neutrino flavors. The "2 + 1" and "1 + 2" model curves are intermediate to the degenerate neutrino cases. The data are the same as in Fig. 1.

E. Sensitivity to cosmological parameters

In the previous sections we have plotted results for specific values of the Hubble constant, initial density fluctuation spectrum, and Ω_{baryon}. We have not drawn strong conclusions about the neutrino mass scenarios, because the results are quite sensitive to the specific values used for these cosmological parameters. We will discuss the effects of these parameters on the cosmological structure

formation and give specific examples of models from each of the three mass scenarios.

(1) H_0. The value of the Hubble constant has been the subject of a long and ongoing observational campaign (see [63] for some recent measurements). The experiments find that $0.4 < h < 1.0$. This uncertainty is quite large when seen from the point of view of the required neutrino masses. Rewriting Eq. (14) for degenerate neutrinos we find

$$m_\nu = 4.7 \frac{1}{\mathcal{N}_\nu} \frac{\Omega_\nu}{0.2} \left(\frac{h}{0.5}\right)^2 \text{ eV,} \qquad (18)$$

which emphasizes that the neutrino masses scale as h^2. In Figs. 1 and 2 we have used $h = 0.5$. Some of the recent measurements [63] seem to imply a larger value of h. In particular, if $h = 0.6$, then the masses used in the previous examples are increased by a factor of 1.44. For example, Ref. [4] advocates using $\mathcal{N}_\nu = 2$ and $\Omega_\nu = 0.2$ with $\Delta m^2 = 6$ eV^2, $h = 0.5$. If instead $h = 0.6$ with the other parameters fixed, then the oscillation signal at Los Alamos would be consistent with $\Delta m^2 = 12$ eV^2.

The scale which corresponds to the horizon size at matter domination also depends on the Hubble constant. If we use a larger value of h, then matter domination occurs earlier, when the horizon size was smaller, which means that the radiation-dominated era is less effective at arresting the growth of small-scale fluctuations. Such an effect exacerbates the small-scale problems in CDM models and favors using a larger value of Ω_ν [3,30].

(2) Initial Mass Fluctuation Spectrum. It has long been known that inflation predicts a power spectrum of density fluctuations with a spectral index n close to unity. Since the amount of deviation is strongly model dependent, many investigators are content to use $n = 1$. In Figs. 1 and 2 we have also used the value of $n = 1$ for the spectral index. However, even small deviations from $n = 1$ can lead to significant changes in the conclusions. To see this we give the initial spectrum in terms of rms mass fluctuations ($\frac{\Delta M}{M}$):

$$\frac{\Delta M}{M} \propto M^{-(3+n)/6}. \qquad (19)$$

In the simplest models of inflation, particularly those based on GUT's (see, e.g., Refs. [29,30]), $n \sim 0.94$–0.98, although other values of n are certainly possible [64]. From Eq. (19) we see that decreasing (increasing) n from unity decreases (increases) the small-scale power. Since the models are normalized at large scales to COBE observations the mass fluctuation curves in Figs. 1 and 2 will "pivot" around a very large mass scale $\sim 10^{21} h^{-1} M_\odot$. The "pivot" mass scale here is somewhat smaller than the horizon mass, because the best-fit COBE quadrupole anisotropy scales as e^{1-n} [53]. (We have fit our spectra by normalizing to the seventh multipole moment of the Sachs-Wolfe anisotropy, as recommended in Ref. [53].)

(3) Ω_{baryon}. In the past few years comparisons of the primordial light element abundances inferred from observations with those calculated have led to strict limits [52] on the amount of baryonic material in the universe,

$0.04 < \Omega_{\text{baryon}}(0.5/h)^2 < 0.06$. We have used the central value of this range for the models in Figs. 1 and 2. Improvements in the observations of deuterium and ^{3}He and determinations of the neutron lifetime have now led to a situation where all of the light element abundances are not consistent in a universe where the three known neutrino flavors have the standard number densities (derived from thermal equilibrium) at nucleosynthesis [65]. If, however, the ^{4}He abundances derived from observations have been systematically underestimated [66] by about 5%–10%, then big bang nucleosynthesis would be made consistent with the three flavors of neutrinos, and would also allow for a larger baryon density than previously thought. This latter effect would go in the right direction to explain why the baryon to dark matter ratio in cluster cores is so much larger than Ω_{baryon} [67].
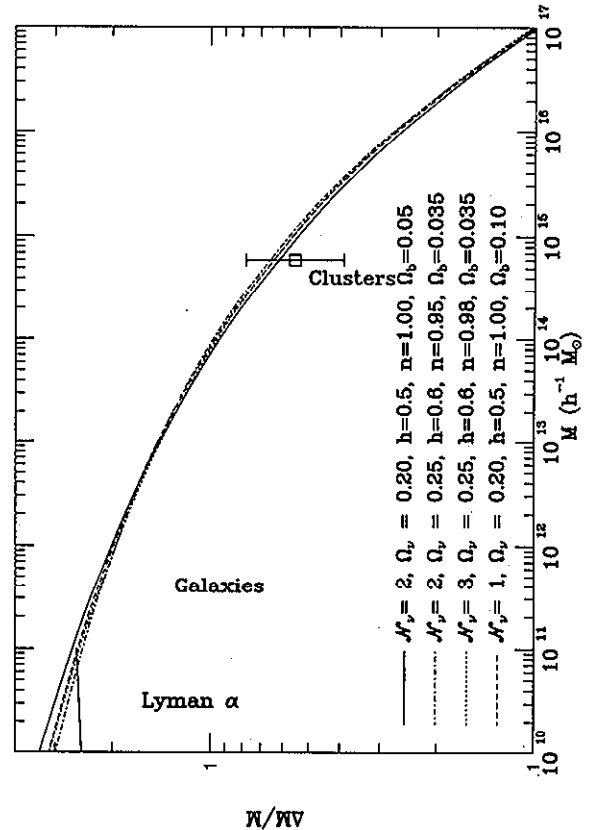

FIG. 3. The rms mass fluctuation in some selected models with varying cosmological parameters. We plot curves for two models from scenario (iii) with $\mathcal{N}_\nu = 2$ for ($\Omega_\nu = 0.20$, $h = 0.5$, $n = 1$), as advocated in ref. [4], and for ($\Omega_\nu = 0.25$, $h = 0.6$, $n = 0.95$). These two models would predict $\delta m^2 = 5.5$ and 18 eV^2 at the LSND and KARMEN experiments. The third curve is for a model with $\mathcal{N}_\nu = 3$ ($\Omega_\nu = 0.25$, $h = 0.6$, $n = 0.98$). Last, we show a curve from scenario (ii) with $\mathcal{N}_\nu$ ($\Omega_\nu = 0.20$, $h = 0.5$, $n = 1.00$). Here we have increased Ω_{baryon} to reduce the amplitude at cluster scales. As can be seen, it is difficult to determine which neutrino mass scenario is correct based on current cosmological data alone. The data are the same as in Fig. 1.

If we allow for larger baryon fractions, this will also change the amplitude of the mass fluctuations for a given Ω_ν and n. We can understand this through the following. Baryons (mostly protons) after nucleosynthesis are coupled to the photon field through the scattering of free electrons. Baryonic density fluctuations cannot grow until the photon temperature decreases to allow the stable formation of neutral hydrogen. Because this does not happen until after matter domination, the baryons are prevented from falling into the gravitational wells supplied by the cold dark matter. This leads to a damping in the growth of density fluctuations (relative to a case with no baryons). The amount of damping (on scales smaller than the horizon size at photon baryon decoupling) in the final amplitude of density fluctuations is constant, and so the effect is quite similar to changing the number of degenerate neutrino flavors.

These uncertainties mean that one cannot currently use the cosmological data to determine which of our three scenarios is correct, or even what the value of Ω_ν is. One must do a systematic study of the available parameter space, taking into account the full range of these parameters. Studies of the interplay between these cosmological parameters have been done in various contexts [13,29,68], with more in progress [57].

As a graphic illustration of this uncertainty, and also to demonstrate use of the formulas presented in Secs. III A– III C, we show four models in Fig. 3 that give nearly identical predictions of structure formation, but use quite different solutions of the neutrino oscillation data. The choices of parameters are as follows.

(1) The solid curve [scenario (iii)]. This model with $\mathcal{N}_\nu = 2$, $\Omega_\nu = 0.20$, $n = 1$, and $h = 0.5$ is advocated as a good fit to cosmological observations by Ref. [4] consistent with the Los Alamos LSND experiment. We present this model for comparison.

(2) The dot-dashed curve [also scenario (iii)]. We increase h to 0.6 in this $\mathcal{N}_\nu = 2$ model, but now there will be too much small-scale power. To compensate for this, we increase Ω_ν to 0.25, and decrease n to 0.95, a value which actually is more in line with standard inflationary predictions. The $\bar{\nu}_e$-$\bar{\nu}_\mu$ experiments would then be expected to see a signal corresponding to $\Delta m^2 = 18$ eV^2. The baryon fraction has been scaled from the first model as h^2 consistent with the nucleosynthesis constraints.

(3) The short dashed curve [scenario (i)]. Again using $h = 0.6$ and $\Omega_\nu = 0.25$, we now consider $\mathcal{N}_\nu = 3$ degenerate flavors. Because the extra flavor provides an additional damping factor over the two-flavor model of $(2/3)^{0.16} = 0.94$ [Eq. (16)], to get a curve similar to the previous one we can increase n to 0.98, which increases the amplitude at the cluster mass by $(10^{21}/10^{15})^{0.0075} = 1.10$, [Eq. (19)]; so we get very nearly the same fit at the cluster scale. The value $n = 0.98$ happens to be the value

predicted in a particular supersymmetric (SUSY) GUT inflation model [30].

(4) The long dashed curve [scenario (ii)]. Here we return to $h = 0.5$, $\Omega_\nu = 0.2$, and $n = 1$ as in the solid curve. To compensate for the increase in amplitude by going back to one flavor $(2/1)^{0.13} = 1.09$ in the solid curve, we increase Ω_{baryon} to 0.10. This value of Ω_{baryon} is consistent with three relativistic neutrino flavors during nucleosynthesis if the ^{4}He abundances have been systematically underestimated.

We have now shown that allowing for more than one degenerate (in mass) neutrino state, as indicated by solar and atmospheric oscillation experiments, produces a "degeneracy" in the predictions of structure formation for C+HDM models, given the uncertainty in cosmological parameters. In order to break this latter "degeneracy" we need a convincing detection of neutrino oscillations in an accelerator experiment or an improvement in the determination of cosmological parameters, or both. In the meantime, we just note that there is a very rich structure contained in C+HDM models of structure formation, which still allows considerable flexibility in fitting astrophysical data.

IV. CONCLUSIONS

We have investigated the impact of neutrino oscillations, as indicated by a number of solar and atmospheric neutrino experiments, on the "cold plus hot" dark matter scenario of large-scale structure formation. We are led to three distinct scenarios for neutrino masses and mixings with interesting predictions for the ongoing or planned experiments. We note in particular the expectations for $\bar{\nu}_\mu$-$\bar{\nu}_e$ and ν_μ-ν_τ oscillations being currently searched for. The cosmological implications of the three scenarios are explored in some detail. For some choices of the cosmological parameters (particularly $h = 0.5$, $n = 1$, and $\Omega_{\text{baryon}} = 0.05$), the two- and three-neutrino flavor scenarios provide a somewhat better fit to the present data than the single- (neutrino-) flavor case. However, as we show, this need not hold for a different parameter choice. It is too early to pick out the best model but it is remarkable that taking account of the oscillation experiments, the C+HDM models can still provide a good fit to the large-scale structure data within the context of the simplest inflation models.

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